— SOFTWARE REVIEW —

Phasing into PHASER

JAMES T. TOWNSEND
Indiana University, Bloomington, Indiana

Differential and difference equations through computer experiments

By Hüseyin Koçak. 2nd ed. 1989, New York: Springer Verlag. 224 pp. \$49.95. (Manual, with software PHASER diskette)

The guts of this package consist of the software contained in the diskette that accompanies the text. However, the text is necessary until one becomes familiar with the operations. A subtitle expands as follows: *With a Supplementary Diskette Containing PHASER: An Animator/Simulator for Dynamical Systems for IBM Personal Computers*. This tells the potential reader and user basically what to expect. Without some qualifiers, that individual may be led to expect too much, but for the price, the package delivers a lot.

Deterministic—that is, nonstochastic—dynamics are based on differential equations for modeling continuous time phenomena, whereas difference equations are employed to model discrete time data. PHASER was developed by Hüseyin Koçak at Brown University to: (1) illustrate some of the most famous continuous and discrete dynamic systems through their difference or differential equations and (2) permit users to explore their own dynamic systems.

An important aspect of (1) is that PHASER is designed to handle nonlinear dynamics. Linear dynamics admit well-known solutions, and in the event of time-invariant, linear systems, the solutions can usually be expressed in closed form. In any event, the theory is almost completely worked out and there are no real surprises. With nonlinear dynamics, the situation is quite the opposite: Very little of a general nature can be said about arbitrary classes of nonlinear differential equations and their systemic behavior. Indeed, in many such systems, chaotic behavior can occur (see, e.g., Devaney, 1986; Townsend, in press).

Although mathematicians and other scientists have begun to learn much about the nature of nonlinear systems and such phenomena as chaos over the past few decades,

it has been difficult, if not impossible, to incorporate this knowledge into general-purpose software. PHASER instead takes the time-honored path of using numerical means of approximating solutions to the equations. Obviously, numerical methods are by nature subject to error. Here, the user is helped by the fact that three, somewhat distinct, methods can be employed to provide a partial check on the results. Also, users are free to select their time points of computation, or step size, and to attempt to achieve a reasonably accurate depiction of the system behavior.

A "Read Me" chapter first tells the reader that PHASER is intended for almost everyone; the reader is then offered vital information about hardware requirements, the diskettes, installation on hard disks, and printer support. The hardware requirements are modest, but a math coprocessor will greatly increase the speed of computation. The software comes in a single 5.25-in. diskette or two 3.5-in. diskettes. (One diskette is for CGA displays and the other for EGA/VGA.) Basic instruction in installation is then given; it is easy. Printer support is somewhat limited, especially in the combination of the current DOS facility for printing with EGA/VGA resolution. However, the Epson/IBM dot matrix and HP LaserJet+ printers are covered, and wider printer support is promised by the author.

Part I of the text begins with a quite terse review (in chapters 1 and 2) of the topic "What is a Differential Equation?" which is followed by an equally brief but useful statement about the numerical methods employed by the software. It is to his credit that the author provides several caveats about potential dangers of such numerical procedures. Chapter 3 teaches the reader a little about difference equations, which are not subject to the time step-size error problem associated with differential equations.

Part II includes a set of tutorial lessons on the use of PHASER, in chapters 4 and 5, and ends with a helpful "Reference Guide to Menus" in chapter 6. During the lessons, the reader is taught how to enter the equations of his or her own dynamic equations.

The actual "in-house" dynamic systems, already contained in the software, are described in Part III, chapters

Correspondence should be addressed to James T. Townsend, Department of Psychology, Indiana University, Bloomington, IN 47405.

7 and 8. The differential systems are presented first, followed by the difference systems. Each presentation shows the dynamic equations, the parameter notation, and the values of the parameters used in the default mode—that is, the parameter values assumed by the software, if left unaltered by the user. A qualitative description and a little mathematical detail accompany the presentation. Some reference to additional information about each particular system follows the individual discussion. Finally, after an epilogue entitled “What remains to be done?” two appendices provide a PHASER quick reference section and a library of equations, respectively. An index completes the text.

How useful is this package for teaching and research? In my opinion, it should be extremely useful for teaching people about nonlinear dynamics. It contains a number of sophisticated ways of graphically depicting behavior, which for lack of space I cannot deal with here. Even in the context of linear systems—not to mention nonlinear dynamics—it is astounding to those of us who have cut our teeth with minutes to hours or days of arduous calculations and “error correcting,” to see the behavior of a complex system unfold before our eyes. Of course, as Koçak admonishes, this type of numerical work with nonlinear systems should be accompanied wherever possible with analytic cross-checks.

The repertoire includes many of the most chaotic systems, some of which are not well understood to this day. Hence, the user gains the excitement both of the illustration of “weird” systems and of adventuring over little trodden ground, in some cases. A good deal of mathematical research in chaotic dynamics has come about in recent years through much numerical exploration, which has served to uncover certain regularities; this has been followed by intense analytic research in the attempt to es-

tablish formal and closed-form mathematical developments and principles.

However, the package falls a little short, in my view, with respect to the ease with which users are allowed to program their own equations. There is some awkwardness in what is in view during the programming phase, and there are limits to what one can correct in one's own program without starting over. Also, as the author admits, new and complicated systemic equations, especially of the differential variety, can be quite time-intensive for the computer to carry out. Thus, the package's value for the investigation of new and quite complex systems is somewhat circumscribed. Probably, more powerful software oriented especially toward numerical and analytic solutions of ordinary differential equations is called for in such cases. Too, as with any such system—the often excellent packages available for connectionist modeling are a good example—there is some danger in using PHASER to generate predictions for psychological models: Those who fail to delve into the actual mathematics behind the program on their own may simply bubble out theoretical froth, without much psychological modeling substance.

These are minor cavils. I highly recommend PHASER for teaching about dynamic systems and for light numerical research of new nonlinear systems. Considering the cost, it is likely within the reach of most researchers and students alike.

REFERENCES

- DEVANEY, R. (1986). *An introduction to chaotic dynamical systems*. New York: Addison Wesley.
- TOWNSEND, J. T. (in press). Chaos theory: A brief tutorial and discussion. In *Festschrift for W. K. Estes*. Hillsdale, NJ: Erlbaum.