METHODS & DESIGNS

Fourier components of moving gratings

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In designing experiments in which the proximal stimulus is a moving grating (including cases in which the distal stimulus is stationary but eye movements play a significant role), one must consider the effects of the motion of the stimulus on its Fourier components in the spatiotemporal frequency domain. Some of these effects are unexpected and counterintuitive. For example, the Fourier components of a moving grating do not include its stationary (or "instantaneous") spatial frequency. Thus there is no linear filter that can extract a stationary grating from a moving one. Several useful relations are given for analyzing such stimuli.

Grating patterns moving at a constant velocity are widely used as visual stimuli in both neurophysiological (e.g., Andrews & Pollen, 1979) and psychophysical (Van Nes, Koenderink, Nas, & Bouman, 1967) studies. With the aid of image stabilization, a drifting sinusoidal grating permits uniform stimulation of every point in the field by known, controlled, spatial and temporal frequencies, independent of the subject's eye movements (Kelly, 1979b). In addition, moving gratings are the underlying components of the (also widely used) counterphase-flickering grating, as shown by the trigonometric identity,

$$2 \cos \alpha x \cos \omega t = \underbrace{\cos \alpha(x + Vt)}_{\text{leftward}} + \underbrace{\cos \alpha(x - Vt)}_{\text{rightward}}, \quad (1)$$

where α is the (radian) spatial frequency, ω is the (radian) temporal frequency, and $V = \omega/\alpha$.

The purpose of this note is to draw attention to certain properties of the Fourier components of these stimuli, which may add some insight into their applications. The relation given by Equation 1 is depicted graphically in the spatiotemporal frequency domain by Figure 1. In these coordinates, each of the moving components is represented by a pair of singularities, equidistant from the origin and lying on a line whose slope is proportional to the velocity, V .

The fact that the same result can be obtained either by adding two oppositely moving components or by modulating (i.e., multiplying) a spatial pattern by a temporal waveform is true only of sinusoidal stimuli. For example, a square-wave grating flickered in counterphase has the spatiotemporal singularities shown in Figure 2a: All spatial harmonics occur at the same temporal frequency. A moving square-wave grating, on the other hand, has the spatiotemporal singularities

Figure 1. Spatiotemporal frequency domain, showing the locations of the singularities for a sinusoidal, counterphase-flickering grating or its component moving gratings; α = spatial frequency, ω = temporal frequency, and $\tan \theta$ = velocity. (Amplitudes and phases cannot be represented in this diagram.)

shown in Figure 2b: All spatial harmonics lie on the line through the origin whose slope is determined by their common velocity. Clearly, adding together two oppositely moving square-wave gratings will not produce a counterphase-flickering square-wave grating.

In ordinary unstabilized vision, the retinal image is constantly in motion with respect to the receptor mosaic, and at any given instant, all the Fourier components of this image are moving at the same velocity.¹ Thus, the constant-velocity stimulus of Figure 2b is more "natural" in this sense than the constant-flicker stimulus of Figure 2a.

It is important to note that neither the instantaneous spatial frequency of the stimulus, α , nor its local temporal frequency, ω , considered alone may be regarded as a Fourier component of either the moving or flickering grating. This counterintuitive result follows from the fact that none of the singularities for either stimulus lies on the spatial or temporal frequency axis.² All of the Fourier components of Figures 1 and 2 are moving gratings.

In other words, there is no linear spatial, temporal, or spatiotemporal filtering operation that can extract either a stationary nonflickering grating or a flickering uniform field from the stimuli of Equation 1 or Figure 1

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Figure 2. Spatiotemporal Fourier components of (a) a square-wave grating flickering in counterphase and (b) the same grating moving at constant velocity.

or 2. Only moving gratings can be extracted from these stimuli by linear operations. Thus, a response to a given moving grating does not logically permit the experimenter to conclude that a subject is sensitive to the spatial frequency of that grating without specifying its velocity (or temporal frequency). In particular, this result is applicable to unstabilized psychophysical experiments, in which the retinal image of a nominally stationary grating is constantly in motion because of normal eye movements.

How are the Fourier components of a moving grating related to those of a stationary one? If we took an instantaneous snapshot of a moving or flickering grating, its spatial frequency components would be those obtained by projecting the singularities of Figure 1 or 2 onto the spatial frequency axis. Let us examine these instantaneous spatial frequency components at regular

intervals in time (corresponding to an orderly succession of snapshots of the grating). In this type of representation, (shown schematically in Figure 3), the spatial properties of the stimulus remain in the Fourier domain, but the temporal ones are now depicted in the time domain. The left-hand and right-hand columns of Figure 3 represent the spatial frequency components for rightward- and leftward-moving gratings, respectively; the center column represents the components for a counterphase-flickering grating. The rows in Figure 3 represent successive instants of time; temporal phase has been advanced by 45 deg between each adjacent pair of rows.

A number of useful relations are apparent in Figure 3: (1) At any time, the two components of the counterphase grating always equal the sums of the components of the moving gratings. That is, the additive relation given in Equation 1 also holds in the Fourier domain. (This necessarily follows from the linearity of the Fourier transform.) (2) Although each of the six singularities in Figure 3 varies at the same temporal frequency, the two components of each moving grating are 90 deg apart in temporal phase (while both components of the counterphase grating are in the same temporal phase). (3) The spatial phase of either moving grating determines the instantaneous amplitudes of its two Fourier components, not just at the familiar sine and cosine phases (0 deg, 90 deg, 180 deg, and 270 deg), but everywhere; hence, two components are necessary

Figure 3. Any row of this diagram represents the instantaneous amplitude as a function of spatial frequency for a rightward-moving grating (first column), a leftward-moving grating (third column), and their sum, a counterphase-flickering grating (second column). In each successive row, the spatial phase of the moving gratings (and hence the temporal phase of all three stimuli) has advanced by 45 deg.

for a complete representation in the frequency domain. Note that there are four spatial phases (45 deg, 135 deg, 225 deg, and 315 deg) at which one component vanishes. (4) Not only the position but also the direction of the moving gratings can be read from the Fourier components in Figure 3. That is, the Fourier components for the leftward-moving grating cannot be obtained from those for the rightward-moving one merely by shifting the temporal phase of one or the other. Only if one of the columns is taken in reverse order (like running a movie backward) can the leftward and rightward columns be made equivalent.

Only at the instant of zero (temporal) phase are the spatial frequency components of a moving grating equal to those of a stationary one. Note that this confirms our earlier conclusion: No linear filtering operation can extract a stationary pattern from a moving or flickering one.³

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NOTES

1. Of course, if there is relative motion between objects in a scene, this statement must be qualified.
2. This result is exactly analogous to the fact that a checkerboard contains neither horizontal nor vertical components in the two-dimensional spatial frequency domain. (See Kelly, 1976, 1979a.)
3. This implies that the reported percept of a stationary pattern, when the stimulus is actually a flickering grating of high spatial frequency, must be the result of a much subtler process than the simple filter previously assumed. (See, e.g., Kulikowski & Tolhurst, 1973.)

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