PAM: A double-precision FORTRAN routine for the parallel analysis method in principal components analysis

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Parallel analysis (Horn, 1965) is a Monte Carlo method of establishing the number of components in principal components analysis, and suggests retention of only those components from observed data whose eigenvalues are larger than the corresponding eigenvalues from random data. It can be seen as a correction of the eigenvalue-one rule for sample, rather than population, characteristics. PAM is a mainframe computer program that implements the parallel analysis method by providing mean and upper-percentile eigenvalues from multiple replications of random data.

Both Monte Carlo studies (e.g., Zwick & Velicer, 1986) and studies with established data sets (e.g., Hubbard & Allen, 1987) have found that parallel analysis yields accurate results. Unfortunately, parallel analysis is not widely used, because at least one replication of random data must be generated. Typically, eigenvalues from several replications of random data are averaged for each ordinal position to provide criterion eigenvalues, which are then compared to observed eigenvalues.

To increase the accessibility of parallel analysis for researchers, regression equations have been developed to predict mean expected eigenvalues from random data (Allen & Hubbard, 1986; Lautenschlager, Lance, & Flaherty, 1989; Longman, Cota, Holden, & Fekken, 1989). As well, Longman et al. (1989) provide regression weights for predicting 95th-percentile eigenvalues from random data. This is suggested as a potential remedy for the tendency of parallel analysis to overestimate slightly the number of components to retain (Lautenschlager, in press; Zwick & Velicer, 1986). These regression equations have been implemented in microcomputer programs, which calculate expected eigenvalues (e.g., Hays, 1987; Holden, Longman, Cota, & Fekken, in press).

There are drawbacks to the use of regression strategies, however. First, they yield only *estimates* of criterion

eigenvalues. Second, they are valid for only a limited range of subjects and variables (e.g., no equations exist for applications of principal components analysis with more than 50 variables or 1,000 subjects). Third, there are conditions under which both the Allen and Hubbard (1986) procedure and the Lautenschlager et al. (1989) procedure yield anomalous results for applications within the nominal range of these techniques (Cota, Longman, Holden, & Fekken, 1989; Lautenschlager, in press; Longman et al., 1989).

PAM is a mainframe program that circumvents these deficiencies. Designed for parallel analysis of principal components, it calculates both mean and upper-percentile eigenvalues from a specified number of correlation matrices generated directly from random normal deviates. The procedure eliminates regression errors, and calculations are done in double-precision arithmetic to minimize rounding errors. Moreover, PAM does not restrict the number of subjects and variables that can be specified. Because it can be set to perform any number of replications, 95th- or other upper-percentile eigenvalues can be calculated for specified subject and variable parameters. As mentioned above, this may be more useful than relying solely on mean eigenvalues.

Program description. PAM is a mainframe computer program written in VS FORTRAN, using double-precision subroutines from IMSL version 10 (International Mathematical and Statistical Libraries, 1987). If necessary, subroutines from other sources (e.g., Numerical Algorithms Group, 1984) could be substituted for those from IMSL. As well, PAM can be implemented using the microcomputer versions of the IMSL subroutines described in the listing. Caution is advised on implementing PAM with microcomputers, however; even powerful microcomputers may take a great deal of time to complete Monte Carlo simulations (Lautenschlager, in press), and mainframe implementation may be more convenient.

Limitations. Two limitations exist for this program. First, the magnitude of eigenvalues from random data is constrained by the distribution of those data (Fang & Krishnaiah, 1982). Because the random number generator currently provides random normal deviates, only data from multivariate normal or near-normal distributions should be compared with the output from PAM as implemented here. If the data being compared have substantially nonnormal distributions, some similarly distributed source of random numbers should be used (Armstrong & Soelberg, 1968). IMSL version 10 has routines that can generate random data from a variety of distributions (e.g., RNLNL for log-normal distributions, or RNUN for uniform distributions). Second, when the number of subjects is less than the number of variables, inaccurate or anomalous results may occur. These results, of course, may also emerge with actual data when there are more variables than subjects.

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Table 1
Example with 50 Subjects and 10 Variables

Item	Source of Eigenvalues						
	Observed Data	PAM (100 trials)		PAM (1,000 trials)		Regression*	
		Mean	95th Percentile	Mean	95th Percentile	Mean	95th Percentile
1	4.336	1.785	2.017	1.779	2.020	1.753	1.975
2	1.484	1.510	1.700	1.512	1.668	1.501	1.647
3	1.042	1.335	1.458	1.314	1.442	1.319	1.432
4	0.756	1.166	1.261	1.156	1.260	1.134	1.233
5	0.729	1.010	1.115	1.020	1.114	1.010	1.097
6	0.439	0.873	0.966	0.888	0.982	0.896	0.979
7	0.423	0.760	0.853	0.764	0.859	0.788	0.869
8	0.341	0.638	0.759	0.644	0.734	0.686	0.765
9	0.262	0.526	0.621	0.524	0.617		
10	0.188	0.397	0.484	0.398	0.498		

*These values were derived from the Longman et al. (1989) regression procedure. Eigenvalues for the last two ordinal positions should not be estimated when using this procedure.

Program listing and sample output. In the program listing provided in the Appendix, PAM is set up to run an analysis with 50 subjects, 10 variables, and 100 replications, and to output mean and 95th-percentile eigenvalues. Any of these values may be changed by replacing the current values with the preferred values, as described in the program comments. We recommend that PAM first be run to these specifications (including the same seed for the random number generator), and the output compared to Table 1 to ensure proper implementation.

Results from PAM with 100 and 1,000 replications, and from a regression equation (Longman et al., 1989), are shown in Table 1, along with actual eigenvalues derived from 50 subjects' responses to a 10-item handedness questionnaire. Using both mean and 95th-percentile criterion eigenvalues, all three methods converge to indicate that only one component should be retained, because only the first eigenvalue from the observed data is greater than the corresponding eigenvalue from random data. In this example, PAM (100 replications) is slightly (although not significantly) more accurate than the regression method in predicting the eigenvalues based on 1,000 replications.

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APPENDIX

Program Listing

PAM

```
C 'NV' IS NUMBER OF VARIABLES 'NS' IS NUMBER OF SUBJECTS
C 'I' IS NUMBER OF TRIALS (REPLICATIONS) USED
  INTEGER NV, NS, I, NF
C THE USER MUST PROVIDE THE VALUES OF NV, NS, AND I BELOW
  PARAMETER (NV=10, NS=50, I=100)
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APPENDIX (Continued)

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C THE USER SHOULD SET NF AT THE APPROPRIATE LEVEL
C (E.G. 95 IF 100, OR 19 IF 20 TRIALS)
C NF MUST BE LESS THAN I
  PARAMETER (NF = 95)
C OUTPUT IS SENT TO UNIT 7. THIS MUST BE DEFINED BEFORE RUNNING PAM.
  DOUBLE PRECISION R(NV,NV), S(NV), D(NV), A(I,NV), C(I)
  DOUBLE PRECISION X(NS,NV), XM(NV), MEAN(NV)
  DIMENSION Y(NS)
C THE USER CAN SET ISEED TO ANY NON-NEGATIVE INTEGER
  ISEED = 973
C SETTING THE SEED FOR RANDOM NUMBER GENERATOR USING RNSET
  CALL RNSET (ISEED)
  DO 40 L=1,I
  DO 20 K=1,NV
C GENERATING MATRIX OF RANDOM NORMAL DEVIATES USING RNNOA
  CALL RNNOA (NS, Y)
  DO 10 J=1,NS
    X(J,K) = DBLE(Y(J))
10  CONTINUE
20  CONTINUE
C GENERATING AN (NV BY NV) CORRELATION MATRIX USING DCORVC
  CALL DCORVC (0,NS,NV,X,NS,0,0,0,2,XM,R,NV,IX,1,IB,IC,ID)
C FINDING THE EIGENVALUES OF THE CORRELATION MATRIX USING DEVLFSF
  CALL DEVLFSF (NV,R,NV,D)
  DO 30 K=1,NV
    A(L,K) = D(K)
30  CONTINUE
40  CONTINUE
  DO 70 K=1,NV
  DO 50 L=1,I
    C(L) = A(L,K)
50  CONTINUE
C SORTING EIGENVALUES USING DSVRGN; UPPER PERCENTILES FOUND THIS WAY
  CALL DSVRGN (I,C,C)
  DO 60 L=1,I
    A(L,K) = C(L)
60  CONTINUE
70  CONTINUE
  DO 110 K=1,NV
    MEAN(K)=0.
  DO 100 J=1,I
    MEAN(K)=MEAN(K)+A(J,K)
100 CONTINUE
    MEAN(K)=MEAN(K)/I
110 CONTINUE
  WRITE (7,65) NS,NV,I
  WRITE (7,75)
  WRITE (7,85) (MEAN(K),K=NV,1,-1)
  WRITE (7,95) NF,I

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APPENDIX (Continued)

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      WRITE (7,85) (A(NF,J),J=NV,1,-1)
      WRITE (7,65) NS,NV,I
65    FORMAT (I5,' SUBJECTS ',I5,' VARIABLES ',I5,' TRIALS')
75    FORMAT ('MEAN EIGENVALUES ARE GIVEN BELOW')
85    FORMAT (10X,F10.7)
95    FORMAT ('THE VALUES BELOW RANK ',I4,'TH OUT OF',I5,' TRIALS')
      STOP
      END
```

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