PROGRAM ABSTRACTS/ALGORITHMS

A BASIC program for the generation of Latin squares

MICHAEL J STRUBE

Washington University, St. Louis, Missouri

In experimental research, it is often desirable to expose subjects to more than one stimulus or experimental condition to provide for the greater power inherent in repeated measures designs, the economical use of available subjects, or the within-subjects test that conceptual hypotheses may specifically require. A potential problem in these designs is the possibility of carryover effects or the influence of a particular order of presentation. One popular counterbalancing strategy involves the selection of k orders from all possible orders by construction of a Latin square, with subjects assigned randomly to one of the k orders. The advantage of the Latin square is that a relatively small number of orders is required (i.e., k) and that the orders are selected in such a way as to guarantee the existence of certain features. Specifically, each stimulus appears once in every ordinal position, and the set selected is chosen randomly from among all possible Latin squares of size k . Furthermore, specific statistical analyses have been developed for Latin square designs, which allow for the testing of certain order effects (see, e.g., Kirk, 1968; Myers, 1972; Winer, 1971).

Construction of a Latin square requires four basic steps: (1) selection of a standard square of size k , (2) random reordering of the columns of the standard square, (3) random reordering of the rows of the square resulting from steps 1 and 2, and (4) random shuffling of the numerical indicators (e.g., Myers, 1972). The resulting rows (or columns) of the final square constitute the k orders, to which each subject in the study is randomly assigned.

The major drawback to using the Latin square approach is that it can be very time-consuming to implement. As k increases, the number of standard squares increases dramatically, and one square must be chosen at random as the starting point for the reordering of rows, columns, and indicators (each standard square gives rise to a unique subset of all possible Latin squares). For example, although there is only 1 standard square for $k=2$ and $k=3$, and only 4 standard squares for $k=4$, there are 56 standard squares for $k=5$, 9,408 standard squares for $k=6$,

and 16,942,080 standard squares for $k=7$ (see Kirk, 1968). The ability to enumerate efficiently the population of standard squares and to select one at random becomes virtually impossible for $k > 5$ (for small k , standard squares have been tabulated; see Fisher & Yates, 1963). This limitation has led to the use of procedures that are less time consuming, but which also deviate from the standard selection procedure described previously (e.g., specifying a small number of the possible standard squares and randomly selecting one).

The present program allows adherence to the standard Latin square generation process for large k , and capitalizes on the speed of the computer to make the procedure time efficient. The program is written in Commodore BASIC (Version 7.0) for the Commodore C-128 computer, although it can be modified easily to run on other machines. For example, the program will run on the Commodore C-64 computer provided that PRINT USING commands are changed (they are not supported by the C-64). C-128 users should note that the program will run considerably faster using the C-128's FAST operation mode.

Input. The user simply specifies the number of elements to be ordered (i.e., the size, k , of the Latin square).

Output. The program constructs one random standard square from among all possible standard squares of size k . The time necessary to carry out this construction increases as k increases. For example, sample runs for $k = 3, 4, 5, 6, 7, 8, 9$, and 10 required 1.1, 2.6, 11.7, 16.8, 29.6, 88.1, 164.8, and 207.1 sec, respectively, to execute construction of the standard square. The resulting standard square represents a random (technically quasi-random, since the computer's pseudorandom number generator is used) selection of one square from among all possible squares for a given k . After the random standard square is printed, the program randomly rearranges the columns of the standard square, randomly rearranges the rows of the column-modified square, and then randomly shuffles the indicators (cf. Myers, 1972). The resulting Latin square represents a random selection from the population of all possible squares of size k . This random Latin square is printed.

For very large k , the program can take a considerable amount of time to construct the standard square. For example, one sample run with $k=20$ required 1 h and 35 min to execute. Note, however, that the equivalent standard selection process would be virtually impossible to carry out without the aid of a computer, because the number of possible standard squares is astronomical in size. Furthermore, the typical number of stimuli to be or-

The author's mailing address is Department of Psychology, Washington University, St. Louis, MO 63130.

dered in most studies is not likely to be large enough to require such lengthy execution time.

Availability. A listing of the program can be obtained from Michael J Strube, Department of Psychology, Washington University, St. Louis, MO 63130. Alternatively, an appropriately formatted disk and return envelope can be mailed to the author, and the program will be copied to the disk.

REFERENCES

- FISHER, R. A., & YATES, F. (1963). *Statistical tables for biological, agricultural and medical research*. Edinburgh: Oliver & Boyd.
- KIRK, R. E. (1968). *Experimental design: Procedures for the behavioral sciences*. Belmont, CA: Brooks/Cole.
- MYERS, J. L. (1972). *Fundamentals of experimental design* (2nd ed.). Boston: Allyn & Bacon.
- WINER, B. J. (1971). *Statistical principles in experimental design* (2nd ed.). New York: McGraw-Hill.

(Revision accepted for publication June 7, 1988.)