PROGRAM ABSTRACTS/ALGORITHMS

FRACTION: A computer program for fractional factorial design

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Factorial designs are used efficiently and effectively to carry out experiments involving multiple independent variables needed to explain complex human behavior. These designs are not feasible, however, when a study requires a large and, in some cases, prohibitive number of measurements on the same subjects (within-subjects designs) or on groups of different subjects (between-subjects designs). For example, a 2^{10} factorial design, not unusual in some behavioral research, produces 1,024 treatment combinations.

If the experimenter can assume, on theoretical and empirical grounds, that higher order interactions are negligible, only a fraction of the complete factorial design is needed to estimate main effects and lower order interactions. For example, in a 2^{10} factorial experiment, if the highest order interaction is considered to be negligible, the number of observations or groups can be reduced in half, from 1,024 to 512. If another interaction can be sacrificed, the number of observations can be halved again, to 256 ($\frac{1}{4}$ fraction), and so on. Moreover, "in cases where ambiguity is present because of confounding, the initial experiment may be supplemented by follow-up experiments specifically designed to clarify such ambiguities" (Winer, 1971).

A factorial design with n factors, each with two levels, contains 2^n treatment combinations in its full replication. However, in a $\frac{1}{2}^p$ fraction of a 2^n design (where $p=1$ in a $\frac{1}{2}$ fraction, $p=2$ in a $\frac{1}{4}$ fraction, and so on), a total of 2^{n-p} treatment combinations are actually tested with $2^{n-p}-1$ degrees of freedom among them. In such fractional designs, 2^p-1 out of the total of 2^n-1 effects and interactions are confounded with the grand mean. Thus, effects will be mutually confounded in groups of 2^p (Kempthorne, 1952, p. 405). That is, in a half replicate, each effect is confounded with one other effect: the confounding is in sets of two; in a quarter replicate, each effect is confounded with three others: the confounding is in sets of four; and so on.

The effects that are confounded with each other are called aliases, because the same source of variation has more than one name due to confounding of sources of variation. "The problem of deciding to which alias an effect is to be attributed is one that the experimenter must face

when he comes to interpret the results" (Cochran & Cox, 1957, p. 246).

The crucial part of designing a fractional factorial experiment is the choice of relationship (fundamental identity), namely, the selection of the interaction to be confounded. The experimenter must be extremely cautious in selecting appropriate interactions for confounding. The goal is to sacrifice the least useful information and to keep important information, usually contained in main effects and lower order interactions. It is, therefore, "imperative to determine the basis for any proposed fractional design in order to avoid confusion of important effects" (Davies, 1956, pp. 451-452).

The Program. FRACTION is a program for generating fractional factorial designs with a maximum of 16 factors with a minimum of $\frac{1}{256}$ observations in a full 2^{16} design. The program can also perform analysis of variance on a complete or fractional factorial design. If the complete feature is chosen, the program performs an analysis of variance on data in the way other statistical programs usually do. There are, however, some differences: FRACTION is faster than other existing computer programs, such as SAS, SPSSX, and BMDP, and requires less memory than other existing programs.

The program also computes analyses of variance and F ratios for main effects and appropriate interactions for fractional factorial designs. This fraction could be $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, ..., to $\frac{1}{256}$ of the total number of observations in the full 2^{16} design.

The user decides which interaction(s) to confound to achieve a fraction of the complete design. This decision is extremely important and difficult. By selecting an inappropriate interaction for confounding, the main effects may even be confounded with each other; thus, this is poor practice with fractional designs.

If users are confident that they can identify unimportant interaction terms to be used as fundamental identities, the program provides an option to specify these interaction terms. Otherwise, the program provides the appropriate fundamental identities, following the rule that main effects are to be confounded with highest order interactions, first-order interactions are to be confounded with the next highest order interactions, and so on.

As mentioned earlier, each effect could have one, two, three, or more aliases (up to 255 aliases in a 2^{16-8} fractional design). The program provides a list of aliases that enables users to attribute each effect to one of these aliases.

Computer and Language. This program was written originally in FORTRAN for the IBM 3090, and was translated into BASIC for the IBM PC (fixed disk not required). The FORTRAN version of the program is standard FOR-

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TRAN, which can be used by almost any computer with a FORTRAN compiler.

Core Requirement. For the maximum—a 2^{16-8} fractional factorial design—the program requires less than 1K of memory (for dimensions). However, if a complete factorial design is run, the program requires a maximum of 70K for a complete 2^{16} factorial design.

Input. The program input includes a parameter card, fundamental identity card(s) (optional), variable names card(s) (optional), and data cards. The parameter card calls for the following information: (1) number of factors, (2) number of fractions, (3) number of observations, (4) data set option (low level, or high level of interactions), (5) data identification option (data with or without group membership), (6) fundamental identity option, (7) variable names option, (8) alias output option, and (9) treatment combinations output option.

Output. The program outputs (1) a description of the input parameters, (2) fractional treatment combination

(optional), (3) tables of effects and aliases (optional), (4) an analysis of variance summary table including significance level and η (proportion of total variance for each effect), and (5) a graph that plots η s against effects.

Limitations. Each factor must have 2 levels, and the number of factors may not exceed 16.

Availability. The source listing and its documentation may be obtained from the author.

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