POLYLIN: A FORTRAN IV program for the analysis of multiplicative (multilinear) trend components of interactions

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Many psychological models and theories, when examined algebraically, reduce to a multiplicative form. Thus, the response can be stated as a product of two (or more) psychological variables. Such models can be found in perception (Anderson, 1974), risky decision making (Shanteau, 1974; Tversky, 1967), social judgment (Graesser & Anderson, 1974), and information acquisition (Shanteau & Anderson, 1972). A listing of theories that lead to a multiplying model can be found in Krantz, Luce, Suppes, and Tversky (1971).

Instances of multiplicative models are widespread, but there appear to be no widely useful techniques for analyzing such models. Krantz et al. (1971) developed conjoint measurement procedures, but these ordinal techniques can only be applied in limited situations and lack tests of significance to evaluate deviations from multiplicativity. These shortcomings are avoided by Tversky (1967), who proposes a log procedure to reduce multiplicative data to additivity; ordinary analysis of variance procedures can then be used. The Tversky procedure, however, requires ratio scaled data and does not permit analysis of combination adding-multiplying (multilinear) models.

A procedure proposed by Anderson and Shanteau (1970) bypasses these difficulties. A multiplicative model, $D_{ij} = A_i \times B_j$, can be tested by varying the stimulus factors in a factorial design. This model predicts that the data should plot as a diverging fan of straight lines. In analysis of variance terms, there should be a significant A by B interaction. Moreover, the interaction should be concentrated in the linear by linear (bilinear) component. Examination of deviations from this component leads to a test of the adequacy of the multiplicative model.

Program Logic. POLYLIN employs a least squares procedure which maximizes the sum of squares (SS) in the multilinear component of each interaction. This is accomplished by deriving trend coefficients which maximize linearity. These coefficients permit computation of the multilinear SS. The test of the multiplicative

model results from testing the residual SS, that is, the difference between the total interaction SS and the multilinear SS.

If the model is successful, then the derived coefficients provide estimates of the psychological values along each factor. These values are generally on an interval scale, although ratio scale estimates can also be obtained depending on the experimental design (see Shanteau, 1974). The logic of this estimation procedure is based on the functional measurement approach (Anderson, 1974). The basis for the statistical test comes from developments of Mandel (1961) and Tukey (1949).

Program Operation. POLYLIN is designed to extract the multilinear sum of squares from each interaction in up to a four-factor design. To extract each term, coefficients are calculated based on the marginal means:

$$\begin{aligned} c_i &= R_{i.} - R_{..} \\ c_j &= R_{.j} - R_{..} \\ c_{ij} &= c_i c_j = (R_{i.} - R_{..})(R_{.j} - R_{..}). \end{aligned}$$

The c_i is the deviation of the Row i means from the grand mean; the same holds for c_j and the Column j mean. These are essentially the linear orthogonal polynomial coefficients for the row and column factors, respectively. The c_{ij} provide the coefficients for the linear by linear component of the interaction. The sum of squares for the bilinear component is computed from:

$$SS_{\text{Bilinear}} = \frac{(\sum c_{ij} T_{ij})^2}{n \sum c_{ij}^2}$$

T_{ij} is the total of the n scores in each cell of the design; the sums are over all cells. The program extracts similar multilinear sums of squares for all two-, three-, and four-way interactions.

The same data must also be run through an ordinary analysis of variance program to yield the SS-interaction terms along with an appropriate MS-error term. The multiplying model predicts that the bilinear component with its one degree of freedom should be significant. More importantly, the SS-residual can be calculated from the difference between SS-interaction and SS-bilinear. Multiplying predicts that this should be nonsignificant on one less than the interaction degrees of freedom.

The program is primarily intended for analysis of single-subject data. In such designs, each subject is run through the entire set of stimuli several times. The within-cells variability can then be used as an error term for all the tests. The program can also be applied to group data, although the resulting repeated-measurements analysis is considerably more complicated (see Graesser & Anderson, 1974).

Program development was supported in part by a Public Health Service Grant MH 22610 from the National Institute of Mental Health. Modifications suggested by David J. Weiss have been incorporated to make the program more efficient and widely compatible. All requests or comments on the program should be sent to James Shanteau, Department of Psychology, Kansas State University, Manhattan, Kansas 66506.

Input. Besides a factorial array of data specified through a variable format card, two other types of input are required. The first is a specification of the design parameters and the various program options. The second is a listing of the specifications for the interval scale, that is, the values to be arbitrarily assigned to establish an interval scale of the subjective values. A separate experiment title and subject number is allowed for each set of data.

Output. Standard output includes a table of marginal means and rescaled (interval scale) values for each factor. In addition, the multilinear sum of squares is listed for each interaction. Several options are also available. A table of cell means can be obtained. Means for both marginal and rescaled values can be listed for a group of subjects. Finally, the multilinear sum of squares can be obtained as punched output.

Language. The program was written in FORTRAN IV for an IBM 370/158 computer, and has been checked out on a number of other machines, including a CDC 3300. The program requires 120K of memory and completes a full four-factor analysis in under 5 sec on an IBM 370.

Limitations. The program is restricted to designs with four or fewer factors. There can be no more than 10 levels on any one factor, and no more than 200 total cells in the design. These limits can, however, be modified by changing various array limitations within the program.

Availability. A copy of the program instructions, listing of the deck, and sample output can be obtained

free of charge. A card copy of the program along with sample data will be sent if postage is reimbursed. All requests should be addressed to James Shanteau, Department of Psychology, Kansas State University, Manhattan, Kansas 66506.

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(Received for publication March 31, 1977.)