PROGRAM ABSTRACTS/ALGORITHMS

Probabilities and Critical Values for z, chi square, r, t, and F

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The requirements of psychological research often necessitate considerable familiarity with various probability functions, most notably the z, chi-square, r, t, and F distributions. In most cases, experimenters employ numbers of groups, numbers of subjects, and a nominal alpha level such that the significance (or lack thereof) of a particular test statistic can be readily determined by reference to any available statistical table. However, this is not always the case. For instance, in using the F test for homogeneity of variance, reference must be made to the F distribution on $n_1 - 1$ and $n_2 - 1$ degrees of freedom, where n refers to the number of subjects in the respective groups. When n becomes even moderately large, a substantial majority of statistical tables are of minimal value; at best, linear interpolation using the available information can be attempted. Alternately, experimenters may have statistics for frequently tabled degrees of freedom but may need a probability value other than the "traditional" .05, .01, or .001 levels of significance. For example, researchers who employ a Bonferroni approach (e.g., Harris, 1975) in conducting multiple comparisons or who wish to establish a "familywise" alpha level often end up with a specified level of significance not easily found in any statistical table. The purpose of this paper is to describe a computer program that will aid in the solution of both of these problems.

Description. The program consists of a brief main routine and 10 subprogram functions. The main routine controls all input and output operations and calls the appropriate function(s), as needed. When the user wishes to obtain probability value(s) for a given test statistic, input consists of a short character string, indicating the probability distribution to be referenced (e.g., "F" for the F distribution), degrees of freedom (as appropriate), and the test statistic. Output consists of the test statistic input to the program, degrees of freedom (when appropriate), and the two-tailed significance level associated with this statistic. In the case in which the user wishes to obtain a critical value for a predetermined probability value, input consists of a short character string indicating the distribution to be referenced (e.g., "CT" for a critical value for the t distribution), degrees of freedom (as appropriate), and a two-tailed significance level. Output consists of the alpha level input to the program, the associated critical

value, and the degrees of freedom (when appropriate). Multiple requests of any sort may be made in any one program execution.

Algorithms for z, chi square, r, t, and F. Five functions are responsible for calculating the various probability estimates. The z probabilities are calculated by PROBZ, according to a fifth-degree polynomial approximation (Abramowitz & Stegun, Note 1, Equation 26.17, p. 932). Chi-square probabilities are computed by PROBXI according to one of two approximations. In the case in which $\chi^2/2$ is less than either 13 or $df/2$, a convergent series approximation is employed (Abramowitz & Stegun, Note 1, Equation 6.5.29, p. 262); otherwise, an asymptotic expansion approximation is used (Abramowitz & Stegun, Note 1, Equation 6.5.32, p. 263). F probabilities are calculated by PROBF according to a convergent series approximation (Abramowitz & Stegun, Note 1, Equation 26.5.4, p. 944). The t probabilities are computed by referring t^2 on 1 and df degrees of freedom to PROBF. The r probabilities are computed by PROBR, which first converts r to t using the equation,

$$t = (r * \sqrt{df}) / \sqrt{(1 - r^2)},$$

and then refers t^2 to PROBF. PROBXI and PROBF are based on the prior work of Clyde, Cramer, and Sherin (Note 2) and make use of DLOGG, which calculates the natural logarithm of the gamma function of a number by means of an asymptotic approximation (Abramowitz & Stegun, Note 1, Equation 6.1.41, p. 257).

Estimation Procedure. Five functions—Z, XI, R, T, and F—are utilized to estimate the critical values of their respective probability distributions for a desired significance level. All five of these functions employ an iterative technique that converges on the critical value through repeated calls to the respective probability function. The iterative procedure involves the continuous reestimation of its critical value and a stepping factor. The initial estimate of the critical value is based on the natural logarithm of the inverse of the desired significance level (multiplied by some function of the degrees of freedom, as appropriate); the initial stepping factor equals the initial estimation of the critical value. This initial critical value estimate is then referred to its respective probability function for evaluation. If the probability level returned is too large, the initial estimate of the critical value is increased by the stepping factor; if the probability level is too small, the stepping factor is halved, and the initial estimate of the critical value is reduced by this revised stepping factor. This process

continues until the probability value returned by the probability function agrees to 14 digits with the desired significance level.

Accuracy and Efficiency. Because the errors in the estimates of both the probability levels and critical values largely depend on the accuracy of **PROBZ**, **PROBXI**, and **PROBF**, their accuracy was evaluated over a wide range of values, at various significance levels, and for various degrees of freedom (as appropriate). The overall performance of these three functions was excellent, with the "worst-case" errors in computed probability, even at extreme degrees of freedom or significance levels, occurring in the fourth decimal place.

The computing time required for any single call to **PROBZ**, **PROBXI**, and **PROBF** should be only several milliseconds on any modern computer. (Because **PROBXI** and **PROBF** solve series equations, computing time will vary for these functions.) In the estimation of a critical value, length of computing time depends on the accuracy of the initial estimate, the desired significance level, and the degrees of freedom (when appropriate), with more time required for more extreme estimates. In the typical or average situation, several hundredths of 1 sec are needed; in the most extreme case, perhaps .5 sec might be required on any modern large-scale computer.

Language. The program is written in ANSI standard FORTRAN IV. All calculations are done in double-precision arithmetic, and core storage requirements on an IBM 3032 are approximately 17K bytes. No use is made of peripheral storage devices, and only limited use is made of array storage in the main routine only.

Availability. Reprints of this paper, detailed tables concerning the accuracy of the program, and source listings can be obtained at no charge from Kevin E. O'Grady, Department of Psychology, University of New Mexico, Albuquerque, New Mexico 87131.

REFERENCE NOTES

1. Abramowitz, M., & Stegun, I. A. (Eds.). *Handbook of mathematical functions* (Applied Mathematics Series 55) Washington, D.C: National Bureau of Standards, 1964.
2. Clyde, D. J., Cramer, E. M., & Sherin, R. J. *Multivariate statistical programs* Coral Gables, Fla: University of Miami, Biometric Laboratory, 1966.

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