A BASIC subroutine to generate random Greco-Latin squares

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Greco-Latin squares can be used to provide experimental arrangements when the experimenter wishes to provide a highly efficient test of main effects and can assume no interaction effects exist (cf. Winer, 1971, p. 709ff for a more complete discussion of the assumptions underlying the analysis of data derived from this type of design). Most available procedures are not random, thereby allowing for the possibility of bias in the choice of a square. The routine described here generates random Greco-Latin squares up to size 5 by 5 conveniently.

After initial dimensioning, this routine forms three vectors, X1, Y1, and E1. The pair (X1, Y1) is the Cartesian product of the integers 1 to N, where N is the size of the square. N1 is an ordinal number (1 to N^2) assigned uniquely to each pair of the product. An ordinal number (N1) is chosen at random and checked to see if it is an unused element (Statements 690-710). If it is an unused element, X1 (N1) is tested for Latin square properties in a matrix A and Y1 (N1) is tested for Latin square properties in a matrix B. Note that the use of the Cartesian product assures that the matrices A and B will be orthogonal. Therefore, the concatenation of A and B will be a Greco-Latin square.

To test for Latin square properties in A and B, elements are compared to all other elements in a row and column; if it is a permissible element (has not occurred before that row or column), it is kept; if not, the pool of elements is resampled. If an acceptable element is not found in a reasonable¹ number of trials, the entire column is reset and elements resampled [see Scheirer (1976) for a more complete description of this procedure]. If an acceptable column has not been found in J + 1 tries (J for a 4 by 4 square), where J is the column number, the entire matrix is reset and the procedure starts again. This is necessary since it is possible, in a Greco-Latin square, to have no permissible elements for the completion of later columns after earlier columns have been determined.

The Greco-Latin square is output as a matrix of two-digit numbers, where the first digit corresponds to the item for matrix A and the second element for matrix B. On the average, 3 by 3 Greco-Latin squares require 68.77 (standard deviation = 33.83) resamplings, 4 by 4 squares require 1,070.92 (standard deviation = 774.46), and 5 by 5 squares require 16,730.16 (standard deviation = 10,049.18) resamplings to reach a solution as determined empirically.

Support. This program was developed on an Interdata Model 70 minicomputer. It is written in INTERACTIVE BASIC but is easy to convert to BASIC or FORTRAN. The program requires 1,628 bytes of core on the Model 70 before dimensioning. The program as written is 125 statements long, but more than 50 are remarks for documentation purposes.

Availability. Copies of this program are available from C. James Scheirer, Department of Psychology, State University of New York, Binghamton, New York 13901, without cost.

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10 REM                                     GENERATION OF A RANDOM GRECO-LATIN SQUARE
20 DIM A$(1)
30 "ENTER RANDOM NUMBER"
40 INPUT R2
50 REM                                     INITIALIZE RANDOM NUMBER GENERATOR
60 FOR I=1 TO R2
70 X=RND(0)
80 NEXT I
90 "WHAT SIZE SQUARE?"
100 INPUT N
110 L9=1
120 IF N=4 THEN LET L9=0
130 IF N=2 OR N=5 GOTO 1180
140 DIM E1(N*N+1)
150 DIM C(N,N)
160 DIM A(N,N),B(N,N)
170 DIM X1(N*N+1),Y1(N*N+1)
180 FOR L1=1 TO N
190 FOR L2=1 TO N
200 X1(N*(L1-1)+L2)=L1
210 Y1(N*(L1-1)+L2)=L2
220 NEXT L2
230 NEXT L1
240 REM
250 REM                                     (X1,Y1) CONTAINS ALL COMBINATIONS OF THE INTEGERS
260 REM                                     1 TO N.
270 REM
280 J
290 FOR J=1 TO N
300 T1=0
310 FOR I1=1 TO N
320 A(I1,J)=0
330 B(I1,J)=0
340 NEXT I1
350 T1=T1+1
360 REM                                     T1 COUNTS THE NUMBER OF TIMES A COLUMN HAS
370 REM                                     BEEN RESET
380 REM
390 FOR I=1 TO N
400 N1=N*(J-1)+1
410 REM
420 REM                                     THE PAIRS (X1,Y1) ARE ASSIGNED ORDINAL NUMBERS
430 REM                                     BETWEEN 1 AND N*N
440 REM
450 T=0
460 T=T+1
470 REM
480 REM                                     T IS THE NUMBER OF UNACCEPTABLE ELEMENTS SAMPLED.
490 REM
500 E1(N1)=0
510 REM
520 REM                                     E1(I) IS A VECTOR KEEPING TRACK OF ALL OF THE
530 REM                                     N1 USED TO DATE.
540 REM
550 IF T>N*N-N1+1 GOTO 310
560 REM
570 REM                                     RESET COLUMN IF AN ACCEPTABLE ELEMENT HAS NOT
580 REM                                     BEEN FOUND IN N*N-N1+1 TRIES.
590 REM
600 IF T1>J+L9 GOTO 290
610 REM
620 REM                                     IF J RESETS OF THIS COLUMN HAVE OCCURRED, RESET
630 REM                                     THE ENTIRE MATRIX AND START AGAIN.
640 REM
650 P5=INT(RND(0)*N*N)+1
660 REM
670 REM                                     ONE OF THE N1 IS CHOSEN RANDOMLY
680 REM
690 FOR K1=1 TO N1
700 IF P5=E1(K1) GOTO 650
710 NEXT K1
720 REM
730 REM                                     IF THIS N1 HAS BEEN USED, RESAMPLE
740 REM
750 E1(N1)=P5
760 REM
770 REM                                     TRY THE VALUES (X1,Y1) FOR LATIN SQUARE PROPERTIES
780 REM
790 A(I1,J)=X1(P5)
800 B(I1,J)=Y1(P5)
810 FOR Y=1 TO (I-1)
820 IF Y=1 GOTO 890
830 IF A(I,Y)=A(I1,J) GOTO 460
840 IF B(I,Y)=B(I1,J) GOTO 460
850 REM
860 REM                                     IF EITHER X1 OR Y1 HAVE APPEARED IN THIS ROW OF
870 REM                                     A OR B, RESAMPLE THE N1

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880 REM
890 NEXT Y
900 FOR Z=1 TO (J-1)
910 IF Z=J GOTO 950
920 IF A(I,J)=A(I,Z) GOTO 460
930 IF B(I,J)=B(I,Z) GOTO 460
940 REM
950 REM      IF EITHER X1 OR Y1 HAVE OCCURRED IN THE COLUMN
960 REM      OF A OR B, RESPECTIVELY, RESAMPLE THE N1
970 REM
980 NEXT Z
990 NEXT I
1000 NEXT J
1010 FOR I=1 TO N
1020 FOR J=1 TO N
1030 C(I,J)=10*A(I,J)+B(I,J)
1040 REM
1050 REM      THE MATRIX C CONTAINS 2-DIGIT NUMBERS AS ELEMENTS
1060 REM      WITH THE FIRST DIGIT REPRESENTING THE VALUE IN
1070 REM      THE FIRST SQUARE AND THE SECOND DIGIT REPRESENTING
1080 REM      THE VALUE IN THE SECOND SQUARE
1090 REM
1100 NEXT J
1110 NEXT I
1120 MAT PRINT C;
1130 ;
1140 ; "DO YOU WANT ANOTHER SQUARE?"
1150 INPUT AS
1160 IF AS="Y" GOTO 280
1170 STOP
1180 IF N=2 OR N=6 GOTO 1230
1190 ; "LARGE SQUARES REQUIRE MUCH TIME. DO YOU WISH TO CONTINUE?"

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1200 INPUT AS
1210 IF AS="Y" GOTO 140
1220 STOP
1230 ; "GREGO-LATIN SQUARES OF SIZE";N;"DO NOT EXIST!"
1240 STOP
1250 END
BASIC

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FOOTNOTE

1. "Reasonable" is derived empirically. A substantial number of Monte Carlo simulations were run to check a range of these parameters. The parameters in the program are optimal among those tested.

REFERENCES

- SCHEIRER, C. J. A BASIC subroutine to generate Latin squares. *Behavior Research Methods & Instrumentation*, 1976, 8, 31-32.
- WINER, B. J. *Statistical principles in experimental design*. New York: McGraw-Hill, 1971.