A PDP-11/45 program for playing n-person characteristic function games

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NPER II is an on-line PDP-11/45 computer program for studying bargaining and coalition formation processes within the framework of n-person games in characteristic function form. The extensive vocabulary of the program and its mode of operation are presented nontechnically, followed by an examination of a protocol from a sample four-person game. Improvements over a previous version of the program and applications to other experimental paradigms investigating coalition formation and bargaining are discussed in the last two sections of the paper.

NPER II is a program written for the PDP-11/45 computer to study bargaining and coalition formation processes within the framework of n-person games in characteristic function form. This program is a successor to the *coalitions* system (Kahan & Helwig, 1971; Rapoport & Kahan, 1974) written for the PDP-8, which has been employed in a number of studies on bargaining and coalition formation. NPER II differs from its predecessor in that it possesses a wider game vocabulary, has better methods of game definition, data storage and retrieval, and bookkeeping, and provides options for types of games not possible previously. We shall first present a description of NPER II, then a sample game, and shall conclude with a discussion of improvements of the new version over the old, including the generalizability of the basic paradigm that is provided by the program.

PLAYING GAMES IN CHARACTERISTIC FUNCTION FORM

An n-person game in characteristic function form is defined by specifying a number of players and a function that assigns a real number (positive integer in the present implementation) to each possible subset (coalition) of the players. This value assigned to the coalition represents the utility that its members jointly command. The object of the game is for a player to gain for himself the maximum possible utility through one of the various coalitions of which he is a member. A coalition is formed when its members agree on how to allocate its value.

When the players are seated, each at individual Teletypes in a soundproofed cubicle, NPER II starts a game by telling each player his role in the game (by a letter identification) and by printing out the values for that game. Consider an example game for four players defined by the values: $A = 10$, $B = 15$, $C = 15$, $D = 10$,

$AB = 30$, $AC = 35$, $AD = 25$, $BD = 40$, $CD = 35$,
 $ABC = 55$, $ACD = 60$, $ABD = 60$, $ABCD = 85$.

Consider this game from the point of view of Player A. Alone he can realize 10 points. If he forms a coalition with Player B, the two players command 30 points. Similarly, the coalitions AC and AD command 35 and 25 points together, respectively. Or, if Player A joins with both Players B and C, the three command 55 points. Both coalitions ACD and ABD command 60 points. Finally, if four players combine, this grand coalition of all the players controls 85 points. Note that Players B and C cannot form a coalition without the presence of Player A. The value of coalitions BC and BCD are technically zero; empirically they are not allowed to form.

In NPER II, the players communicate by typing various statements about how to divide the value of a coalition among its members, in the context of a bargaining process that contains three stages. The first, or negotiation stage, is when players explore the potentials of various coalitions. Different proposals for forming coalitions and allocating the values are made. In this stage, players gain awareness of their relative strengths and weaknesses, and of the stances and expectations of the other players. The second stage, termed the acceptance stage, begins when a set of players indicates general acceptance of a proposal. This acceptance is not binding, but does indicate that serious consideration is being given to the proposal as an outcome for that coalition. During the acceptance stage, the members of the coalition may be interested in modifications of the agreement, and the nonmembers of the coalition are interested in disrupting the proposal to further their own interests. It is in this stage that various strategies and counterstrategies may become quite subtle and complex; and for this reason, it is a psychologically interesting stage. Finally, the third, or ratification stage, terminates the bargaining process. The members of a tentative coalition have considered a proposal, have seen it through acceptance, and are now willing to make a binding com-

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mitment. If all players agree upon ratification, the coalition is formed, the players receive their payoffs, and the game is terminated for them. The entire game is terminated when each player is in one, and only one, ratified coalition.

Communication in NPER II is via a set of key words. Upon receiving a request for input from the computer, the player types a key word, followed by parameters if necessary. The computer then checks the legality of the message and transmits its content to the players who are supposed to receive it. Table 1 shows the set of key words used in NPER II, along with their required parameters and their effects on the coalitions addressed and on other coalitions.

The basic key words for bargaining are OFFER, where a player proposes an allocation of points; ACCEPT, where a player makes a tentative commitment to the allocation; and RATIFY, where a player initiates termination of the game. Indeed, it is possible to play the game with only this primitive set of three key words. Any member of a coalition may make an offer; the computer maintains each player's most recent statement regarding each coalition. To enter the acceptance stage, all members of a coalition (including the originator of the OFFER) must ACCEPT it (the originator cannot be

the first to ACCEPT his own OFFER). When one player proposes RATIFY, all players in accepted coalitions immediately vote yes or no. Coalitions with unanimous consent are ratified; others are dissolved.

The key words AGREE, REJECT, and AFFIRM give shading to intentions concerning coalitions. AGREE indicates a mild liking of an OFFER and REJECT strong distaste (the proposal can simply be ignored). AFFIRM indicates a reiteration of a player's most recent stand with respect to a coalition. PASS indicates that a player has nothing to say at the present time. SOLO immediately removes a player from the game; he will take what he can get by not joining any coalition. SUGGEST is like OFFER except that (1) it is unofficial and (2) it may be sent to any players, not necessarily including all those in the proposal. REMARK and MESSAGE allow communication outside the NPER II vocabulary. The former allows free text, while the latter sends experimenter-defined standard messages. If players lose track of the proceedings, the key words VALUE and STATUS keep them up-to-date. All key words other than OFFER, ACCEPT, and RATIFY may be disabled at the experimenters option.

Figure 1 presents an example game from the point of view of Player B. The game is an example designed to

Table 1: Key Words and Their Effects

Key word	E.g. of parameter	Primary effect	Other effect on this coalition	Effects outside named coalition
PASS	(none)	Send null message.	(none)	(none)
SOLO	(none)	Immediate ratification of player's one-person coalition.	(none)	Player is removed from further play in this game.
OFFER	A = 20, B = 18, C = 17	Make a proposal.	Supersedes player's previous OFFER, AGREE, & ACCEPT.	(none)
SUGGEST	A = 20, C = 15 to ACD	Make informal, secret proposal	(none)	(none)
REJECT	ACD BY C	Erase an offer.	(none)	(none)
AGREE	ACD BY C	Show liking for an offer.	Same as for OFFER.	Erases any ACCEPT made by this player.
ACCEPT	ACD BY C	Tentative commitment to an offer.	Same as for OFFER.	Erases any AGREE or ACCEPT made by this player.
	(when all players have accepted an offer)	Game moves to acceptance stage.	(none)	Erases all other positions taken by included players. They may make no new OFFER except to subsets or supersets of the accepted coalition.
AFFIRM	ACD	Stand by previous action.	(none)	(none)
RATIFY	(none)	Propose move to ratification.	Form coalition if all members agree to ratification. Coalition is dissolved otherwise.	Concludes game for players in ratified coalitions.
REMARK	AC (text)	Send (text) to named player(s). Counts as a communication.		
MESSAGE	A = 1, C = 3	Send numbered message(s) to named player(s). Counts as a communication.		
VALUE	(none)	Information: Give all values in the game except those involving removed players. Does not count as a communication.		
VALUE	ACD	Information: Give value of that named coalition. Does not count as a communication.		
STATUS	(none)	Information: Give current statements involving every coalition in the game. Does not count as a communication.		
STATUS	ABC	Information: Give current statements regarding named coalition. Does not count as a communication.		
STATUS	ABOUT AC	Information: Give current statements involving coalitions including named player(s). Does not count as a communication.		
STATUS	BY AC	Information: Give current positions taken by named player(s). Does not count as a communication.		

0. IN THE FOLLOWING GAME YOU ARE PLAYER B
 COALITIONS HAVE THE FOLLOWING VALUES:
 A=10; B=15; C=15; D=10; AB=30; AC=35; AD=25; BD=40; CD=35
 ABC=55; ACD=60; ABD=60; ABCD=85.

1. A OFFERS AC: A=15; C=20

2. B, GO AHEAD: OFFER B=25, C=25
 BC IS NOT A VALID COALITION

B, GO AHEAD: OFFER A=25, B=25, D=10
 B OFFERS ABD: A=25; B=25; D=10

3. C PASSES

4. D SAYS: IF I DON'T GET A GOOD DEAL, I MAY SOLO.

5. A AGREES ABD BY B: A=25; B=25; D=10

6. B, GO AHEAD: ACCEPT ABD BY B
 YOU CANNOT ACCEPT YOUR OWN OFFER UNTIL SOMEONE ELSE DOES

B, GO AHEAD: OFFER A=20, B=20
 COALITION DOES NOT SUM TO 30

B, GO AHEAD: REMARK ACD SORRY, I JUST KEEP MAKING MISTAKES
 B REMARKS TO ACD: SORRY, I JUST KEEP MAKING MISTAKES

7. C OFFERS ABC: A=16; B=17; C=22

8. D REJECTS ABD BY B: A=25; B=25; D=10

9. A OFFERS ABC: A=20; B=15; C=20

10. B, GO AHEAD: OFFER C=13, A=17, B=25
 B OFFERS ABC: A=17; B=25; C=13

11. C AFFIRMS HIS OFFER TO ABC: A=16; B=17; C=22

12. D SOLOS AND IS REMOVED FROM PLAY

13. A OFFERS AB: A=13; B=17

14. B, GO AHEAD: VALUE
 A=10; B=15; C=15; AB=30; AC=35; ABC=55

B, GO AHEAD: STATUS
 A OFFERED AC: A=15; C=20
 C OFFERED ABC: A=16; B=17; C=22
 A OFFERED ABC: A=20; B=15; C=20
 B OFFERED ABC: A=17; B=25; C=13
 ** SOLOED D: D=10
 A OFFERED AB: A=13; B=17

B, GO AHEAD: SUGGEST A=15, B=21, C=19, TO A
 B SUGGESTS ABC TO A: A=15; B=21; C=19

17. B, GO AHEAD: AGREE AB BY A
 B AGREES AB BY A: A=13; B=17

18. C AGREES AC BY A: A=15; C=20

19. A ACCEPTS ABC BY C: A=16; B=17; C=22
 REPLACING HIS OWN OFFER TO ABC

20. B, GO AHEAD: STATUS
 A OFFERED AC: A=15; C=20
 C AGREED.
 C OFFERED ABC: A=16; B=17; C=22
 A ACCEPTED.
 B OFFERED ABC: A=17; B=25; C=13
 ** SOLOED D: D=10
 A OFFERED AB: A=13; B=17
 B AGREED.

B, GO AHEAD: OFFER A=16, C=20, B=19
 B OFFERS ABC: A=16; B=19; C=20
 REPLACING HIS PREVIOUS OFFER

21. C ACCEPTS ABC BY B: A=16; B=19; C=20
 REPLACING HIS OWN OFFER TO ABC
 AND HIS AGREE TO A'S OFFER TO AC

22. A ACCEPTS ABC BY B: A=16; B=19; C=20

23. B, GO AHEAD: AC ABC BY B
 B ACCEPTS ABC BY B: A=16; B=19; C=20
 REPLACING HIS AGREE TO A'S OFFER TO AB
 ABC NOW ALL-ACCEPTED, BULLYING ALL OTHER POSITIONS BY THEM

24. B, PLAYER C CALLED FOR A RATIFICATION VOTE. DO YOU RATIFY
 THE COALITION ABC: A=16; B=19; C=20
 (Y OR N) ? Y
 C CALLED FOR RATIFICATION
 ABC VOTED TO RATIFY
 ABC ARE NOW IN RATIFIED COALITION AND ARE REMOVED FROM
 PLAY
 !!GAME OVER!!

25. TODAY YOU HAVE EARNED 19 POINTS

Figure 1. Player B's protocol of a four-person example game in characteristic function form. (B's typed messages are underlined.)

show various features of the program, and does not represent actual data. Except for the numbering of the statements, the figure is an exact replica of the input and output from Player B's teletypewriter. Message 0 is from the computer, and tells the player his own identity and the characteristic function for this game. Our discussion will be guided by the statement numbers in Figure 1,

which correspond to moves in the play of this game.

The first move of the game is an OFFER from Player A to coalition AC, in which both Players A and C receive five more points than each would by participating in no coalition. Player B moves second, and after attempting to address an illegal coalition, makes a three-person OFFER to coalition ACD. Note that, in OFFER statements, the coalition is implied by who is named in the point allocation. Player C chooses to wait one round, so Statement 3 is a PASS on his part. Player D uses his move to send a MESSAGE. Player B receives the message shown on his teletypewriter; he does not know if other players received similar, different, or no messages from Player D. The game continues through move 11, showing the different types of key words and some error messages in response to Player B's errors. Note that in B's REMARK in move 6, the computer reminds him who received his remark; other players do not receive that information.

On move 12, Player D chooses SOLO, which removes him from the game. Player B, on his next move (14), uses the VALUE key word to find out what coalitions remain legal in the game, and then uses STATUS to obtain a record of all current positions in the game. Note that Player B does not see moves 15 and 16; Players A and C have sent communications (SUGGEST, MESSAGE, or REMARK) to each other, but have not included him as a recipient.

In moves 17 through 23, the players again proceed with the game, showing how the computer signals the effect of new moves on previously taken positions and demonstrating how all-accept is signaled. Finally, on move 24, Player C calls for ratification. Both Players A and B vote in favor of ratification (simultaneously and independently) and, with all players now in ratified coalitions, the game terminates. As this game was the only one played in this session, the person who played B is given the sessions' earnings in Statement 25.

COMPUTATIONAL FEATURES

NP-ER II is written for the PDP-11/45 in 11TRAN, a language resembling PL/I written at the L. L. Thurstone Psychometric Laboratory. In the program design, emphasis is placed on the subject interface; for example, the program utilizes extensive error checking and validation, dynamic key word abbreviations, and clear, concise messages in easily readable formats. Because of the large number of players allowed, as well as the total possible number of positions that could theoretically be outstanding at one time (on the order of 300,000), data structures are dynamically allocated from free space within the task and are inserted in appropriate linked lists. The essence of NP-ER II, then, is a computer program composed of a sophisticated parser, an internal structure based on linked lists, and a message formulator and printer.

IMPROVEMENTS OVER PREVIOUS VERSIONS

As stated earlier, NPER II is a revised version of a system used extensively for the past 5 years. Perhaps an indication of the flexibility of the new program can be gained by reiterating three limitations of the previous version discussed by Rapoport and Kahan (1974), and showing how the present version overcomes these limitations.

The first constraint of the previous program was that a player could accept only one coalition at a time. In effect, the ACCEPT key word's function was too narrow. In the present version, acceptance has been broken down into AGREE and ACCEPT statements. An agreement, the weaker version, can now be made to each potential coalition of which a person is a member. Acceptance is now free to have stronger force. A player must now accept his own original offer, once others have indicated acceptance of it. That is to say, where an OFFER was once assumed to be automatically accepted by its author, now it is only assumed to be agreed to. This loosening of acceptance procedures also allows the program to be expanded so that each player's most recent OFFER to a coalition is outstanding, rather than just one OFFER per coalition. Thus, particularly in coalitions of size greater than two, the range of possibilities for a player is considerably expanded.

A second limitation of the previous game was that a ratification of a coalition of a subset of players changed the game, as coalitions with members of that subset were then not available to the remaining players. Thus, only the first coalition ratified could be used as a test of any particular model which assumes simultaneous ratification. In the present version, the experimenter may lock out ratification for an unlimited time, until all players are in accepted coalitions. A single ratification statement then serves all accepted coalitions at once, instead of only those players in a particular accepted coalition.

A third limitation was that only one issue could be negotiated at a time. As present, research trends in the areas of bargaining and decision making indicate increased interest in multi-attribute environments; NPER II has been constructed to play with up to three issues (each specified by its own characteristic function) being negotiated simultaneously. In the multi-attribute version, the issues are named by the experimenter (typically with color names), and the player must specify, in addition to his key word, which issue he wants to address with his message. Thus, an agreement in this version might read: BLUE AGREE ABC BY B. Here a player agrees to the proposal to coalition ABC made by Player B, in the issue labeled "blue."

APPLICABILITY

NPER II can accommodate all of the research done using the previous system. Coalitions was used in the in-

vestigation of games in characteristic function form for three players, both with (Rapoport & Kahan, 1975; Medlin, Note 1) and without (Kahan & Rapoport, 1974, 1975) three-person coalitions, and varieties of four-person games (Funk, 1972; Horowitz & Rapoport, 1974). By simple extension, other work directly investigating n-person games in characteristic function form (e.g., Buckley & Westen, 1973; Maschler, 1965; Riker, 1967; Selten & Schuster, 1968) could be run using NPER II.

In our usage, subjects are paid a linear function of the points they earn. However, by employing special payoff transformations, other experiments that are not already in NPER II format could be replicated. Laing and Morrison (1974) have studied games of status, where a player's payoff depends not on the absolute number of points he has, but rather on the rank order of his total among the totals of all participating players. Their experiments could be replicated by using their characteristic functions as originally presented, but by rewarding subjects on the basis of rank order rather than on absolute quantity.

Much of the social psychological work on coalition formation has used the "Pachisi" paradigm (Caplow, 1968) pioneered by Vinacke and Arkoff (1957). In many of these studies, interest focused on a game where players could combine weights of four, three, and two in a majority-rule constant-sum game. This classical game could be played in NPER II format by using the following characteristic function: $A = 1$, $B = 0$, $C = 0$, $AB = 700$, $AC = 600$, $BC = 500$, $ABC = 900$. The values to the multiperson coalitions represent the 4-3-2 weights multiplied by 100. The values to the one-person coalitions represent the fact that, if no coalition forms, Player A (with the largest weight) wins the game. The multiplicative factor of 100 is to round Player A's payoff to zero if coalition BC forms. Subjects are paid a constant sum, which is split in proportion to the points each has at the end of the NPER II game. Such a technique would allow a test of the social psychological hypotheses without the confounding of restricted communication and the artificial separation of payoff and coalition formation that characterize most of the Pachisi studies.

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