A program of a generalized Bayesian algorithm for quality control and auditing

HERBERT MOSKOWITZ

Krannert School of Industrial Administration
Purdue University, West Lafayette, Indiana 47907

and

RAYMOND K. FINK

School of Aeronautics and Astronautics
Purdue University, West Lafayette, Indiana 47907

The algorithm described below determines the minimum cost parameter values (sample size n^* and acceptance number c^*) for single sample acceptance plans which incorporate inspector errors in quality control and auditing problems when the prior distribution of lot or account quality and sampling distribution are discrete. In quality control, for example, such plans involve: (1) drawing a single sample of n items from a lot of size N , (2) determining the number of defective items, y , in the sample (perhaps erroneously observed and reported), and (3) rejecting the lot if more than c defectives are observed. In principle, three different approaches can be used to determine the parameter values for an error prone (or error free) single sample attribute acceptance plan when a cost criterion is applied. First, Bayesian decision tree methods can be applied to determine an exact optimal solution by complete implicit enumeration of all possible solutions (Raiffa & Schlaifer, 1961). However, this involves substantial computational effort, since all of the branches of the decision tree must be evaluated and the computations of the various probabilities, if inspector error is involved, can become onerous. Second, analytical methods can be used when the distribution of the lot fraction defective can be approximated by a standard density function such as the Beta or Polya distribution (Collins, 1974, and references cited therein). Although this procedure reduces the computational requirements, nonoptimal solutions to the real problem may be produced as a result of these approximations, since the densities may not be sufficiently rich to adequately capture the true prior distribution. Finally, direct numerical search techniques can be used to reduce the computational effort, but such methods may not converge on an optimal solution because of the "bumpy" nature of the cost surface in such problems. The programmed algorithm described below is a general algorithm that determines the optimal parameter values for Bayesian single sampling attribute plans with (or without) inspector error. It has been shown to be clearly superior to the above alternate approaches, producing the optimal solution with minimal computational requirements over a wide range of problem types (Moskowitz & Fink, 1975). The algorithm is applicable to a broad range of acceptance sampling and auditing problems, assuming only that the sampling or auditing cost is either a linear or strictly convex function of the sample size. The algorithm can accommodate both constant as well as nonconstant inspector errors, which, for example, might reflect the effects of fatigue on inspector performance.

Description. The program of the algorithm consists of three principal aspects: (1) recursion routines for generating the *probability matrices* relating the quality states (F), actual defectives (X), and reported or adjudged defectives (Y) for the sample

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sizes considered (i.e., $p_n(X|F)$, $p_n(Y|X)$, $p_n(Y|F)$, $p_n(Y)$, $p_n(F|Y)$); (2) a computational routine which "prunes" branches of the decision tree in *searching* for the region containing the global optimum solution, and (3) the *computation* of the expected total cost for finding the optimal plan in the region of the global optimum.

In generating the probabilities, the program employs three subroutines. Subroutine RECUR is used to recursively generate $p_n(Y|X)$ and subroutine BIREC (or a commonly available system library function such as MDBIN) to recursively generate $p_n(X|F)$. If the cost function is such that these arrays are used only to calculate $p_n(Y|F)$, a substantial reduction in CPU time and core memory requirements can be achieved by simply calculating $p_n(Y|F)$ directly. This is quite straightforward; subroutine BIREC (or its equivalent) will do this if directed to. The probabilities $p_n(F|Y)$ and $p_n(Y)$ can then be calculated; subroutine PPY does this now. Thus, if only $p_n(Y|F)$ is needed, subroutines RECUR and PPY can be eliminated. The results obtained from the algorithm will be *identical*, but CPU time and core memory requirements will be reduced by at least 50%.

Two key notions form the basis of that portion of the algorithm which searches for the region of the global optimum: (1) the determination of the switchover points, namely, where the value of the optimal acceptance number c^* changes as n is increased, and (2) the observance of the cost properties of these switchover points which locates the region of the global optimum.

Input. The input consists of five cards. Card 1 reads in (1) the number of prior states, (2) lot size, (3) maximum sample size, and (4) the cost parameters of the problem (i.e., fixed cost of sampling and the sampling, repair, and defective product costs). Card 2 specifies the prior probability distribution of lot (account) quality. Card 3 inputs the possible states of the lot (account) quality. Cards 4 and 5 specify the initial array of inspector error probabilities for each item sampled [$p_i(Y|X)$]. With no inspector error, $p_i(Y|X)$ will be a 2 by 2 identity matrix.

Output. The printed output includes: (1) a listing of the problem parameters; (2) the value or expected total cost (ETC) of the null sampling plan; (3) the ETC at each switchover point; (4) the ETC for all sampling plans in the region of the global optimum; (5) the optimal sampling plan; and (6) CPU time.

Computer and Language. The program is written in FORTRAN IV for the CDC 6500 computer. Conversion to other computing systems is straightforward.

Restrictions. All arrays are currently dimensioned to handle a maximum of seven prior states and a maximum sample size of 200. As such, the program requires slightly less than 150K (octal) core to load and run. Restrictions on problem size can easily be altered by changing dimension statements.

The 201 by 201 array "PYX" [for $p_n(Y|X)$] takes up most of the space in the program; if it is changed to 101 by 101, the program will fit in 60K (octal). While redimensioning other arrays will further reduce this core requirement, the change will be small compared to the change for "PYX;" thus, if a change is warranted, "PYX" is the only array that really should be bothered with. It is dimensioned twice in the common /PPYRCR/ block specified in subroutines PPY and RECUR. As previously noted, if $p_n(Y|F)$ is calculated recursively and $p_n(Y|X)$ is not used, the "PYX" array can be eliminated entirely.

BIREC is a binomial sampling recursive routine. If the sampling distribution is not binomial, it is a straightforward procedure to replace BIREC (or MDBIN) with other commonly available system functions to obtain hypergeometric or poisson sampling distributions, for example.

Availability. A more detailed description of the program and the algorithm and a complete program listing of the algorithm may be obtained by writing Herbert Moskowitz, Krannert

Graduate School of Industrial Administration, Purdue University, West Lafayette, Indiana 47907. Program descriptions and listings of the decision tree (exhaustive enumeration) program, pattern search program, and a program using an analytical approximation scheme based on a Beta prior distribution (Smith, 1965) are also available.

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ERRATA

Prokasy, W. F. Random control procedures in classical skin conductance conditioning. *Behavior Research Methods & Instrumentation*, 1975, 7 (6), 516-520. Page 516, Column 1, Paragraph 4, Line 9, the sentence should read: This inequality contrasts with that of the two-cue situation in which $P(\text{UCS}/\text{CS}-) = P(\text{UCS}/\text{CS}-, \text{CS}+)$. Page 516, Column 2, beginning with the first sentence in the first complete paragraph, the sentence should read: Rescorla (1967) theorized, and demonstrated with a conditioned suppression paradigm, that, when $P(\text{UCS}/\text{CS}-)$ is less than $P(\text{UCS}/\text{CS}-)$, the $\text{CS}-$ becomes inhibitory.

Lynch, J. J. Use of a generalized analysis of variance program in repeated measures designs having unequal group sizes. *Behavior Research Methods & Instrumentation*, 1975, 7 (1), 54-56. Page 55, Table 1, Column 1, Line 20: $\text{NNW} = \text{NNW} + 1$ should read $\text{NNW} = \text{NW} + 1$; Line 50: $\text{DO } 90 \text{ I} = 1, \text{NCELLS}$ should read $\text{DO } 95 \text{ I} = 1, \text{NCELLS}$; Line 68: $100 \text{ SMEAN} = \text{SSUMSQ}(\text{I}) / \text{NNDF}(\text{I})$ should read $100 \text{ SSMEAN} = \text{SSUMSQ}(\text{I}) / \text{NNDF}(\text{I})$; Line 81: $\text{IX} = 0$ should be moved one space to the right.