SESSION XV MULTIDIMENSIONAL SCALING: A SYMPOSIUM

CYNTHIA NULL, *College of William and Mary, Presider*

Design considerations for multidimensional scaling

CYNTHIA H. NULL

College of William and Mary, Williamsburg, Virginia 23185

This paper begins with a short tutorial on multidimensional scaling. The focus of the remainder of the paper is on the proper designing of research that will use multidimensional scaling analysis techniques and includes suggestions for the interpretation of results.

Multidimensional scaling (MDS) refers to a set of data analysis techniques. These techniques all present the structure of data in a spatial or pictorial way. Most construct a geometric representation of the data in a low-dimensional euclidean space. Some provide additional information about subject differences or changes over time (see Carroll & Chang, 1970; Kruskal, 1964a, 1964b; Takane, Young, & de Leeuw, 1977).

The results of an MDS analysis are related to the data in a way that is very similar to the relationship of a table of distances between cities that often appear on a map and the road map. In this case the table of distances corresponds to the data, and the road map corresponds to the spatial representation usually called the configuration. Clearly, both the table of distances and the road map present the same information (at least if the table is large enough), but the road map is much easier to understand. The relative ease of understanding the map as compared with the table illustrates the basic reason for using MDS procedures.

The data appropriate for MDS analysis is essentially any matrix whose elements represent the relationship between row and column objects. Some common relationships used are distances, correlations, similarities, confusions, and preferences. The matrices may be symmetric or asymmetric, square or rectangular, complete or with missing entries.

Presented in Figures 1 and 2 are the results from a study of color perception looking at differences between the Dani (primitive Indonesian tribe) and Americans (Heider & Olivier, 1972). The stimuli were 10 hues at four intensities. The data were four square confusion

matrices. The configurations for the American observers are approximately color cylinders for both tasks: "name the color" (naming, Figure 1a) and "which one did you just see?" (memory, Figure 2a). The Dani memory results (Figure 2b) are very similar to the American results. However, the naming results (Figure 1b) reflect the different color-naming system of the Dani language. The Dani have but two words for color, which roughly translate into bright and dark. I believe these results illustrate not only how MDS can be used but how useful it can be.

The previous examples have dealt with the analysis of single matrices. Let us briefly turn our attention to replicated matrices and individual differences scaling (Carroll & Chang, 1970; Takane et al., 1977). In addition to the goal of spatially representing large amounts of data in a low-dimensional space, individual differences MDS presents the differences between individual matrices. Let me illustrate with a study of the perception of nations (Wish, Deutsch, & Biener, 1972). In addition to making similarity ratings between pairs of the 12 nations, the subjects stated their opinion regarding the Vietnam War. Results from this study appear in Figures 3 and 4.

The dimensions of the group configuration (Figure 3a) have been interpreted as political alignment and economic development, with U.S.S.R. and U.S.A. differing on political alignment but not substantially on economic development and with U.S.A. and Cuba differing the most on both dimensions. The individual differences information for the subject's weights space appears in Figure 3b. To get a configuration for any particular subject, the group configuration can be multiplied by the individual's weights. Figure 4a contains such an individual solution for a subject using political alignment more than economic development. The results

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for a subject using economic development more than political alignment can be found in Figure 4b. Details on the meaning of these results are available elsewhere (Kruskal & Wish, 1978; Wish et al., 1972).

A good beginning reading in MDS is a monograph by Kruskal and Wish (1978). I will spend the remainder of this paper discussing considerations that should be

made in designing a study using MDS analysis and in interpreting the results.

DESIGN AND INTERPRETATION CONSIDERATIONS

Selection of the stimuli must be done carefully. The

Figure 1. Multidimensional scaling of Dani and American color naming from Heider and Olivier (1972).

stimuli must be representative of the domain of interest. The best situation is, of course, using all stimuli in the domain. However, if one is interested in a map of the United States, using all stimuli is impossible. But the map is less than veridical if only large East Coast cities are used.

Additionally, the set of stimuli must not be too heterogeneous. For example, all living things would be a nearly impossible stimulus domain. The choice of a relatively small number (20-40) of representative stimuli would be impossible. One has to be careful of the other extreme, as well. All Monet paintings of the Rouen

Figure 2. Multidimensional scaling of Dani and American color naming from Heider and Olivier (1972).

Figure 3. Two-dimensional INDSCAL analysis of 18 matrices of similarities among nations from Kruskal and Wish (1978).

Figure 4. Private perceptual spaces for two subjects, 17 and 4, obtained by applying their weights to the group stimulus space (Figure 3) from Kruskal and Wish (1978).

Cathedral, West Facade, is probably much too homogeneous to gain useful or meaningful information from MDS. A stimulus set should not include one or two stimuli that are distinct from the others or have some obvious special distinction(s). For example, including George Washington and Abraham Lincoln in a study along with recent American presidents and presidential candidates would probably yield a very strong alive-dead dimension that might override other considerations. Using two very distinct groups of stimuli in one study may yield a degenerate solution. That is, the differences between the stimuli within each of the groups are overridden by the between-group distinctions (see Shepard, 1974).

In most cases it is not advisable to factorially choose the stimulus set. If an experimenter varies size (large or small), color (red or blue), shape (oval or rectangle), and orientation (horizontal or vertical) of geometric figures, he or she should not be surprised if the subjects use size, color, shape, and orientation alone in making their judgments. Using individual differences MDS one might be able to conclude that color-blind subjects do not make color distinctions or that children use the orientation dimension less than all other subjects. It is not appropriate to conclude that, overall, size is an important dimension in visual perception because the subjects in such a study use size. The subjects in factorially designed studies usually have few dimensions to consider, the distinctions are defined by the experimenter, and most subjects seem willing to "help" an experimenter by using all the defined dimensions.

There have been many misconceptions about the number of stimuli that should be used in a MDS study. The answer is simple: Use as many as feasible. There are some simple guidelines (Young, 1970): If a two-dimensional solution is expected, use at least 12 stimuli; if three-dimensional, use at least 15; and if four, use at least 20. If the data is considered to have very low amounts of error, as few as one more than four times the number of expected dimensions may be an adequate number of stimuli. Of course, the smaller the number, the less generalizable the results. The group configuration from an individual differences MDS should remain fairly stable if the number of stimuli fall below these guidelines; however, the subject weights will suffer greatly and should be used with caution.

If data are collected using similarity ratings of all possible pairs of stimuli, certainly the most common method, the number of judgments per subject can quickly become prohibitive. For example, if the number of stimuli is 25, the number of judgments would be 300, if 30, then 435. Fortunately, many of the programs available for MDS allow missing data entries (INDSCAL by Carroll & Chang, 1970, being a major exception). The total number of judgments obtained should be at least three times the number of expected dimensions times the number of stimuli (Spence, 1977).

Several interactive scaling techniques have been developed to deal with this problem (Green, 1980; Hamer, 1978; Null & Young, 1978; Young, 1980; Young & Cliff, 1972). Other methods have been suggested as well for limiting the work required by each subject, including using sorting instead of similarities, giving subjects small subsets of all possible pairs and combining subjects, and using profiles or rankings (Young, 1975, 1980; Wish, Note 1).

Selection of the subjects is also a consideration. In most cases a heterogeneous population of subjects is desirable. This is especially important in individual differences MDS, where differences between subjects is vital in orienting the group configuration. One notable exception is, if subject data are being combined or averaged to make a single matrix, then homogeneous subjects are desirable.

At this point I will make three additional specific comments on the interpretation of MDS solutions. First, let us consider the labeling of the dimensions. I strongly believe this is not the time for crystal ball gazing. Two types of external unfolding are available to help a researcher in dimension interpretation: vector and point unfolding (Carroll, 1972; Chang & Carroll, 1968). An example of vector unfolding from Rosenberg and Sedlak (1972) is given in Figure 5. Ratings of the 80 personality traits along many properties, including hard-soft, good-bad, and dominant-submissive, were obtained from subjects. These ratings were unfolded into the MDS solution. The numbers in parentheses indicate the correlation between projections on the best-fitting axis and the ratings for the property. Either orthogonal or oblique dimensions can be labeled using such techniques. Note that with an individual differences MDS solution the dimensions interpreted must be orthogonal and lie along the axes obtained in the solution (unless all subject weights lie on the same vector).

The data used in such an unfolding analysis usually come from at least one of two sources. The first source is the physical attributes of the stimuli (where applicable) (e.g., intensity measures for colors or sounds). The second source is ratings by subjects. I suggest that the subjects be asked to suggest properties of the stimuli. Then a subject should choose and rate those properties he or she perceives as most important from personal suggestions and a list provided by the experimenter.

Schönemann and Carroll (1970) have developed a method for rotating two configurations to congruence that has proven to be very helpful in interpretation of MDS solutions. The configurations to be matched could be from two different sets of subjects, the same subjects under two different conditions or at two different times, or one configuration could be based on physical information about the stimuli. For example, the last instance would be used when studying cognitive geographic maps, that is, comparing a true map with the perceived map.

Figure 5. Two-dimensional configuration obtained from scaling 80 personality traits shown with the best-fitting axis for each of five properties from Rosenberg and Sedlak (1972).

Table 1
Subject Weights Example

Subject	Dimension		Variance Accounted for
	1	2	
1	.33	.09	.1170
2	.33	.70	.5989

My last interpretation comment is made because of confusions about the proper way to use weights from individual differences MDS. These subject weights are to be interpreted as vectors or directions in the weight space and not as discrete values or values for a dimension. For example, suppose for a two-dimensional solution we had the weights given in Table 1. These subjects are not using Dimension 1 in the same manner. In fact, 93% $[(.33^2 / (.33^2 + .09^2))]$ of the variance in Subject 1's data is accounted for by the first dimension, while

82% $(.49 / .5989)$ of the variance in Subject 2's data is accounted for by the second dimension.

In conclusion, MDS is a powerful and useful tool if used correctly (not unlike any other statistical procedure). The stimuli must be chosen carefully. The number of stimuli should be large and representative. And, of course, interpretations must be made with great care.

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