COMPUTER TECHNOLOGY

RSCORE-J: Pooled rating-method data: A computer program for analyzing pooled ROC curves

DONALD D. DORFMAN and KEVIN S. BERBAUM

University of Iowa, Iowa City, Iowa

RSCORE-J, a computer program for a signal-detection analysis of pooled rating-method data, is listed and described. RSCORE-J computes jackknife estimates of ROC parameters and their standard errors from rating-method data pooled over a group of observers.

A standard method of estimating signal-detection theory parameters and confidence intervals for a rating ROC curve of an individual observer has been the application of a maximum-likelihood computer algorithm (e.g., see Dorfman, 1982; Dorfman & Alf, 1969). Sometimes, however, an investigator is interested in estimating parameters of a group-rating ROC curve. A group ROC curve is usually obtained by pooling the raw rating data of the individual observers and proceeding as if the data pooled over observers were data obtained from a single observer (Swets & Pickett, 1982). This approach has recently been used to fit a variety of group-rating ROC curves (Swets, 1986). Group (pooled) ROCs are useful for a variety of reasons. For instance, in medical research they are valuable in characterizing diagnostic systems. In general, pooling of individual rating data may be the method of choice when data of individual observers is sparse (Swets & Pickett, 1982). Macmillan and Kaplan (1985) presented a valuable theoretical discussion of group ROC curves obtained by pooling data, and they recommend pooling sparse data from individual observers when sensitivities and response criteria are not very disparate.¹ Unfortunately, there has been no way of performing statistical tests or of computing confidence intervals for parameters estimated from group ROCs.

The current paper lists a computer program, RSCORE-J, which analyzes rating-method ROC data pooled over a group of observers using the unequal-variance Gaussian model. That model seems to give a quite satisfactory fit to a variety of rating-method group ROC curves (Swets, 1986). The program uses the jackknife method in combination with the RSCORE-II maximization algorithm (Dorfman, 1982; Dorfman & Alf, 1969)² to compute estimates and their standard errors of the ROC parameters of a group ROC under the unequal-

variance Gaussian model. Moreover, the jackknife method provides estimates that have reduced statistical bias (Rey, 1978; Tukey, 1958, 1969). The method can be shown to apply to estimates derived from a group likelihood function (see Arvesen & Salsburg, 1975; Brillinger, 1964). Although the justification is based upon asymptotic theory, the method seems to be robust with regard to sample size. Moreover, the results obtained with the jackknife method are impressively good in most cases (Rey, 1978). The program has recently been used in the analysis of detection of lesions in medical images, where only 43 cases per observer were available (Berbaum et al., 1986). It is encouraging that the program gave meaningful results in good agreement with the visual differences among the empirical ROCs. A short mathematical description of the procedure is provided in Appendix A, and an extensive justification through Monte Carlo simulations is given in Dorfman and Berbaum (1986). The Monte Carlo simulations simultaneously evaluated the jackknife method and the computer program for the problem of interest.

The method yields a pseudovalue for each subject. The pseudovalues can be treated as if they are independent and identically distributed (Rey, 1978), and therefore can serve as observations to compute confidence intervals and to test hypotheses with statistical tests derived from standard Gaussian theory. In other words, the pseudovalue estimates of ROC parameters are analyzed in much the same way as individual-subject estimates of ROC parameters.

Appendix B is a listing of RSCORE-J. The first section (through the References) is a user's manual that includes sample data from 4 subjects that are useful in testing implementation of the program, and includes the same sample data used to test implementation of RSCORE-II (Dorfman, 1982). They are actual data from a visual-detection study (see Swets, 1986, pp. 186-187).

The next section (ending after Line 6011) is the main program. Approximately half of the instructions perform the jackknife procedure; the other half are adopted from

Requests for reprints should be sent to Donald D. Dorfman, Department of Psychology, University of Iowa, Iowa City, IA 52242. Kevin S. Berbaum is affiliated with the Department of Radiology.

	a	S.E. of a	Δm	S.E. of Δm	A_z	S.E. of A_z
— Average	1.268	.076	1.669	.079	.842	.012
- · - Pool	1.092	—	1.375	—	.804	—
— Jackknife	1.030	.143	1.307	.137	.793	.026

Figure 1. Binormal ROC curves derived from parameter estimates computed from (1) averaging estimates across observers, (2) fitting a group (pool) ROC as if it were an ROC from a single observer, and (3) fitting a group (pool) ROC by the jackknife method.

RSCORE II and perform the maximization algorithm. The final section lists the program subroutines that include IBM-produced, public domain software.

Appendix C presents output from the sample data provided in the user's manual. The group data pooled over the 4 subjects are presented first, followed by estimates of group ROC parameters and decision criteria, where the group data are treated as if they are the data of a single observer. These data are followed by the jackknife pseudovalues (see Tukey, 1969, and Appendix A) of the parameter estimates for the four subpools (each subject has an associated subpool, consisting of the total pool with his/her data deleted). Finally, jackknife estimates and standard errors, computed in the usual way (see Tukey, 1969, and Appendix A), are displayed.³ Figure 1 presents the binormal ROC curves fitted to the sample data using (1) the averages of the individual-subject ROC parameter

estimates computed from RSCORE-II, (2) the group (pooled) data ROC parameter estimates computed from RSCORE-II, and (3) the jackknife ROC parameter estimates computed from RSCORE-J. Note how much closer the curves from pooling and jackknifing lie than those from pooling and averaging. Also note that there are no standard errors associated with the pooled ROC estimates.

Figure 1 shows that for this set of sample data the visual fit of the theoretical curve from jackknifing appears slightly poorer than the fit of the theoretical curve from direct pooling. In particular, the theoretical curve from jackknifing is slightly lower than the theoretical curve from pooling. This is manifested in the slightly lower values of sensitivity (discrimination accuracy) for jackknifing than for pooling. Notice, however, that the sensitivity estimates from pooling are well within the 95% confidence intervals for the jackknife estimates

($\hat{\theta} \pm 3.18 S_{\hat{\theta}}$). Moreover, in all other sample data sets, we have found the difference between estimates of sensitivity for jackknifing versus pooling to be much smaller than the difference observed in Figure 1 (Dorfman & Berbaum, 1986).

In summary, RSCORE-J will allow a theoretical ROC curve to be fitted to an empirical group ROC curve so as to permit statistical inference. In discussing the jackknife method, Tukey has said (1969, p. 84), "In my personal judgment there are few cases where it is not worth using."

REFERENCES

- ARVESEN, J. N., & SALSBERG, D. S. (1975). Approximate tests and confidence intervals using the jackknife. In R. M. Elashoff (Ed.), *Perspectives in biometrics*. New York: Academic Press.
- BERBAUM, K., FRANKEN, E. A., JR., DORFMAN, D. D., BARLOON, T., ELL, S. R., LU, C. H., SMITH, W., & ABU-YOUSEF, M. (1986). Tentative diagnoses facilitate the detection of diverse lesions in chest radiographs. *Investigative Radiology*, **21**, 532-539.
- BRILLINGER, D. R. (1964). The asymptotic behavior of Tukey's general method of setting approximate confidence limits (the jackknife) when applied to maximum likelihood estimates. *Review of the International Statistical Institute*, **32**, 202-206.
- DORFMAN, D. D. (1982). RSCORE II. In J. A. Swets & R. M. Pickett, *Evaluation of diagnostic systems: Methods from signal detection theory* (pp. 212-232). New York: Academic Press.
- DORFMAN, D. D., & ALF, E., JR. (1969). Maximum likelihood estimation of parameters of signal detection theory and determination of confidence intervals—Rating method data. *Journal of Mathematical Psychology*, **6**, 487-496.
- DORFMAN, D. D., & BERBAUM, K. (1986). *A method for analyzing pooled ROC rating method data using the jackknife technique*. In preparation.
- MACMILLAN, N. A., & KAPLAN, H. L. (1985). Detection theory analysis of group data: Estimating sensitivity from average hit and false-alarm rates. *Psychological Bulletin*, **98**, 185-199.
- QUENOUILLE, M. (1949). Approximate tests of correlation in time series. *Journal of the Royal Statistical Society (Series B)*, **11**, 68-84.
- REY, W. J. J. (1978). *Robust statistical methods*. New York: Springer-Verlag.
- SWETS, J. A. (1986). Form of empirical ROCs in discrimination and diagnostic tasks: Implications for theory and measurement of performance. *Psychological Bulletin*, **99**, 181-198.
- SWETS, J. A., & PICKETT, R. M. (1982). *Evaluation of diagnostic systems: Methods from signal detection theory*. New York: Academic Press.
- TUKEY, J. W. (1958). Bias and confidence in not quite large samples. *Annals of Mathematical Statistics*, **29**, 614. (Abstract)
- TUKEY, J. W. (1969). Analyzing data: Sanctification or detection work? *American Psychologist*, **24**, 83-91.

NOTES

1. In the psychophysical literature, sensitivity is synonymous with discrimination accuracy. In the medical literature, sensitivity is synonymous with the probability of a true positive (hit). In this paper, sensitivity refers to discrimination accuracy and is measured by indices such as A_z or Δm .
2. RSCORE II (Dorfman, 1982) is a revised version of the program used by Dorfman and Alf (1969).
3. Notice that the pseudovalue estimate of Beta can be negative. This is an effect of the bias-reducing property of jackknife.

APPENDIX A The Jackknife Method

The jackknife method was first introduced by Quenouille (1949) to reduce the bias of a parameter estimate, and was generalized by Tukey (1958) to include estimation of confidence intervals and hypothesis testing. The results obtained with the jackknife method have been found to be impressively good in most cases (Rey, 1978).

Let $p_j(\theta_{01}, \theta_{02}, \dots, \theta_{0k})$ be the expected probability of the j^{th} rating category for data pooled over cases and observers. Assume that the standard unequal-variance Gaussian model of signal-detection theory (Swets & Pickett, 1982) holds for the group ROC curve, and that $\theta_{01}, \theta_{02}, \dots, \theta_{0k}$ are the parameters of that model. Let K_j denote the random variable for the total number of occurrences of the j^{th} rating category pooled over cases and observers, and let k_j denote an observed value of that random variable. Define a group log-likelihood function by

$$\ln L = \sum_{j=1}^r k_j \ln p_j(\theta_1, \theta_2, \dots, \theta_k),$$

where $E(K_j/st) = p_j(\theta_{01}, \theta_{02}, \dots, \theta_{0k})$, E denotes expectation taken over cases and observers, $(\theta_{01}, \theta_{02}, \dots, \theta_{0k})$ is some k -dimensional real number, s is the number of cases per observer, and t is the number of observers. Thus, we assume that the unequal-variance Gaussian model fits the group ROC curve for rating-method data.

We now apply the jackknife method (Tukey, 1958, 1969). First, compute the parameter estimate of $(\theta_{01}, \theta_{02}, \dots, \theta_{0k})$ from the group likelihood function by solving $\nabla \ln L = 0$.

Let $\hat{\theta}_{all}$ be a unidimensional real number that is the estimate of the signal-detection parameter of interest derived from that solution. Let $\ln L_{(i)}^{(1)}$ denote the group log-likelihood function with the data of the i^{th} observer removed from the pool. Let $\hat{\theta}_{(i)}$ denote the estimate of the signal-detection parameter of interest derived from the solution to $\nabla \ln L_{(i)}^{(1)} = 0$. Define the i^{th} pseudovalue as $\hat{\theta}_{*i} = t\hat{\theta}_{all} - (t-1)\hat{\theta}_{(i)}$, $i = 1, 2, \dots, t$. The pseudovalues can be treated as approximately independent and normally distributed. The jackknife estimate of the signal-detection parameter for the group ROC curve is

$$\hat{\theta} = t^{-1} \sum_{i=1}^t \hat{\theta}_{*i}.$$

It is, thus, the arithmetic mean of the pseudovalues. The variance of the jackknife estimate of the signal-detection parameter is

$$S_{\hat{\theta}}^2 = \frac{1}{t(t-1)} \sum_{i=1}^t (\hat{\theta}_{*i} - \hat{\theta})^2.$$

Justification

The justification for the application of jackknife to a particular problem is customarily through the evaluation of the results of Monte Carlo studies. Dorfman and Berbaum (1986) performed 10 Monte Carlo simulations to evaluate the jackknife method, with groups (pools) consisting of 5 or 10 hypothetical observers. The results showed that the jackknife estimates of parameters were consistent and unbiased, and that the nominal 95% confidence intervals contained the population parameter of interest about 95% of the time.

APPENDIX B (Continued)

```

C      5
C      1
C      591      174      172      104      92      41
C      597      46      57      66      101      154
C      2
C      591      217      206      97      57      12
C      597      37      54      80      156      165
C      3
C      591      334      99      26      42      43
C      597      81      44      26      32      66
C      4
C      591      513      54      17      4      1
C      597      206      121      107      45      51
C      /*
C      //
C
C      IF YOU HAVE 17 RESPONSE CATEGORIES, THE FORMAT STATEMENT MAY
C      REQUIRE CHANGE.
C
C      REFERENCES
C
C      DORFMAN, D.D. RSCORE II. IN J.A. SWETS AND R.M. PICKETT.
C      EVALUATION OF DIAGNOSTIC SYSTEMS: METHODS FROM SIGNAL
C      DETECTION THEORY. NEW YORK: ACADEMIC PRESS, 1982.
C
C      DORFMAN, D.D. STANDARD ERRORS AND CONFIDENCE INTERVALS FOR MEASURES
C      OF SENSITIVITY. ACOUSTICAL SOCIETY OF AMERICA MEETINGS, MIAMI
C      BEACH, FLORIDA, DECEMBER, 1977.
C
C      DORFMAN, D.D., & ALF, E., JR. MAXIMUM-LIKELIHOOD ESTIMATION OF
C      PARAMETERS OF SIGNAL-DETECTION THEORY AND DETERMINATION OF
C      CONFIDENCE INTERVALS--RATING-METHOD DATA. JOURNAL OF MATHEMATICAL
C      PSYCHOLOGY, 1969, 5, 487-496.
C
C      DORFMAN, D.D., BEAVERS, L.L., & SASLOW, C. ESTIMATION OF SIGNAL-
C      DETECTION THEORY PARAMETERS FROM RATING-METHOD DATA: A COMPARISON
C      OF METHOD OF SCORING AND DIRECT SEARCH. BULLETIN OF THE PSYCHONOMIC
C      SOCIETY, 1973, 1, 207-208.
C
C      GREEN, D.M., & SWETS, J.A. SIGNAL DETECTION THEORY AND PSYCHOPHYSICS.
C      NEW YORK: WILEY, 1966.
C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C      CHARACTER*53 COPYR
C      INTEGER SUBNO,SWT
C      DIMENSION XDUM(200)
C      REAL BETA(16),LBETA(16)
C      REAL LOGL,X(16),R(17),RP(17),SUM(18),SUM1(18),SUM7(18),SUM10(18),
C      *E(16),ML(16),XK(18,18),XP(18,18),CR(16),F(17),FP(17)
C      REAL*8 DSUMM,DSUMN,DSUM(18),DSUM10(18),DF(17),DFP(17)
C      DIMENSION NCURGP(100),A1PLUS(100),A2PLUS(100)
C      REAL RPLUS(100,20),RPPLUS(100,20)
C      REAL ZKKEEP(100,20),ROCPAR(100,6),BEKEEP(100,20),
C      *LBKEEP(100,20)
C      REAL XM(16),ZKK(16),RK(17),RPX(17)
C
C      COPYR='COPYRIGHT (C)1985 BY DONALD DORFMAN AND KEVIN BERBAUM'
C      LO=0
C      READ(5,199) NGP
C      198 LO=LO+1
C      IF(LO.GT.NGP)GO TO 2
C      199 FORMAT(12)
C      READ(5,199) NTHISG
C      READ(5,199) KK
C      KKK=KK+1
C      DO 197 L1=1,NTHISG
C      READ(5,199) SUBNO
C      READ(5,70) A1, (R(I),I=1,KKK)
C      READ(5,70) A2, (RP(I),I=1,KKK)
C      70 FORMAT(8F10.0)
C      A1PLUS(L1)=A1
C      A2PLUS(L1)=A2
C      DO 195 L2=1,KKK
C      RPPLUS(L1,L2)=R(L2)
C      RPPLUS(L1,L2)=RP(L2)
C      195 CONTINUE
C      197 CONTINUE
C
C      KO=-1
C      190 KO=KO+1
C      IF(KO.LE.NTHISG)GO TO 398
C      N=NTHISG
C      NN=NN+1
C      NNN=NN+1
C      DO 405 I=1,6
C      ROCPAR(NN,I)=0.0
C      405 ROCPAR(NNN,I)=0.0
C      DO 407 I=1,KK
C      ZKKEEP(NN,I)=0.0
C      ZKKEEP(NNN,I)=0.0
C      BEKEEP(NN,I)=0.0
C      BEKEEP(NNN,I)=0.0
C      LBKEEP(NN,I)=0.0
C      LBKEEP(NNN,I)=0.0
C      407 CONTINUE
C      DO 403 J=1,N
C      DO 404 I=1,6
C      404 ROCPAR(NN,I)=ROCPAR(NN,I)+ROCPAR(J,I)
C      DO 406 I=1,KK
C      ZKKEEP(NN,I)=ZKKEEP(NN,I)+ZKKEEP(J,I)
C      BEKEEP(NN,I)=BEKEEP(NN,I)+BEKEEP(J,I)
C      406 LBKEEP(NN,I)=LBKEEP(NN,I)+LBKEEP(J,I)

```

APPENDIX B (Continued)

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X(KK)=R(KKK)
CR(KK)=RP(KKK)
DO 12 I=2,KK
  X(KK-I+1)=X(KKK-I+1)+R(KKK-I+1)
12 CR(KK-I+1)=CR(KKK-I+1)+RP(KKK-I+1)
C
C TEST FOR 1.'S AND CORRECT BY SUBTRACTING 1-1/2N AND TEST FOR 0.'S AND CORRECT
C BY CHANGING TO 1/2N THEN CALL SUBROUTINE ZDEV TO TRANSFORM THE CUMULATIVE
C PROBABILITIES IN THE ARRAYS TO STANDARD NORMAL DEVIATES
C
Q=0.
DO 29 I=1,KK
  IF(X(I).EQ.0.) X(I)=1./(2.*A1)
  IF(X(I).GE.1.) X(I)=1.-1./(2.*A1))
  P=X(I)
  CALL ZDEV(P,Q,D,IER)
  X(I)=Q
  IF(CR(I).EQ.0.) CR(I)=1./(2.*A2)
  IF(CR(I).GE.1.) CR(I)=1.-1./(2.*A2))
  P=CR(I)
  CALL ZDEV(P,Q,D,IER)
  CR(I)=Q
29 CONTINUE
I2=KK-1
C
C STARTING FROM THE RIGHT, TEST TO SEE IF SDN(I+1) IS EQUAL TO OR LESS THAN
C SDN(I) AND IF SO MAKE SDN(I+1)=SDN(I)+.1
C
DO 4 I=1,I2
  IF(X(I2-I+1).LE.X(I2-I+2))X(I2-I+1)=X(I2-I+2)+.1
4 IF(CR(I2-I+1).LE.CR(I2-I+2))CR(I2-I+1)=CR(I2-I+2)+.1
C
C CALCULATE LEAST SQUARES SOLUTIONS
C
SUMX=0.
SUMY=0.
SUMXY=0.
SUMX2=0.
YK=KK
DO 13 I=1,KK
  SUMX=SUMX+X(I)
  SUMX2=SUMX2+X(I)**2
  SUMY=SUMY+CR(I)
  SUMXY=SUMXY+CR(I)*X(I)
13 CONTINUE
XMEAN=SUMX/KK
YMEAN=SUMY/KK
B=(KK*SUMXY-SUMX*SUMY)/(KK*SUMX2-SUMX**2)
AA=YMEAN-B*XMEAN
P1=3.1415926
ZA=1./SQRT(2.*PI)
NN=KK+2
SWT=0
LL=0
C
C GET INTEGRALS AND DENSITIES
DO 2000 J=1,NN
  DO 2000 K=1,NN
    XX(J,K)=0.
2000 XP(J,K)=0.
2001 LL=LL+1
  SUM(1)=0.
  SUM(NN)=1.
  SUM7(1)=0.
  SUM7(NN)=0.
  DO 6 I=2,KKK
    Y = -X(I-1)*B - AA
    CALL NDRTR(Y,SUM(I),SUM7(I))
6 CONTINUE
  SUM1(1)=0.
  SUM1(NN)=0.
  SUM10(1)=0.
  SUM10(NN)=1.
  DO 46 I=2,KKK
    Y = -X(I-1)
    CALL NDRTR(Y,SUM10(I),SUM7(I))
46 CONTINUE
19867 FORMAT(10E12.4)
  IF(LL.NE.1) GO TO 601
C
C WRITE INITIAL VALUES
  IF(KO.EQ.0)WRITE(6,6000)
6000 FORMAT(///'1')
  IF(KO.EQ.0)WRITE(6,6400)
6400 FORMAT(T20,'JACKKNIFE ESTIMATES FOR POOLED RATING METHOD DATA')
  IF(KO.EQ.0)WRITE(6,6401)
6401 FORMAT(T33,'-- DORFMAN AND BERBALM')
  IF(KO.EQ.0)WRITE(6,6402)
6402 FORMAT(/,T20,'COPYRIGHT (C)1985 BY D.D.DORFMAN AND K.S.BERBALM')
  IF(KO.EQ.0)GO TO 333
  WRITE(6,299) KO, LO
299 FORMAT(/'OPOOL ALL SUBJECTS EXCEPT SUBJECT ',I2,' IN GROUP ',I2)
  GO TO 334
333 WRITE(6,298) LO
298 FORMAT(/'OPOOL ALL SUBJECTS IN GROUP ',I2)
334 CONTINUE
  WRITE(6,6001) AA,B,KK
6001 FORMAT(/,T10,'INITIAL VALUES',//T20,'A= ',F10.4,'10X','B= ',F10.4
C6001 FORMAT(/,T10,'INITIAL VALUES',//T20,'A= ',F15.8,'10X','B= ',F15.8
  * ,//T20,'NO. OF Z(K)'S = ',I2)
  DO 521 I=1,KK
521 X(I)= -C(X(I))
  WRITE(6,6002) (X(I),I=1,KK)
6002 FORMAT(/,T20,'Z(K)= ',12(F7.3,X))
C6002 FORMAT(T20,'+Z(K)= ',7(E15.8,1X))
  WRITE(6,8000)
  WRITE(6,6003) A1,A2
5003 FORMAT(/,T20,'N1= ',F6.0,5X,'N2= ',F6.0)
  WRITE(6,6004) (I,I=1,KKK)
  WRITE(6,8009) (F(I),I=1,KKK)
  WRITE(6,8010) (FP(I),I=1,KKK)
  WRITE(6,530)
530 FORMAT(/,T10,'CUMULATIVE PROPORTIONS:')
  WRITE(6,6004) (I,I=1,KKK)
6004 FORMAT(/,T19,8X,17(I2,5X))
  RK(KKK+1)=0
  RPK(KKK+1)=0
  DO 535 I=1,KKK
    IK=KKK+1-II
    RK(IK)=R(IK)+RK(IK+1)
    RPK(IK)=RPK(IK)+RPK(IK+1)
535 CONTINUE
  WRITE(6,6009) (RK(I),I=1,KKK)
6009 FORMAT(T19,' N ',17(F6.4,1X))
8000 FORMAT(/,T10,'RESPONSE FREQUENCIES:')
8009 FORMAT(T19,' N ',17(F6.0,1X))
  WRITE(6,6010) (RPK(I),I=1,KKK)
6010 FORMAT(T19,' S+N ',17(F6.4,1X))
8010 FORMAT(T19,' S+N ',17(F6.0,1X))
C
C GET FIRST PARTIALS
C
C W R TO A
601 AAA=0.
  DO 4000 I=2,KKK
4000 AAA=AAA-SUM1(I)*(FP(I-1)/(SUM(I)-SUM(I-1))-FP(I)/(SUM(I+1)-SUM(I))
  1)
C
C W R TO B
  BBB=0.
  DO 3001 I=2,KKK
3001 BBB=BBB-SUM1(I)*X(I-1)*(FP(I-1)/(SUM(I)-SUM(I-1))-FP(I)/(SUM(I+1)-
  SUM(I))1)
C
C W R TO Z'S
  DO 3002 I=2,KKK
    Q1= -SUM1(I)*B*(FP(I-1)/(SUM(I)-SUM(I-1))-FP(I)/(SUM(I+1)-SUM(I)
    1))
    Q2= -SUM7(I)*(F(I-1)/(SUM10(I)-SUM10(I-1))-F(I)/(SUM10(I+1)-SUM1
    10(I)))
3002 E(I-1)=Q1+Q2
C
C GET EXPECTED SECOND AND MIXED PARTIALS
C
C W R TO A
  XX(1,1)=0.
  DO 3003 I=2,KKK
3003 XX(1,1)=XX(1,1)-SUM1(I)*((SUM1(I)-SUM1(I-1))/(SUM(I)-SUM(I-1))-
  1(SUM1(I+1)-SUM1(I))/(SUM(I+1)-SUM(I)))
  XX(1,1)=A2*XX(1,1)
C
C W R TO B
  XX(2,2)=0.
  DO 3004 I=2,KKK
    D=X(I-1)
    IF(I.EQ.KKK)GO TO 26
    IF(I.EQ.2) GO TO 25
    DD=X(I-2)
    DDD=X(I)
    GO TO 3004
25 DD=0.
    DDD=X(I)
    GO TO 3004
26 DD=X(I-2)
    DDD=0.
3004 XX(2,2)=XX(2,2)-SUM1(I)*X(I-1)*((SUM1(I)*D-SUM1(I-1)*DD)/(SUM(I)-
  1SUM(I-1))-SUM1(I+1)*DDD-SUM1(I)*D)/(SUM(I+1)-SUM(I)))
  XX(2,2)=A2*XX(2,2)
C
C W R TO A AND B
  XX(1,2)=0.
  DO 3005 I=2,KKK
3005 XX(1,2)=XX(1,2)-SUM1(I)*X(I-1)*((SUM1(I)-SUM1(I-1))/(SUM(I)-
  1SUM(I-1))-SUM1(I+1)-SUM1(I))/(SUM(I+1)-SUM(I)))
  XX(1,2)=A2*XX(1,2)
  XX(2,1)=XX(1,2)
C
C W R TO A AND Z'S
  DO 3006 I=2,KKK
    XX(I,1+I)=-(A2*B*SUM1(I)*((SUM1(I)-SUM1(I-1))/(SUM(I)-SUM(I-1))-
    1(SUM1(I+1)-SUM1(I))/(SUM(I+1)-SUM(I))))
3006 XX(I+1,1)=XX(I,1+I)
C
C W R TO B AND Z'S
  DO 3007 I=2,KKK
    XIL2 = 0.0
    IF(I.GT.2) XIL2 = X(I-2)
    XX(2,I+1) = -(A2*SUM1(I)*B*
    1((SUM1(I)*X(I-1)-SUM1(I-1)*XIL2)/(SUM(I)-SUM(I-1))-
    2 (SUM1(I+1)*X(I)-SUM1(I)*X(I-1))/(SUM(I+1)-SUM(I))))
3007 XX(I+1,2)=XX(2,I+1)
C
C W R TO Z'S AND MIXED W R TO Z'S
  DO 3010 I=2,KKK
    IF(I.EQ.KKK) GO TO 3008
    XX(I+1,I+2) = (A2*SUM1(I)*SUM1(I+1)*B**2)/
    1 (SUM(I+1)-SUM(I)) + (A1*SUM7(I)*SUM7(I+1))/
    2 (SUM10(I+1)-SUM10(I))
    XX(I+2,I+1) = XX(I+1,I+2)
3008 XX(I+1,I+1)=-(A2*(SUM1(I)*B)**2*(1./(SUM(I)-SUM(I-1))
  1+1./SUM(I+1)-SUM(I)))+(A1*SUM7(I)**2*(1./SUM10(I)-SUM10(I-1))
  2+1./SUM10(I+1)-SUM10(I)))
3010 CONTINUE
C
C INVERT MATRIX
  DO 200 I=1,NN
    DO 200 J=1,NN
      XX(I,J) = -XX(I,J)
19866 FORMAT(7E15.6)
  CALL MSTR(XX,XXDUM,18,0,1)
  CALL SIN(XXDUM,NN,.001,IER)
167 IF(IER.EQ.-1) STOP 1
  IF(IER.GT.0) WRITE(6,201) IER
201 FORMAT('LOSS OF SIGNIFICANCE. STEP ',I5,'+1')
  CALL MSTR(XXDUM,XP,18,1,0)
  IF(SWT.EQ.1) GO TO 11
C
C FORM SOLUTION VECTOR

```

APPENDIX B (Continued)

```

      H=AAA*XP(1,1)+BBB*XP(1,2)
      DO 130 I=1,KK
130  H=H+E(I)*XP(1,I+2)
      HK=AAA*XP(2,1)+BBB*XP(2,2)
      DO 131 I=1,KK
131  HK=HK+E(I)*XP(2,I+2)
      DO 2200 I=1,KK
      HL(I)=AAA*XP(I+2,1)+BBB*XP(I+2,2)
      DO 2200 K=3,NN
2200  HL(I)=HL(I)+E(K-2)*XP(I+2,K)
C   ITERATE
      AA = AA + H
      B = B + HK
      DO 140 I=1,KK
140  X(I)=X(I)+HL(I)
      IF(LL.EQ.1) GO TO 11
      IF(LL.GT.200) GO TO 2011
C   CHECK FOR MAXIMIZATION
      TNET=ABS(H)+ABS(HK)
      DO 4500 I=1,KK
4500  TNET=TNET+ABS(HL(I))
191  IF(TNET.LT.(.001)) SVT=1
      GO TO 2001
11  IF(LL.EQ.1) GO TO 2001
C   WRITE OUT FINAL VALUES, ETC., IF LAST ITERATION.
      WRITE(6,555) LL
555  FORMAT(//,T10,'PROCEDURE CONVERGES -- ',I3,' ITERATIONS.')
556  WRITE(6,6005)
6005  FORMAT(//,T10,'FINAL VALUES')
      IF(KO.NE.O) GO TO 465
      WRITE(6,7020)
7020  FORMAT(//,T37,'ESTIMATES OF ROC PARAMETERS')
      GO TO 466
465  WRITE(6,467)
467  FORMAT(//,T35,'PSEUDOVALUES OF ROC PARAMETERS')
466  CONTINUE
C   COMPUTING PARAMETER ESTIMATES
      DTAM=AA/B
      DE=2*AA/(B+1)
      V=AA/SQRT(B**2+1)
      DA=V*SQRT(2.O)
C   CALLING SUBROUTINE NDTR FOR PROBABILITY AND DENSITY FUNCTIONS OF V
      CALL NDTR(V,P,D)
      DO 610 I = 1,KK
      BETA(I) = SUM1(I+1)*B/SUM7(I+1)
      LBETA(I) = ALOG(BETA(I))
610  CONTINUE
      IF(KO.EQ.O) GO TO 400
      N=NTHISG
      AA=(N*ROCPAR(100,1))-((N-1)*AA)
      B=(N*ROCPAR(100,2))-((N-1)*B)
      DTAM=(N*ROCPAR(100,3))-((N-1)*DTAM)
      DE=(N*ROCPAR(100,4))-((N-1)*DE)
      P=(N*ROCPAR(100,5))-((N-1)*P)
      DA=(N*ROCPAR(100,6))-((N-1)*DA)
400  CONTINUE
      WRITE(6,7000)AA
7000  FORMAT(//,T20,'A= ',F25.5)
      WRITE(6,7001)B
7001  FORMAT(T20,'B= ',F25.5)
      WRITE(6,7003)DTAM
7003  FORMAT(T20,'DELTA M= ',F19.5)
      WRITE(6,7004)DE
7004  FORMAT(T20,'D(E)= ',F22.5)
      WRITE(6,7005)P
7005  FORMAT(T20,'A(2)= ',F22.5)
      WRITE(6,7006)DA
7006  FORMAT(T20,'D(A)= ',F22.5)
C   WRITE OUT RESPONSE BIAS ESTIMATES (BETA,LOGBETA)
      IF(KO.NE.O) GO TO 468
      WRITE(6,8003)
8003  FORMAT(//,O',T41,'ESTIMATES OF DECISION CRITERIA')
      GO TO 469
468  WRITE(6,470)
470  FORMAT(//,T35,'PSEUDOVALUES OF DECISION CRITERIA')
469  CONTINUE
      IF(KO.EQ.O) GO TO 6405
      N=NTHISG
      DO 401 I=1,KK
      X(I)=(N*ZKKEEP(100,I))-((N-1)*X(I))
      BETA(I)=(N*BEKEEP(100,I))-((N-1)*BETA(I))
      LBETA(I)=(N*LBKEEP(100,I))-((N-1)*LBETA(I))
401  CONTINUE
6405  CONTINUE
      WRITE(6,8001) (BETA(I),I=1,KK)
8001  FORMAT(//,T20,'BETA(K)= ',12(F7.3,3X))
C8001  FORMAT(//,T20,'BETA(K)= ',7(E15.8,1X))
      WRITE(6,8002) (LBETA(I),I=1,KK)
8002  FORMAT(T20,'LNBETA(K)= ',12(F7.3,3X))
C8002  FORMAT(T20,'LNBETA(K)= ',7(E15.8,1X))
      DO 522 II=1,KK
522  XH(II)= -(X(II))
      WRITE(6,6002)(XH(II),I=1,KK)
      KQ=KO
      IF(KO.EQ.O) KQ=100
      ROCPAR(KQ,1)=AA
      ROCPAR(KQ,2)=B
      ROCPAR(KQ,3)=DTAM
      ROCPAR(KQ,4)=DE
      ROCPAR(KQ,5)=P
      ROCPAR(KQ,6)=DA
      DO 1010 I=1,KK
      ZKKEEP(KQ,I)=X(I)
      BEKEEP(KQ,I)=BETA(I)
      LBKEEP(KQ,I)=LBETA(I)

```

```

1010 CONTINUE
GO TO 190
2011 WRITE(6,6011)
6011 FORMAT('DOES NOT CONVERGE -- 200 ITERATIONS')
GO TO 556
2  CALL EXIT
END

```

```

C   NDTR 10
C   NDTR 20
C   NDTR 30
C   NDTR 40
C   NDTR 50
C   NDTR 60
C   NDTR 65
C   NDTR 70
C   NDTR 80
C   NDTR 90
C   NDTR 100
C   NDTR 110
C   NDTR 120
C   NDTR 130
C   NDTR 140
C   NDTR 150
C   NDTR 160
C   NDTR 170
C   NDTR 180
C   NDTR 190
C   NDTR 200
C   NDTR 210
C   NDTR 220
C   NDTR 230
C   NDTR 240
C   NDTR 250
C   NDTR 260
C   NDTR 270
C   NDTR 280
C   NDTR 290
C   NDTR 295
C   NDTR 300
C   NDTR 310
C   NDTR 320
C   NDTR 330
C   NDTR 340
C   NDTR 350
C   NDTR 360
C   NDTR 370

```

```

SUBROUTINE NDTR(X,P,D)
  AX = ABS(X)
  T = 1.0/(1.0+0.2316419*AX)
  D = 0.3989423*EXP(-X*X/2.0)
  P = 1.0 - D*T*(((1.330274*T-1.821256)*T+1.781478)*T-
    *0.3565638)*T + 0.3193815)
  IF(X) 1,2,2
1  P = 1.0 - P
2  RETURN
END

```

```

C   SINV 10
C   SINV 20
C   SINV 30
C   SINV 40
C   SINV 50
C   SINV 60
C   SINV 70
C   SINV 80
C   SINV 90
C   SINV 100
C   SINV 110
C   SINV 120
C   SINV 130
C   SINV 140
C   SINV 150
C   SINV 160
C   SINV 170
C   SINV 180
C   SINV 190
C   SINV 200
C   SINV 210
C   SINV 220
C   SINV 230
C   SINV 240
C   SINV 250
C   SINV 260
C   SINV 270
C   SINV 280
C   SINV 290
C   SINV 300
C   SINV 310
C   SINV 320
C   SINV 330
C   SINV 340
C   SINV 350
C   SINV 360
C   SINV 370
C   SINV 380
C   SINV 390
C   SINV 400
C   SINV 410
C   SINV 420
C   SINV 430
C   SINV 440
C   SINV 450
C   SINV 460
C   SINV 470
C   SINV 480
C   SINV 490
C   SINV 500
C   SINV 510

```

```

C   SUBROUTINE SINV
C   PURPOSE
C   INVERT A GIVEN SYMMETRIC POSITIVE DEFINITE MATRIX
C   USAGE
C   CALL SINV(A,N,EPS,IER)
C   DESCRIPTION OF PARAMETERS
C   A - UPPER TRIANGULAR PART OF THE GIVEN SYMMETRIC
C   POSITIVE DEFINITE N BY N COEFFICIENT MATRIX.
C   TRIANGULAR MATRIX.
C   ON RETURN A CONTAINS THE RESULTANT UPPER
C   N - THE NUMBER OF ROWS (COLUMNS) IN GIVEN MATRIX.
C   EPS - AN INPUT CONSTANT WHICH IS USED AS RELATIVE
C   TOLERANCE FOR TEST ON LOSS OF SIGNIFICANCE.
C   IER - RESULTING ERROR PARAMETER CODED AS FOLLOWS
C   IER=0 - NO ERROR
C   IER=-1 - NO RESULT BECAUSE OF WRONG INPUT PARAMETER N OR BECAUSE SOME RADICAND IS NON-POSITIVE (MATRIX A IS NOT POSITIVE DEFINITE, POSSIBLY DUE TO LOSS OF SIGNIFICANCE)
C   IER=K - WARNING WHICH INDICATES LOSS OF SIGNIFICANCE. THE RADICAND FORMED AT FACTORIZATION STEP K+1 WAS STILL POSITIVE BUT NO LONGER GREATER THAN ABS(EPS*A(K+1,K+1)).
C   REMARKS
C   THE UPPER TRIANGULAR PART OF GIVEN MATRIX IS ASSUMED TO BE STORED COLUMNWISE IN N*(N+1)/2 SUCCESSIVE STORAGE LOCATIONS. IN THE SAME STORAGE LOCATIONS THE RESULTING UPPER TRIANGULAR MATRIX IS STORED COLUMNWISE TOO.
C   THE PROCEDURE GIVES RESULTS IF N IS GREATER THAN 0 AND ALL CALCULATED RADICANDS ARE POSITIVE.
C   SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C   MFSO
C   METHOD
C   SOLUTION IS DONE USING THE FACTORIZATION BY SUBROUTINE MFSO
C   SUBROUTINE SINV(A,N,EPS,IER)
C   DIMENSION A(1)

```



```

1      +0.001308* $\pi^2$ 2)
      IF(P-0.5)10,10,11
10     X=X
11     D=0.3989423*EXP(-X*X/2.0)
12     RETURN
      END
.....
SUBROUTINE LOC
.....
PURPOSE
      COMPUTE A VECTOR SUBSCRIPT FOR AN ELEMENT IN A MATRIX OF
      SPECIFIED STORAGE MODE
.....
USAGE
      CALL LOC (I,J,IR,N,M,MS)
.....
DESCRIPTION OF PARAMETERS
      I - ROW NUMBER OF ELEMENT
      J - COLUMN NUMBER OF ELEMENT
      IR - RESULTANT VECTOR SUBSCRIPT
      N - NUMBER OF ROWS IN MATRIX
      M - NUMBER OF COLUMNS IN MATRIX
      MS - ONE DIGIT NUMBER FOR STORAGE MODE OF MATRIX
           0 - GENERAL
           1 - SYMMETRIC
           2 - DIAGONAL
.....
REMARKS
      NONE
.....
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
      NONE
.....
METHOD
      MS=0 SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N*M ELEMENTS
           IN STORAGE (GENERAL MATRIX)
      MS=1 SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N*(N+1)/2 IN
           STORAGE (UPPER TRIANGLE OF SYMMETRIC MATRIX). IF
           ELEMENT IS IN LOWER TRIANGULAR PORTION, SUBSCRIPT IS
           CORRESPONDING ELEMENT IN UPPER TRIANGLE.
      MS=2 SUBSCRIPT IS COMPUTED FOR A MATRIX WITH N ELEMENTS
           IN STORAGE (DIAGONAL ELEMENTS OF DIAGONAL MATRIX).
           IF ELEMENT IS NOT ON DIAGONAL (AND THEREFORE NOT IN
           STORAGE), IR IS SET TO ZERO.
.....
SUBROUTINE LOC(I,J,IR,N,M,MS)
.....
      IX=I
      JX=J
      IF(MS-1) 10,20,30
10     IRX=N*(JX-1)+IX
      GO TO 36
20     IF(IX-JX) 22,24,24
22     IRX=IX+(JX-JX-JX)/2
      GO TO 36
24     IRX=JX+(IX*IX-IX)/2
      GO TO 36
30     IRX=0
      IF(IX-JX) 36,32,36
32     IRX=IX
36     IR=IRX
      RETURN
      END

```

Sample Output from Data Appearing in the User's Manual

	1	2	3	4	5	6
N 1.0000	0.4763	0.2517	0.1485	0.0660	0.0250	
S+N 1.0000	0.8451	0.7293	0.6126	0.4728	0.2902	

APPENDIX C (Continued)

PROCEDURE CONVERGES -- 4 ITERATIONS.

FINAL VALUES

ESTIMATES OF ROC PARAMETERS	
A=	1.09221
B=	0.79425
DELTA M=	1.37514
D(E)=	1.21745
A(Z)=	0.80380
D(A)=	1.20953

ESTIMATES OF DECISION CRITERIA

BETA(K)=	0.464	0.821	1.301	2.371	5.685
LN BETA(K)=	-0.767	-0.197	0.263	0.863	1.738
+Z(K)=	0.068	0.639	1.030	1.481	2.057

POOL ALL SUBJECTS EXCEPT SUBJECT 1 IN GROUP 1

INITIAL VALUES

A=	1.2018	B=	0.8833				
NO. OF Z(K)'S =	5						
+Z(K)=	0.253	0.851	1.183	1.552	1.900		

RESPONSE FREQUENCIES:

N1= 1773. N2= 1791.

	1	2	3	4	5	6
N	1064.	359.	140.	103.	56.	51.
S+N	324.	219.	213.	233.	282.	520.

CUMULATIVE PROPORTIONS:

	1	2	3	4	5	6
N	1.0000	0.3999	0.1974	0.1184	0.0603	0.0288
S+N	1.0000	0.8191	0.6968	0.5779	0.4478	0.2903

PROCEDURE CONVERGES -- 4 ITERATIONS.

FINAL VALUES

PSEUDOVALUES OF ROC PARAMETERS

A=	0.88710
B=	0.63209
DELTA M=	1.39623
D(E)=	1.10233
A(Z)=	0.77940
D(A)=	1.08325

PSEUDOVALUES OF DECISION CRITERIA

BETA(K)=	0.159	0.152	0.297	1.254	6.376
LN BETA(K)=	-1.361	-0.918	-0.424	0.426	1.862
+Z(K)=	-0.518	0.147	0.643	1.309	2.217

POOL ALL SUBJECTS EXCEPT SUBJECT 2 IN GROUP 1

INITIAL VALUES

A=	1.0432	B=	0.8003				
NO. OF Z(K)'S =	5						
+Z(K)=	0.191	0.703	1.003	1.405	1.851		

RESPONSE FREQUENCIES:

N1= 1773. N2= 1791.

	1	2	3	4	5	6
N	1021.	325.	147.	138.	85.	57.
S+N	333.	222.	199.	178.	271.	588.

CUMULATIVE PROPORTIONS:

	1	2	3	4	5	6
N	1.0000	0.4241	0.2408	0.1579	0.0801	0.0321
S+N	1.0000	0.8141	0.6901	0.5790	0.4796	0.3283

PROCEDURE CONVERGES -- 3 ITERATIONS.

FINAL VALUES

PSEUDOVALUES OF ROC PARAMETERS

A=	1.22758
B=	0.76617
DELTA M=	1.59164
D(E)=	1.38652
A(Z)=	0.83677
D(A)=	1.37528

PSEUDOVALUES OF DECISION CRITERIA

BETA(K)=	0.210	0.539	1.193	3.272	10.516
LN BETA(K)=	-1.270	-0.300	0.181	1.270	2.737
+Z(K)=	-0.302	0.470	1.026	1.794	2.658

POOL ALL SUBJECTS EXCEPT SUBJECT 3 IN GROUP 1

INITIAL VALUES

A=	1.0421	B=	0.7553				
NO. OF Z(K)'S =	5						
+Z(K)=	0.025	0.685	1.158	1.784	2.470		

RESPONSE FREQUENCIES:

N1= 1773. N2= 1791.

	1	2	3	4	5	6
N	904.	432.	218.	153.	54.	12.
S+N	289.	232.	253.	302.	370.	345.

CUMULATIVE PROPORTIONS:

	1	2	3	4	5	6
N	1.0000	0.4901	0.2465	0.1235	0.0372	0.0068
S+N	1.0000	0.8386	0.7091	0.5678	0.3992	0.1926

PROCEDURE CONVERGES -- 4 ITERATIONS.

FINAL VALUES

PSEUDOVALUES OF ROC PARAMETERS

A=	1.30518
B=	0.98908
DELTA M=	1.29979
D(E)=	1.32659
A(Z)=	0.82916
D(A)=	1.33752

PSEUDOVALUES OF DECISION CRITERIA

BETA(K)=	0.530	0.914	0.956	-0.482	-19.514
LN BETA(K)=	-0.622	-0.081	0.009	-0.148	-0.984
+Z(K)=	0.185	0.553	0.628	0.642	0.482

POOL ALL SUBJECTS EXCEPT SUBJECT 4 IN GROUP 1

INITIAL VALUES

A=	1.2344	B=	0.8220				
NO. OF Z(K)'S =	5						
+Z(K)=	-0.230	0.462	0.863	1.364	1.851		

RESPONSE FREQUENCIES:

N1= 1773. N2= 1791.

	1	2	3	4	5	6
N	725.	477.	227.	191.	96.	57.
S+N	164.	155.	172.	289.	385.	626.

CUMULATIVE PROPORTIONS:

	1	2	3	4	5	6
N	1.0000	0.5911	0.3221	0.1940	0.0863	0.0321
S+N	1.0000	0.9084	0.8219	0.7259	0.5645	0.3495

PROCEDURE CONVERGES -- 4 ITERATIONS.

FINAL VALUES

PSEUDOVALUES OF ROC PARAMETERS

A=	0.69982
B=	0.76400
DELTA M=	0.93902
D(E)=	0.80293
A(Z)=	0.72609
D(A)=	0.79493

PSEUDOVALUES OF DECISION CRITERIA

BETA(K)=	0.924	1.495	2.309	3.567	6.754
LN BETA(K)=	0.433	0.762	1.159	1.415	1.932
+Z(K)=	0.910	1.284	1.651	1.885	2.288

JACKKNIFE ESTIMATES

ROC PARAMETERS

ESTIMATES OF
ROC PARAMETERS

STANDARD ERRORS

A =	1.02992	0.14265
B =	0.78783	0.07405
DELTA M =	1.30667	0.13676
D(E) =	1.15459	0.13222

APPENDIX C (Continued)

A(Z) =	0.79286	0.02564
D(A) =	1.14774	0.13430

LIKELIHOOD RATIOS

	ESTIMATES OF LIKELIHOOD RATIOS	STANDARD ERRORS
BETA(1) =	0.45582	0.17624
BETA(2) =	0.78014	0.28481
BETA(3) =	1.18878	0.41863
BETA(4) =	1.90275	0.94655
BETA(5) =	1.03289	6.91232

LOG LIKELIHOOD RATIOS

	ESTIMATES OF LOG LIKELIHOOD RATIOS	STANDARD ERRORS
LNBETA(1) =	-0.70504	0.41337
LNBETA(2) =	-0.18442	0.35868

LNBETA(3) =	0.23151	0.33437
LNBETA(4) =	0.74063	0.36795
LNBETA(5) =	1.38671	0.81478

CUTPOINTS

	ESTIMATES OF CUTPOINTS	STANDARD ERRORS
+Z(1) =	0.06874	0.31663
+Z(2) =	0.61354	0.24009
+Z(3) =	0.98690	0.23964
+Z(4) =	1.40752	0.28481
+Z(5) =	1.91131	0.48620

(Manuscript received May 16, 1986;
revision accepted for publication August 11, 1986.)