A MINITAB macro for testing marginal homogeneity in contingency tables

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Matched-pairs designs sometimes produce nominally scaled (categorical) data, as illustrated by the following hypothetical example. Ninety students each take two courses. Each student is given a choice as to how he/she should be assessed on each course. Assessment can be by examination (E), by written work (W), or by oral presentation (O). It is considered useful to know whether the students' choice of assessment procedure differs for the two courses. The data are nominal, comprising three categories, E, W, and O, which cannot meaningfully be ordered, and a matched-pairs design is used, with each student making a choice for each course.

The resulting data can be tabulated in the conventional form for matched-pairs designs, as in Figure 1, or more conveniently in a square contingency table, as in Figure 2. In the latter format, the question to be asked is whether each row total in Figure 2 is essentially the same as the corresponding column total. This is a question of *marginal homogeneity*.

Figure 2 can also be viewed as an agreement matrix of the kind for which Cohen's (1960) kappa coefficient could be calculated. In some circumstances, interpretation of kappa may be aided by knowledge of the degree of dissimilarity between the base rates with which the two observers assign items to the various categories: this, too, is a question of marginal homogeneity (Collis, 1985).

Test Statistics. When there are only two categories, McNemar's procedure is appropriate for testing homogeneity of the margins of the resulting 2×2 table. Stuart (1955) generalized McNemar's test to the case of $k \geq 2$ categories and the resulting $k \times k$ contingency table. Fleiss (1981) gives a useful account of Stuart's test which emphasizes its applicability to matched-pairs designs. Maxwell (1970) illustrates the use of Stuart's test in the analysis of agreement matrices.

A standard notation for contingency tables is used: n_{ij} denotes the frequency in the cell in the i th row and j th column, $n_{i.}$ and $n_{.j}$ denote the i th row and column totals respectively, and $n_{..}$ denotes the grand total. $\mathbf{d}$ is a vector of differences $d_i = n_{i.} - n_{.i}$. Stuart (1955) proposed estimating the elements of $\mathbf{V}$, the covariance matrix of $\mathbf{d}$, as $V_{ii} = n_{i.} + n_{.i} - 2n_{ii}$ and $V_{ij} = -(n_{ij} + n_{ji})$. To avoid problems associated with a singular covariance matrix, one d_i is dropped from the calculations, together with the corresponding row and column of the covariance matrix.

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student	course 1	course 2
1	W	E
2	W	W
3	E	O
:	:	:
:	:	:
90	O	E
frequency of:		
E	40	25
W	20	30
O	30	35
	90	90

Figure 1. Choice of assessment procedure by students taking two courses.

		course 2			
		E	W	O	total
course	E	20	12	8	40
1	W	3	15	2	20
	O	2	3	25	30
total		25	30	35	90

Figure 2. Hypothetical data from a matched-pairs design with nominal data, in the form of a contingency table.

Stuart's statistic is the quadratic form $Q = \mathbf{d}' \mathbf{V}^{-1} \mathbf{d}$. Q is referred to tables of the χ^2 distribution with $k - 1$ degrees of freedom.

Bhappkar (1966) introduced a test statistic which differs from Stuart's only in that the null case is not assumed in estimating the covariance matrix. Specifically, for Bhappkar's test $V^*_{ii} = n_{i.} + n_{.i} - 2n_{ii} - d_i^2/n_{..}$ and $V^*_{ij} =$

$-(n_{ij}+n_{ji}) - d_i d_j / n \dots$ Thus Bhapkar's statistic may be written in terms of Stuart's estimator V as $Q^* = d'(V - n^{-1}dd')^{-1}d$. Q^* is also referred to the χ^2 distribution with $k-1$ degrees of freedom. Unlike Stuart's Q , Bhapkar's statistic does not reduce to McNemar's chi square in the 2×2 case.

When $k \leq 4$, it is possible to calculate Stuart's statistic without resorting to matrix inversion (Fleiss & Everitt, 1971). For the general case, programs are not difficult to write, provided a matrix inversion routine is available. Dunlap (1984) provides a FORTRAN program for Stuart's test. However, it is very convenient to be able to perform Stuart's and Bhapkar's tests in the environment of the MINITAB package (Ryan, Joiner, & Ryan, 1982), which is a versatile and friendly package for data analysis and statistics. MINITAB's matrix manipulation commands are well suited to the necessary calculations.

The Macro. Table 1 contains a listing of a MINITAB macro MARGHOMO.MTB. As it stands, this macro will deliver Bhapkar's statistic Q^* , but it may easily be con-

verted to deliver Stuart's Q by deleting the five commands in Lines 18-23 of the listing.

Before running the macro, it is necessary to assign values to the three MINITAB constants K1, K2, and K3. K1 is set to k , the number of categories. K2 points to the first of a set of k consecutive columns that contain the data in the form of a $k \times k$ contingency table, and K3 points to the first of k columns that can be used for workspace (these will be overwritten when the macro is run). Thus, if $k=3$ and the 3×3 contingency table is stored in Columns C8-C10, K1 should be set to 3, and K2 should be set to 8. Assuming Columns C21-C23 do not contain data that the user requires, K3 may be set to 21.

The macro is run using the command EXEC 'MARGHOMO'. The MINITAB Constant K4 and Matrices M1-M7 are used by the macro, and any prior contents will be lost. At the termination of the macro run, M1 contains the contingency table, M2 the Q^* (or Q) statistic, M3 the row totals, M4 the column totals, M5 the first $k-1$ d_i s, M6 the corresponding covariance matrix, and M7 its inverse. MINITAB prints out ANSWER= followed by the value of the Q^* statistic (or Q). (For the special case of a 2×2 table, MINITAB prints out more than one ANSWER. This is because MINITAB automatically does this whenever a matrix multiplication yields a scalar product, which happens for intermediate steps in the 2×2 case. The last ANSWER is the Q^* or Q statistic.) For the data given in Figure 2, $Q^* = 10.1124$.

The macro requires MINITAB Release 82.1 or later. It normally is possible to analyze quite large contingency tables, limited only by the size of matrices allowed, and the number of columns that may be copied with one COPY command, in a particular implementation of MINITAB.

Table 1

MARGHOMO.MTB: A MINITAB Macro to Test Marginal Homogeneity in a Square Contingency Table

note	Stuart/Bhapkar tests of marginal homogeneity	
note	number of categories in K1	
note	first column of contingency table in CK2	
note	first column available for workspace : CK3	
noecho		
let	K4=K2+K1-1	
copy	CK2-CK4 into M1	# data matrix
transpose	M1 into M2	
set	CK3	
K1(1)		
mult	M1 by CK3 into M3	# row totals
mult	M2 by CK3 into M4	# col totals
add	M1 M2 into M2	
add	M3 M4 into CK3	
diag	CK3 into M6	
subtract	M2 from M6 into M6	# Stuart's covar
subtract	M3 from M4 into M5	# differences
#	convert Stuart's covar to Bhapkar's	
transpose	M5 into M7	
mult	M5 by M7 into M7	
let	K4=2/sum(CK3)	
mult	K4 by M7 into M7	
subtract	M7 from M6 into M6	# Bhapkar's covar
#	eliminate singularity	
generate	1 to K1 into CK3	
let	K4=K3+K1-1	
indicator	CK3 into CK3-CK4	
let	K4=K4-1	
copy	CK3-CK4 into M2	
mult	M6 by M2 into M6	
transpose	M2 into M2	
mult	M2 by M6 into M6	# reduced covar
mult	M2 by M5 into M5	# reduced diffs
#	calculate quadratic form	
invert	M6 into M7	# inverse
mult	M7 by M5 into M2	
transpose	M5 into M5	
mult	M5 by M2 into M2	# chisquare
note	the final ANSWER is chisquare	
note	with K1-1 degrees of freedom	

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