A computer program to generate signal-detection theory values for sensitivity and response bias

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The theory of signal detection was originally proposed by Tanner and Swets (1954) and has come to serve as an alternative to classical psychophysical methodologies. Signal detection's greatest advantage over classical psychophysical methodologies is the separate determination of parameters reflecting sensory and nonsensory factors.

The sensitivity of the decision-making process is a function of the variable d' , which is equal to the distance between the noise and signal-plus-noise distribution and which is monotonically related to the signal strength. Nonsensory factors are reflected in the response criterion. The most commonly used measure of response criterion is β , which is defined as the ratio of the signal-plus-noise distribution at the criterion to the ordinate of the noise distribution. Gardner, Dalsing, Reyes, and Brake (1984) recently published a table of β values corresponding to various hit and false-alarm rates.

Traditionally, researchers in psychophysics have been primarily interested in measures of sensory sensitivity. The matter of how a subject sets or maintains a criterion at the appropriate value and adjusts that value to take account of relevant intercurrent events has been largely ignored (Treisman & Williams, 1984).

One disadvantage of β as a measure of response criterion is that it forms an asymmetric distribution such that values cannot be easily averaged over subjects, trial blocks, and so forth. By taking the natural logarithm of β , one derives a measure of response criterion that can be averaged. In addition, $\ln(\beta)$ is invariant over sensitivity under maximization of expected value. The conversion to $\ln(\beta)$ is helpful when the researcher wishes to average β or to do other parametric analyses of β values.

We describe a program (see Appendix), written for the Apple computer, that generates measures of d' , β and $\ln(\beta)$ from a given pair of hit and false-alarm rates. The ZGEN program is written in Applesoft and uses an approximation for the inverse normal integral function, which is defined as the value of X such that for a given Q :

$$Q = \int_x^\infty \frac{1}{2\pi} e^{-t^2/2} dt.$$

The following rationale approximation for this function is used:

$$X = t - (c_0 + c_1 t + c_2 t^2) / (1 + d_1 t + d_2 t^2 + d_3 t^3),$$

where $t = \sqrt{\ln[1/(Q^2)]}$, $c_0 = 2.515517$, $d_1 = 1.432788$, $c_1 = .802853$, $d_2 = .189269$, $c_2 = .010328$, and $d_3 = .001308$. This approximation is valid for $0 < Q < 0.5$ and is accurate to $\pm 4.5 \times 10^{-4}$. Brophy (1985) describes in detail procedures for other approximations of the inverse normal distribution function.

The present program breaks the function down to numerator and denominator sections and then expands the approximation, using the symmetry of the function to allow values ranging between 0 and 1. The expanded function called FN Z in the program, along with the function FN G (the standardized normal distribution curve), can then be used to calculate the desired quantities. The FN R function is used to round the resulting values to three decimal places.

An additional feature of this program is that it calculates values of d' , (β) , and $\ln(\beta)$, even if individual subjects' hit or false-alarm rates contain proportions of 0 or 1, which correspond to infinite d' s. In these circumstances, the program follows the procedure outlined by Macmillan and Kaplan (1985), in which 0 is replaced by $1/(2N)$ and 1 is replaced by $1 - 1/(2N)$, where N is the number of trials on which the relevant stimulus was presented. When a 0 or 1 is entered as a data point, the program requests N . The resulting output is marked by an asterisk as a reminder that this procedure was used for those values.

An advantage of the program is that it allows computation of sensitivity and response bias values with greater accuracy than do existing tables of those values. Hit and false-alarm proportions of three-decimal accuracy may be entered, and all outputs are also to three decimals. In addition, the program calculates $\ln(\beta)$, which should prove helpful to researchers who wish to average response bias values across trials or across subjects. This program should prove helpful to individuals wishing to rapidly calculate measures of sensitivity and response bias where the signal detection paradigm has been followed.

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APPENDIX

Program to Calculate Signal Detection Values of d' , β , and $\ln(\beta)$.

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10 TEXT
20 NORMAL
30 C0 = 2.515517
40 C1 = 0.802853
50 C2 = 0.010328
60 D1 = 1.432788
70 D2 = 0.189269
80 D3 = 0.001308
90 DEF FN L(X) = SQR ( LOG ( 1 / (X * X)))
100 DEF FN N(X) = C0 + C1 * FN L(X) + C2 * FN L(X) ^ 2
110 DEF FN D(X) = 1 + D1 * FN L(X) + D2 * FN L(X) ^ 2 + D3 * FN L(X) ^ 3
120 DEF FN A(X) = FN L(X) - FN N(X) / FN D(X)
130 DEF FN Z(X) = FN A(1 - X) * (X > .5) - FN A(X) * (X < .5)
140 PI = 4 * ATN (1)
150 K = SQR (1 / (2 * PI))
160 DEF FN G(X) = K * EXP ( - (X * X) / 2)
170 DEF FN R(X) = INT ((X * 1000) + .5) / 1000
180 DEF FN X(X) = (X * 35) + 140
190 DEF FN Y(X) = 160 - X * 350
200 HOME
210 PRINT "THIS PROGRAM CALCULATES THEN PLOTS THE"
220 PRINT "SIGNAL DETECTION THEORY VALUES"
230 PRINT "BETA AND D PRIME"
240 PRINT
250 PRINT
260 PRINT "THIS PROGRAM IS BY:"
270 PRINT
280 PRINT "    RUSSEL L. BOICE"
290 PRINT "    UNIVERSITY OF SOUTHERN COLORADO"
300 PRINT "    2200 NORTH BONFORTE BLVD."
310 PRINT "    PUEBLO, COLORADO 81001"
320 PRINT
330 PRINT "PRESS RETURN TO CONTINUE";
340 INPUT " ";X$
350 HOME
360 INPUT "ENTER HITS PROPORTION      ";H
370 IF H < 0 OR H > 1 THEN GOSUB 1000: GOTO 360
380 IF H = 0 OR H = 1 THEN GOSUB 2000
390 IF H = 0 THEN H = 1 / (2 * N)
400 IF H = 1 THEN H = 1 - 1 / (2 * N)
410 INPUT "ENTER FALSE ALARM PROPORTION ";F
420 IF F < 0 OR F > 1 THEN GOSUB 1000: GOTO 410
430 IF F = 0 OR F = 1 THEN GOSUB 2000
440 IF F = 0 THEN F = 1 / (2 * N)
450 IF F = 1 THEN F = 1 - 1 / (2 * N)
460 HOME
470 VTAB 24: PRINT "WORKING..."
480 Z1 = FN Z(H)
490 Z2 = FN Z(F)
500 VTAB 22: HTAB 1
510 PRINT "HIT PROP. ";H;
520 BE = FN G(Z1) / FN G(Z2)
530 HTAB 25
540 PRINT "BETA      = "; FN R(BE);
550 VTAB 23: HTAB 1
560 PRINT "F/A PROP. ";F;
570 HTAB 25
580 DP = Z1 - Z2
590 PRINT "D PRIME = "; FN R(DP);
600 VTAB 24: HTAB 25
610 PRINT "LN(BETA)= "; FN R( LOG (BE));
620 GOSUB 5000
630 VTAB 24: HTAB 1
640 INVERSE
650 INPUT "PRESS <RETURN>";X$
660 NORMAL
670 TEXT
680 GOTO 350
1000 PRINT "ALL PROPORTIONS MUST BE >= 0 AND <= 1 "
1010 RETURN
2000 REM CREATE ADJUSTED PROPORTION
2010 PRINT
2020 PRINT "IF A PROPORTION OF 1 OR 0 EXISTS, A"
2030 PRINT "PROPORTION WILL BE ASSUMED BASED ON THE"
2040 PRINT "NUMBER OF TIMES THE STIMULUS WAS"
2050 PRINT "PRESENTED."
2060 PRINT
2070 INPUT "ENTER THE NUMBER OF TIMES THE STIMULUS WAS PRESENTED ";N

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APPENDIX (continued)

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2080 IF N < 2 THEN GOTO 2070
2090 PRINT
2100 RETURN
5000 REM PLOT ROUTINE
5010 HCOLOR= 7
5020 HGR
5030 HPLOT 0,0 TO 279,0 TO 279,159 TO 0,159 TO 0,0
5040 FOR I = - 6 TO 6
5050 X1 = FN X(I): GOSUB 5300
5060 HPLOT X1,0 TO X1,5
5070 NEXT I
5080 HPLOT 140,0 TO 140,159
5085 VTAB 21: HTAB 20: PRINT "CR";
5090 X1 = FN X(Z1)
5100 Y1 = FN Y( FN G(0))
5110 HPLOT X1,0 TO X1,Y1
5112 Y1 = FN Y( FN G(Z1))
5114 HPLOT 140,Y1 TO 279,Y1
5116 FL = - Z1 - 4: FH = FL + 8
5120 FOR Z = FL TO FH STEP .1
5130 X1 = FN X(Z + Z1)
5140 Y1 = FN Y( FN G(Z))
5150 GOSUB 5300
5160 IF Z = FL THEN HPLOT X1,Y1
5170 HPLOT TO X1,Y1
5180 NEXT Z
5190 X1 = FN X(Z2)
5200 Y1 = FN Y( FN G(0))
5210 HPLOT X1,0 TO X1,Y1
5212 Y1 = FN Y( FN G(Z2))
5214 HPLOT 140,Y1 TO 279,Y1
5216 FL = - Z2 - 4: FH = FL + 8
5220 FOR Z = FL TO FH STEP .1
5230 X1 = FN X(Z + Z2)
5240 Y1 = FN Y( FN G(Z))
5250 GOSUB 5300
5260 IF Z = FL THEN HPLOT X1,Y1
5270 HPLOT TO X1,Y1
5280 NEXT Z
5290 RETURN
5300 IF X1 < 0 THEN X1 = 0
5310 IF X1 > 279 THEN X1 = 279
5320 IF Y1 < 0 THEN Y1 = 0
5330 IF Y1 > 159 THEN Y1 = 159
5340 RETURN

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