Behavior Research Methods & Instrumentation
1974, Vol. 6 (4) 403-406

An extension of the Riggs projection colorimeter

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A method for the construction and calculation of CIE chromaticity coordinates from a simple and inexpensive projection colorimeter is presented. The device allows variation along the dimensions of hue, saturation, and brightness and specification of these variations in terms of chromaticity coordinates, dominant wavelength, relative luminance, and excitation purity.

In a previous paper (Murch, 1972), a design for a projection colorimeter proposed by Riggs (1964) was presented in a modified version to allow determination of the CIE x,y coordinates of obtained color matches. The present paper extends the design to allow variation of hue and saturation and brightness of a given colorimetric match, and modifies the measurement technique such that any combination of chromatic filters may be used to produce the hues.

In the original design, the condensing lenses from a Viewlex Model V 27 projector are removed and two achromatic lenses are placed in the projection tube. A light-blocking filter with a ¼-in. opening is placed in the slide holder of the projector. Three chromatic filters are mounted in the path of the beam at the focal point of the tungsten filaments of the projection lamp, and the projector tube is adjusted to produce a homogeneous circle of color on a screen located between 8 and 10 ft from the projector. By varying the proportionate contribution of the three filters, various hues can be produced within the range of the three filters chosen. Details of the design may be found in Riggs (1964) and Murch (1972).

To allow variation of brightness, a neutral density wedge is placed behind the mounting stage containing the three filters. Thus, the amount of light passing through the wedge can be varied continuously. A wedge, variable over the range of 0.0-2.0 log units, has proven satisfactory. The variation of saturation is accomplished by mounting a second Viewlex V 27 projector (with a light-blocking filter containing a ¼-in. opening in the slide holder) at a right angle to the colorimeter and by combining the beams for each projector at a beam splitter (Edmund Scientific Co. No. 578 nonabsorbing beam splitter), as shown in Fig. 1. A second mounting stage containing the same three filters (red, green, and blue) is mounted in the path of the second projector such that the tungsten filaments from the projector are focused upon the filters.

As with Projector 1, the condensing lens should be removed and two Edmund Scientific Co. achromatic lenses (No. 6313) mounted in the projection tube. In the Viewlex Model V 27 projector, the first achromatic lens can be placed in the tube and moved forward until it rests securely in the tube. The second lens should be mounted at the point where the film strip holder normally sits. The mounting stage containing the three filters is then moved to a point in front of the projection tube until the filaments of the bulb are in focus upon it. The trifilter should be adjusted to white. Once the correction values for the three filters are determined (Steps 1-6 are noted below), the trifilter can be adjusted to produce an x = .447 and a y = .407, and be secured in that position.

A second neutral density wedge (variable over the range of 0.0-2.0 log units) is mounted to interrupt the beam of the second projector so that white light may be added or subtracted from the color produced by Projector 1 (colorimeter). Thus, any color match involves setting the Projector 1 mounting stage (containing the three colored Wratten filters) to a given proportionate contribution of each filter, adjusting the brightness by increasing or decreasing the density of the wedge interrupting the beam of the first projector, and adjusting the saturation by adding or subtracting white light through the netural density wedge interrupting the beam of the second projector. Naturally, any change in the amount of white changes the luminance of the mixture which requires a compensatory change in the wedge setting of the colorimeter.

If CWA (G.E.) projection lamps are used, the color temperature will be close to the ideal Source A value of 2,854°K. This value may be obtained by increasing or decreasing the voltage to the lamps via a voltage regulator so that 2,854°K is obtained. The color temperature can be judged with an optical pyrometer by setting the pyrometer on 2,854°K and varying the voltage to the lamps until a satisfactory match is obtained. We have found the color temperature of the CWA with the heat filter in place to be 2,840°K ± 30.5°K, the color temperature of Source A.[1]

In a previous paper (Murch, 1972), six steps were outlined to obtain the proper weighting values for the transformation of the amounts of red, green, and blue contributing to a color match. This can be extended to the use of any photosensor and filter combination by the techniques described below.

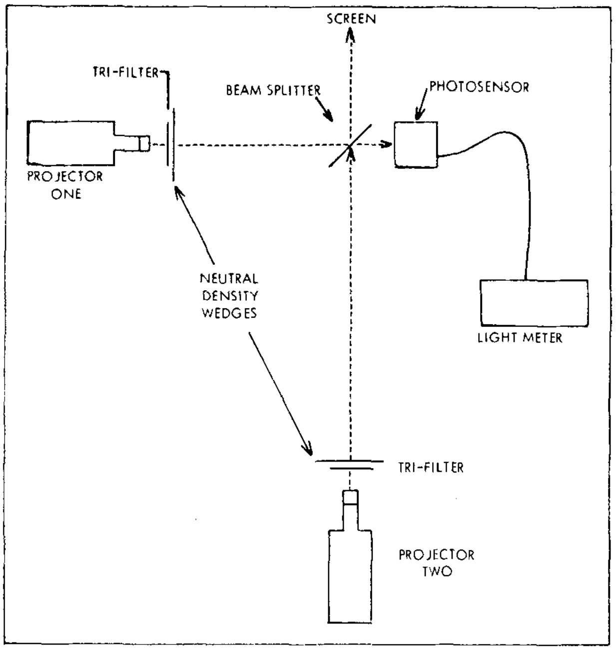

Fig. 1. Diagram of projection colorimeter.

SETUP OF THE PHOTOSENSOR

Mount a photosensor or any light detector capable of picking up light throughout the visible spectrum of 400 to 740 nm at the position indicated in Fig. 1 such that it picks up the combined mixture from Projectors 1 and 2. Use a filter wheel or filter mount to modify the hood of the photosensor so that each of the three filters used in the mounting stage can be individually placed over the entrance slit of the photosensor. The photosensor must be positioned to pick up all the light from both projectors. If both mounting stages are set to white and the wedges of both projectors are adjusted to the same density, the amount of each color measured at the photosensor for the saturation projector should be 50% of the value obtained for the colorimeter projector. (This is because the beam splitter reflects 33% of the light and transmits 66%.) Thus, 66% of the light from Projector 1 reaches the photosensor, while only 33% of the light from Projector 2 reaches the sensor. At the projection screen, these values are reversed. When the two projectors are equated for intensity at the screen, the values for the saturation projector measured at the photosensor should be 4 times the values for the color projector.

THE WEIGHTING FACTORS OF SINGLE FILTERS

The calculation of the weighting factors X_r, X_g, X_b, Y_r, $\cdots$, Z_b is described in the paper by Murch (1972), Eqs. 6a-8c. The weighting factors are calculated as described, except for deletion of the correction for spectral sensitivity of the photosensor. The following calculations take the spectral sensitivity of the photosensor into account in determining the weight assigned to each of the factors in a given mixture.

WEIGHTS OF THE FILTERS IN A MIXTURE

Weights for the contributions of the red, green, and blue filters to a given mixture are determined as follows. Let v_{mr}, v_{mg}, v_{mb} represent the photosensor readings for the mixture to be refiltered through the respective filters at the entrance to the photosensor. Set the brightness of the colorimeter projector to its maximum and turn the saturation projector off. Let v_{rr}, $\cdots$, v_{bb} (in the following matrix) represent the readings obtained when the trifilter mounting stage is set entirely on each filter in turn, so that the beam from the colorimeter projector passes only through the red, green, or blue part of the trifilter.

		Filter in front of entrance to photosensor		
		Red	Green	Blue
Trifilter mounting stage on pure color	Red	v_{rr}	v_{rg}	v_{rb}
	Green	v_{gr}	v_{gg}	v_{gb}
	Blue	v_{br}	v_{bg}	v_{bb}

Let w_r, w_g, and w_b be the unknown weights. Then,

$$v_{mr} = w_r v_{rr} + w_g v_{gr} + w_b v_{br}$$

$$v_{mg} = w_r v_{rg} + w_g v_{gg} + w_b v_{bg}$$

$$v_{mb} = w_r v_{rb} + w_g v_{gb} + w_b v_{bb}.$$

When the photosensor readings have been obtained and substituted in these equations, they can readily be solved for w_r, w_g, w_b. The v_{mr}, v_{mg}, v_{mb} readings are obtained for each match, but the other vs need to be obtained once for a given set of filters.

If the spectral energy distribution of each filter overlaps the others, then all of the readings are positive, and the general solutions for the weights are uniformative algebraic combinations of these readings. If the spectral energy distributions for the two filters do not overlap, then the reading for that combination of filters is 0 and the equations can be simplified. In the extreme case of no overlap between any of the filters, the solutions for the unknown weights are

$$w_r = v_{mr}/v_{rr}$$

$$w_g = v_{mg}/v_{gg}$$

$$w_b = v_{mb}/v_{bb}.$$

The latter equations could be substituted for Formulas 1a, 1b, and 1c in Murch (1972). The blue and green filters employed in that study overlap, but the other combinations do not. The corrected formulas become:

$$w_r = v_{mr}/v_{rr}$$

$$w_g = \frac{v_{mg} - \dfrac{v_{mb}v_{bg}}{v_{bb}}}{v_{gg} - \dfrac{v_{gb}v_{bg}}{v_{bb}}}$$

$$w_b = \frac{v_{mb} - \dfrac{v_{mg}v_{gb}}{v_{gg}}}{v_{bb} - \dfrac{v_{bg}v_{gb}}{v_{gg}}}.$$

For any given color mixture (adjustment of hue, brightness, and saturation), let v_{cr}, v_{cg}, v_{cb} be the photosensor readings for the amount of light passed by the red, green, and blue filters, respectively, for the colorimeter projector, and v_{sr}, v_{sg}, v_{sb}, the photosensor readings for the amount of light passed by the filters at the photosensor for the saturation projector. Both sets of readings need to be corrected for spectral sensitivity and overlap of the filters. Let w_{cr}, w_{cg}, w_{cb}, and w_{sr}, w_{sg}, w_{sb} represent the corrected weights. Recall that 1/3 of the light from the colorimeter projector is going to the screen and 2/3 to the photosensor, with the fractions reversed for the saturation projector, and that the readings for combinations of pure filters used in determining the weights are obtained with the saturation projector off. Thus, the sum of the weights with both projectors on maximum brightness is 3/2. In order to arrive at the correct weights for a mixture with the saturation projector on, all weights should be multiplied by 2/3, so the weights will add to 1 or less; the weights for the colorimeter projector must be divided by 2 and the weights for the saturation projector multiplied by 2 in order to reflect their relative weights at the screen, instead of at the photosensor. This is all accomplished by taking the weights for the mixture to be

$$w_{mr} = w_{cr}/3 + 4w_{sr}/3$$

$$w_{mg} = w_{cg}/3 + 4w_{sg}/3$$

$$w_{mb} = w_{cb}/3 + 4w_{sb}/3.$$

Finally, the CIE x,y,z coordinates are obtained from the color factors for single filters and the weights by substituting $R = w_{mr}$, $G = w_{mg}$, and $B = w_{mb}$ in

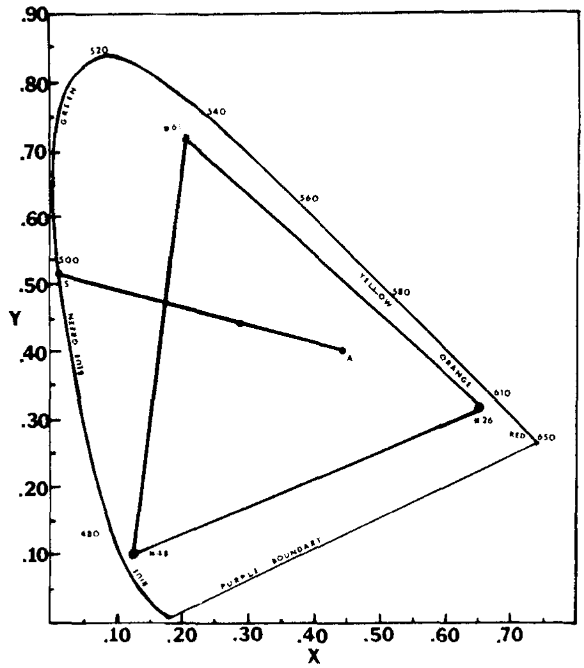

Fig. 2. CIE chromaticity space diagram indicating positions of the red No. 26, green No. 61, and blue No. 48 filters as well as Source A white.

$$X_m = R(X_r) + G(X_g) + B(X_b)$$

$$Y_m = R(Y_r) + G(Y_g) + B(Y_b)$$

$$Z_m = R(Z_r) + G(Z_g) + B(Z_b).$$

These are then translated into CIE x,y,z coordinates by

$$x = X_m/(X_m + Y_m + Z_m)$$

$$y = Y_m/(X_m + Y_m + Z_m)$$

$$z = Z_m/(X_m + Y_m + Z_m).$$

For the filters employed here, the values of X_r, X_g, $\cdots$, Z_g, Z_b are as follows:

$$X_m = R(43.0288) + G(4.8167) + B(1.3330)$$

$$Y_m = R(19.0850) + G(13.7125) + B(.7312)$$

$$Z_m = R(.0082) + G(1.0857) + B(8.0215).$$

When the colorimeter is set to pure white or the saturation projector alone is on, the Source A values for white of x = .447 and y = .407 should be obtained.[2]

CALCULATION OF RELATIVE LUMINANCE, EXCITATION PURITY, AND DOMINANT WAVELENGTH

Thus far, we have referred to hue, brightness, and saturation as varied by an observer to match a chromatic stimulus. Hue is defined in terms of the x,y coordinates described above. The calculation of the dominant wavelength (λ_d or λ_c) involves plotting the x,y coordinates of an obtained match on a chromaticity space diagram and extending a line from Source A though the match to the borders of the spectrum. This is depicted in Fig. 2. The resulting x,y coordinates are then used in Table 3.3 of Wyszecki and Stiles (1967) to read off the dominant wavelength (λ_d) of the match. If the resulting line intersects the purple boundary, then it is extended in the opposite direction in order to specify the dominant wavelength of the match in terms of the compliment (λ_c).

The dominant wavelength may be calculated analytically following a method described in Wyszecki and Stiles (1967) on pages 325-333.

Excitation purity is used to define the observer's judgment of saturation in a mixture:

$$p_e = (y - y_a)/(y_s - y_a)$$

or

$$p_e = (x - x_a)/(x_s - x_a),$$

where x,y are the obtained color coordinates for a given match, x_a = .447, and y_a = .407 (Source A white), and x_s,y_s the coordinates representing the dominant wavelength. If the dominant wavelength was obtained by graphic representation, these values may be read off the chromaticity chart directly. If the dominant wavelength was calculated via the method of Wyszecki and Stiles, these coordinates are obtained by reference to Table 3.3 of their publication.

Because mixtures beyond the borders of the triangle formed by interconnecting the defining points of the applied filters are not possible, excitation purity values will always be less than 1.0.

The final calculation to represent the observer's brightness setting is of relative luminance, and is given as:

$$A = \frac{Y_m}{\Sigma \bar{y}_\lambda H \Delta_\lambda},$$

where $\bar{y}_\lambda H\Delta_\lambda$ represents the CIE spectral sensitivity function weighted by Source A, which for the apparatus described here solves to:

$$A = \frac{Y_m}{1078.96}.$$

This value is actually a measure of relative luminance rather than brightness, so that the size of A for any two matches determines the luminance ratio of the two matches.

In summary, the apparatus and calculations described here allow the production of colors over a wide range of the visible spectrum, whose properties can be defined in terms of hue (x,y), dominance wavelength (λ_d or λ_c), relative luminance (A), and excitation purity (p_e).

REFERENCES

Murch, G. M. CIE x,y coordinates from an inexpensive projection colorimeter. Behavior Research Methods & Instrumentation, 1972, 4, 3-5.

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Wyszecki, G., & Stiles, W. S. *Color science*. New York: Wiley, 1967.

NOTES

1. Setting the saturation projector to 2,854°K and adjusting the trichromatic filter to produce Source A guarantees that the Source A contribution from the saturation projector produces the same relative readings at the photosensor as a Source A setting for the colorimeter would produce. It is possible to vary saturation without using the saturation projector, but the extra projector allows independent variation of luminance at the same time.

2. The accuracy of the above calculations is based on the following assumptions: (1) The maximum output of both projectors is equal. (2) The reflectance/transmittance ratio of the beam splitter is 3/1, although other values can be substituted. (3) The beam splitter and neutral density wedges are spectrally neutral. (4) The transmittance characteristics of the Wratten filters approximate the values provided by Kodak. As these filters fade after prolonged exposure to light, they should be changed often. A maximal exposure period of 15 h should be observed. (5) The photosensor is not sensitive to polarization of light by the beam splitter.

(Received for publication April 8, 1974; revision received May 15, 1974.)