Program Abstracts/Algorithms

SEREX2:

A program to generate exhaustive series

CAMILLE-AIME POSSAMAI and GUY REYNARD*

*Institut de Neurophysiologie et de Psychophysiologie
C.N.R.S., Marseille, France*

Since Gellermann's work (1933), it has been generally accepted that it is better, for repeated testing of a single S, to use a series constructed to have certain definite properties rather than a sequence of randomly chosen terms. Gellermann constructed series of two events in which the two events occurred with equal probability and each event occurred equally often in the two halves of the series; also, the length of the sequences and the score chance for simple and double alternation responses were controlled. Fellows (1967) and Durup (1967) have criticized Gellermann's series. One of the main criticisms is that the conditional probability of occurrence of an event, given the two or three previous ones, is not the same for each event of the series. These criticisms led Durup (1967) to construct exhaustive series, i.e., series in which the n elements are organized so that each of the n^m different m -tuples occurs n times followed once and only once by each element. Obviously, in these series, sequential effects are exhaustively balanced up to the m^{th} order. Moreover, it is possible to control the effects of slow trend by selecting, among the different exhaustive series, the series whose gaps are as short as possible, the gap for an element being the number of terms intervening between two occurrences of this element. In these series, sequential effects can be investigated by means on a simple type of analysis of variance (Durup, 1969).

DESCRIPTION OF THE PROGRAM

SEREX2 generates an exhaustive series and computes the greatest gap for each element. The series can be a simple exhaustive series [each $(m+1)$ -tuple being present once] or a block of k series [each $(m+1)$ -tuple being present k times]. The program can generate several series and eventually choose the best one (this can be done several times to obtain more than one series). Two criteria are used for this choice: (1) the size of the greatest gap (as short as possible), and (2) the total of these maximum gaps summed over all elements (as small as possible). The two criteria are applied in this order.

LIMITATIONS

The program allows any values for n and m . The limitations are due to computer memory capacity. With 64K, it is possible to obtain series the lengths of which do not exceed 10,000 terms.

The quality of the series depends on the size of the sample. When n and m increase, the number of different exhaustive series and the construction time increase. For example, with $n = 4$ and $m = 2$, the running time to generate 1,000 series is 22 sec; with $n = 4$ and $m = 3$, it is 75 sec.

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LANGUAGE

The program was written in FORTRAN V and developed on UNIVAC 1110.

The construction of the series requires a function which generates a sequence of pseudorandom numbers. When the choice of the series is performed through the program, an auxiliary storage unit is necessary.

OUTPUT

SEREX2 prints either all the series generated, or the best one, and the value of the greatest gap for each element. The best series can also be punched.

AVAILABILITY

A write-up for use and a FORTRAN V listing may be obtained by writing to Guy Reynard, C.N.R.S.-INP4, 31, chemin J. Aiguier, 13274-Marseille Cedex 2-France.

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TREBIG: A 360/75 FORTRAN program for three-mode factor analysis designed for big data sets

HOWARD WAINER

The University of Chicago, Chicago, Illinois 60637

GUNNAR GRUVAEUS

Princeton University, Princeton, New Jersey 08540

and

MICHAEL BLAIR

Mathematica, Princeton, New Jersey 08540

In the analysis of multifactor, longitudinal data, it has long been the practice of investigators to factor analyze the data in each time period separately and then compare the factor structures by eye. This has obvious problems and various measures of similarity were devised, none of which really worked properly.

In recent years, other procedures have been proposed to better handle these sorts of data (i.e., Corballis, 1970; Jöreskog, 1970; Tucker, 1966). Tucker's solution is termed "3-mode factor analysis," which is essentially the Eckart-Young decomposition of a set of matrices that are derived by collapsing a tensor of order three. It has the advantage of combining the factor analyses of each time period and indicating how the changes occur.

THE METHOD OF ANALYSIS

The following is a capsule description of the kind of three-mode factor analysis done by the program; for a more detailed description, the interested reader is referred to Tucker's (1966) article, *Method 2* (pp. 298-299).

Define

X_{ijk} = a tensor of order 3

${}_iX_{(jk)}$ = a matrix of order i by $j \times k$, which is formed by collapsing X_{ijk} over the j, k modes.

Compute

$${}_jP_j = {}_jX_{(ik)} X_j \quad (1)$$

[Postmultiply ${}_jX_{(ik)}$ by its transpose.]

$${}_kQ_k = {}_kX_{(ij)} X_k \quad (2)$$

[Postmultiply ${}_kX_{(ij)}$ by its transpose.]

$${}_jP_j = {}_jB_{p_1} P_{p_1} B_j, \quad (3)$$

where ${}_jP_j$ is a diagonal matrix of positive roots of ${}_jP_j$ and ${}_jB_{p_1}$ is a matrix of unit length vectors associated with the roots in ${}_jP_j$.

$${}_pX_{(ik)} = {}_pB_j X_{(ik)}, \quad (4)$$

where ${}_pX_{(ik)}$ is a "filtered" ${}_jX_{(ik)}$ matrix. [It is hypothesized that because of some inherent latent structure in ${}_jX_{(ik)}$, ${}_p < j$.]

$${}_kQ_k = {}_kC_{q_1} Q_{q_1} C_k, \quad (5)$$

where ${}_kQ_k$ is a diagonal matrix of the positive roots of ${}_kQ_k$ and ${}_kC_{q_1}$ is a matrix of the associated unit length vectors.

$${}_qX_{(ip_1)} = {}_qC_k X_{(ip_1)}. \quad (6)$$

Once again, it is hypothesized that $q_1 < k$, which can be rewritten as

$${}_iX_{(p_1 q_1)}. \quad (7)$$

which, when premultiplied by its transpose, yields

$$({}_{p_1 q_1})S_{(p_1 q_1)} = ({}_{p_1 q_1})X_i X_{(p_1 q_1)}. \quad (8)$$

This represents the factor-combination mode, which can be decomposed

$$({}_{p_1 q_1})S_{(p_1 q_1)} = ({}_{p_1 q_1})V_{m_1} S_{m_1} V_{(p_1 q_1)}. \quad (9)$$

Borrowing from Tucker Method III, we then create the variable-combination mode

$$({}_{jk})F_{(m_1)} = {}_jB_{p_1} \otimes {}_kC_{q_1} ({}_{p_1 q_1})V_{m_1} S_{m_1}^{-1/2}, \quad (10)$$

where $\otimes$ denotes the Kronecker product. Since these are principal axis factors, we normalize them to find the roots. Finally, we define

$${}_iA_{m_1} = {}_iX_{(p_1 q_1)} V_{m_1} S_{m_1}^{-1/2}. \quad (11)$$

The matrices A, B, and C are the factor loadings in each of the three modes. The G or "core" matrix that interconnects them is a factor matrix of S and can be determined from V.

PROGRAM CAPACITY

Number of variables for Mode 1 = unlimited; number of variables for Mode 2 = 10; number of variables for Mode 3 = 10; and number of factors allowable on each mode = 10.

COMPUTER

IBM 360/75.

PROGRAM LANGUAGE

FORTRAN IV.

COMPUTATIONAL PROCEDURE

The computation of the eigenvalues and eigenvectors is done using a modified HOW method. It uses Householder's method (see Wilkinson, 1960) to reduce the matrix to codiagonal form, the QR algorithm for computing the eigenvalues (see, e.g., Ortega & Kaiser, 1963), and Wilkinson's (1958) method to compute a specified number of eigenvectors.

AVAILABILITY

The program and its associated description is distributed by National Educational Resources, Inc., 215 Kenwood Ave., Ann Arbor, Mich. 48103. At present, the cost for deck and documentation is \$100 for educational institutions and \$200 for all others.

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