SESSION V

CONTRIBUTED PAPERS: PERCEPTION AND JUDGMENT

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Using a laboratory computer for identification of nonlinear behavioral and electrophysiological systems*

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A system for mathematically identifying unknown nonlinear systems based upon Wiener kernels and G-functionals is presented as an example of the sophisticated analyses of electrophysiological and behavioral data made possible by the introduction of powerful digital computers to the laboratory environment. The applicability of these methods to a wide variety of systems is emphasized. By measuring the response of the unknown system to appropriate white noise stimulation, data may be obtained which are sufficient for the characterization of the system under test. The only restrictions upon the unknown system is that it be approximately stationary during the period of measurement and that its output depend upon its input in the finite past. Various pretests to evaluate the applicability of the method are presented. Simple FORTRAN programs that clarify the methods of computation are given for the preprocessing of data, the analysis of lower-order kernels, the synthesis of system response from the computed kernels, and the computation of the proportion of the variance in the systems output which is accounted for by the kernels zero, one, and two. These programs may be implemented on a digital laboratory computer of moderate size. Analysis may be performed either on-line or after data collection, depending upon the temporal characteristics of the system under test. This method represents one of the few analytical techniques presently available for the characterization of both linear and nonlinear systems. For this reason, it constitutes a uniquely powerful tool for many branches of psychology. Successful applications of this method to electrophysiological and behavioral data are mentioned. Limiting factors in the use of Wiener kernel analysis are also discussed.

The development of statistical communication theory has provided a set of methods and models for the analysis of the function and behavior of biological systems. Psychologists are perhaps most familiar with the application of statistical decision theory to problems of perception proposed by Tanner and Swets (1954) in the form of signal detection theory. However, there are

other numerical tools in statistical communication theory which have been successfully used in engineering applications, but which have not been widely applied to biological and psychological problems, even though the need for such techniques is clear. In this paper, we shall describe a general method for the analysis of nonlinear systems drawn from the work of N. Wiener and his colleagues, Y. W. Lee and M. Schetzen.

Wiener kernel analysis has been applied to biological systems in recent years by a number of investigators. DeBoer (1967; DeBoer & Jongkees, 1968) has studied the linear impulse response of single units in the auditory system and drawn interesting conclusions about their bandwidth. Stark (1969) has obtained estimates of the linear and quadratic kernels of the pupil control system in man. These methods have been employed by Lopes de Silva (1970) and Beatty (1971) in studies of visual system input and cortical EEG activity. Marmarelis and Naka (1972) have published first- and second-order kernels of the horizontal-bipolar ganglion system of the catfish retina, from which they were able to determine the sequence of signal transforms occurring in this system. In each case, Wiener kernel analysis permitted functional identification of a biological system by measurement of the system response to white noise stimulation. Wiener kernel analysis is equally appropriate for the study of some continuous behavioral phenomena, such as tracking, although results of such investigations have not yet been published.

IDENTIFYING UNKNOWN NONLINEAR SYSTEMS

In each of these cases, the problem of system identification is one of determining the input-output relations for that system under all conditions of stimulation. Identification in this sense involves the construction of a model of that system capable of simulating the performance of the unknown system with an acceptable degree of accuracy. This model is not a structural model; it does not necessarily utilize the same internal processes as does the system. Instead, it is a functional model, reproducing only the input-output relation, or system transfer function. Since there are

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potentially a great number of physical systems with a given transfer function, a functional model is not unique. But it captures the rules governing the unknown system's operations, and is therefore highly informative.

Wiener, in his lectures on nonlinear problems in random theory (1958), has argued that such a model might be constructed from a set of orthogonal functionals of increasing order. These functionals (G_n) sum to give the output of the system based upon its past input.

$$y(t) = \sum_{n=0}^{\infty} G_n \cdot h_n \cdot x(t)$$

In this formulation, h_n is a member of a set of orthogonal kernels, which together may be thought of as the generalized impulse response of the system. Wiener developed these kernels in terms of Laguerre functions and Hermite polynomials, largely because they could be realized with existing analog equipment, but the actual computation of the kernels by this method is extremely time-consuming and difficult.

A more suitable method of kernel estimation has been advanced by Lee (1960) and Schetzen (Lee & Schetzen, 1965). They have shown that the Wiener kernels may be digitally computed using cross-correlational techniques.

$$h_n(\sigma_1, \dots, \sigma_n) = \frac{1}{n! P^n} E \left\{ \left[y(t) - \sum_{k=0}^{n-1} G_k(h_k, x(t)) \right] x(t-\sigma_1) \dots x(t-\sigma_n) \right\}$$

This conceptualization of the kernels permits the application of Wiener nonlinear systems analysis to the study of biological systems in a computer-based laboratory.

Critical to the development of an orthogonal set of kernels, computed in either fashion, is the use of Gaussian white noise as the input signal. Wiener (1958) has argued that Gaussian white noise is the ideal test signal for the analysis of nonlinear systems, since it may represent any conceivable signal. By its very nature, there exists some probability that some portion of this signal will approximate with sufficient accuracy any function which may be applied to the system. Thus, the actual test signal is drawn from the universe of all possible signals, not from an artificially limited subset thereof. This is important in system identification, since any two systems which respond in the same way to all possible input signals are necessarily functionally identical. Thus, use of a Gaussian white noise input signal forms a good test of this hypothesis.

EVALUATING THE WIENER KERNELS

An understanding of this method and its application may be gained by examining the processes of system identification and synthesis. Figure 1 presents a

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00001      DIMENSION X(1000),Y(1000),-100,100,-100,100,2,100
00002      DIMENSION G(1000),G2(1000)
00003      N=1000
00004      L=100
00005
00006      C COMPUTE & REMOVE 0TH ORDER KERNEL & STIMULUS MEAN INTENSITY
00007      C
00008      SMIR=0.
00009      -0.0.
00010      DO 1 I=1,N
00011      SMIR=SMIR+X(I)
00012      -0.40+Y(I)
00013      SMIR=SMIR/N
00014      -0.00/N
00015      G=0.0
00016      DO 2 I=1,N
00017      X(I)=X(I)-SMIR
00018      Y(I)=Y(I)-G
00019      C
00020      C COMPUTE 1ST ORDER KERNEL
00021      C
00022      DO 4 K=0,L
00023      S=0.
00024      DO 3 I=K+1,N
00025      S=S+X(I-K)*Y(I)
00026      -0.00
00027      C
00028      C COMPUTE & REMOVE 1ST ORDER SYNTHESIS
00029      C
00030      DO 6 I=L+1,N
00031      S=0.
00032      DO 5 K=0,L
00033      S=S+H1(K)*X(I-K)
00034      G1(I-L)=S/(L+1)
00035      DO 7 I=L+1,N
00036      Y(I)=Y(I)-G1(I-L)
00037      C
00038      C COMPUTE 2ND ORDER KERNEL
00039      C
00040      DO 9 K=0,L
00041      DO 8 J=0,K
00042      S=0.
00043      DO 8 I=L+K+1,N
00044      S=S+X(I-J)*X(I-K)*Y(I)
00045      H2(J,K)=S/(L+1)*(N-L-K)
00046      DO 9 K=0,L
00047      H2(K,J)=H2(J,K)
00048      C
00049      C COMPUTE 2ND ORDER SYNTHESIS
00050      C
00051      T=0.
00052      DO 10 K=0,L
00053      T=T+H2(K,K)
00054      DO 12 I=L+1,N
00055      S=0.
00056      DO 11 K=0,L
00057      DO 11 J=0,K
00058      S=S+2*(X(I-K)*X(I-J)*H2(K,J)-T)
00059      G2(I-L)=S/(L+1)*H2(I-L)
00060      C
00061      C RESTORE ORIGINAL INPUT & OUTPUT
00062      C
00063      DO 13 I=1,L
00064      X(I)=X(I)-SMIR
00065      Y(I)=Y(I)-G
00066      DO 14 I=L+1,N
00067      X(I)=X(I)-SMIR
00068      Y(I)=Y(I)-G+G1(I-L)
00069      C
00070      C COMPUTE PROPORTIONS OF VARIANCE UNACCOUNTED FOR BY SYNTHESIS
00071      C
00072      V=0.
00073      PV=0.
00074      P=0.0
00075      DO 15 I=L+1,N
00076      V=V+(Y(I)-G+G1(I-L)-G2(I-L))**2
00077      PV=PV+(Y(I)-G+G1(I-L))**2
00078      P=PV*(V+G2(I-L)-G2(I-L))**2
00079      PV=PV/V
00080      PV=PV/V
00081      V=V/(N-L)
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single main program for conceptual clarity, not computational efficiency. However, it may be streamlined, segmented, or otherwise modified to fit the needs of a particular laboratory computer. The dimensions and parameters of the program should be adjusted to fit the temporal properties of the system under test.

The process of identification begins with the estimation of the relevant Wiener kernels. For a linear system, only the first two kernels, h_0 and h_1 , are needed for complete characterization. Nonlinear systems require additional kernels.

Input to the program consists of two arrays, X and Y, and several parameters. X is the digitized record of the Gaussian white noise stimulus. Y is the system's response to that stimulus. X and Y may also be timelocked averages of the system input and output if the same white noise stimulus is presented to the system several times.

In the initial segment of the program, the means of arrays X and Y are computed and then removed. The mean of X is the average intensity of the stimulus signal (SMI), and is stored as a parameter of the experiment. Since the dynamic range of the testing and measurement system is typically less than the range of the biological system, only a portion of its dynamic range may be investigated in a single experiment. Slightly different values may be obtained for the Wiener kernels for different levels of SMI. The X array is demeaned to simplify the calculation of the G-functionals.

The mean of the response array, Y, is the value of the zero-order kernel, h_0 , and the expected output of the system, G_0 :

$$G_0 = h_0 = \bar{Y}$$

Since the computation of higher-order kernels must be independent of the lower-order kernels, G_0 is removed from the Y output array in Lines 16-18.

$$x'_i = x_i - \bar{x}, \quad i = 1, \dots, N$$

$$y'_i = y_i - G_0, \quad i = 1, \dots, N$$

The value y'_i is simply the i th value of the measured output with G_0 removed.

The linear kernel, h_1 , is computed in Lines 22-26.

$$h_1(k) = \frac{1}{N-k} \sum_{i=k+1}^N y'_i x'_{i-k}, \quad k = 0, \dots, L$$

It can be seen that this kernel is proportional to the conventional cross-correlation function between the

adjusted system input and output, where L is the maximum lag for which the function is computed. The value of L should exceed the presumed memory of the system.

These two kernels, h_0 and h_1 , completely characterize a linear system. The linear kernel, h_1 , corresponds to the impulse response of the system, or the response of the resting system to a single pulse of energy. If, however, the system is nonlinear, these kernels will not provide complete identification. Instead, they constitute a characterization accurate to the first order, regardless of the nature of the nonlinearity involved. This results from the fact that the linear kernel is made orthogonal to the higher-order kernels.

To evaluate the second-order kernel, linear effects must first be computed and then removed from the output array, Y' . This is done in Lines 30-36 by the following equations

$$G_1(i) = \frac{1}{L+1} \sum_{k=0}^L h_1(k) x'_{i-k} \quad i = L+1, \dots, N$$

$$y''_i = y'_i - G_1(i), \quad i = L+1, \dots, N$$

The linear response of the system, $G_1(i)$, is computed for all points for which sufficient data are available, from Y'_L to Y'_N . This results from the necessity for utilizing the first L points in the stimulus array to predict the linear response for Point Y'_L in the output array. The synthesis takes place by simply convolving the input array, X' , with linear kernel, h_1 . This estimated value of the linear transfer is removed from the output array, Y' , to form an array devoid of both zero and first-order influences, Y'' . It is with this residual that the second-order kernel is computed.

The second-order kernel resembles closely a second-order cross-correlation function. It is formed in Lines 40-46 from the averaged triple product of an output value, Y'' , and two input values of X' , each at lags between 0 and L. The result is a weighting surface symmetric about its diagonal. It reflects the extent to which the system's transfer function deviates from linearity when the inputs at two points of past time interact.

$$h_2(j, k) = \frac{1}{2} \left[\frac{1}{N-L-k} \sum_{i=L+k+1}^N y''_i x'_{i-j} x'_{i-k} \right], \quad \begin{matrix} k = 0, \dots, L \\ j = 0, \dots, k \end{matrix}$$

$$n_2(k, j) = h_2(j, k)$$

It is a measure of cross-talk between two impulses as a function both of time and of the distance between the impulses in time (Marmarelis, 1972). The greater the values in this kernel, the more second-order cross-talk characterizes the unknown system.

The second-order kernel, like the first, is used as a weighting function for system synthesis.

$$G_2(i) = \frac{1}{(L+1)^2} \sum_{k=0}^L \sum_{j=0}^L h_2(j, k) x'_{i-j} x'_{i-k} = \frac{1}{L+1} \sum_{k=0}^L h_2(k, k),$$

$i = L+1, \dots, N$

For each and every point on this weighting surface, a triple product is formed between a value of the kernel, h_{jk} , and two values of the stimulus array, x'_{i-j} and x'_{i-k} . The resulting sum is then averaged. The diagonal is removed, since at the diagonal two temporally distinct pulses cannot exist. This gives the second-order response of the system.

If a third-order kernel, a weighting volume, were to be computed, the second-order effects would be removed from the response array before computation. In Lines 50-59, the second-order synthesis is computed (as Array G2) but not removed from the output array. Instead, the input and output arrays are restored in Lines 63-68.

TESTING THE ADEQUACY OF SYSTEM SYNTHESIS

Testing the adequacy of system identification may be accomplished by comparing the output of the unknown system with that of the Wiener model, when the same stimulus is applied to both. Any input stimulus may be used, since the model is assumed to be general. As before, a Gaussian white noise stimulus is preferable, since it samples the entire set of possible stimuli randomly. Further, the correspondence of the system and the model may be tested on the very data used for model construction. The response of the model at various orders of completeness may be compared with the actual output to determine the adequacy of simulation.

This is the method for synthesis evaluation employed in this program. We ask simply what proportion of the variance of the actual system's output, Y , may be accounted for by our model. This is accomplished by computing the variance of Y and the expected value of the squared differences between the actual and predicted values of the output for syntheses of orders zero, one, and two.

$$\begin{aligned} \text{VAR} &= E(y - \bar{y})^2 \\ \text{PVO} &= E(y - G_0)^2 / E(y - \bar{y})^2 \\ \text{PV1} &= E(y - G_0 - G_1)^2 / E(y - \bar{y})^2 \\ \text{PV2} &= E(y - G_0 - G_1 - G_2)^2 / E(y - \bar{y})^2 \end{aligned}$$

Ratios are formed between these expected values and the variance of Y . It should be noted that if the synthesis were computed on another set of data, $E(y - G_0)^2$ would not necessarily equal $E(y - \bar{y})^2$. When the same data are used both to estimate and to test the kernels, PVO must be one. To the extent the

linear synthesis accounts for the variance of the output, the ratio PV1 will be less than one. If the synthesis is based on insufficient data, if there are no linear components in the actual system, but there are nonzero values in the linear kernel, or a number of other undesirable states hold, the variance will increase and the ratio PV1 will exceed one.

Similarly, if the second-order kernel successfully models a quadratic component of the physical system, the ratio PV2 will be less than PV1.

Thus, the Wiener method of system identification provides not only methods for analysis, but also methods for synthesis from which the adequacy of analysis may be estimated. This check greatly increases the power of the method as a scientific tool, and increases the confidence of the investigator in his results.

PRETESTING THE SYSTEM FOR PARAMETER SETTING

In performing an actual experiment, realistic values must be placed on such parameters as noise bandwidth, number of lags, and the order of the highest kernel to be computed. Marmarelis (1972) has outlined simple experiments which may be carried out to aid in the selection of these values.

The first assumption of Wiener analysis, stationarity, may be tested by performing a simple evoked response experiment at several points in time. If the results are reasonably comparable, then stationarity is not likely to be seriously violated.

System memory or the period of dependence of the system's output upon past input may be estimated by performing a two-pulse experiment. Two pulsed stimuli are presented to the system. The interval between the pulses is increased until the response of the system to each pulse is identical. The memory of the system probably does not exceed this interval in most cases. Such an experiment is useful in determining the number of lags necessary to cover the memory of the system. The number of lags is taken to be the length of memory in units of time, divided by the period of the sample interval, plus a small safety factor of 10% to 20%.

The bandwidth of the white noise signal should be greater—by about an octave—than the bandwidth of the system under test. The empirical bandwidth can be determined by an experiment with sine-wave stimulation. The noise should be approximately Gaussian, and not clipped at less than three standard deviations from its mean (Marmarelis, 1972).

Sine-wave stimulation may also be used to determine the nonlinearity of the system and the order of the highest kernel that needs to be computed. In general, the order of the highest harmonic in the response to a sine-wave stimulus is the order of the highest nonlinearity in the system. Thus, the analysis of a system showing only a second harmonic may be terminated by the second-order kernel.

The power of the white noise stimulus (P) over the

bandwidth of the system should be set to 1.0 in most experiments. This minimizes error in the computation of the linear kernel, which is desirable, since the linear kernel often accounts for a major portion of the systems output. If the computation of the quadratic kernel were to be optimized, then P would be set to 2 (Marmarelis, 1972).

LIMITS TO WIENER ANALYSIS

Despite the generality of the Wiener method, there are limits to its utility. The two assumptions of approximate stationarity and dependence upon the finite past of the input have already been mentioned. Wiener analysis of nonstationary systems would be inappropriate. Similarly, systems which do not depend upon past input in any important way, such as oscillators or pacemaker cells, cannot be studied using these methods.

Systems with low signal-to-noise ratios give less than satisfactory results when Wiener analysis or any other form of system identification is employed. Much of the output of such systems is not attributable to input, so that no transfer function could completely specify system output. Further, the introduction of a large error variance increases the amount of data necessary to characterize accurately the input-output relation.

Systems with low-order nonlinearities are more suited to Wiener analysis than are systems with important higher-order kernels. This results from the fact that computation time increases greatly in the higher kernels. Nevertheless, Wiener analysis may be applied to highly nonlinear systems, even if only the first few kernels are estimated. Such an incomplete representation nonetheless constitutes the best characterization of the unknown system to that order, in the mean squared error sense.

The particular strength of the Wiener method lies in the analysis of systems with prominent nonlinearities. For systems known to be linear, other simpler methods of linear analysis may be preferred.

Wiener kernel analysis is a powerful and general method for the identification and simulation of unknown systems. It is likely that it will receive more widespread application in the study of behavioral and biological problems, for which it is well suited, as digital computers become increasingly available.

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