An individual differences analysis of interdimensional additivity in multidimensional scaling*

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Comparative judgments of the dissimilarity between schematic faces varying on 1, 4, 7, or 10 binary attributes were obtained under two instructional sets. Using the Tucker-Messick procedure, three subgroups of like-perceiving Ss were isolated and the nature of cue-utilization by each described. Perceptual independence of attributes was demonstrated both within and between instructional sets for Subgroup 1 and within instructional sets for Subgroup 2. The Subgroup 3 results indicated an interaction of the attributes. Data from the Shor cognitive elements test differentially characterized Subgroups 1 and 2, but did not uniquely characterize Ss requiring more complex models for the combination of attributes. The study illustrates the possible usefulness of an individual differences approach to the study of selective attention and information processing.

Emphasis by many perceptual theorists (e.g., Gibson, 1966, 1969; Garner, 1970) that perception involves an active search for and selection of distinctive features of a stimulus array has led to a rapidly increasing literature on processing information from multidimensional stimuli. Forsyth and Brown (1967) presented research supporting their proposition that there is a hierarchy of physical dimensions to which an individual attends in the perceptual process. It seemed to us that if weights could be attached to each physical dimension or stimulus attribute to reflect its relative utility, extremely good predictability of perceptual judgments could be achieved. The work by Tversky and Krantz (1969) suggested that an interdimensional additivity model might be appropriate for combining dimensions in a prediction scheme for perceptual judgments. This model posits that the perceived dissimilarity between stimulus objects can be decomposed into the additive components of each attribute differing between the two stimuli. Tversky and Krantz (1969) provided some validation of this hypothesis using schematic faces varying on three features. Several aspects of this research led us to question the general utility of the model for perceptual judgments about faces. First, the small number of features permitted Tversky and Krantz's Ss to count the number of differing features to make a judgment. Second, Ss scaled stimulus pairs for overall dissimilarity rather than the more complex and more typical perceptual process of judging faces on a specific expression. A third concern was the substantial individual differences typical of multidimensional scaling tasks. Wender (1971), using a different stimulus domain and varying S's set for testing, failed to validate the

interdimensional additivity hypothesis for about half of his Ss. This further emphasized the need for an individual differences approach for a careful examination of the model.

Taking into account these concerns, the present study sought to examine the adequacy of the interdimensional additivity hypothesis with a larger number of component dimensions and with the requirement that Ss scale faces for dissimilarity on specific expressions rather than simply overall dissimilarity. We expected that these two modifications would provide a more rigorous test of the additivity hypothesis, and also hoped to determine how the salience of the attributes changed as the expression scaled was changed. A second purpose of the study was to address the problem of individual differences. Wender (1971) was willing to reject the interdimensional additivity hypothesis because half of his Ss violated predictions of that model when he varied S's set for testing. Some Ss, however, continued to demonstrate interdimensional additivity despite these variations in set. We hoped with the individual differences approach (Tucker & Messick, 1963) to identify homogeneous subgroups of Ss with regard to the scaling data, describe differences in the predictive systems between subgroups, then identify individual differences variables which would differentially characterize the subgroups. Earlier work on selective feature utilization (e.g., Forsyth & Brown, 1968; Forsyth, Forsyth, & Pinsince, 1970) stressed the need for this individual differences approach. Because the Shor Cognitive Elements Test provides a number of individual difference measures in tasks requiring active selective attention (Shor, 1971), it was used in the search for variables to characterize the subgroups.

METHOD

Subjects

Thirty students enrolled in introductory psychology at the

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Fig. 1. Exemplars of schematic face pairs.

University of New Hampshire served as Ss in this study in partial fulfillment of course requirements.

Stimuli and Cognitive Elements Test

Faces used in this study were selected from a stimulus domain of schematic faces consisting of all possible combinations of 10 binary attributes. These attributes are: pupil dilation, pupil placement, eye separation, eyebrow length, eyebrow curvature, mouth length, mouth curvature, mouth placement, nose size, and nose placement. Four sets of 10 stimulus pairs each were selected such that 1, 4, 7, and 10 attributes differed in each set. Exemplars of these four sets of stimulus pairs are presented in a top-bottom arrangement in Fig. 1. From left to right, the four stimulus pairs differ on 1, 4, 7, and 10 attributes, respectively. Ten pairs were selected randomly from all possible pairings meeting the criterion number of attributes differing for each set. An additional constraint on the selection of the single-attribute set pairs was that each of the 10 pairs differ on a different attribute.

The Shor Cognitive Elements Test administered to each S is an extrapolation from the logic of the Stroop color-word test. The general nature of the test is described by Shor (1971). The specific concepts of the test included in this study were the spatial direction concepts of up, down, right, and left. The concepts were symbolically coded as outline drawings of arrows and linguistically coded as the word names for the four directions. In a pure condition, 24 arrows were placed on the stimulus sheet. In a compounded condition, a mismatched word name was printed inside each arrow (e.g., DOWN within an arrow pointing to the right). On half of the trials of each condition, S was given a verbal task of reading off the spatial directions of the arrows. On the other half, S was given a motoric task of moving a four-way toggle switch to indicate the spatial directions of the arrows.

Procedure

Following the presentation of instructions and practice trials, each S made dissimilarity ratings of the 40 schematic-face pairs under two instructional sets for a total of 80 ratings. In the first instructional set, S rated the dissimilarity between the schematic faces of each pair on "how interested in something" the faces appeared to be. In the second instructional set, S rated dissimilarity of the same pairs on "how afraid of something" the faces appeared to be. The order of presentation of the 40 pairs was random, but the same order was used for both instructional sets. Each S rated the expression dissimilarity on a scale from 0 (no dissimilarity) to 10 (maximum dissimilarity) and reported which face showed more the expression being scaled. The Shor Cognitive Elements Test was administered following the completion of the ratings.

RESULTS

The analysis of the data followed a procedure

suggested by Tucker and Messick (1963) and described by Forsyth (1973) and Richards (1972). The analysis involved: (1) a principle components factor analysis of the scaling data to obtain a rotated factor matrix with loadings on Ss, (2) selecting homogeneous subgroups of Ss on the basis of loadings on the rotated factor matrix, (3) obtaining the average dissimilarity ratings for each subgroup on each stimulus pairing under each expression scaled, (4) using each subgroup's expression dissimilarity ratings on the 10 stimulus pairs differing on a single attribute to predict within an expression the dissimilarity ratings on the remaining 30 pairings, (5) computing correlations between the predicted ratings and the actual ratings both within and across expressions for each subgroup, (6) comparing predicted and actual ratings within and across expressions and within and across subgroups, and (7) searching for individual differences variables which differentially characterize the subgroups.

The correlation matrix for the principle components factor analysis represented the intercorrelations of the 30 Ss across the 80 perceptual judgments. Using a criterion of eigenvalues greater than unity, three factors resulted. The proportion of variability accounted for by these three factors was .48, .14, and .07, respectively. This factoring permitted the selection of homogeneous subgroups of Ss having similar profiles of perceptual judgments. Table 1 presents the median factor loading for each subgroup on each of the rotated factors. The 10 Subgroup 1 Ss had a median intercorrelation with one another of .72, the 8 Subgroup 2 Ss had a median intercorrelation with one another of .70, and the median intercorrelation of the 5 Subgroup 3 Ss was .51.

The average dissimilarity judgment for each expression by stimulus pair combination was obtained for each subgroup. These judgment averages on the 10 pairs differing on a single attribute under each expression scaled were used to derive predictions of the judgments on the remaining 30 pairs for each subgroup. For example, suppose that for one subgroup the judged average difference between the pair of faces differing only in pupil dilation is 3, with the dilated-pupil face rated as more interested in something. Let the average difference between the pair differing only on mouth in direction be 4, with mouth-up rated as more interested for this subgroup. Using only these two attributes, we would predict that the judged difference between the third pair of faces in Fig. 1 would be 7, with the top face more interested. The predicted difference for the fourth

Table 1
Median Factor Loadings for Each Subgroup
on Each Rotated Factor

		Rotated Factor		
		1	2	3
Homogeneous Subgroups	1	.81	-.20	.15
	2	.19	-.77	.09
	3	.30	-.15	.60

pair in Fig. 1 would be 1, with the top face more interested. Thus, the predicted dissimilarity judgment for a pair differing on 10 attributes would involve combining the dissimilarity predictions based on each of the 10 attributes in this additive manner. These predicted values were then correlated with the actual dissimilarity judgments on each expression for each subgroup separately. Table 2 presents the resulting correlations.

As Anderson (1972) and Birnbaum (1973) have emphasized, correlations between theoretical predictions and obtained data do not serve as an adequate test of a model. The degree of disagreement between the predictions and the data must also be considered in testing the fit of a model. Accordingly, an alternative approach was taken with Subgroups 1 and 2 to further examine the correlational support for interdimensional additivity. For purposes of contrast, comparisons between predicted and actual ratings were made within and between expressions and within and between Subgroups 1 and 2. Since Ss rated the expression dissimilarity on a scale from 0 to 10 and also reported which face showed more of the expression being scaled, the absolute difference between predicted and obtained ratings could vary from 0 to 20. Table 3 presents the mean discrepancies between predicted and obtained ratings within and between expressions and within and between Subgroups 1 and 2. The entries represent the means across 30 stimulus pairings.

The proximity of the predicted ratings to the obtained ratings as well as the correlations for Subgroups 1 and 2 demonstrate substantial support for the interdimensional additivity hypothesis when the predictions and observed ratings are based on the same expression. All correlations for these combinations were .92 or greater ($p < .01$). The mean absolute discrepancy between predicted and obtained ratings for the four conditions in which predicted and observed ratings both were taken for the same expression within a subgroup was 1.37. Within-subgroup comparisons of each of these four conditions were made with mean discrepancies in conditions in which predicted ratings were based on a different expression than were obtained ratings (see Table 3). Each of these comparisons resulted in a significantly lower mean discrepancy ($p < .01$) when

Table 2
Correlations of Predicted Judgments With Observed Judgments on Each Expression for Each Subgroup

	Predictions	Observed Judgments	
		Afraid Expression	Interested Expression
Subgroup 1 (N = 10)	Afraid	.97	-.94
	Interested	-.91	.93
Subgroup 2 (N = 8)	Afraid	.97	.36
	Interested	.33	.92
Subgroup 3 (N = 5)	Afraid	.80	-.35
	Interested	.26	.00

Table 3
Mean Discrepancies Between Predicted and Observed Ratings Within and Between Expressions and Within and Between Subgroups 1 and 2

Predicted	Ratings	Observed Ratings			
		Subgroup 1		Subgroup 2	
		Inter- ested	Afraid	Inter- ested	Afraid
Subgroup 1	Interested	1.55	10.82	5.52	9.90
	Afraid	10.85	1.27	5.42	1.14
Subgroup 2	Interested	5.98	5.18	1.49	4.92
	Afraid	10.20	1.08	4.75	1.18

predicted and observed ratings were based on the same expression.

What did differentiate Subgroups 1 and 2 were the correlations of judgments on one expression with predictions based on a different expression (see Table 2). Violations of strict independence for Subgroup 1 involves little more than sign dependence or a reversal in ordering. Krantz (1972) suggests that such sign dependence generalizes the notion of independence and interdimensional additivity. Subgroup 1 gave approximately the same absolute weight to each attribute in both the fear and interest scaling judgments, but reversed the sign for those weights. For example, Subgroup 1 utilized the mouth direction (up vs down) attribute most in both the fear and interest judgments, with mouth up reflecting more interest and mouth down reflecting more fear. Similarly, the second most utilized attribute under both expressions was mouth placement, with the centered mouth reflecting more fear and the off-center mouth expression more interest. In contrast, the utilization of an attribute varied as a function of expression for Subgroup 2. In making interest judgments, these Ss attended most to pupil dilation, with larger dilation reflecting more interest. Almost no weight was given to pupil dilation for the fear judgments by Subgroup 2 Ss. While mouth direction was the most utilized attribute for fear judgments by Subgroup 2, this feature ranked fifth in their interest ratings. The discrepancies between predicted and obtained ratings (Table 3) also reflect this difference between Subgroups 1 and 2. When predicted and obtained ratings were based on different expressions, the mean discrepancy was significantly higher ($p < .01$) within Subgroup 1 than within Subgroup 2. Further, Table 3 indicates that Subgroups 1 and 2 were alike in use of attributes for judging fear, but differed in use of attributes in judging interest.

While moderate support for interdimensional additivity obtained on the afraid judgments for Subgroup 3, $r(29) = .80$, $p < .01$, these Ss provided no support for the model in their interest judgments ($r = .00$). Examination of the Subgroup 3 interest judgments indicated interactive contributions of the physical features. Specifically, the pupil dilation prediction

Table 4
Verbal and Motoric Interference Means for Each Subgroup

Group	Verbal	Motoric
1	2.75	1.85
2	2.64	1.22
3	1.55	.55

weight was determined with two schematics with mouths curving up. This attribute received the greatest weight in the Subgroup 3 interest predictions, with dilated pupils indicating greater interest. Whenever dilated pupils were coupled with a large mouth curving down, that face was judged less interested than another face having nondilated pupils. In addition, whenever a face with dilated pupils and mouth curving up was compared to one with nondilated pupils and mouth curving down, the former was judged more interested in something than predicted.

Correlations between predicted and observed judgments were calculated separately for the remaining seven Ss not in one of the three subgroups. While correlations of afraid predictions with obtained afraid judgments weakly supported the interdimensional additivity hypothesis ($r > .68$ for each S), significant correlations ($p < .01$) were obtained for only two Ss on the interested expression. No consistent pattern of nonadditivity of the 10 attributes obtained for the remaining five Ss.

The search for individual differences variables which differentially characterize the subgroups was limited to measures obtained from the Shor Cognitive Elements Test. Both verbal and motoric interference scores were obtained for each S. These interference scores are defined as the difference in response speed under the pure condition of responding to arrow directions and the compounded-mismatched condition of responding to arrow directions. Table 4 presents the verbal and motoric interference score means for each subgroup.

Using the three homogeneous subgroups as one variable and the verbal and motoric task interference measures as a two-level repeated-measures variable, an unweighted means analysis (Winer, 1971) demonstrated significantly less interference in the motoric than in the verbal task for all three groups, $F(1,20) = 18.80$, $p < .001$. An examination of a violation of the assumption of homogeneity of variance-covariance matrices for this analysis revealed differences between the three homogeneous subgroups in flexibility across modes of response. Specifically, the high predictability of motoric from verbal interference scores for Subgroup 1 ($r = .89$, $p < .01$) showed little flexibility in strategy used across these two modes of responding. This relationship between the verbal and motoric modes of responding existed neither for Subgroup 2 ($r = .06$, $p > .05$) nor for Subgroup 3 ($r = .37$, $p > .05$). The high correlation for Subgroup 1 was interpreted as reflecting a tendency to use the same strategy for responding

whether a verbal or a motoric response was demanded. The lack of significant correlation between interference scores suggested a more flexible shifting of strategies under the different modes of response for Subgroups 2 and 3.

DISCUSSION

The study of selective attention in perceptual process has emphasized the need for obtaining a physics of stimulus domains studied and models for weighting the utility of these physical dimensions (Egeth, 1967). The results of the present study indicate the importance of an individual differences multidimensional scaling approach to the study of cue utilization in information processing. The prediction of multidimensional results from unidimensional results assumes no interactions of these dimensions. The difficulties for models making this assumption of perceptual independence of dimensions have been discussed by Anderson (1970), Garner and Morton (1969), and Kaufman and Levy (1971). With the more complex task of scaling dissimilarity of specific expressions and the larger number of attributes, strong support for interdimensional additivity such as offered by Tversky and Krantz (1969) was not expected in the present study. The results offered more support for the interdimensional additivity hypothesis than had been anticipated. The scaling of the stimuli varying on multiple attributes was highly predictable from the unidimensional results for the first two homogeneous subgroups. Further, one of these subgroups also demonstrated additivity. Subgroup 3 and some of the Ss not placed into one of the three homogeneous subgroups demonstrated the need for more general models for the identification and combination of perceptual dimensions such as the one proposed by Kaufman and Levy (1971). However, the individual differences approach indicates that specific parsimonious models can be developed for different homogeneous information processors, permitting a compromise between complete idiography and the attempt to write a single prediction system for a large number of Ss with diverse points of view.

The Tucker-Messick approach used appears to be the most widely used method for dealing with individual differences. Carroll and Chang (1970) have proposed a more general model and method of analyzing individual differences in multidimensional judgments. Carroll and Chang (1970) noted that comparisons of the subject spaces from these two methods show them to be strongly related. The critical difference between these two approaches for the present study is that the Tucker-Messick procedure does not address communalities of dimensional structures of the different idealized individuals. An examination of Table 3 and a characterization of the subgroups in terms of attributes utilized for each expression reveal a strong communality in the points of view of Subgroups 1 and 2. While the attributes used in judging the interest expression differed

markedly for the two subgroups, the attributes used for judging the fear expression was almost identical. The fear expression predictions based on either Subgroup 1 or Subgroup 2 predicted the obtained fear ratings equally well for both Subgroups 1 and 2 (see Table 3). Thus, while the Tucker-Messick method initially ignores communality in the configurations of different idealized individuals, a careful examination or description of the pattern of responses should reveal communalities attended to within the method proposed by Carroll and Chang.

The search for person variables to characterize the subgroups was limited to cognitive measures obtained from the Shor Cognitive Elements Test. While this test discriminated the cognitive style of Subgroup 1 from those of Subgroups 2 and 3, it did not differentially characterize Ss for whom perceptual independence of attributes was apparent and Ss requiring more complex models for the combination of attributes. The differential performances of Subgroups 1 and 2 across the two tasks of the Shor Cognitive Elements Test was quite consistent with the differences between these two subgroups on the schematic face scaling tasks. Specifically, Subgroup 1 Ss appear to rigidly adhere to the same strategy across tasks, while Subgroup 2 Ss appear more flexible in altering strategies with minor alterations in the nature of the task. This individual differences variable may have been helpful in Wender's (1971) study, which tested the independence of dimensions with modifications in context effects. The rating scale data for only about half of the Ss in that study violated the independence of dimensions assumption across contexts. Further research using the individual differences approach must be undertaken with a search of personality as well as cognitive variables which may help to characterize the subgroups. In addition, these studies must be followed by designs in which the subgroups are determined a priori on the basis of the individual differences variables suggested from studies such as the present one. Richards (1972) suggests that the extent of individual differences in cue utilization increases as the complexity of the stimulus domain increases. Whether the same individual differences variables will be valuable in characterizing subgroups as the stimulus domain changes is also a problem for future research.

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