Notes and Comment

Polynomial perception of exponential growth

GREGORY V. JONES
University of Oxford
Oxford OX1 3UD, England

In a recent article entitled "Misperception of exponential growth," Wagenaar and Sagaria (1975) have investigated the way in which people extrapolate from digitally or graphically presented information. Wagenaar and Sagaria conclude that people are systematically erroneous in their extrapolations. It is argued here, however, that this conclusion may be misleading, and an alternative interpretation of these data is suggested.

The methods used in each of Wagenaar and Sagaria's experiments were similar. Subjects were shown, digitally or graphically, series of five numbers, said to represent indices of pollution (y) in the upper air space in successive years ($x = 1, 2, \dots, 5$), and asked to predict future indices. Each set of five numbers conformed to an exponential series, either $y = a \exp(x)$, $a = 1, 2, 4, \dots, 128$; $y = \exp(bx)$, $b = 1.0, 1.1, \dots, 1.7$; or $y = \exp(x) + c$, $c = 0, 100, \dots, 700$. As an example, the first member of each of the three different types of exponential series is $y = \exp(x)$. In this case, the subjects were asked to extrapolate from the series 3, 7, 20, 55, 148. Each subject had to predict one of three different types of result. These were the index value for Year 10; the index value for Years 6, 7, $\dots$, 10; or the year in which the index would reach a specified value (in fact, this value was that of the exponential series with $x = 10$).

The way in which subjects choose to extrapolate from such series is a matter of legitimate psychological interest. Wagenaar and Sagaria, however, assume that there is one particular form of extrapolation (namely, exponential extrapolation) which is mathematically correct. This leads them to conclude that "The most prominent result is the considerable underestimation of growth by all groups in all conditions" (p. 419), and to represent such "underestimation" by a model in which subjects systematically underestimate exponential indices (i.e., x or bx). Alternative models for the data are encouraged,

The author's research is supported by the British Medical Research Council. Requests for reprints should be sent to Gregory V. Jones, University of Oxford, Department of Experimental Psychology, South Parks Road, Oxford OX1 3UD, England.

Figure 1. Polynomial and exponential extrapolation from initial values given by $y = \exp(bx)$, compared with subjective extrapolation (1st, 2nd, and 3rd quartiles shown dashed).

however, by the fact that for any set of starting values there is not a uniquely correct form of extrapolation, as is pointed out in many statistics texts (e.g., Kendall & Stuart, 1973, p. 376). In the present case, although the five numbers that a subject is provided with may conform to an equation such as $y = \exp(bx)$, this does not alter the fact that they also conform to an infinite number of other equations. Failure to extrapolate exponentially may not represent "gross misperception" (Wagenaar & Sagaria, 1975, p. 417), but instead the choice of a different mathematical function.

As an example, perhaps intuitively the most likely way in which subjects might choose to extrapolate would be along the line of the *collocation polynomial* (i.e., the lowest order polynomial to perfectly fit the given data). Quartic collocation polynomials were calculated (using orthogonal polynomials) for each of the series of five numbers used by Wagenaar and Sagaria, and they proved to yield extrapolations much closer than exponential extrapolations to the data. Figure 1, for example, shows the extrapolated values for Year 10, on a logarithmic scale, as a function of the different values of b in the equation

$y = \exp(bx)$ used to generate the initial five values. It can be seen that the polynomial extrapolation is not very far from the second quartile (i.e., median) data.

It is thus concluded that not only is the characterization of subjects' performance in Wagenaar and Sagaria's experiments as "misperception of exponential growth" theoretically arguable, but also that it is possible that the subjects' performance may usefully be described in terms of its conformity to collocation polynomial extrapolation.

REFERENCES

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(Received for publication November 22, 1976;
accepted November 30, 1976.)