Counting processes in human timing

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A subject reproducing a long duration, t , may time out either a single interval of duration t or a succession of n intervals, each of duration t/n . It is shown that a class of timing models obeying Weber's law predicts the variance of reproductions of t to be a decreasing function of the number of subdivisions, n . In contrast, a second class of proportional variance models, which includes Creelman's pulse counter model (1962), predicts no change in the variance as a function of n . Data are presented from a duration reproduction experiment in which subjects counted silently at a specified rate up to a given number and then responded. Several statistics involving the variance of the reproduced durations are shown to be predicted significantly better by the Weber's law class of models than by the proportional variance class of models.

A person faced with the task of timing a duration on the order of several seconds often subdivides the interval into a sequence of short durations which he or she counts rhythmically: "one-thousand one, one-thousand two . . ." Subjects in duration discrimination and duration reproduction experiments frequently attempt to use this same type of strategy if they are not explicitly instructed not to tap, count, or in any other way to subdivide the presented durations. These informal observations suggest that subjects may perceive some advantage in subdividing a long duration into subintervals. One possible advantage in using a subdivision strategy may be a decrease in the variability of the produced durations. That is, for a fixed duration t , the variability in repeated productions of t may be smaller if the subject times out n intervals of duration t/n than if he times out a single interval of duration t . This paper presents a theoretical investigation of this problem based on two different classes of timing models, and compares several predictions made by the two model classes with the results of an experiment involving a silent counting task.

It is reasonable to suppose that the variance associated with the production of a duration which consists of a sequence of subintervals is related in a lawful manner to the variance associated with the timing of each subinterval. The next section will describe an analysis of this relationship.

The Silent Counting Task

Each trial of the experiment reported here began

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with the subject counting silently from 1 to 10 in synchrony with a sequence of 10 periodic auditory clicks. Upon completing this synchronization sequence, the subject continued counting at the same rate in the absence of the synchronization clicks, starting again at 1. In this second phase of the trial, the subject counted up to either 5 or 10, depending upon the condition, and pressed a key in synchrony with the terminal count. The datum recorded was the multicount duration, T_n , between the start of the production segment (the last click of the synchronization sequence) and the keypress response.

The interval, T_n , can be partitioned into the n subintervals between successive counts, and a synchronization error interval. This latter interval represents the combined error in synchronizing the start of the first production count with the final click of the synchronization sequence, and the error in synchronizing the keypress response with the final production count. Thus, we can decompose the random variable T_n into a sum of random variables

$$T_n = C_1 + C_2 + \cdots + C_n + S, \quad (1)$$

where C_i ($i = 1, n$) are random variables denoting the successive intercount durations and S is a random variable, with mean μ_s and variance σ_s^2 , denoting the synchronization error. If we assume that the successive intercount durations are identically distributed as a random variable C , with mean μ_c and variance σ_c^2 , then the mean value of T_n is

$$E(T_n) = n\mu_c + \mu_s. \quad (2)$$

Assuming further that the intercount durations are independent of one another and independent of the synchronization error, then the variance of T_n is given by

$$\text{Var}(T_n) = n\sigma_c^2 + \sigma_s^2. \quad (3)$$

There is some reason to question the assumption of mutual independence of the intercount durations. However, it can be argued that the magnitude of this dependence, if it does in fact exist in this task, is small and that Equation 3 is therefore a good first approximation. This issue will be raised again in the discussion section.

Equations 2 and 3 describe the relationship between the mean and variance of the distribution of multicount durations and the corresponding mean and variance of the distribution of intercount durations. What remains to be specified is the relationship between the variance of the intercount distribution and the duration of the count. The next two sections will describe two classes of timing models which predict two different forms of this relationship.

Proportional Variance Models

Models belonging to this class predict that the variance of the intercount duration is proportional to the duration of the count:

$$\sigma_c^2 = k_p c. \quad (4)$$

It can be shown that Creelman's (1962) pulse counter model is a member of this class of timing models. The pulse counter model assumes that the internal representation of a time interval is the number of pulses emitted by a stochastic source at random times during that interval at a fixed mean rate, λ . Consequently, the total number of pulses accumulated in an interval t is described by the Poisson distribution with mean and variance both equal to λt . In his treatment, Creelman added other assumptions from signal detection theory to allow the model to be applied to two-alternative forced choice duration discrimination paradigms. While that particular development of the pulse counter model is not applicable to a duration production paradigm, the necessary extension is straightforward and given below.

We may suppose that during the synchronization segment, the subject obtains a sample pulse count for each of the 10 presented synchronization intervals, each of duration c . Let M denote the average of these 10 samples, a random variable with mean λc and variance $\lambda c/10$. Assume that the subject is able to store and reference this averaged pulse count. To time a multicount duration during the production segment, the subject repeatedly accumulates pulses, incrementing his count each time the sum reaches the reference value. For a particular reference value, $M = m$, the waiting time to the m th pulse, when pulses occur randomly in time, has a gamma distribution with mean m/λ and variance m/λ^2 (Parzen, 1962, pp. 133-134). The unconditional mean intercount duration may be derived from the con-

ditional mean intercount duration as follows:

$$\mu_c = E[E(C | M)] = E(M/\lambda) = \lambda c/\lambda$$

$$\mu_c = c.$$

More important, the unconditional variance of the intercount duration may also be determined, using the following relationship (Parzen, 1962, p. 55):

$$\sigma_c^2 = E[\text{Var}(C | M)] + \text{Var}[E(C | M)]$$

$$= E(M/\lambda^2) + \text{Var}(M/\lambda)$$

$$= \lambda c/\lambda^2 + \lambda c/10\lambda^2$$

$$\sigma_c^2 = 1.1c/\lambda.$$

This equation corresponds to Equation 4 with the proportionality constant $k_p = 1.1/\lambda$. Thus, the pulse counter model is a member of this class of timing models which predict the intercount variance to be proportional to the intercount duration.

For models satisfying Equation 4, the variance of the multicount duration T_n is predicted by substituting Equation 4 in 3:

$$\text{Var}(T_n) = nk_p c + \sigma_s^2. \quad (5)$$

This equation provides the basis for deciding whether or not the proportional variance models predict an advantage to the subject in subdividing a duration t . For a given duration t , we wish to determine how the variance of T_n varies with n , subject to the constraint that nc is equal to t . That is, if the number of counts, n , is increased, then the count duration, c , is decreased such that the total duration, $t = nc$, remains constant. Substituting this expression in Equation 5, we obtain

$$\text{Var}(T_n) = k_p t + \sigma_s^2. \quad (6)$$

Thus, the total variance depends linearly on the total duration, t , but is independent of the number of subdivisions and, consequently, independent of the intercount duration. For a fixed total duration, the proportional variance models predict no change in variability as the intercount duration varies.

Weber's Law Models

Models in this class share the Weber's law prediction that the standard deviation of the distribution of intercount durations is proportional to the intercount duration:

$$\sigma_c = k_w c.$$

Timing models exhibiting the Weber's law property may differ considerably in terms of their formal structure (Getty, 1975, Note 1). However, the point of interest here is in discriminating between the two general classes of models rather than in discriminating among models within a class.

If Weber's law holds, the predicted variance for a single count is given by

$$\sigma_c^2 = k_w^2 c^2. \quad (7)$$

This nonlinear growth of variance with count duration may be contrasted with the linear growth predicted by the proportional variance models (Equation 4). Substituting Equation 7 in 3, we obtain the predicted variance for a multicount duration T_n :

$$\text{Var}(T_n) = nk_w^2 c^2 + \sigma_s^2. \quad (8)$$

We may investigate the effect on variance of varying the duration of a count, c , while holding the total duration, $t = nc$, constant. Substituting $n = t/c$ in Equation 8, we obtain

$$\text{Var}(T_n) = k_w^2 ct + \sigma_s^2. \quad (9)$$

Thus, the Weber's law models predict that for a fixed duration t , the total variance is an increasing function of the intercount duration. According to these models, the performance of a subject producing a given duration would improve as the intercount duration decreases.

Equations 6 and 9 provide the basis for a strong test between the two models. The Weber's law models predict that for a given value of c , the variance of T_n is a linear increasing function of total duration. Moreover, the slope, $k_w^2 c$, of the linear function increases proportionally with c . In contrast, recall that the pulse counter model predicts that the slope, k_p , of the linear function relating the variance of T_n to total duration is a constant, independent of c (Equation 6). In order to test these predictions, subjects were run in a silent counting experiment in which the intercount duration was varied from 300 to 900 msec.

METHOD

Subjects

The four subjects in the experiment consisted of the author, another staff member, a graduate student, and an undergraduate. The two students were naive regarding the purpose of the experiment.

Apparatus

The experiment was controlled by an on-line software control system operating in a 4K Digital Equipment Corporation PDP-8/S computer. Subjects were run one at a time in a small, isolated room partially lined with acoustic panels. The ambient room noise level was 51 dB SPL (weighting network A), including

masking noise provided by continuous operation of an air-conditioner fan. The subject listened to stimuli presented binaurally through a pair of Telephonics TDH-39 earphones. The electrical waveform of the acoustic clicks was a 100- μ sec square wave whose amplitude had been previously adjusted to produce a comfortable listening level.

Stimulus and response durations were timed by a programmable clock controlled by the computer and driven by a 1,000-Hz timebase. Overall accuracy of timing was ± 1 msec.

The subject responded on a Micro Switch 75SW12 keyboard interfaced to the computer. The keyboard, positioned on a table in front of the subject, contained a row of lights above the keys to provide information to the subject.

Procedure

Each trial of the silent counting task consisted of two segments: (a) a *synchronization segment*, followed by (b) a *production segment*. In the synchronization segment, the subject counted silently from 1 to 10 in synchrony with a sequence of 10 periodic auditory clicks which began at the end of a 1-sec warning light flash. This segment served both to inform the subject of the rate at which he was to count in the production segment and to synchronize the temporal phase of his counting with the beginning of the production segment. After completing the tenth count, the subject was instructed to continue counting, starting again at 1, at the same rate established in the synchronization segment and in phase with his previous counts. The subjects counted up to either 5 or 10, depending upon the condition specified for that block of trials, and pressed the space bar on the keyboard in synchrony with his terminal count. The response produced a click in the subject's earphones. The next trial followed after a 5-sec intertrial interval. The subject heard no synchronization clicks within the production segment. The datum recorded for each trial was the multicount duration, T_n , between the start of the production segment (the last click of the synchronization sequence) and the subject's keypress response. If the subject lost track of his count or felt that his timing was in error, he could abort the trial by pressing an error key. This recycled the trial, beginning again with the intertrial interval.

Five values of the intercount duration, c , were used: 300, 450, 600, 750, and 900 msec. For each intercount duration, two values of the terminal count, 5 and 10, were used, for a total of 10 conditions. Each subject participated in five experimental sessions. All 10 conditions were presented within a session, each for a block of 13 trials. The end of each block of trials was signaled to the subject by pause lights on the keyboard. When the subject was ready to begin the next block of trials, he pressed the space bar. This turned off the pause lights and turned on one of two labeled condition lights which indicated the integer to which the subject was to count throughout the next block of trials. The condition light remained on for reference throughout the block of trials. The order of the 10 conditions within a session was randomized separately for each subject and experimental session.

RESULTS AND MODEL EVALUATION

Data from the first 3 trials of each 13-trial block were excluded from analysis to minimize variability arising from the subject's initial adjustments to the new condition presented in each block.

Intercount Duration and Synchronization Error

For each subject the mean multicount duration was obtained for each condition by averaging together the 10 trials for that condition from each of the five sessions. For a given target count duration, c , the expected multicount duration, given for the

general model in Equation 2, is a function of both μ_c , the mean intercount duration, and μ_s , the mean synchronization error. It is possible to determine estimates of both parameters since two different values of n were used in the experiment, giving two equations in two unknowns:

$$E(T_5) = 5\mu_c + \mu_s$$

and

$$E(T_{10}) = 10\mu_c + \mu_s.$$

Estimating expected values by means and then solving for μ_s we obtain

$$\hat{\mu}_s = 2\bar{T}_5 - \bar{T}_{10}. \quad (10)$$

Since μ_s is assumed to be constant over all values of c , a single estimate of μ_s was obtained by averaging the values estimated from Equation 10 across all values of c . The estimated value of μ_s is given for each subject in Table 1. The magnitude of the estimated synchronization error varies considerably across subjects, suggesting fairly large individual differences in synchronizing counts with the preliminary train of clicks and/or in synchronizing the motor response with the final count.

Having estimated μ_s , an estimate of μ_c is obtained for each value of c by solving Equation 2 for μ_c :

$$\hat{\mu}_c = (\bar{T}_n - \hat{\mu}_s)/n. \quad (11)$$

The deviation of the estimated intercount duration from the target count duration, $\hat{\mu}_c - c$, is shown for each subject in Figure 1 for each value of c . It should be noted that the deviations are all quite small, never exceeding 5.3% of the target count duration for any subject. The pattern of error as a function of target count duration appears to be highly idiosyncratic. It seems clear that the pattern of error for a given subject is not due simply to sampling error since the patterns for both the 5-count and 10-count conditions are highly similar for each subject. The factors which determine the pattern of error for a given subject are not apparent from this study.

Variance of the Multicount Duration

For each subject, the variance of the multicount

Table 1
Estimated Mean Synchronization Error (Milliseconds)

Subject	$\hat{\mu}_s$
R.C.	26
D.G.	126
K.H.	62
S.R.	165

Figure 1. Deviation of the estimated intercount duration, $\hat{\mu}_c$, from the target count duration, c , as a function of the target count duration for each subject. Deviations for the 5- and 10-count conditions are plotted separately by the dashed and solid curves, respectively.

duration for a given condition was calculated separately for each of the five sessions, and then the median of these five sample variances was determined. The median variance is shown for each subject as a function of target count duration in the left-hand panels of Figure 2. The reason for calculating variance separately for each session, rather than by pooling all scores, was to minimize the inflationary effect on variance that would be produced by small changes in the mean multicount duration occurring across sessions. The variance data provide the basis for testing the differential predictions of the two model classes derived in the introduction.

Proportional variance models. These models pre-

Figure 2. Median variance of the multcount durations plotted for each subject as a function of the target count duration, c , in the left column, and as a function of c^2 in the right column. Observed variance functions for the 5- and 10-count conditions are plotted separately by the dashed and solid curves, respectively. Variance functions predicted by the proportional variance model class are shown in the left column; those predicted by the Weber's law model class are shown in the right column.

dict that the variance of T_n grows linearly with the value of the intercount duration, c (Equation 5). For each subject, estimates of the two parameters, k_p and σ_s^2 , were obtained by fitting a least-squares straight line to the variance data for each of the two count conditions, subject to the constraint that σ_s^2 must be greater than or equal to zero. The estimated parameter values and the proportion of

variance accounted for by the best fitting line are given for each subject in Table 2. This model class accounts for only a moderate proportion of the variability in the data. A comparison of the predicted straight lines with the data, both shown in the left-hand panels of Figure 2, suggests that the variance of T_n increases with c considerably faster than the predicted linear rate.

Weber's law models. It was shown in Equation 8 that the Weber's law model predicts that the variance of T_n should increase linearly with the square of c . Thus, the two parameters, k_w and σ_s^2 , may be estimated by fitting a straight line to the variance data as a function of c^2 , again subject to the constraint that σ_s^2 must be greater than or equal to zero. The estimated parameter values for each subject and count condition are given in Table 2. In every case, the proportion of variance accounted for by the Weber's law model class is considerably greater than that accounted for by the proportional variance model class. The observed and predicted variance of the multi-count durations are plotted in the right-hand panels of Figure 2 as a function of c^2 . The straight lines appear to provide a reasonably good description of the data, in contrast to the systematic discrepancies between the variance data and the lines predicted by the proportional variance models.

Variance of T_n as a Function of Total Duration

It was shown in the introduction that, for a given value of c , both the proportional variance and Weber's law model classes predict the variance of T_n to be an increasing linear function of the total duration, t ($= nc$). However, the Weber's law models predict that the slope of the line is $k_w^2 c$, a linear increasing function of c (Equation 9), while the proportional variance models predict that the slope is a constant, k_p , independent of c (Equation 6).

For each of the five values of c , a least-squares straight line with intercept (σ_s^2) greater than or equal to zero was fit to the pair of data points corresponding

Table 2
Estimates of Model Parameters and Proportion of Variance Accounted for by Each Model Class

Subject	Count Condition	Proportion of Variance Models			Weber's Law Models		
		k_p	σ_s^2	Proportion of Variance	k_w	σ_s^2	Proportion of Variance
R.C.	5	1.35	0.	.72	.043	0.	.97
	10	1.83	0.	.66	.051	0.	.91
D.G.	5	2.53	0.	.65	.060	0.	.92
	10	3.11	0.	.51	.067	0.	.77
K.H.	5	1.87	0.	.71	.051	0.	.96
	10	1.88	0.	.68	.051	0.	.94
S.R.	5	7.00	0.	.69	.099	0.	.94
	10	10.54	0.	.65	.121	0.	.85

to the two values of n , the data being plotted as a function of total duration, t . The set of five slopes estimated in this way are plotted in Figure 3 as a function of c . The functions predicted by the two model classes are also plotted in Figure 3, using the estimates of the parameters k_w and k_p obtained in the earlier analysis (Table 2). It is quite clear that the slope of the obtained function increases with c , contrary to the constancy predicted by the proportional variance models. Moreover, the agreement between the Weber's law predictions and the data is reasonably good.

DISCUSSION

While the general agreement between the data and the Weber's law model class is quite good, there is one curious discrepancy. Estimates of the mean synchronization error are positive for all subjects, indicating the probable existence of this component, yet the estimates of the variance of the error are zero for all subjects (Table 2). In fact, if straight lines are fit to the variance data with no constraint on the intercept, the estimated intercepts have small negative values for all subjects. The simplest explanation of this result is based on the hypothesis, mentioned in the introduction, that the successive intercount durations produced by a subject are not independent. In a task similar to that used in the present experiment, Wing and Kristofferson (1973) found a small, but significant, negative correlation between adjacent intertap durations. While their task involved repeated tapping on a Morse key rather than silent counting, it seems quite possible that successive counting responses would also be negatively correlated. Based on their own data and that of Michon (Note 2), Wing and Kristofferson suggest that the magnitude of the intertap correlation is probably maximal for very short intertap durations, decreasing towards zero with increasing duration.

If there is mutual dependence between counts, then the expression for the variance of T_n should be

$$\text{Var}(T_n) = n\sigma_c^2 + 2 \sum_{i=1}^{n-1} \sum_{j=1}^n \text{Cov}(C_i \cdot C_j) + \sigma_s^2. \quad (12)$$

Assuming that all of the covariances are either negative or zero, the middle term of Equation 12 will be negative for small values of c , decreasing towards zero for increasing c . Consequently, we would expect the observed variance of T_n to be less than that predicted by the Weber's law model as given by Equation 8, which assumes independence. Moreover, we would expect the deviation between

Figure 3. Slope of the linear function relating variance of the multcount duration to total duration, $t (= nc)$, as a function of the target count duration, c . For each subject, the horizontal line labeled PV is the constant slope predicted by the proportional variance models; the line labeled WL is the increasing slope predicted by the Weber's law models.

predicted and observed values to decrease with increasing intercount duration, predicting a concave upward function for the observed values. A comparison of the predicted and observed functions in the right-hand panels of Figure 2 appears consistent with this prediction. According to this argument, the zero (actually negative) values obtained in estimating the intercept of the Weber's law model are the result of the negative covariance whose magnitude decreases with increasing c . Unfortunately, it is not possible to obtain a more accurate estimate of σ_s^2 without more specific information regarding the covariance function.

CONCLUSIONS

Both tests of the two model classes presented in the analysis section favor the predictions made by the Weber's law models over those made by the proportional variance models. First, the observed growth of intercount variance as a function of count duration is considerably faster than the linear increase predicted by the proportional variance models, but in good agreement with the increase as the square of count duration predicted by the Weber's law models. Second, the observed multicount variance was found to depend upon the number of subintervals into which it is divided, contrary to the prediction made by the proportional variance models. On the other hand, the data are in good agreement with the prediction of the Weber's law models that the variance increases linearly with the duration of the subintervals, *c.* Thus, the Weber's law class of models predict, and the data from this experiment are consistent with, the initial speculation that it is to a subject's advantage when producing a long

duration to subdivide it into a number of shorter durations.

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