Pattern discrimination of Gaussian distributions of orientation and line length parameters

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The two pattern parameters, line length (L) and orientation (θ), have been salient in microelectrode studies and adaptation experiments with the vertebrate visual system, but have not been systematically examined in a pattern discrimination paradigm. It was argued that, because of their physiological importance, the detection of pattern differences should be predictable with known distributions of L and θ , using naive Ss. It was also argued that the discriminability of the concatenation of L and θ (L θ) could be predicted from the individual L and θ performance results. Results indicated support for both hypotheses and also implied the L discriminations were more difficult than θ , using signal detection theory.

Quantitative approaches to the modeling and prediction of visual pattern discrimination are centered around two basic problems. The initial difficulty is that of procuring a basic representation for a pattern which embraces the known physiology of the vertebrate visual system. The second problem is that of characterizing such basic representations in terms of a metric, n-dimensional vector, or similar measures. For example, Zusne (1965) characterizes a pattern by means of moments about the X and Y axes. Such techniques have proven effective in predicting discrimination performance. However, it is of primary importance that the representation problem be investigated independently of this characterization problem, since inappropriate characterizations can distort the predictive value of a given representation (Caelli & Keats, 1973).

In order to isolate both these problems, and deal with complex shapes, known distributions of parameters have to be employed such that a unique characterization can be obtained for the pattern representation. Consequently, the use of random polygons, etc., is not appropriate.

This experiment was concerned with investigating the representation of linear patterns in terms of line length and orientation parameters for the following reasons.

Considering the results of Hubel and Wiesel's (1962, 1968) research on vertebrates' visual cortex, recent researchers have been investigating the detectability of simple line segments in various orientations. Gilinsky and Cohen (1972) and Woll, Eriksen, and Hake (1970) have both demonstrated how the comparative orientations of mask and test

stimulus elements affect the detectability of the test stimulus, using striated line patterns. Similarly, Campbell and Kulikowski (1966), also using a masking paradigm, found that by increasing the angle between the mask and test gratings, the masking effect fell exponentially. These findings, together with those of Leibowitz and Harvey (1973), indicate that for shapes consisting of line segments orientation (θ) is a relevant representation parameter.

Although line-length distributions are a corequisite for studying the effect of orientation sensitivities, they have not been examined as the orientation parameter has been. However, it has been noted as an important pattern parameter, both in striated line segments (Breitmeyer, 1973) and random polygons (Zusne, 1970, p. 208). Since any linear shape (spatial union of line segments) is, to a great extent, determined by line length (L) and orientation (θ) parameters, it would seem important to establish their relative detectabilities in isolation, and in combination (L θ), in order to construct a pattern representation consistent with the types of parameters the visual system detects.

A more elementary type of complex pattern which may vary only in orientations (θ) and line lengths (L) is the class of radial star patterns (Fig. 1). Figure 1a demonstrates how θ may vary alone, and Fig. 1b how L may vary alone. Figure 1c illustrates variations in both L and θ (L θ).

By using Gaussian distributions of L and θ , the basic pattern functions (frequency functions of L and θ) can be uniquely characterized by means of (μ_x , σ_x) (Hogg & Craig, 1965, p. 96). This property of Gaussian functions avoids the characterization problem mentioned above.

Since L and θ are independently varied in radial star patterns, they may be classified as independent random variables, and it can be shown that:

$$\sigma^2_{L\theta} = \sigma^2_L + \sigma^2_{\theta}.$$

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Fig. 1. (a) An example of a Gaussian sample of Θ , for $\sigma_{\Theta} = 1.16$. (b) A Gaussian sample of L , for $\sigma_L = 1.16$. (c) The $L\Theta$ case for $\sigma_{L\Theta} = 1.64$.

If the patterns are equated on means (μ_X), then $[\sigma_L, \sigma_{\Theta}, \sigma_{L\Theta} | \sigma_{L\Theta} = (\sigma_L^2 + \sigma_{\Theta}^2)^{1/2}]$ uniquely characterizes the three classes of patterns illustrated in Fig. 1.

Weintraub and Hake (1962) have developed a model for the application of signal detection theory (SDT) to the visual discrimination of geometric squares and rectangles. They have experimentally demonstrated, by the use of the Gaussian assumption (Green & Swets, 1966, p. 58), that the model has favorable application to visual perception. Following Weintraub and Hake, SDT may also be applied to visual pattern discrimination where the sensitivity function (d') can be directly related to pattern characterizations.

With Gaussian patterns equated on means, d' would measure the scale of discrimination between patterns differing in σ_X , while β would represent the response criterion adopted by each S for such discriminations.

In this experiment, the detection paradigm was constructed as follows. A "noise" stimulus consisted of two identical patterns of given σ_X (σ_{XN}), while a "signal" stimulus consisted of two different patterns, one identical to those used in the noise stimulus, the other having a different σ_X (σ_{XS}).

Consequently, for different values of σ_{XS} and σ_{XN} , false-alarm and hit probabilities could be calculated over S s from "different" responses on the noise and signal stimuli, respectively. Similarly, for each pattern type (L , Θ , or $L\Theta$), a hit and false-alarm rate could be calculated for each S over the various signal and noise stimuli presentations. From these hit and false-alarm

responses, d' and β could be calculated in line with the method used by Weintraub and Hake (1962).

By calculating both individual d' and β measures for each pattern type, and group d' and β measures for each signal and noise intensity, it was possible to both analyze for significant differences between discriminations of pattern types over S s, as well as investigate the various relationships between pattern types and changes in signal and noise intensities.

Considering the psychophysiological importance of these L and Θ parameters, as well as the experimental paradigm described above, the detection task was that of detecting "signals specified exactly [Green & Swets, 1966, p. 162]." In this case, it has been shown that, for the ideal detection:

$$d' = (2ES | N_0)^{1/2}, \quad (1)$$

where ES = signal intensity (σ_{XS}) and N_0 = noise intensity (σ_{XN}).

Using log transforms of both sides of Eq. 1, this becomes in general:

$$\log(d' + k) = a[\log(\sigma_{XS}) - \log(\sigma_{XN})] + \beta. \quad (2)$$

That is, the log of the detectability function (d') is linearly related to the difference in the log values of the signal and noise intensities.

With this characterization of patterns, it was expected that discrimination performance (d') would increase with differences in σ_{XS} and σ_{XN} for L , Θ , and $L\Theta$ patterns in accordance with Eq. 2. Yet no a priori predictions could be related to the case of detecting

Table 1
Standard Deviations for the Six Levels of Pattern "Intensities"

Variable	N ₁	N ₂	N ₃	S ₁	S ₂	S ₃	Description
Orientation (θ)	1.41	1.27	1.16	0.97	0.89	0.82	σ_{θ}
Line Length (L)	1.41	1.27	1.16	0.97	0.89	0.82	σ_L
Both θ , L ($L\theta$)	1.99	1.80	1.64	1.37	1.26	1.16	$\sigma_{L\theta}$

differences between L and θ distributions, since these two parameters have not been compared in this way before.

The integration model (Green & Swets, 1966, p. 238) contends that the detections of two independent signals, under the Gaussian assumption, combine to form an additive detection function of the form:

$$d'_{1,2} = (d'^2_1 + d'^2_2)^{1/2}. \quad (3)$$

Performance results for L and θ pattern parameters in the form of Eq. 2 support the Gaussian assumption (Green & Swets, 1966, p. 58), and L, θ are independent variables. It was thus hypothesized that L and θ discrimination functions (d'_L , d'_θ) would combine according to this model, where, in log form:

$$\log d'_{L\theta} = a \log (d'^2_L + d'^2_\theta) + b \quad (4)$$

The properties of β , response criterion, for the ideal observer, in a pattern discrimination task as defined above, are more simply defined than those of d' . For ideal detectors, it is expected that decisions are made according to likelihood ratios, as, for example, occurs with the Neyman-Pearson objective (Green & Swets, 1966, p. 21-23). That is, β would remain constant while d' varies.

METHOD

Subjects

The Ss were 40 undergraduate students at the University of Newcastle, who volunteered for the experiment. All reported no known visual defects. They were allocated into two groups of 20 Ss.

Stimuli and Apparatus

Six different Gaussian patterns, consisting of 16 lines each, were generated for L, θ , and $L\theta$ pattern types (Fig. 1), giving a total of 18 patterns. Each pattern was characterized by its standard deviation (σ_L , σ_θ , $\sigma_{L\theta}$) as all were equated on means (Table 1).

These intensity levels, for each pattern type, were divided into three noise levels (N₁, N₂, N₃) and three signal levels (S₁, S₂, S₃) (Table 1). Each noise pattern was duplicated and both identical patterns were photographed on a 35-mm slide. Five duplicates were made from each such noise slide. The signal slides were made from pairing each signal pattern (S₁, S₂, S₃) with each noise pattern (N₁, N₂, N₃) and photographed on a 35-mm slide. This process gave a total of 9 signal slides and 15 noise slides for each L, θ , and $L\theta$ pattern type. The slides were left in negative form, with a black background and white figure.

Group I received three presentations of each noise level and Group II received the total five duplicates of the three noise levels. Both groups received the whole nine signal slides for each pattern type. The aim of varying the number of noise slides was to endeavor to induce different criteria in Ss as well as to check the reliability of d' and β .

The slides were presented to each S by means of a Kodak Carousel (S) slide projector onto the back of a small screen. This screen was fixed at the end of a viewing box, such that each S saw each slide at a visual angle of 3 deg. The exposure time for each slide was controlled by means of a Lafayette tachistoscopic shutter, and was 150 msec. A PDP-12 computer controlled the order of slide presentation, exposure time, and interstimulus interval. Responses ("same" or "different") were recorded manually.

Procedure

The various slides were allocated randomly, for σ_{LN} and $\sigma_{\theta S}$ differences and pattern type, over presentations. Ss were told they would receive slides which contained two patterns per slide. These patterns would be either same or different in form features, and they were to respond by saying either "same" or "different." They were also told that each slide would be shown briefly, after which they were to respond quickly within a 10-sec interval. Each S was given four pretest trials.

Table 2
Means and Standard Deviations of Percentage Correct, d' , and β Values for Each Subject Over the Two Groups

	Pattern Type					
	L		θ		$L\theta$	
	Mean	SD	Mean	SD	Mean	SD
Group I						
Percent Correct	76.60	6.90	88.60	6.90	79.40	10.96
d'	0.47	0.58	2.27	0.40	1.63	0.37
β	1.60	0.69	1.50	0.64	1.93	1.01
Group II						
Percent Correct	58.60	7.53	85.00	5.73	76.95	9.06
d'	0.54	0.44	2.14	0.42	1.63	0.57
β	1.39	0.47	1.05	0.29	1.33	0.53

Table 3
Averaged d' Values Over Signal and Noise
Increases for Both Groups

		S_1	S_2	S_3	N_3	N_2	N_1
Group I	L	-0.03	0.33	0.86	-0.06	-0.15	1.38
	θ	2.49	2.56	2.81	2.18	2.54	3.13
	L θ	1.13	1.92	2.11	0.81	1.66	2.69
Group II	L	0.18	0.63	1.20	0.39	0.55	1.07
	θ	2.63	2.81	3.02	2.59	2.57	3.30
	L θ	1.58	1.72	2.25	1.27	1.99	2.30

Table 4
Means and Standard Deviations of d' and β Values for Each
Signal and Noise Intensity Over Subjects for the Two Groups

		Pattern Type					
		L		θ		L θ	
		Mean	SD	Mean	SD	Mean	SD
Group I	d'	0.39	0.85	2.62	0.52	1.72	0.97
	β	1.31*	0.61	0.92*	0.45	1.19*	0.71
Group II	d'	0.67	0.55	2.82	0.43	1.85	0.63
	β	1.23*	0.29	0.94*	0.50	0.96*	0.60

* $p > .05$

RESULTS

Percentage correct scores were calculated for each S over the various pattern types for both groups. Means and standard deviations for the groups are shown in Table 2.

Hit and false-alarm probabilities were calculated in two ways. In order to obtain individual d' and β values per S, hit and false-alarm probabilities were calculated for each pattern type, over signal and noise intensity differences. Means and standard deviations are shown in Table 2. A two-way repeated measures analysis of variance was computed on individual Ss' d' over pattern types for each group. Using Keppel's (1973) notation, this was a Gp x (L θ x Ss) design [2 x (3 x 20)].

The results indicated an insignificant group difference, but a significant pattern type (L θ) difference [$F(2,57) = 181.75$, $p < .01$]. Scheffé post hoc analyses (Winer, 1962) were computed between L, θ , and L θ levels of the L θ factor. Results indicated that L differed significantly from the L θ and θ data ($p < .05$).

Hit and false-alarm probabilities were then calculated over Ss for each signal and noise intensity combination and pattern type. From these values, d' and β scores were tabulated, and averaged scores are shown in Tables 3 and 4. An analysis of variance was not computed on this data as it would have been a single-entry-per-cell design with repeated measures (Winer, 1962, p. 354) requiring quasi F ratios, and would not have contributed extra information to what had been gained from the above analysis. These group d' scores, and signal and noise intensities, were

transformed to log values in $\log_e (d' + 1)$ and $\log_e (\alpha_X)$ forms. The $\log_e (d' + 1)$ scores were then correlated with differences in log signal and noise intensities for each pattern type, over the various signal and noise values (Table 5), for each group.

Standard errors of the correlation coefficients (σ_{yx}) and coefficients of determination were calculated (Table 5; Hays, 1963). Considering the proportional reduction of variance on the dependent variable, given the linear regression rule (r^2 ; Hays, 1963, p. 502), and the significant correlations, these results indicated support for a linear regression between $\log_e (d' + 1)$ and differences between logs of signal and noise intensities.

Log d' values for L and θ were then formed as $\frac{1}{2}\log_e (d'^2_L + d'^2_\theta)$ and correlated with $\log d'_{L\theta}$ values over signal and noise intensities, for each group. For Group I, $r = 0.67$ ($p < .025$), and for Group II, $r = 0.65$ ($p < .05$).

It has been shown by Blair, O'Connor, and Pollatsek (1970, p. 118) that the log of the likelihood ratio is normally distributed. Since β is a specific variate of this ratio, log β values were calculated for each observed β , over the various signal and noise intensities and pattern types, for both groups. By means of this log transformation of β differences between β values could be tested by Student t tests. Results indicated that none of the observed β values differed significantly over increases in signal and noise intensities (Table 4).

DISCUSSION

The analysis of variance indicated that in terms of detectability (d') of differences, the L-type patterns were more difficult to discriminate than the L θ - and θ -type shapes. This conclusion is also supported in terms of the percentage correct scores (Table 2) over Ss for both groups.

This ordering of pattern type difficulty was preserved when d' and β values were calculated over Ss, and not over various presentations of pattern types (Table 4). It may well be argued that this difference was purely a property of the sensitivity to the various parameters, since β was not found to differ although

Table 5
Correlations Between $\log_e(\sigma_{x_N}) - \log_e(\sigma_{x_S})$ and $\log_e(d' + 1)$
[$x = L, \theta, L\theta$ Pattern Types] Over Groups, Standard Errors of
Correlations (σ_{yx}), and Coefficients of Determination (r^2)

		L	θ	L θ
Group I	r	0.89**	0.80**	0.94**
	σ_{yx}	0.05	0.07	0.04
	r^2	0.79	0.64	0.88
Group II	r	0.89**	0.85**	0.77*
	σ_{yx}	0.03	0.06	0.07
	r^2	0.79	0.72	0.60

* $p < .05$

** $p < .01$

d' increased over signal and noise intensity differences (Table 4).

The insignificant group differences on both d' and β measures indicate that the percentages of signal and noise intensities did not affect performance and criteria adopted by Ss for detecting differences. This result may well be expected if the parameters used are ideal characterizations of the stimuli (and related to the physiology of the receiver), since detection of differences on such stimuli would be virtually "induced" on the Ss, and consequently the effect of changes in stimulus situation would be minimal. However, different techniques of inducing different criteria would have to be considered before this conclusion could be affirmed in any absolute sense.

The significant correlations between $\log_e (d' + 1)$ and signal and noise deviation differences (Table 5) have not been recorded in past research (Zusne, 1970, Chap. 5). This contrast is indicative of the necessity for isolating the two problems of pattern representation and characterization mentioned previously. By the choice of Gaussian distributions, this second problem has been successfully avoided.

A formal language, or mathematical model, for the representation of visual patterns should contain facilities and support for how the various parameters combine. The significant correlations, for both groups, between log transforms of $d'_{L\theta}$ and $(d'^2_L + d'^2_{\theta})^{1/2}$, indicate support for the integration model, or an additive concatenation of these parameters.

It would seem, then, that a major reason why orientation and line length parameters have not been successful in predicting discrimination performance with random polygons (Zusne, 1970, p. 208), may lie in the basically uncontrolled number of parameters used in such shapes. Indeed, it would seem that, by using the isolation technique described in this experiment, knowledge concerning the discriminability of such parameters can be improved, and basic interactions between them can be predicted and observed.

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