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Stochastic catastrophe analysis of switches in the perception of apparent motion

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Dynamical phenomena such as bistability and hysteresis have been found in a number of studies on perception of apparent motion. We show that new developments in stochastic catastrophe theory make it possible to test models of these phenomena empirically. Catastrophe theory explains discontinuous changes in responses caused by continuous changes in experimental parameters. We propose catastrophe models for two experimental paradigms on perception of apparent motion and present experiments that support these models. We test these models by using an algorithm for fitting stochastic catastrophe models. We derive from catastrophe theory the prediction that a dynamical phenomenon called *divergence* is necessary when hysteresis is found. This new prediction is supported by the data.

An important objective when the dynamics of visual perception are studied is to discover the nonlinear basis for cooperative phenomena such as bistability and hysteresis, phenomena that are not readily explained by a linear model. Bistability means that more than one percept is possible for a single stimulus. Hysteresis means that what is perceived at any moment depends on the preceding state of the visual system. As a result, perception depends on the direction of parameter change. When a parameter is gradually increased, changes in percept will occur at a different point from changes in percept that occur when the parameter is gradually decreased. That is, the initial percept is likely to persist, although the parameter has changed to a value that favors the alternative percept.

It is clear that a linear model ($y = ax + b$) cannot explain bistability and hysteresis. According to a linear model, one stimulus should provoke one and only one percept (i.e., in the formula of the linear model, every value of x gives a single value of y , not two or more different values). A nonlinear model is needed to deal with these dynamical phenomena. A logistic model is not sufficient; it can explain rapid changes in responses, but not the occurrence of two different percepts for the same stimulus. We will propose a

subset of dynamical system models, so-called *stochastic catastrophe* models, to explain phenomena such as bistability and hysteresis. A new prediction, that hysteresis must be accompanied by another dynamical phenomenon called *divergence*, is derived. The derivation of this new prediction is one of the main advantages of using catastrophe theory to explain nonlinear phenomena. Other theories do not have this mathematical link between hysteresis and divergence. We will show on a conceptual level why hysteresis must go together with divergence, and we will present the first empirical evidence of the existence of divergence in perception. We also show that stochastic catastrophe theory offers indices for model selection, which is seen as a major problem in psychology, and we show that we can test the fit of different catastrophe models efficiently.

We use two existing paradigms based on apparent motion (Hock, Kelso, & Schöner, 1993; Hock, Kogan, & Espinoza, 1997) that clearly show that hysteresis and bistability do exist in visual perception. Apparent motion is an important subject in the study of the nonlinear dynamics of perception because, contrary to static stimuli, it has an extended time scale and parameters, such as speed and direction, that are relatively easy to manipulate. A lot of studies on dynamics have been done with static stimuli like Fisher's man's-face/woman's-figure-picture (Fisher, 1967). This drawing can be interpreted either as a man's face or as a woman's figure (i.e., the percept is bistable). These studies show that the dynamics of visual perception can be studied with static stimuli. However, by changing qualitative parts of pictures, it is not exactly clear which parameter is changed. The changing parameter cannot be measured quantitatively. It is not possible to single out and system-

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Figure 1. Two examples of motion patterns for the motion quartet paradigm. Example A shows a quartet with a relatively small aspect ratio, with which the percept of vertical motion is favored. Example B shows a quartet with a relatively large aspect ratio, with which the percept of horizontal motion is favored.

atically manipulate individual parameters. These possibilities can be explored by using an apparent-motion paradigm.

Several studies of cooperative phenomena in the perception of apparent motion have been done with random-dot cinematograms (Chang & Julesz, 1984; Nawrot & Sekuler, 1990; Williams, Phillips, & Sekuler, 1986). Although hysteresis has been found, random-dot cinematograms are complicated stimuli, which are hard to interpret. Hock et al. (1993) studied hysteresis in a motion quartet paradigm. The stimulus in this paradigm was very simple. Two point lights were presented at the same time, first one pair from two of the diagonally opposite corners of the rectangle, and then a second pair from the other two diagonally opposite corners of the rectangle. Either vertical or horizontal apparent motion was seen, and by gradually increasing or decreasing the aspect ratio (i.e., the ratio of height to width of the quartet), switches between the two percepts took place. Hysteresis was found, since changes happened at different points for ascending and descending aspect ratios.

Hock et al. (1997) found hysteresis in a luminance contrast paradigm. When two point lights were presented simultaneously, each with different, greater-than-background luminance, and their values were exchanged on successive frames, the brighter of the two point lights could be perceived as moving through the space between them. However, when the luminance contrast was too low, stationarity, rather than motion, was seen. By gradually increasing or decreasing the luminance contrast, a switch took place from perceiving stationarity to perceiving motion and vice versa. Again, hysteresis effects were found.

We propose stochastic catastrophe models for the motion quartet paradigm and the luminance contrast paradigm. First, we describe the motion quartet paradigm and the luminance contrast paradigm in more detail, and we explain the advantages of these two paradigms for studying dynamical aspects of apparent motion. Second, we give a short introduction to catastrophe theory (for more extended overviews, see Arnold, Afrajmovich, Il'yashenko, & Shil'nikov, 1999; Castrigiano & Hayes, 1993; Gilmore, 1981; Poston & Stewart, 1978; van der Maas & Molenaar,

1992), we explain how *stochastic* catastrophe theory is different from *deterministic* catastrophe theory, and we derive the prediction that hysteresis goes together with divergence. Third, we review three studies on catastrophe theory and visual perception (Stewart & Peregoy, 1983; Ta'eed, Ta'eed, & Wright, 1988; Yelen, 1980), and we explain the limitations of these studies. Then we describe the two catastrophe models proposed for the motion quartet paradigm and the luminance contrast paradigm. We present experiments that have been done to find empirical evidence for the models and for the existence of divergence, and finally, we analyze the results of the experiments by using an algorithm based on stochastic catastrophe theory.

The Motion Quartet Paradigm and the Luminance Contrast Paradigm

Both the motion quartet paradigm (Hock et al., 1993) and the luminance contrast paradigm (Hock et al., 1997) are based on the finding that apparent motion is seen when two or more neighboring images are presented in quick succession (Exner, 1888, as cited in Smith & Snowden, 1994). For the motion quartet paradigm, lights are placed in the corners of an imaginary rectangle. One first presents one pair of diagonally opposite lights and then, immediately, the other pair of diagonally opposite lights (see Figure 1). Apparent motion can be seen in either the vertical or the horizontal direction (Hoeth, 1968; Ramachandran & Anstis, 1985; von Schiller, 1933), depending on the aspect ratio (i.e., the ratio of height to width) of the quartet. When the aspect ratio is relatively small, the perception of vertical motion is favored. When the aspect ratio is relatively large, the perception of horizontal motion is favored. At intermediate aspect ratios, either horizontal or vertical motion is seen, but never at the same time—that is, perception is bistable.

Hock et al. (1993) gradually increased and decreased the aspect ratio within a sequence of presentations of the motion quartet and found that switches occur from the percept of vertical motion to the percept of horizontal motion and vice versa. However, the switches did not occur near the aspect ratio of 1.0, but the initial percept persisted although the aspect ratio favored the alternative percept. Thus, the moment of change in percept depended on the direction of stimulus change, indicating hysteresis.

In the luminance contrast paradigm, two lights are presented simultaneously, each with different, greater-than-background luminance. When their luminance values are exchanged on successive frames, the brighter of the two lights can be seen as moving through the space between them (Johansson, 1950). Hock et al. (1997) found that whether or not motion was perceived depended on the background-relative luminance contrast (BRLC). This is the luminance difference between the two lights ($L_1 - L_2$) divided by the difference between the average luminance (L_m) and the background luminance (L_b). When the BRLC is low, stationarity, rather than motion, is perceived. It was also found that for middle-range values of BRLC, the percepts of stationarity and motion are bistable. Thus, sub-

jects did not report perceiving a gradual change between stationarity and motion but, rather, an abrupt change. Also, they never reported uncertainty about their percept, although the BRLC had middle-range values. They reported seeing either motion or stationarity, and not something in between. Hock et al. (1997) also found hysteresis effects when the BRLC was gradually increased or decreased within a trial.

These two paradigms have several advantages over the other studies on cooperative phenomena in vision that were mentioned. The main advantage is the introduction of the modified method of limits (Hock et al., 1993). In the classical method of limits, subjects respond after each change in parameter value. With this classical method, however, it is often difficult to distinguish between perceptual hysteresis and hysteresis that is due to a delay in the subject's response. Hysteresis in a subject's response refers to a possible persistence in response even though the actual percept has been altered. When parameter values change too quickly, subjects might perceive a change in percept, but they systematically respond too late to this change. Williams et al. (1986) tried to reduce this problem by changing the parameter values very slowly, in order to give subjects more time to react. However, the rate of parameter change influences not only hysteresis in a subject's response, but also the amount of perceptual hysteresis. Thus, slowing down the rate of parameter change is not a satisfying solution to this problem.

By using the modified method of limits, this problem is minimized. Subjects do not respond during a trial, but only after the entire trial. By varying the length of trials, one can conclude at which parameter value subjects perceive a switch. For example, in the motion quartet paradigm, trials with an increasing aspect ratio always start with a vertical separation of 2 pixels and end with a vertical separation of 3, 4, 5, 6, 7, or 8 pixels (the horizontal separation is fixed at 4 pixels, resulting in endpoint aspect ratios of 0.75, 1.0, 1.25, 1.5, 1.75, or 2.0). Trials with a decreasing aspect ratio always start with a vertical separation of 8 pixels and end with a vertical separation of 7, 6, 5, 4, 3, or 2 pixels (resulting in endpoint aspect ratios of 1.75, 1.5, 1.25, 1.0, 0.75, or 0.5). Trials vary in duration with regard to the endpoint aspect ratio. The shortest ascending trial has the following series of vertical separations: 7 6. The longest ascending trial has the following series of vertical separations: 7 6 5 4 3 2. Subjects respond after each trial, without speed stress, as to whether they saw a switch. For one block, subjects indicate whether they saw vertical motion or not. In the second block, subjects indicate whether they saw horizontal motion or not. Because subjects respond only after the entire trial (and not after each parameter change), the change in percept and the response do not interfere. The hysteresis effects found are perceptual, and not the result of hysteresis of the subject's response. The modified method of limits has also been used in the luminance contrast paradigm.

Another advantage of these two paradigms is the clear distinction between the percepts that are favored by the

parameter and the percepts that are actually seen. In the luminance contrast paradigm, when trials with a constant BRLC are presented, stationarity is seen for low BRLCs, motion is seen for high BRLCs, and both percepts are seen for middle-range BRLCs, but never at the same time. However, when BRLC is increased or decreased within a trial, the initial percept persists, and stationarity can be seen even for high BRLCs, and motion can be seen for low BRLCs. BRLC has a nonspecific influence on the percept. In the motion quartet paradigm, aspect ratio also has a nonspecific influence on the percept. For example, Williams et al. (1986) and Nawrot and Sekuler (1990) did not show whether their hysteresis effect in random-dot cinematograms was due to a directional component in their display (a specific influence) or not. If there was a directional component, the hysteresis effects they found might be artifacts.

A third advantage is that subjects are never uncertain about what they see. In the motion quartet paradigm, they see either horizontal motion or vertical motion, even in the bistable region. They never have a decisional problem. In the luminance contrast paradigm, subjects perceive either nonmotion or motion; they never report any uncertainty. In random-dot cinematograms, the direction of motion can be perceived, but the percepts remain rather fuzzy.

Catastrophe Theory

Well-known statistical techniques, such as regression analysis and multivariate analysis, have been developed to analyze smooth, *continuous* change. Catastrophe theory (Thom, 1975) is a mathematical theory of nonlinear, *discontinuous* phenomena. It describes *discontinuities* in dependent variables as a function of *continuous* changes in independent variables. An example of such a situation is a bridge that can handle a certain increase of stress, but when the stress increases too much, it suddenly collapses. Another example is the change from water into ice. This is a sudden qualitative change caused by a gradual decrease of temperature. In the Hock et al. (1993; Hock et al., 1997) paradigms, the changes in percept are sudden and qualitative, owing to continuous changes of a parameter. Thus, it is important to note that catastrophe theory does not deal with discontinuous changes in dependent variables caused by discontinuous changes in independent variables.

In catastrophe theory, dependent variables are called *behavioral variables*, and independent variables are called *control variables*. In general, control variables do not prescribe the behavior but lead the system through the variety of possible patterns or states. Catastrophe theory provides a qualitative mathematical relationship between control variables and behavior. This relationship can be seen as representing a potential system that optimizes some function under given values of the control variables. That is, the behavior will change until it reaches equilibrium (i.e., a stable state). Mathematically, an equilibrium is found by setting the first derivative of the potential equation to zero.

An important aspect of catastrophe theory is structural stability, which refers to the fact that a lot of phenomena have the same pattern (the same qualitative aspects), although

the quantitative aspects (e.g., time, location, and magnitude) may differ. Thom (1975) called the qualitative aspects *recurrent identifiable elements*. For example, the quantitative aspects of two different caterpillars turning into butterflies are different, but the processes, or the qualitative aspects, are identical. This process is a recurrent pattern. In catastrophe theory, structural stability means that various perturbations will not change the type of relation between control variables and behavior.

Thom (1975) proved that for a wide range of structurally stable system functions showing discontinuous behavior (up to two behavioral variables and four control variables), only seven stable unfoldings of patterns, the seven elementary catastrophes, are possible. This means that there are only seven *qualitatively* different discontinuous changes. Of course, there are infinite *quantitative* differences possible, but only seven qualitative differences. Which one of the seven elementary catastrophes will happen depends on the number of behavioral and control variables. That is, before doing any experimental work, by knowing the number of variables, one knows which catastrophe model is appropriate. This is a major advantage to using catastrophe models. Other dynamical models (e.g., Giese, 1999; Kelso, 1995) do not use the concept of structural stability. However, both catastrophe theory and the other dynamical approaches are strongly related to synergetics (Haken, 1983). The different approaches do not have competing theories but can best be seen as complementary. For example, Giese's dynamic neural field theory is an excellent example of showing how to combine dynamics, neural

networks, computational models, and motion perception. Stochastic catastrophe theory, however, provides a statistical test of dynamical models, which will be explained later. Another advantage of catastrophe theory is based on the mathematical derivation that hysteresis must go together with divergence. This leads to a new prediction that when one finds hysteresis, one also has to find divergence. For example, Hock et al. (1993; Hock et al., 1997) found hysteresis in both the motion quartet paradigm and the luminance contrast paradigm. However, they did not study divergence, since they did not use catastrophe modeling as their theoretical framework. We state that studies on hysteresis that do not study divergence are incomplete. We follow Thom, who proved mathematically that the so-called cusp model (one of the seven elementary catastrophes) is the most parsimonious model when hysteresis is studied. The cusp model is described by two control variables and one behavioral variable. When we look closely at the motion quartet paradigm and the luminance contrast paradigm, we see that, in both paradigms, only one control variable has been modeled. In the motion quartet paradigm, this is the aspect ratio, and in the luminance contrast paradigm, this is the background-relative luminance contrast. According to Thom, there should be another control variable. This variable has not been modeled, which is why Hock et al. (1993) and Hock et al. (1997) only found hysteresis, and not divergence as well. What is missing is a so-called *splitting variable*. The two control variables mentioned by Thom can be classified as the normal and the splitting variables. The normal variable is associated with

Figure 2. An illustration of the plastic spoon example. The horizontal push on the middle of the spoon (F_h) is the normal variable. The vertical pressure on the ends of the spoon (F_v) is the splitting variable.

hysteresis and causes relatively small changes in behavior at low and high values, but at middle values it might cause relatively large changes (sudden jumps) in behavior. The splitting variable is associated with divergence and causes a splitting of states when it is increased from low to high. An example is the bending of a plastic spoon, adapted from Poston and Stewart (1978, chap. 13). Imagine holding a disposable plastic spoon upright in your right hand, with the thumb at one end and the index finger at the other end of the spoon (see Figure 2). It is now possible to bend the spoon to the left or to the right with a push of the index finger of the left hand. In Figure 2, this force is indicated as F_h (horizontal force). However, the spoon will not bend when you only push softly. When you push a little bit harder, it also will not bend. However, as the push gets harder and harder, the spoon will suddenly bend to the other side. Thus, pushing the spoon causes a sudden jump, indicating that pushing is the normal variable in this example. But this is not the whole story. When you release the pressure on the ends of the spoon, the jump gets less strong. When this pressure is close to zero, bending from left to right and vice versa takes place continuously. By increasing the pressure on the ends, the spoon bends either to the left or to the right, indicating divergence. Thus, the pressure on the ends of the spoon is the splitting variable, indicated as F_v (vertical force) in Figure 2. We will explain later how we can model the motion quartet paradigm and the luminance con-

trast paradigm so they are consistent with catastrophe theory. First, we will describe the cusp model more deeply.

Mathematically, the cusp potential function is

$$V(z; \alpha, \beta) = \frac{1}{4}z^4 - \frac{1}{2}\beta z^2 - \alpha z,$$

with normal variable α , splitting variable β , and behavioral variable z . Setting the first derivative with respect to z to zero gives

$$z^3 - \beta z - \alpha = 0.$$

A three-dimensional representation of the solution of this equation—for example, the equilibria of the cusp—is given in Figure 3. We first will explain this figure by using the example of the plastic spoon. Later, we explain how the motion quartet paradigm can be modeled, by using the cusp model in Figure 4.

In Figure 3, all the behavior points together form the *cusp surface*. For some combinations of values of control variables, more than one mode of behavior is possible. This is indicated by the folded part of the cusp surface. The middle sheet on the cusp surface represents unstable behavior. Therefore, this sheet is called the *inaccessible region*.

In Figure 3, some important aspects of the cusp model can be observed. For example, when the splitting variable is relatively low, behavior changes continuously with the normal variable. However, when the splitting variable is rel-

Figure 3. The smooth folded surface of the cusp in three dimensions (z represents values of the behavioral variable; α represents the normal variable; β represents the splitting variable), with the important properties of inaccessibility, sudden jump, bimodality, hysteresis, and divergence. It is shown that varying the normal variable causes hysteresis and the sudden jump; varying the splitting variable causes divergence. When hysteresis, the sudden jump, and divergence occur, behavior will be bimodal. No other behavior modes will happen, because other modes are inaccessible.

atively high, a *sudden jump* happens, which refers to a large change in behavior (i.e., a catastrophe) owing to a small change in the normal variable. In the example of the plastic spoon, when the pressure on both ends of the spoon is high enough, a sudden jump in bending from one side to the other can occur by a small increase of a push of the other hand. A second important aspect is *bimodality*, which means that for one value of the control variable, two possible behavior points exist. That is, when the pressure is high, the spoon bends either in the right or in the left direction. It is impossible to stay in the middle (the inaccessible region).

Hysteresis can also be seen in Figure 3. When moving along the normal axis when the splitting variable is high, the jump from the upper sheet to the bottom sheet of the cusp surface occurs at a different value of the normal variable than does the sudden jump from the bottom to the upper sheet. That is, when the pressure on the ends of the spoon is high, the point of switch in bending to the right or to the left depends on the direction of change. The fourth aspect is *divergence*. Moving from the neutral point along the splitting axis into the bimodality zone, the behavior should go one way or the other. Thus, the splitting variable should cause divergence—that is, stable behavior becomes unstable and changes to either one of two possible new kinds of behavior. Which way the behavior will go depends on the value of the normal variable. That is, when no pressure is put on the ends of the spoon, the spoon will be straight. When the pressure continuously increases, either the left or the right side is selected.

The cusp model is the most simple catastrophe model to describe hysteresis. As can be seen in Figure 3, hysteresis goes together with divergence. Divergence is caused by increasing the splitting variable, but the splitting variable is also involved when hysteresis occurs. Hysteresis will occur only when the splitting variable is relatively high. Thus, both the existence of divergence and the existence of hysteresis are influenced by the splitting variable. But divergence and hysteresis are also both influenced by the normal variable. Hysteresis takes place when the normal variable is gradually increased or decreased. Thus, the normal variable is important when analyzing hysteresis, but it is also involved when divergence occurs. Divergence will occur only when the normal variable has a neutral value. Thom (1975) proved mathematically that hysteresis and divergence must go together. Thus, it is important to know that when hysteresis has been found experimentally by manipulating a normal variable, one also must be able to find divergence by manipulating a splitting variable.

Originally, catastrophe theory was deterministic. Thus, criticism arose that catastrophe theory is not applicable to the social sciences, where data contain a lot of noise (Sussmann & Zahler, 1978; Zahler & Sussmann, 1977; Guastello, 1981, and Oliva & Capdevielle, 1980, reply to this criticism). Cobb (1978, 1980; Cobb, Koppstein, & Chen, 1983; Cobb & Watson, 1980; Cobb & Zacks, 1985) tried to solve this problem by developing *stochastic* catastrophe theory. Other approaches to catastrophe modeling (Guastello's,

1982, 1992, polynomial regression technique; Oliva, Desarbo, Day, & Jedidi's, 1987, GEMCAT technique) have met with some criticism (Alexander, Herbert, Deshon, & Hanges, 1992; Hartelman, 1997; Hartelman, van der Maas, & Molenaar, 1998). Cobb showed that, by using stochastic differential equations, there is a cusp family of probability density functions. In his theory, a stable equilibrium corresponds to a mode and an unstable equilibrium corresponds to an antimode of the probability density function. That is, a stable equilibrium state is a point of high probability. A change in the number of equilibrium states corresponds to a change in the number of modes and antimodes of the probability density function. Given the probability density functions of the cusp model, the control variables can be estimated by using maximum likelihood estimation. In this procedure, the most likely cusp surface is estimated, given empirical data. The form of the cusp surface in raw score form in stochastic catastrophe theory is

$$f(z|\alpha, \beta) = C(\alpha, \beta) \exp\left(\alpha y + \frac{1}{2} \beta y^2 - \frac{1}{4} y^4\right),$$

with $y = (z - \lambda)/\sigma$, where λ and σ scale the observed behavioral variable z to y . α and β are linear functions of the independent variables x_1 to x_n , with $\alpha = a_0 + a_1x_1 + a_2x_2 + \dots + a_nx_n$ and $\beta = b_0 + b_1x_1 + b_2x_2 + \dots + b_nx_n$. The parameters λ , σ , $a_0 \dots a_n$, and $b_0 \dots b_n$ have to be estimated. $C(\alpha, \beta)$ is an integration constant.

Cobb (1980) developed a computer program for this catastrophe model fitting technique. However, this program often breaks down for nonapparent reasons. Hartelman (1997) solved some computational and practical problems of Cobb's (1980) cusp fitting algorithm and called his program Cusffit.¹ Cobb's (1980) algorithm calculates whether the cusp model or the linear model gives the best description of its relationship between the independent and the dependent variables. Hartelman added the comparison with a logistic model that makes it possible to distinguish an arbitrarily fast acceleration from a catastrophic change. In Cobb's (1980) algorithm, it is concluded that the cusp model has a better fit when its log likelihood is higher than the log likelihood of the linear model. The cusp model, however, uses more parameters, and the better fit can be ascribed simply to having a model with more parameters. Hartelman added two fit measures—Akaike's information criterion (AIC; a goodness-of-fit index that takes into account the number of parameters [Loehlin, 1992]) and Bayes's information criterion (BIC, which also takes into account the number of data points [Schwarz, 1978])—to solve this problem. When the AIC and the BIC of the cusp model are lower than the AIC and the BIC of the logistic and the linear models, the cusp model has the best fit. Different cusp models can also be compared by means of these fit measures. For example, in a situation with two independent variables and one dependent variable, the parameters λ , σ , a_0 , a_1 , a_2 , b_0 , b_1 , and b_2 are estimated. It is, however, likely that one independent variable serves as the normal variable and the other as the splitting

variable. If the first independent variable is expected to be the normal variable and the second independent variable is expected to be the splitting variable, it is possible to fix a_2 and b_1 to zero. In addition, in this case, it is expected that the first independent variable loads highly on a_1 and the second independent variable on b_2 , so we will not fix those parameters. Those loadings can be compared with factor loadings in a factor analysis. One expects that some variables load highly on one factor and other variables on another factor. Thus, in catastrophe analysis, if one expects that one or more of the independent variables do not contribute to the normal or the splitting variable, it is possible to fix parameters to zero. Only the nonfixed parameters are estimated. With two independent variables, it is possible to construct and to calculate the fit of 16 different cusp models. The fit measures indicate which of the 16 cusp models is most appropriate and which of the independent variables contribute to the normal and the splitting variables of the model. We will show several examples in the Results sections of the experiments. First, we review three earlier applications of catastrophe theory in visual perception.

Three Applications of Catastrophe Theory in Visual Perception

Yelen (1980) presented an application of catastrophe theory in visual perception. In a discrimination task, subjects were to indicate which of two lights was brighter. There were two conditions, based on Weber's law. In the easy condition, the standard light was relatively easy to discriminate from the other light, because the two lights were relatively dim. In the difficult condition, the difference between the two stimuli was more ambiguous, because the lights were relatively bright. Yelen found evidence for a cusp model, with the brightness of standard light (either 26% or 92% of 120V) as the splitting variable and the brightness of the other light as the normal variable. When the splitting variable was low (i.e., in the easy condition), subjects' judgments changed continuously with the normal variable, and the change in judgment occurred at the same point for ascending and descending trials. However, when the splitting variable was high (i.e., in the difficult condition), hysteresis effects were found.

Stewart and Perego (1983) presented a catastrophe model of switches in the percept of ambiguous drawings. They used Fisher's (1967) picture in which both a man's face and a woman's figure can be seen. Fisher created a drawing in which subjects see the man in 50% of the cases and the woman in the other 50% of the cases. Then he made drawings that are more or less biased toward the man or toward the woman. If one puts the pictures in order, it is expected that at some point a sudden jump occurs from perceiving the man's face to perceiving the woman's figure and vice versa. Stewart and Perego proposed a cusp model, with the amount of the drawing's bias as the normal variable and the amount of details presented in the pictures as the splitting variable. By using Cobb's (1980) program based on sto-

chastic catastrophe theory, they found that the cusp model had a good fit and that the proposed normal and splitting variables were appropriate.

Ta'eed et al. (1988) proposed a catastrophe model of switches in the percept of a Necker cube. The Necker cube is a bistable figure, since it is possible to see the cube as either solid or hollow. Ta'eed et al. proposed that the normal variable is the shape of the cube. It can be changed continuously by varying perspective cues. The splitting variable is the amount of shading of the cube. They hypothesized that if the splitting variable is low (i.e., shading is minimized), the perceptual switch from the hollow cube to the solid cube is continuous. However, when shading is maximized and the bias reaches middle values, there is a sudden jump from the hollow cube to the solid cube. They also used Cobb's (1980) algorithm and found that the two manipulated variables influenced both the normal and the splitting variable. Thus, varying the shape of the cube and varying the shading are not very distinctive variables.

All three studies show how to develop a cusp model in visual perception and how to conduct experiments to test the model. However, there are some limitations to these studies. Yelen (1980) used discontinuous rather than continuous control variables. Especially his splitting variable, which was either the standard light of 26% of 120V or the standard light of 92% of 120V, is a discrete variable, not a continuously changing variable. Also, he did not use the modified method of limits to avoid hysteresis in the subject's response. Another disadvantage is that he did not fit the model by using stochastic catastrophe theory, but at that time this technique was not available. So we do not know in which amount the variables influenced the splitting or the normal variable.

Stewart and Perego (1983) presented a much more extended overview of the possibilities of applying catastrophe theory, and they used Cobb's (1980) algorithm to analyze the data (although this algorithm is not robust). However, they used drawings with many details, and the drawings can be changed continuously in many different ways. It is hard to control experimentally which parts of the pictures influence the percept or do not. Stewart and Perego also did not use the modified method of limits. Subjects indicated whether they saw the man's face or the woman's figure after each change in picture.

Ta'eed et al. (1988) also presented a good example of how to formulate a cusp model, but their proposed model did not fit well. In a new study on catastrophe theory and Necker cubes, different variables should be chosen. They also did not use the modified method of limits, and they also used the nonrobust Cobb algorithm.

Catastrophe Models for the Motion Quartet Paradigm and the Luminance Contrast Paradigm

It is possible to overcome the problems mentioned above by applying stochastic catastrophe theory to the motion quartet paradigm (Hock et al., 1993) and the luminance

Figure 4. The proposed catastrophe model of the motion quartet paradigm. Aspect ratio is the normal variable, background-relative luminance contrast (BRLC) is the splitting variable, and the percept of horizontal or vertical motion is the behavioral variable. Bifurcation lines are drawn to show the bimodal region. Path A crosses the bimodal region (BRLC is relatively high). It is predicted that at first only horizontal motion is seen; a sudden jump to perceiving vertical motion happens only when the right bifurcation line is passed. When Path A is followed from the other direction, it is predicted that at first only vertical motion is seen. A sudden jump to horizontal motion happens only when the left bifurcation line is passed. Path B crosses the unimodal region (BRLC is relatively low). It is predicted that no motion will be seen.

contrast paradigm (Hock et al., 1997). Previous experiments with both paradigms have resulted in clear indications of cooperative phenomena. The cusp model of the motion quartet paradigm is shown in Figure 4. The aspect ratio is modeled as the normal variable, since it causes small changes at low and high values but large changes in the middle region. That is, when the aspect ratio is small, vertical motion is seen almost always. When the aspect ratio is large, horizontal motion is seen almost always. However, when the aspect ratio is gradually increased or decreased to values in the middle region, sudden jumps can happen from the percept of horizontal motion to the percept of vertical motion and vice versa. Hock et al. (1993) showed that hysteresis occurred, also indicating bimodality (i.e., at the same aspect ratio, both percepts are possible). However, Hock et al. (1983) did not show that divergence could occur, since they did not manipulate a splitting variable. Following catastrophe theory, the presence of hysteresis is an indication of other phenomena such as divergence. This means that there must also be a splitting variable. This variable still has to be identified empirically.

In Figure 4, bifurcation lines have also been drawn. This is done to make more clear whether behavior is bistable or not. When behavior is between the two bifurcation lines, it is bistable. When it is outside the two bifurcation lines, it is not. When we present the data, we will make use of these bifurcation lines to show which data points are bistable and which are not.

First, we replicated the hysteresis experiment of the motion quartet paradigm. Second, in a new experiment, we tried to find an appropriate splitting variable. As was mentioned above, the splitting variable causes divergence—that is, a single percept splits into one of two possible different percepts. We propose to use background-relative luminance contrast as the splitting variable in the motion quartet paradigm. Hock et al. (1997) showed in the luminance contrast paradigm that when BRLC is low no motion will be seen. We applied this finding to the motion quartet paradigm. When the BRLC of the lights is relatively high, horizontal or vertical motion is seen. When BRLC is relatively low, it is expected that no motion will be seen. We gradually increased the BRLC of the lights of the

quartets, and we found a switch from a single percept (i.e., stationarity) to one of two different percepts (i.e., horizontal or vertical motion).

In the cusp model of the luminance contrast paradigm (Hock et al., 1997), BRLC is the normal variable, causing a sudden jump from perceiving stationarity to perceiving motion at values in the middle region. However, in this study, hysteresis occurred also, and it follows that there must be a splitting variable. First, we replicated the hysteresis experiment of the luminance contrast paradigm. Second, we conducted an experiment in which the distance between the two point lights was manipulated as a potential splitting variable. In a pilot study, we found that motion is easier to detect when the point lights are closer together. So, we predicted that BRLC would influence the seeing of motion or stationarity only when the distance between the point lights was relatively small. When the distance was relatively large, no motion would be seen, independent of the value of the BRLC.

Experiments

We present two main experiments: one on the motion quartet paradigm and one on the luminance contrast paradigm. Both experiments contained two subexperiments: one in which only the normal variable was manipulated, as in the original paradigms, and one in which a potential splitting variable was manipulated also. We report the results of the stochastic catastrophe analysis and show how this analysis indicates which model is the best description of the data and which variables are relevant or are not. In the first part of the first experiment, modified motion quartets were used to study hysteresis effects. In the second part, a divergence experiment was done to test luminance contrast as a splitting variable for the motion quartets. When the luminance contrast is low, only one percept is stable—that is, the percept of stationarity.

In the second experiment, a splitting variable for the luminance contrast paradigm (Hock et al., 1997) was studied. This time, no divergence experiment was done, but within the original paradigm, distance between the point lights was studied as a second control variable.

EXPERIMENT 1

General Method

Motion quartets as in Hock et al. (1993) were used. Stimuli were point lights presented in the corners of an imaginary rectangle. Two

point lights were presented at a time—one pair for 195 msec from the diagonally opposite corners of the rectangle and then the second pair for 195 msec from the other two diagonally opposite corners of the rectangle. In contrast to Hock et al. (1993), no blank frame was shown between the presentations of the point lights. Pilot studies showed that using a blank frame did not affect the results. We decided to implement no blank frame in order to keep the experiments shorter. The refresh rate of the computer was 16 msec. The subjects viewed the point lights from a distance of 30 cm in a darkened room.

Experiment 1A

Method

Aspect ratio was the only control variable. Stimuli were 1-pixel point lights (visual angle, 4.2 min) with a luminance value of 12.8 cd/m² presented on a dark screen (1.5 cd/m²). We created two different blocks of motion quartets. In the first block, the horizontal separation between the point lights was fixed at 4 pixels (as in Hock et al., 1993). The vertical separation was 2, 3, 4, 5, 6, 7, or 8 pixels, resulting in aspect ratios of 0.5, 0.75, 1.0, 1.25, 1.5, 1.75, and 2.0. In the second block, the vertical separation between the point lights was fixed at 4 pixels. The horizontal separation was 2, 3, 4, 5, 6, 7, or 8 pixels, resulting in aspect ratios of 2.0, 1.33, 1.0, 0.8, 0.67, 0.57, and 0.5.

As in Hock et al. (1993), motion quartets were presented in ascending and descending trials, following the modified method of limits. For example, in the blocks with a fixed horizontal separation, all the descending trials started with an aspect ratio of 2.0 and ended with an aspect ratio of 1.75, 1.5, 1.25, 1.0, 0.75, or 0.5. All the ascending trials started with an aspect ratio of 0.5 and ended with an aspect ratio of either 0.75, 1.0, 1.25, 1.5, 1.75, or 2.0.

Six subjects participated in four sessions, each containing one block with a fixed horizontal separation between the point lights and one block with a fixed vertical separation. In both blocks, all conditions (6 endpoint aspect ratios $\times$ 2 directions) were presented eight times, resulting in 96 randomly ordered trials in a block. The order of the blocks was alternated on successive sessions. After each trial, subjects indicated first, by pressing a key on the keyboard of the computer, whether they saw horizontal or vertical motion at the beginning of the trial and, second, whether they saw a switch in motion pattern at any time during the trial.

Results

In all the experiments, we found at least one cusp model that had a better fit than the linear and the logistic models. In the tables in which we present the results of the analyses, we report the parameter values and the fit measures of all possible cusp models and the fit measures of the unconstrained linear and logistic models. In the present catastrophe analysis, one independent variable (i.e., aspect ratio) and one behavioral variable (i.e., the mean proportion of perceived switches of each block for each endpoint aspect ratio) were taken into account. We present the re-

Table 1
Catastrophe Analysis of Experiment 1A (Motion Quartets With a Changing Horizontal Separation)

Model	a_0	a_1	b_0	b_1	λ	σ	Log Likelihood	Parameters	AIC	BIC
Cusp 1	-0.10	0.00	3.32	0.00	0.18	0.59	-230.5	4	469.0	483.7
Cusp 2	-0.14	0.00	3.47	-0.63	0.24	0.58	-210.8	5	431.5	449.8
Cusp 3	-0.15	0.45	3.17	0.00	0.18	0.59	-177.0	5	364.1	382.4
Cusp 4	-0.12	0.42	3.36	-0.52	0.22	0.58	-163.8	6	339.5	361.5
Linear							-464.3	3	934.5	945.5
Logistic							-320.0	2	644.0	651.3

Note— a_0 , constant of the normal variable; a_1 , loading weight of the normal variable; b_0 , constant of the splitting variable; b_1 , Bayes's loading weight of the splitting variable; λ , location; σ , scale; log likelihood, log likelihood value; AIC, Akaike's information criterion; BIC, Bayes's information criterion. Zeros are fixed parameters. The fit measures of the linear and the logistic models are given only for the unconstrained model.

Figure 5. The bottom part of Figure 4 is shown with the bifurcation lines, the axis of the normal variable, and the axis of the splitting variable. The data points of Experiment 1A (the motion quartets with a fixed horizontal separation), represented by AR (aspect ratio), are all between the bifurcation lines. This indicates that all the values of the aspect ratio were bistable. That is, sometimes the subjects never saw a switch during trials, even though the aspect ratio got relatively large or small.

sults for the different blocks separately. For both blocks, 6 subjects completed four sessions, and there were six end-point aspect ratios in both the ascending and the descending trials, resulting in 288 data points for each block.

The results of the block with the motion quartets with a changing horizontal separation can be seen in Table 1. The parameter weights and the fits of four different models are shown.

In the first model, the weights of both the normal variable and the splitting variable were fixed to zero. As was expected, this model had the worst fit. Both the AIC and the BIC were relatively large. In the second model, the normal variable was fixed to zero. This model also did not fit well. The third model, with the splitting variable fixed to zero, was expected to have the best fit. It had a good fit, but the totally unconstrained model (the fourth model in

Figure 6. An example of the large hysteresis effect of Experiment 1A (the motion quartets with a fixed horizontal separation). In 20% of the trials, the subjects kept on perceiving vertical motion during ascending trials, even when the aspect ratio got large. The same happened in descending trials: 20% of the subjects continued to see horizontal motion, even when the aspect ratio got small.

Table 2
Catastrophe Analysis of Experiment 1A (Motion Quartets With a Changing Vertical Separation)

Model	a_0	a_1	b_0	b_1	λ	σ	Log Likelihood	Parameters	AIC	BIC
Cusp 1	0.17	0.00	3.34	0.00	-0.29	0.59	-217.1	4	442.3	456.9
Cusp 2	0.18	0.00	3.47	0.26	-0.32	0.59	-213.5	5	437.0	455.3
Cusp 3	0.48	0.88	3.21	0.00	-0.31	0.59	-141.1	5	292.2	310.5
Cusp 4	0.62	1.12	2.96	-0.41	-0.28	0.60	-136.2	6	284.4	306.4
Linear							-464.9	3	935.9	946.9
Logistic							-348.6	2	701.1	708.4

Note— a_0 , constant of the normal variable; a_1 , loading weight of the normal variable; b_0 , constant of the splitting variable; b_1 , Bayes's loading weight of the splitting variable; λ , location; σ , scale; log likelihood, log likelihood value; AIC, Akaike's information criterion; BIC, Bayes's information criterion. Zeros are fixed parameters. The fit measures of the linear and the logistic models are given only for the unconstrained model.

Table 1) had the best fit, even when the number of parameters was taken into account (which both the AIC and the BIC do). Both the AIC and the BIC had the smallest values in comparison with the other three models. Because the unconstrained model had the best fit, it can be concluded that the aspect ratio loads on both the normal variable and the splitting variable. An illustration of this result can be found in Figure 5. In this figure, a graphical representation of the data is given. The data points are all within the bifurcation set, which means that all values of the aspect ratio were bistable. This is also shown in Figure 6, in which we plotted the hysteresis effects, and it can be seen that the effects were very large.

Because the lower and higher values of the aspect ratio also cause changes in the percept, the variable aspect ratio is not seen solely as a normal variable, which explains why it also contributes to the splitting variable. In Figure 5, this can be seen because the data points do not follow a straight horizontal line (which would be the case when the indepen-

dent variables contribute only to the normal variable), but they do not follow a straight vertical line either (which would happen when they contribute only to the splitting variable).

Similar results are found for the block with motion quartets with a changing vertical separation. This can be seen in Table 2. Again, the model with no constraints has the best fit. The large hysteresis effects can also explain this result. A graphical representation of the data is given in Figure 7. As can be seen, almost all the data are within the bifurcation set. Most subjects switched only at an aspect ratio of 2 from perceiving vertical motion to perceiving horizontal motion.

Experiment 1B

Method

Both aspect ratio and BRLC are variables in the motion quartet paradigm. The main purpose of the subexperiment is to find out whether BRLC is a likely candidate for the splitting variable. As was shown in Hock et al. (1993) and in Subexperiment 1A, aspect ratio is

Figure 7. A graphical representation of the data points of Experiment 1A (the motion quartets with a fixed vertical separation). The data points, represented by AR (aspect ratio), cross the second bifurcation line.

Table 3
Catastrophe Analysis of Experiment 1B

Model	a_0	a_1	a_2	b_0	b_1	b_2	λ	σ	Log Likelihood	Parameters	AIC	BIC
1	0.01	0.00	0.00	2.04	0.00	0.00	-0.01	0.77	-4933	4	9,873	9,899
2	0.01	0.00	0.00	2.04	0.00	0.02	-0.01	0.77	-4932	5	9,875	9,906
3	0.02	0.00	0.00	2.84	1.64	0.00	-0.02	0.61	-3274	5	6,557	6,589
4	0.02	0.00	0.00	2.84	1.64	-0.02	-0.03	0.61	-3273	6	6,559	6,597
5	0.01	0.00	-0.33	1.93	0.00	0.00	-0.01	0.77	-4600	5	9,209	9,241
6	0.01	0.00	-0.33	1.93	0.00	0.02	-0.01	0.77	-4599	6	9,211	9,249
7	0.02	0.00	-0.27	2.77	1.64	0.00	-0.02	0.61	-2938	6	5,887	5,925
8	0.02	0.00	-0.27	2.77	1.64	-0.11	-0.03	0.61	-2938	7	5,889	5,934
9	0.01	0.04	0.00	2.03	0.00	0.00	-0.01	0.77	-4928	5	9,866	9,898
10	0.01	0.04	0.00	2.03	0.00	0.02	-0.01	0.77	-4928	6	9,868	9,906
11	0.00	0.03	0.00	2.84	1.64	0.00	-0.02	0.61	-3270	6	6,553	6,591
12	0.00	0.03	0.00	2.84	1.64	0.02	-0.20	0.61	-3270	7	6,555	6,599
13	0.01	0.04	-0.34	1.93	0.00	0.00	-0.01	0.77	-4594	6	9,201	9,239
14	0.01	0.04	-0.34	1.93	0.00	0.02	-0.01	0.77	-4594	7	9,202	9,247
15	0.00	0.03	-0.27	2.77	1.64	0.00	-0.02	0.61	-2934	7	5,881	5,926
16	0.00	0.03	-0.27	2.77	1.64	-0.01	-0.02	0.61	-2934	8	5,883	5,934
Linear									-6274	4	12,550	12,580
Logistic									-5544	3	11,090	11,110

Note—The first 16 models are cusp models. The last 2 models are the unconstrained linear and logistic models (a_1 and b_1 are parameters for background-related luminance contrast and a_2 and b_2 are parameters for aspect ratio). λ , location; σ , scale; log likelihood, log likelihood value; AIC, Akaike's information criterion; BIC, Bayes's information criterion. Zeros are fixed parameters.

the independent variable causing hysteresis—that is, the normal variable. In the present subexperiment, we tested whether changes in BRLC would cause divergence in the motion quartet paradigm. In order to find divergence, BRLC was increased during each trial, whereas the aspect ratio varied from trial to trial (but was constant during each trial). Each trial started with a static presentation of all four point lights (2×2 pixels, so that they actually looked like small squares; visual angle, 8.5 min) for 900 msec with a luminance value of 7.0 cd/m². The luminance corresponded to the lower luminance value when the BRLC was changed to 0.1 during the next pair of frames. The point lights were presented against a darker, gray background with a luminance of 3.8 cd/m². After the first static frame, all four point lights remained visible simultaneously. One pair of diagonally opposite point lights had a different BRLC from the other pair of diagonally opposite point lights, resulting in 195-msec frames with BRLC values of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, or 0.9. Within each trial, BRLC increased in steps of 0.1, following the modified method of limits (Hock et al., 1993). Thus, the ascending series always started with a BRLC of 0.1 and ended with a BRLC of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, or 0.9. The aspect ratio was 0.75, 1.0, 1.25, or 1.5, with a constant horizontal separation of 16.2 min.

Two subjects participated in eight blocks, 1 in nine blocks, and 1 in four blocks of 144 trials (four presentations of nine trials with an increasing BRLC value for four different aspect ratios). The subjects were instructed to focus on the middle of the quartet. After each trial, they indicated first (by pressing a button on the keyboard) whether they saw motion at any time during the trial and second whether the first motion pattern they saw was either horizontal or vertical (when they indicated seeing no motion, the second response was not included in the analysis).

Results

In the present catastrophe analysis, two independent variables (i.e., aspect ratio and BRLC) and one behavioral variable (i.e., whether stationarity, vertical motion, or horizontal motion was seen) were taken into account. The behavioral variable was 0 when stationarity was perceived, -1 when vertical motion was seen, and 1 when horizontal motion was seen. Since there were 144 trials in each block and 29 blocks were fulfilled, the analysis was done

for 4,176 data points. The results of model parameter fitting can be seen in Table 3.

Since two different control variables were manipulated in Experiment 1B, 16 different cusp models are possible. Because of the two different variables, we did not expect that aspect ratio contributes to both the normal and the splitting variable (as it did in Experiment 1A, since it was the only variable that was manipulated). It was now expected that aspect ratio would serve as the normal variable and BRLC as the splitting variable. BRLC was the first variable, and aspect ratio was the second. Thus, we expected the 7th model with $a_1 = b_2 = 0$ in Table 3 to have the best fit. In this model, the contribution of BRLC to the normal variable and the contribution of aspect ratio to the splitting variable were fixed to zero. The 7th model indeed has the lowest BIC. By looking at the AIC, the 13th and the 14th models have a slightly better fit, but the difference is deniable. We can conclude that aspect ratio served only as a normal variable and BRLC served only as a splitting variable. The hypothesis that we have a complete cusp model with two control variables is confirmed. A graphical representation of the data is shown in Figure 8. Figure 8 shows the location of data points in the control plane. Aspect ratio is related to the normal variable, and BRLC is related to the splitting variable. The percepts are indicated in italics.

EXPERIMENT 2

General Method

The proposed cusp model for the luminance contrast paradigm (Hock et al., 1997) was tested. As was mentioned above, two point lights were presented simultaneously, each with different, greater-than-background luminance, and their values were exchanged on successive frames. The brighter of the two point lights could be perceived as moving through the space between them. However, when the luminance contrast was too low, stationarity, rather than motion, was per-

Figure 8. A graphical representation of the data points of Experiment 1B. The data points, represented by AR (aspect ratio) and BRLC (background-relative luminance contrast) are both between and outside the bifurcation lines, indicating divergence.

ceived. Hysteresis effects were found when luminance contrast was gradually increased or decreased during each trial.

Experiment 2A

Method

This experiment was exactly the same as Experiment 3 in Hock et al. (1997). The distance between the point lights was always 42 min, and the separation was always horizontal. Trials consisted of 195-msec frames with gradually increasing or decreasing BRLC values, following the modified method of limits (Hock et al., 1993). Two subjects indicated whether they saw a switch from motion to nonmotion and vice versa or did not. Both subjects participated in four blocks.

Results

In the original paradigm, there was only one changing variable—that is, luminance contrast. The behavioral variable was the mean proportion of perceived motion of each block for each endpoint BRLC value for each subject. Since there were 2 subjects who completed four blocks

and there were eight endpoint BRLC values in both ascending and descending trials, the analysis was done for 128 data points.

Table 4 displays the results. If BRLC served as the normal variable in the present experiment, it was expected that the third model would have the best fit. In this model, the contribution of BRLC to splitting variable was fixed to zero. The third model is indeed the best-fitting model. BRLC contributed only to the normal variable, not to the splitting variable.

A graphical representation of the data is given in Figure 9. The data lay on a horizontal line, indicating that the variable BRLC indeed contributes only to the normal variable.

Experiment 2B

Method

Stimuli were two 1-pixel point lights, horizontally or vertically separated by 2, 4, 6, or 8 pixels. The point lights were presented in the middle of a gray box ($5.6^\circ \times 5.6^\circ$) on the dark screen (1.5 cd/m^2)

Table 4
Catastrophe Analysis of Experiment 2A

Model	a_0	a_1	b_0	b_1	λ	σ	Log Likelihood	Parameters	AIC	BIC
Cusp 1	0.12	0.00	2.92	0.00	-0.18	0.64	-120.0	4	247.9	259.3
Cusp 2	0.78	0.00	1.55	2.04	-0.99	0.97	-115.1	5	240.2	254.5
Cusp 3	0.33	1.10	2.34	0.00	-0.19	0.64	-668.0	5	143.6	157.9
Cusp 4	0.30	1.06	2.44	0.28	-0.23	0.64	-659.8	6	144.0	161.1
Linear							-239.7	3	485.4	494.0
Logistic							-121.5	2	247.1	252.8

Note— a_0 , constant of the normal variable; a_1 , loading weight of the normal variable; b_0 , constant of the splitting variable; b_1 , loading weight of the splitting variable; λ , location; σ , scale; log likelihood, log likelihood value; AIC, Akaike's information criterion; BIC, Bayes's information criterion. Zeros are fixed parameters.

Figure 9. A graphical representation of the data points of Experiment 2A. The data points, represented by BRLC (background-relative luminance contrast), cross the second bifurcation line.

of a Macintosh II RGB monitor. The subjects viewed the stimuli from a distance of 30 cm (maintained by a head restraint) in a darkened room. The luminance value of the gray box was always 6.8 cd/m². The mean luminance value of the two point lights was always 9.2 cd/m². The luminance difference between the two point lights was 0.2, 0.5, 0.7, 1.0, 1.2, 1.4, 1.6, 1.8, or 2.1 cd/m². This resulted in nine different BRLC values of 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 0.9. The BRLC values of the two point lights were exchanged on alternating 195-msec frames. A trial consisted of eight frames. Six subjects participated in eight blocks with vertically separated point lights and in eight blocks with horizontally separated point lights. In each block, all conditions (4 different distances $\times$ 9 different BRLC

values) were presented four times, resulting in 144 randomly ordered trials per block. The order of the blocks was alternated on successive sessions. The subjects were instructed to indicate after each trial, by pressing a button on the keyboard, whether they saw motion at any time during the trial.

Results

In the present subexperiment, beside BRLC, there was a second variable: distance between the point lights. The behavioral variable was the mean proportion of perceived motion on each combination of BRLC and distance be-

Table 5
Catastrophe Analysis of Experiment 2B (Horizontal Presentation of the Luminance Contrast Paradigm)

Model	a_0	a_1	a_2	b_0	b_1	b_2	λ	σ	Log Likelihood	Parameters	AIC	BIC
1	-0.17	0.00	0.00	3.39	0.00	0.00	0.30	0.60	-166.9	4	341.8	355.3
2	-0.17	0.00	0.00	3.39	0.00	0.07	0.31	0.60	-166.7	5	343.3	360.2
3	1.94	0.00	0.00	-2.39	4.24	0.00	-1.07	1.23	-162.4	5	334.7	351.6
4	1.82	0.00	0.00	-2.03	4.39	-0.89	-1.06	1.15	-147.1	6	306.1	326.4
5	-0.18	0.00	-0.11	3.37	0.00	0.00	0.30	0.60	-163.3	5	336.7	353.6
6	-0.18	0.00	-0.11	3.37	0.00	0.03	0.31	0.60	-163.3	6	338.6	358.8
7	-0.27	0.00	-0.12	3.47	-0.93	0.00	0.46	0.62	-148.0	6	308.1	328.3
8	-0.27	0.00	-0.10	3.49	-1.05	0.20	0.49	0.62	-146.3	7	306.6	330.2
9	-0.71	1.44	0.00	2.68	0.00	0.00	0.32	0.60	-59.2	5	128.4	145.2
10	-0.72	1.45	0.00	2.67	0.00	0.10	0.33	0.60	-58.7	6	129.4	149.7
11	-0.68	1.39	0.00	2.77	-0.14	0.00	0.34	0.60	-58.9	6	129.8	150.1
12	-0.68	1.38	0.00	2.80	-0.20	0.12	0.35	0.60	-58.2	7	130.4	154.0
13	-0.88	1.79	-0.49	2.52	0.00	0.00	0.33	0.60	-44.8	6	101.6	121.8
14	-0.88	1.79	-0.49	2.52	0.00	0.03	0.33	0.60	-44.7	7	103.4	127.1
15	-0.86	1.76	-0.48	2.56	-0.07	0.00	0.33	0.60	-44.7	7	103.4	127.1
16	-0.86	1.74	-0.48	2.59	-0.10	0.05	0.34	0.60	-44.6	8	105.2	132.2
Linear									-432.4	4	872.7	886.2
Logistic									-227.9	3	461.8	471.9

Note—The first 16 models are cusp models. The last 2 models are the unconstrained linear and logistic models (a_1 and b_1 are parameters for background-related luminance contrast and a_2 and b_2 are parameters for distance). λ , location; σ , scale; log likelihood, log likelihood value; AIC, Akaike's information criterion; BIC, Bayes's information criterion. Zeros are fixed parameters.

Figure 10. A graphical representation of the data points of Experiment 2B. The data points, represented by BRLC (background-relative luminance contrast) and Distance (distance between point lights), cross the first and the second bifurcation lines.

tween point lights for each subject. Since there were 6 subjects, nine different BRLC values, and four different distances, the analysis was done for 216 data points. The results for the horizontal presentation of the point lights and the vertical presentation are reported separately.

The fit measures of models for the horizontal presentation of the point lights can be found in Table 5. It is the 13th model, indicating a model with the splitting variable fixed to 0 for both BRLC and distance between point lights.

This is not as was expected. First, it was expected that BRLC would serve as the normal variable, and second, that distance between point lights would serve as the splitting variable. The first expectation is confirmed; the second expectation is not. A graphical representation of the data is given in Figure 10. Both distance and BRLC relate to the normal variable. Most data points are located in the bifurcation set, the area between the bifurcation lines, where behavior is bimodal.

Table 6
Catastrophe Analysis of Experiment 2B (Vertical Presentation of the Luminance Contrast Paradigm)

Model	a_0	a_1	a_2	b_0	b_1	b_2	λ	σ	Log Likelihood	Parameters	AIC	BIC
1	-0.10	0.00	0.00	3.28	0.00	0.00	0.17	0.60	-176.0	4	360.0	373.5
2	-0.10	0.00	0.00	3.28	0.00	0.05	0.18	0.60	-175.9	5	361.7	378.6
3	1.46	0.00	0.00	-0.60	4.03	0.00	-1.09	1.06	-142.3	5	294.6	311.5
4	1.41	0.00	0.00	-0.39	4.22	-0.72	-1.09	1.02	-129.4	6	270.8	291.0
5	-2.24	0.00	-0.36	2.74	0.00	0.00	5.00	2.67	-302.7	5	615.4	632.2
6	-0.10	0.00	-0.09	3.28	0.00	0.03	0.17	0.60	-173.4	6	358.8	379.1
7	2.02	0.00	-0.69	-1.02	4.26	0.00	-1.17	1.09	-132.7	6	277.4	297.7
8	1.45	0.00	-0.20	-0.45	0.42	-0.59	-1.09	1.02	-128.9	7	271.7	295.3
9	-0.51	1.76	0.00	2.38	0.00	0.00	0.19	0.60	-52.5	5	1,15.0	131.8
10	-0.52	1.77	0.00	2.38	0.00	0.07	0.19	0.60	-52.3	6	1,16.5	136.8
11	-0.53	1.82	0.00	2.31	0.20	0.00	0.16	0.60	-52.1	6	1,16.1	136.4
12	-0.53	1.82	0.00	2.32	0.17	0.05	0.17	0.60	-52.0	7	1,17.9	141.6
13	-0.61	2.11	-0.49	2.23	0.00	0.00	0.19	0.59	-40.2	6	92.3	112.6
14	-0.61	2.11	-0.48	2.23	0.00	0.03	0.19	0.59	-40.1	7	94.2	117.8
15	-0.64	2.20	-0.50	2.12	0.26	0.00	0.16	0.59	-39.4	7	92.9	116.5
16	-0.64	2.20	-0.50	2.12	0.27	-0.01	0.16	0.59	-39.4	8	94.9	121.9
Linear									-452.8	4	913.6	927.1
Logistic									-141.7	3	289.4	299.5

Note—The first 16 models are cusp models. The last 2 models are the unconstrained linear and logistic models (a_1 and b_1 are parameters for background-related luminance contrast and a_2 and b_2 are parameters for distance). λ , location; σ , scale; log likelihood, log likelihood value; AIC, Akaike's information criterion; BIC, Bayes's information criterion. Zeros are fixed parameters.

The best-fitting model for the vertical presentation of the point lights is similar. The fit measures can be found in Table 6.

DISCUSSION

We applied stochastic catastrophe theory to two experimental paradigms based on the perception of apparent motion: the motion quartet paradigm and the luminance contrast paradigm. Earlier studies showed that dynamical phenomena such as hysteresis and bistability can be found with both paradigms. We showed that a cusp catastrophe model gives the best description of the results of experiments based on the two paradigms. The cusp model describes hysteresis and bistability, but it also predicts divergence when hysteresis occurs. We presented a new experiment in which BRLC was used as a splitting variable in the motion quartet paradigm. The results of an algorithm based on stochastic catastrophe theory showed that BRLC indeed is an appropriate splitting variable. The cusp model with aspect ratio as the normal variable of the model and BRLC as the splitting variable has the best fit. Our cusp model of the motion quartet paradigm is supported by the data.

The analysis of the experiments with the luminance contrast paradigm shows that distance between point lights was not the appropriate splitting variable. It is necessary to look for an alternative parameter. It is especially important, when hysteresis has been found, to look for variables that might provoke divergence. We have not yet done experiments to test possible alternatives. The size of the point lights is a likely candidate. A consideration is that, in the luminance contrast paradigm, there are two possible percepts: motion and no motion. In contrast, in the motion quartet paradigm, there are three possibilities: vertical motion, horizontal motion, and no motion. To find divergence, one needs three different possible perceptual modes. In the luminance contrast paradigm, it is not clear which third perceptual mode is possible. When the only possible percepts in the luminance contrast paradigm are motion and no motion, this paradigm would be described better by a fold catastrophe model. This is a more simple catastrophe model than the cusp model. In the fold model, only the emergence of a new state (e.g., motion) is described. This hypothesis is supported by the fact that the cusp is a composition of fold catastrophes, whereas the motion quartet paradigm consists of coupled luminance contrast paradigms. However, the fold model does not give rise to hysteresis. Since we have found hysteresis in the experiment related to the luminance contrast paradigm, final conclusions on the best model for this paradigm can not be drawn yet. New experiments in which the normal variable is changed very slowly might give insight into this issue.

Although we found a likely splitting variable in the motion quartet paradigm, it is necessary to continue looking for alternative parameters. In every research that contains model testing, it is always possible that other variables than the ones that have been experimentally controlled influence the process that one is looking for. In our case, it might be that a different parameter than BRLC (or the as-

pect ratio) contributes equally or even more to the splitting variable (or to the normal variable)—for example, stimulus duration, interstimulus interval, or stimulus size. We first did an experiment with presentation time of the point lights as a splitting variable. Trials started with a long presentation time (i.e., 4 sec), and we slowly speeded up the presentation time. When presentation time is very slow, it is also possible to see circular motion. When the presentation time is speeded up, the percept of circular motion loses stability and changes to the percept of either horizontal or vertical motion, depending on the aspect ratio of the quartet. We found divergence, but since not all the subjects managed to see circular motion, we decided to do the experiment with BRLC as the splitting variable. In this experiment, all the subjects reported seeing clear stationarity, horizontal motion, or vertical motion. The new experiments show that stochastic catastrophe theory is not only a useful tool to describe and analyze perception data, but also the basis for new hypotheses and predictions. When research is done on nonlinearities in perception, we suggest stochastic catastrophe theory should be used and that other possible dynamical phenomena besides bistability and hysteresis should be looked for.

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NOTE

1. Hartelman (1997) replaced the expectation-maximization routine that Cobb (1980) used by the more advanced and reliable quasi-Newton optimization routine. The gradient of the objective function is determined by a finite difference approximation. Hartelman also replaced the numerical integrator of Cobb (1980) by the numerical integrator D01AMF of the NAG library.

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