

More power to you: Simple power calculations for treatment effects with one degree of freedom

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Although numerous computer programs for statistical power analysis are available, power is an underused aspect of experimental analysis, perhaps because of the perceived difficulty of performing the necessary calculations or because existing computer software can be expensive or complicated to learn. For single-degree-of-freedom tests, however, it is possible to calculate power in a straightforward manner, using the t distribution. Because these calculations are based on t , they use easily understood and readily available quantities. These calculations can be performed with a desk calculator; we also present a simple-to-use program called *MorePower* that will perform the necessary calculations. The straightforward nature of the calculations potentially will enable more researchers to consider issues of power when planning and reporting their experiments.

There are many sophisticated computer programs available for power analysis, but these may be expensive, complicated to learn, or limited in their application (Thomas & Krebs, 1997). Here, we present simple formulas for calculating power and related statistics for any analysis of variance (ANOVA) test with one degree of freedom. We explain how to do the calculations by hand, as well as by using a simple-to-use freeware program for Microsoft Windows, called *MorePower* (<http://duke.usask.ca/~campbellj/work/MorePowerx.html>). Our formulas and program have several distinctive features. First, because they are based on the familiar and relatively simple t test, the formulas and program are easy to use and understand; nonetheless, they apply to any single-degree-of-freedom test (main effect or interaction) from within-subjects, between-subjects, or mixed designs of arbitrary complexity (assuming equal n s for between-subjects factors). Second, effect size is estimated (or provided by the user) in the original units of measurement, rather than in less intuitive variance-based ratios or small-medium-large approximations [the *MorePower* program also calculates η^2 , however; i.e., $MST/(MST + MSE * df)$]. Third, to specify effect size, one may enter either MST or the difference in original units; to specify error variance, one may enter either MSE or the variance of the difference. Thus, given the F -ratio and MSE provided in most research reports, one can easily

compute observed power and effect size for any ANOVA effect with one degree of freedom. This makes it a trivial task to assess the power associated with published null effects or to use previous results to estimate the required effect size or sample size for planned experiments.

Calculating Power for the Between-Subjects t Test

Our power analysis extends the approach developed by Cochran and Cox (1957) for the between-subjects t test (also, see Hays, 1994, for an analogous derivation for z). Ordinarily, one rejects the null hypothesis when the observed difference between means (d_c) is large enough such that t exceeds the value set a priori to represent the Type I error rate (t_α):

$$d_c > t_\alpha \sqrt{\frac{2 \cdot s_d^2}{n_t}} \quad (1)$$

In Equation 1, s_d^2 = variance of the difference between means and n_t = number of observations per treatment (equal n s assumed). Power is the probability of drawing a sample in which d_c satisfies the above requirement from a sampling distribution whose mean (d_β) is greater than 0; power, therefore, is the probability that $t > t_\beta$:

$$t_\beta = \frac{d_c - d_\beta}{\sqrt{\frac{2 \cdot s_d^2}{n_t}}} \quad (2)$$

For our analysis, we combined Equations 1 and 2 and then solved for power (probability $t > t_\beta$), effect size (d_β), and sample size (n_t).

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Power:

$$t_{\beta} = t_{\alpha} - \frac{d_{\beta}}{\sqrt{\frac{s_d^2}{n_t}}} \quad (3)$$

Effect Size:

$$d_{\beta} = \sqrt{\frac{(t_{\alpha} - t_{\beta})^2 \cdot s_d^2}{n_t}} \quad (4)$$

Sample Size:

$$n_t = \frac{(t_{\alpha} - t_{\beta})^2 \cdot s_d^2}{d_{\beta}^2} \quad (5)$$

Extension to multifactor designs. We extend this analysis to multifactor ANOVA designs, including between-subjects, within-subjects, and mixed designs. This extension is possible because s_d^2 can be derived from MSE (see below) and because F and t^2 are equivalent for any 2^k effect in a multifactor design. That is, any F test with one degree of freedom in the numerator corresponds to a pairwise comparison (e.g., a 2×2 interaction corresponds to a pairwise comparison of the difference of differences), even when the 2^k effect is embedded in a design that includes factors with more than two levels.

Quantities Needed for Calculations

Power, sample size, and effect size can be calculated by hand using Equations 3, 4, and 5, or by using the MorePower program, which is described in the next section. In either case, the researcher needs to have an estimate of error variance (s_d^2 or MSE) and a value for α , as well as two of the remaining three variables: sample size (n_t), effect size (d_{β}), or power (one can calculate the third, given values for the other two).

Computing s_d^2 . One can estimate s_d^2 either from an estimate of individual cell variances or from MSE (MorePower will calculate s_d^2 given a value of MSE). Equation 6 calculates s_d^2 from individual cell variances. For between-subjects designs, the intercell correlations are all zero; thus, the value of the second half of Equation 6 is reduced to zero:

$$s_d^2 = s_1^2 + s_2^2 - 2r_{12}s_1s_2 \quad (6)$$

Equation 7 computes s_d^2 from MSE :

$$s_d^2 = MSE \cdot 2^B \cdot \frac{2^{W \cdot 2}}{y} \quad (7)$$

In this equation, B is the number of two-level between-subjects factors in the relevant test, W is the number of two-level within-subjects factors in the relevant test, and y is the total number of within-subjects cells in the experiment. This equation can be used to calculate s_d^2 for any comparison, including those having both between- and

within-subjects factors. Note that when the effect of interest involves only between-subjects or only within-subjects factors, Equation 7 can be simplified to Equations 7a and 7b, respectively:

$$s_d^2 = MSE \cdot 2^B \quad (7a)$$

and

$$s_d^2 = MSE \cdot \frac{2^{W \cdot 2}}{y} \quad (7b)$$

Calculating d_{β} (effect size). The quantity d_{β} refers to the size of the difference one is interested in and is equal to the absolute difference between treatment conditions. For pairwise comparisons and main effects, therefore, d_{β} equals the absolute value of the difference between the means for the two levels of the relevant factor, averaged over the levels of the other factors. For 2×2 interactions, d_{β} is the absolute value of the difference of differences, $(M_{11} - M_{12}) - (M_{21} - M_{22})$ (again, averaged over the levels of other factors), and for $2 \times 2 \times 2$ interactions, it is the absolute value of the difference between the difference of differences $[(M_{111} - M_{112}) - (M_{121} - M_{122})] - [(M_{211} - M_{212}) - (M_{221} - M_{222})]$ and so on. For example, if M_{11} , M_{12} , M_{21} , and M_{22} were equal to 50, 30, 15, and 5, then d_{β} for the 2×2 interaction would equal $(50 - 30) - (15 - 5)$, or 10.

The observed effect size for a specific main or interaction effect can also be calculated using MST from the corresponding ANOVA test. Specifically,

$$d_{\beta} = \sqrt{\left(MST \cdot 2^B \cdot \frac{2^{W \cdot 2}}{y} \right) / n_t} \quad (8)$$

where B , W , and y are defined as above. The MorePower program allows the user to enter a value for either d_{β} or MST to estimate power or effect size; d_{β} must be supplied to estimate sample size.

Calculating n_t . This quantity is the number of observations that contribute to each treatment level in the effect of interest. Equal n_s are assumed for between-subjects factors. For 2^k effects composed entirely of within-subjects factors (i.e., no between-subjects factors), n_t is equal to the total N . If the 2^k effect of interest includes one between-subjects factor, then n_t is the number of observations contributing to each level of the between-subjects factor ($n_t = N/2$); if the effect includes two between-subjects factors (e.g., a $2 \times 2 \times 2$ interaction involving two between-subjects and one within-subjects factor), then n_t is the number of observations in each of the four between-subjects cells ($n_t = N/4$), and so on.

Calculating Power, Effect Size, and Sample Size With MorePower

The MorePower program was created using VisualBasic 5.0 for Microsoft Windows and should run under any version of the Windows operating system when installed with

the self-extracting MPSetup.exe program available at <http://duke.usask.ca/~campbelj/work/MorePowerx.html>. MPSetup installs (if necessary) all the Windows system and VisualBasic 5.0 files (5.4 MB) required for MorePower, but we found that most of the ≈ 50 computers tested already had the necessary Windows components. Consequently, usually one requires only the MorePower.exe file (100K), which can be downloaded separately. The program requires a screen resolution of 800×600 pixels or higher.

The MorePower program interface is depicted in Figure 1. Point and click the mouse on the appropriate label in the Estimate panel to select the quantity one wishes to estimate: power, d_β (Effect Size on the program interface), or n_i (n /treatment on the program interface). This clears and highlights the window for that quantity and moves the cursor to set alpha, which is .05 by default. Set alpha to α^*2 for a one-tailed test. Pressing Enter on one's keyboard moves the cursor to the next appropriate box, or one may move between boxes by pointing and clicking the mouse.

In the Effect Size panel, point and click the mouse on the label or data entry field to select MST or d_β ; d_β must be >0 to estimate n_i . Similarly, in the Error Variance panel, point and click the mouse to select MSE or s_e^2 .

One must also specify information in the Effect and Design panel (see Figure 1). Here, one specifies the effect of interest and the design of the total experiment. Figure 1 illustrates the case in which the researcher is interested in estimating the sample size required (i.e., n /treatment) for a 2×2 mixed interaction (i.e., one between- and one within-subjects factors) in the context of a $2 \times 2 \times 2 \times 2$ experiment (two within- and two between-subjects factors).

Effect Betwn and Effect Within refer to the number of two-level between- and within-subjects factors in the effect of interest. In the case of the 2×2 mixed interaction described above, these values are 1 and 1, respectively (i.e., there is one within- and one between-subjects factor in the interaction), as is illustrated in Figure 1. For a within-subjects main effect, the values would be 0 (or blank) and 1; for the interaction of two between-subjects variables with one within-subjects variable, these values would be 2 and 1, and so on.

Betwn Factors and Within Factors specify the design of the complete experiment. Specifically, the user must enter the number of levels associated with each of the between- and within-subjects factors in the complete design, including those involving three or more levels. For example, if the experiment consists of two between- and two within-subjects factors, enter 22 for Betwn Factors and 22 for Within Factors, as is illustrated in Figure 1. For a $2 \times 2 \times 3$ within-subjects design, the sequence 223 (or 232 or 322) would be entered for Within Factors and Betwn Factors would be set to 0 or left blank. For a between-subjects t test, enter 2 for Betwn Factors and 0 (or blank) for Within Factors.

Clicking the Calculate button or pressing Enter on the keyboard with the cursor in the Within Factors window initiates calculation based on the data entered. If the Calculate button is pressed with missing or incorrect quanti-

The screenshot shows the 'MorePower: power calculator for df=1 effects' window. It features several input panels and a large output window. The 'Estimate' panel has three radio buttons: 'Power' (unselected), 'Effect Size' (unselected), and 'n/treatment' (selected). The 'Effect Size' panel has two radio buttons: 'MST' (unselected) and 'd-Beta' (selected). The 'Error Variance' panel has two radio buttons: 'MSE' (unselected) and 'Sd^2' (selected). The 'Effect and Design' panel contains a table with columns for 'Effect Betwn', 'Effect Within', 'Betwn Factors', and 'Within Factors'. The values entered are 1, 1, 22, and 22 respectively. The output window displays the following text:

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required sample size/treatment = 20
mean difference (d-Beta) = 100.
desired power = .8
std. dev. of diff. (d-Beta) = 158.113883
Eta^2 = .47059
effect = 2-way: 2 between by 2 within
design = 2b X 2b X 2w X 2w
n/treatment for the effect = 20
total N = 40, df error = 36
test of Ho: t = 2.82843, F = 8.
p = .0076

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Figure 1. MorePower program interface.

ties, a prompt appears in the Output panel, and the cursor is moved to the appropriate data entry window on the interface. The desired estimate will appear in two places: (1) in the highlighted box on the interface (i.e., if one is calculating power, the value for power will appear in the upper right-hand box) and (2) in the Output window at the bottom of the calculator. As is illustrated in Figure 1, the Output window also displays other summary information for the analysis; this information may be copied and pasted into a word processor to be printed or saved.

To begin the next calculation, click the Clear Values button. At any point, one can change any quantity and recalculate. Positioning the mouse pointer over any label or data entry window on the MorePower interface displays a short description or instruction about the corresponding quantity.

The probability value entered or appearing in the Power panel of MorePower corresponds to the upper tail of the t_β distribution (see Equation 3). The probability associ-

ated with the lower tail usually is trivial from a practical standpoint; nonetheless, MorePower displays the exact power in the Output window, including both upper and lower tail probabilities. Probabilities are calculated using the BETAI algorithm for the incomplete beta function presented in Sprott (1996, p. 105).

There are 10 example sets of values for various between, within, and mixed effects that can be accessed by clicking the Test Example Values button on the lower left corner. Enter a number for the example one wishes to view and then click on the Calculate button.

Examples

Example 1: Estimating power and sample size for a t test. Suppose one measures reaction time (RT) for each of 25 people to make true/false decisions about a series of statements. True statements yield a mean of 1,000 msec, and false statements yield a mean of 1,050 msec. The standard deviation of the 50-msec difference is 200 msec. To use the MorePower program to assess power, click Power in the upper left panel and press the Enter key on one's keyboard. Then enter 25 for n_t (n/treat on the MorePower interface), enter the mean difference of 50 msec for d_β (d-Beta on the interface), and enter 40,000 (i.e., 200^2) for s_d^2 . To specify the effect and design (i.e., a paired-difference t test), enter 1 for Effect Within (i.e., there is one two-level factor in the effect of interest) and 2 for Within Factors (i.e., the design consists of two levels in one within-subjects factor). The Effect Betwn and Betwn Factors boxes may be left blank or zeroes entered. Click on the Calculate button. The program displays an observed power of .21 for the 50-msec difference, with $t(24) = 1.25, p = .223$, for the two-tailed test of the null hypothesis.

To estimate the sample size required to detect a difference ≥ 50 msec with a probability of .8, click on $n/\text{treatment}$ in the Estimate panel, which clears and highlights the n/treat box. Press the Enter key (or point and click) to move the cursor to the Power input box, enter .8, and click on the Calculate button. The estimated sample size required (128) appears in the n/treat box and in the output window.

Example 2: Estimating power in multifactor designs. Consider a $2 \times 3 \times 3$ between-subjects design with 10 observations in each of the 18 cells ($N = 180$). The dependent measure is proportion of correct responses, and one wants to estimate the power to detect a main effect of .10 for the two-level factor. There are 90 observations contributing to each level of the main effect; thus, n_t is 90. One is interested in finding a difference of .10 between the two levels; thus, d_β is .10. For the main effect of the two-level factor, $F(1,162) = 2.05, MS_e = 0.22, p > .05$.

In the MorePower program, select Power in the upper left panel and then enter .05, 90, .10, and .22 in the data entry windows labeled Alpha, $n/\text{treatment}$, d-beta, and MSE, respectively. The effect of interest is a between-subjects main effect (i.e., an effect involving a single factor), so enter 1 for Effect Betwn and either enter a 0 for

Effect Within or leave it blank. The complete design involves three between-subjects factors, and the values corresponding to the number of levels in each factor (i.e., 233, 323, or 332) should be entered for Betwn Factors. Within Factors should be set to 0 or left blank. Click on Calculate. The computed value for power should appear in the top right-hand corner, in the box labeled Power. Power for this example is computed to be .29, meaning that the probability is about 30% of finding a difference of $\geq .10$ between the two groups.

One can also use Equation 3 to perform the calculations by hand. Here, power is defined as the probability of finding a value of t greater than t_β . We have included a table of t values in the Appendix. If more precise values of the t distribution are required, accurate probability calculators are available as stand-alone freeware (e.g., <http://www.ncss.com/download.html>).

To illustrate Equation 3, first, compute s_d^2 using Equation 7a:

$$s_d^2 = MSE \cdot 2^B = 0.22 \cdot 2^1 = 0.44.$$

Substituting the remaining values into Equation 3, one gets

$$t_\beta = 1.98 - \frac{.10}{\sqrt{\frac{.44}{90}}} = 0.55.$$

From the Appendix, the probability that $t(162)$ exceeds t_β is about .30; thus, the experiment had 30% power to detect a difference of .10 between levels of the main effect.

Example 3: Estimating effect size. Consider a 2×2 within-subjects experiment with an N of 60. The dependent variable is percentage correct, and one wishes to determine the size of the interaction that one could find with power equal to .8. For the interaction, $F(1,59) = 191.36, MSE = 27.15$.

To use MorePower, select Effect Size in the upper left box. Enter .05, 60, .80, and 27.15 in the boxes labeled Alpha, $n/\text{treatment}$, Power, and MSE, respectively. The effect of interest is a within-subjects interaction (i.e., an effect involving two factors), so enter 2 in the Effect Within box, and leave the Effect Betwn box blank. The complete design involves two within-subjects factors; enter 22 in the Within Factors box, and leave Betwn Factors empty. Click on Calculate. The computed value for d_β should appear in the top Effect Size panel; this value is equal to 3.83, meaning that there is 80% power to find an interaction in which the difference of differences is equal to or greater than 3.83.

One can also use Equation 4 to calculate effect size. For this equation, one needs t_β , which is the t value associated with a given level of power (e.g., for power equal to .80, one needs to find the value of t above which .80 of the sampling distribution lies). In our example, with df equal to 59, t_β is $-.849$ (from the Appendix). For α at .05, $t_\alpha = 2.00$.

Equation 7b will be used to calculate s_d^2 , where W is the number of within-subjects factors associated with the relevant test (i.e., 2) and y = the total number of cells in the design ($2 \times 2 = 4$). Thus,

$$s_d^2 = 27.15 \cdot \frac{2^{2 \cdot 2}}{4} = 108.6.$$

Substituting these values into Equation 4:

$$d_\beta^2 = \sqrt{\frac{(2.00 - -.849)^2 \cdot 108.6}{60}} = 3.8.$$

Example 4: Estimating sample size. Imagine planning a $2 \times 2 \times 2$ within-subjects RT experiment in which the three-way interaction is theoretically important. One wants to estimate the sample size required to detect a three-way effect (i.e., a difference between the difference of differences of 10 msec or greater (i.e., $d_\beta = 10$) with a power of .9. On the basis of previous research, we estimate s_d^2 for the three-way effect to be 50² msec (this can be estimated from the variance of a single cell, using Equation 7).

To use MorePower, select n/treatment in the upper left box. Enter .05, .90, 10, and 2500 in the boxes labeled Alpha, Power, d-beta, and Sd², respectively. The effect of interest is a within-subjects interaction (i.e., an effect involving three factors), so enter 3 in the Effect Within box; leave the Effect Betwn box blank. The complete design involves three within-subjects factors; enter 222 in the Within Factors box, leaving Betwn Factors empty. Click on Calculate. The computed value for n_t (i.e., 264) should appear in the top right-hand corner, in the box labeled n/treatment.

One can also use Equation 5 to calculate sample size. Again, one needs values of t_α and t_β ; because these values depend upon sample size, Equation 5 may have to be solved iteratively. This will be an issue, however, only when one is concerned with small sample sizes, for which the values of t_α and t_β may vary substantially as a function of n_t . In this case, however, one can anticipate that n

will be quite large and, therefore, use the asymptotic values of $t_\alpha(\infty) = 1.960$ for $\alpha = .05$, and $t_\beta(\infty) = -1.282$ for power = .9 (from the Appendix). Substituting into Equation 5,

$$n_t = \frac{(1.96 - -1.28)^2 \cdot 50^2}{10^2} \approx 263.$$

Conclusions

Our approach simplifies several features of doing a power analysis. Specifying effect sizes is typically one of the most difficult features of doing a power analysis (Cohen, 1990, 1992). Effect sizes usually are expressed as a function of an estimate of error (e.g., Cohen, 1988; Howell, 1997) or in terms of a subjective evaluation of what constitutes a small, medium, or large effect size (Cohen, 1992). A convenient feature of our analysis is that it allows researchers to quantify effect sizes in terms of easy to understand, experimentally meaningful units. Similarly, the other quantities required are easily understood and accessible; researchers can enter values for n_t , rather than having to compute df , and use quantities such as MST and MSE from an ANOVA to perform the calculations.

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APPENDIX
Power Levels and Corresponding *t* Values

Following are *t* values associated with various power levels (i.e., upper-tail probabilities of the *t* distribution). Note that the tabled values for tail probabilities greater than .5 (in bold) are negative. For example, to find the *t* value associated with power equal to .8 and *df* equal to 30, the tabled value would be $-.854$. These values were calculated using a Hewlett-Packard 21S calculator.

<i>df</i>	Power Levels									
	.025 .975	.05 .95	.10 .90	.15 .85	.20 .80	.25 .75	.30 .70	.35 .65	.40 .60	.45 .55
1	12.706	6.314	3.078	1.963	1.376	1.000	0.727	0.510	0.325	0.158
2	4.303	2.920	1.886	1.386	1.061	0.816	0.617	0.445	0.289	0.142
3	3.182	2.353	1.638	1.250	0.978	0.765	0.584	0.424	0.277	0.137
4	2.776	2.132	1.533	1.190	0.941	0.741	0.569	0.414	0.271	0.134
5	2.571	2.015	1.476	1.156	0.920	0.727	0.559	0.408	0.267	0.132
6	2.447	1.943	1.440	1.134	0.906	0.718	0.553	0.404	0.265	0.131
7	2.365	1.895	1.415	1.119	0.896	0.711	0.549	0.402	0.263	0.130
8	2.306	1.860	1.397	1.108	0.889	0.706	0.546	0.399	0.262	0.130
9	2.262	1.833	1.383	1.100	0.883	0.703	0.543	0.398	0.261	0.129
10	2.228	1.812	1.372	1.093	0.879	0.700	0.542	0.397	0.260	0.129
11	2.201	1.796	1.363	1.088	0.876	0.697	0.540	0.396	0.260	0.129
12	2.179	1.782	1.356	1.083	0.873	0.695	0.539	0.395	0.259	0.128
13	2.160	1.771	1.350	1.079	0.870	0.694	0.538	0.394	0.259	0.128
14	2.145	1.761	1.345	1.076	0.868	0.692	0.537	0.393	0.258	0.128
15	2.131	1.753	1.341	1.074	0.866	0.691	0.536	0.393	0.258	0.128
16	2.120	1.746	1.337	1.071	0.865	0.690	0.535	0.392	0.258	0.128
17	2.110	1.740	1.333	1.069	0.863	0.689	0.534	0.392	0.257	0.128
18	2.101	1.734	1.330	1.067	0.862	0.688	0.534	0.392	0.257	0.127
19	2.093	1.729	1.328	1.066	0.861	0.688	0.533	0.391	0.257	0.127
20	2.086	1.725	1.325	1.064	0.860	0.687	0.533	0.391	0.257	0.127
21	2.080	1.721	1.323	1.063	0.859	0.686	0.532	0.391	0.257	0.127
22	2.074	1.717	1.321	1.061	0.858	0.686	0.532	0.390	0.256	0.127
23	2.069	1.714	1.319	1.060	0.858	0.685	0.532	0.390	0.256	0.127
24	2.064	1.711	1.318	1.059	0.857	0.685	0.531	0.390	0.256	0.127
25	2.060	1.708	1.316	1.058	0.856	0.684	0.531	0.390	0.256	0.127
26	2.056	1.706	1.315	1.058	0.856	0.684	0.531	0.390	0.256	0.127
27	2.052	1.703	1.314	1.057	0.855	0.684	0.531	0.389	0.256	0.127
28	2.048	1.701	1.313	1.056	0.855	0.683	0.530	0.389	0.256	0.127
29	2.045	1.699	1.311	1.055	0.854	0.683	0.530	0.389	0.256	0.127
30	2.042	1.697	1.310	1.055	0.854	0.683	0.530	0.389	0.256	0.127
40	2.021	1.684	1.303	1.050	0.851	0.681	0.529	0.388	0.255	0.126
50	2.009	1.676	1.299	1.047	0.849	0.679	0.528	0.388	0.255	0.126
100	1.984	1.660	1.290	1.042	0.845	0.677	0.526	0.386	0.254	0.126
∞	1.960	1.645	1.282	1.036	0.842	0.674	0.525	0.386	0.254	0.126

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