Perception of two-body center of mass

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Participants estimated the perceptual center of mass between two horizontally oriented black dots varying in size and distance. Experiment 1 showed that estimates, measured as distance from the larger dot's center, decreased with an increase in size ratio between the dots and a decrease in the distance between them, as predicted by the physical center-of-mass equation. The results were replicated and extended in further experiments with different ratios and distances. In all experiments, the true center was consistently overestimated, either because the stimuli were perceived in a low dimensionality or because of a hesitancy to place estimates near the larger dot's edge. In Experiment 4, the center was deliberately placed near this edge for one- and two-dimensional solutions. Participants still overestimated, indicating an "edge effect" as responsible.

A perceptual center of mass in general terms is the point about which the bulk of an object or objects is seen to be evenly distributed. Most studies investigating this phenomenon have used only one object or coherent clusters of small point-like objects, when in fact a center of mass can exist between multiple large-scale objects. In the present study, we investigated perceived center of mass between two large objects.

The perceptual estimation of center of mass has been shown to be useful in a variety of tasks. Numerous experiments indicate that center of mass is used in estimating relative location (Badcock, Hess, & Dobbins, 1996; Hirsch & Mjolsness, 1992; Morgan, Hole, & Glennerster, 1990; Ward, Casco, & Watt, 1985; Whitaker & Walker, 1988). The stimuli employed in these studies were clusters of small elements, either dots or squares. The task was to judge whether the cluster was aligned to another or to a reference pattern. In most cases, the center of mass, based on the luminance distribution of the clusters or the individual positions of elements within them, provided the best description of the data. Similar results have been obtained using Gaussian blobs, Gabor, and texture patches (Whitaker, McGraw, Pacey, & Barrett, 1996), as well as outline contour figures (Bocheva & Yakimoff, 1996).

Center of mass is additionally useful for grasping and manipulation of objects. Iberall, Bingham, and Arbib (1986) showed that subjects will grasp mechanical parts at this point when assembling them together. According to Bingham and Muchisky (1993a), the center of mass is used to estimate where to place the fingers in grasping, since the object is stable and more easily handled from this point. The task in their study was to grasp, using tongs, triangular and quadrilateral planar shapes cut from

wood. Overall, judgments were quite accurate and corresponded well to each shape's center of mass. In a similar study, Bingham and Muchisky (1993b) found a decrease in errors with an increase in a shape's rotational symmetry. Errors were also reduced in shapes with a greater number of reflective symmetry axes. Since the center of mass falls along axes of symmetry, more axes should increasingly specify its location.

The formula for computing center of mass for round objects with spherical symmetry is $C = (D r_S^N) / (r_S^N + r_L^N)$, where C is the center of mass measured as the distance from the center of the larger body, D is the center-to-center distance between the two bodies, and r_S and r_L are the radii of the smaller and larger bodies, respectively. The superscript N stands for the dimensionality of the bodies. When $N = 3$, the body is a sphere, and when $N = 2$, the body is a disk. Center of mass moves closer to the center of the larger body as the size ratio between it and the smaller body increases. It moves away from this center with an increase in distance. Increasing dimensionality of the bodies increases their mass, moving the center of mass of the two bodies closer to the center of mass of the larger body.

The center of a two-dimensional figure or pattern is also called a *center of gravity*, or *centroid*. It is a geometric center determined as the mean of all the pattern's points (i.e., the average of its x and y coordinates; Yakimoff, Bocheva, & Mitrani, 1990). If the points all have equal mass, then the term *center of mass* and the physical equation discussed above may be used. A centroid calculation can be based on a number of different stimulus properties. These include contours, area, luminance distribution, and the positions of individual points. If the luminance of each point in a figure is equal, then position and luminance information are identical. Proffitt, Thomas, and O'Brien (1983) asked their participants to locate the centroid in bounded shapes. They varied the luminance distribution within the boundary, so that the luminance centroid differed from the boundary centroid. The perceived locations

The results of these experiments were presented at the 39th Annual Meeting of the Psychonomic Society, November 19–22, 1998, in Dallas, TX. Correspondence should be addressed to J. Friedenberg, Department of Psychology, Manhattan College, Manhattan College Parkway, Riverdale, NY 10471 (e-mail: jfrieden@manhattan.edu).

Figure 1. Dot pairs used in Experiment 1 shown at actual size. The vertical marks indicate the true center of mass and were not included in the actual stimuli. The top pair has a size ratio of 1.0 and a distance between centers of 2.0 cm. Size ratio doubles for the middle pair, reducing the center of mass measured as distance from the larger dot's center. Distance increases to 6.0 cm in the bottom pair, increasing center of mass.

were shifted in the direction predicted by luminance but stayed relatively close to boundary-determined positions, suggesting that in these stimuli boundary information was a more powerful determinant of the center.

The results of several studies suggest that the perceived center of dot patterns is influenced more by the distribution of dots within an area than by contour-based or edge-based information. Vos, Bocheva, Yakimoff, and Helsper (1993) studied accuracy in judging the centers of irregular shapes made from dots, bounded contours, and filled areas. Center estimation could be based on dot position, contour, and area, respectively. The data showed a closer fit to area centers than contour centers for contour and filled figures. For dotted figures, the fit was closest to dot centers. In a third experiment, the center of concave angular and U-shaped patterns were judged outside of the bounded region containing the dots. In a final experiment, they varied the density of dots within a right-triangle-shaped region. Estimates were "pulled" to areas of higher density, away from centers determined by contour or area alone.

At least two models have been proposed that specify a mechanism for the determination of a centroid based on a distribution of elements. Bocheva and Mitrani (1993) argue against a centroid computed from contour information. In their model, a Gaussian filter with different spreads is applied to the image. The maximum of the resulting image function is taken as the centroid. Their model predicts shifting of the center toward regions of greater density or luminance. Morgan et al. (1990) describe a centroid extraction process in which large "eclectic unit" receptive fields receive inputs from smaller fields. These smaller fields respond to elements of differing shape, size, and contrast. Position information from these elements are used in computing a center for the cluster or region.

The purpose of the present study was to measure the perceptual estimation of a simple two-body center of mass. We used as our basic stimuli two distinct black dots,

varying in size and distance. In this case, the center of the system always lies on an imaginary line between dot centers. Figure 1 shows some examples of dot pairs used in the first experiment. The true centers of mass are indicated for each with a short vertical line.

An advantage of these stimuli is their simplicity. By varying the size ratio of the dots, we can shift the overall luminance or density distribution of the pair unidimensionally in one direction or another. Similarly, because there are just two dots, position information is reduced to the distance separating each. This should allow us to more accurately assess the contribution that these factors make to a center-of-mass estimate. We can also fit experimentally derived data to the general center-of-mass equation to better understand the way these two factors interact.

There is another reason to investigate center of mass using such stimuli. Most people have extensive practice grasping and manipulating single objects. It is not surprising then that, in some studies using single objects, fairly accurate veridical perception of a center has been found. However, it is not physically possible to manipulate or grasp two spatially separated objects. The closest case to this would be single objects with bimodal mass distributions, such as dumbbells, or two objects resting on a common connecting surface, such as glasses on a tray. Our lack of manipulative experience with these objects may result in lowered accuracy for perceived center of mass relative to the one-object case.

If the perceived center is affected by size ratio, distance, and dimensionality according to the physical center-of-mass equation, then a straightforward set of predictions follow. Center of mass should move in toward the larger dot when size ratio increases. This effect should be stronger when the two bodies are near if distance affects perceived center of mass in the same way as a physical system. When perceived dimensionality is high, the perceived mass of the objects should be higher, moving the center in. If the dots are perceived three-dimensionally, this is equivalent to a volumetric interpretation in which participants see the dots as filled spherical masses. Perceived center of mass measured from the larger dot's center will be small in this case. A two-dimensional interpretation corresponds to perceiving the dots as flat disks lying on the picture plane, in which case the resulting estimates will be somewhat greater. A one-dimensional interpretation corresponds to perceiving the dots as lines the length of their diameters. If this occurs, estimated center values will be largest. We have no a priori expectations that these stimuli will be perceived in anything other than a two-dimensional interpretation, since the dots are inherently two-dimensional and presented on a flat surface.

EXPERIMENT 1

Method

Participants. Ninety undergraduate Manhattan College students voluntarily participated for extra class credit. There were 50 fe-

males and 40 males, with a mean age of 19 years. The students were run in a classroom setting. All participants had normal or corrected-to-normal vision.

Apparatus. The stimuli were pairs of horizontally oriented black-filled dots. The smallest dot measured 0.15 cm in radius. The other dot in a pair was either the same size or larger, with radius size ratios of 1.0, 2.0, or 4.0 times the size of the smaller. Each size ratio occurred at distances of 2.0, 6.0, and 10.0 cm, measured between dot centers. At a viewing distance of about 40 cm, the smaller dot subtended about 0.5° of visual angle. The range of distances was from approximately 3.0° to 14.0° . Unequal-sized pairs were presented with the larger dot on the left and the right. These, together with the equal-sized pairs, made up 15 unique pairs. Three repetitions of unique pairs resulted in 45 total stimuli.

Dot pairs were presented in random order on standard 21.5 × 28 cm sheets of white paper and were spaced apart vertically from dot centers by 5.0 cm. This vertical separation was considered large enough to ensure that other pairs on a page did not interfere with judgments. Pairs were randomly displaced to the left or the right from the page center by 1.0, 2.0, or 3.0 cm to prevent the participants from comparing one pair with another. A minimum margin of 2.5 cm separated dot edges from all page edges. There were 5 pairs per page, resulting in 9 complete stimulus pages.

Procedure. Every participant received a test packet with a consent form, instructions, stimulus sheets, and a debrief form. The written instructions described the stimuli and task. The participants were asked to judge the center of mass by drawing a short vertical line. Center of mass was defined in the instructions as follows:

Your task is to judge the balance point between the dots in each pair. The balance point is a position of equilibrium. One way to think of this point is to imagine the two black dots on either end of a see-saw. The balance point would then be the location of the fulcrum which would balance the see-saw so that it does not tip one way or the other. The balance point can be located anywhere between the centers of the two dots.

A sample dot pair not used in the experiment followed. This illustrative example indicated the center of both dots and the true center of mass. The researcher then answered any questions concerning the experiment, although it was evident that most of the participants intuitively understood the task. All participants were instructed to keep the page upright while viewing.

Following the center-of-mass estimation, the participants filled out a debrief form, in which they were asked several questions concerning the task. These included the dimensionality in which they perceived the dots, whether the dots looked like familiar objects, a description of how they estimated the balance point, and what factors influenced their placing of the lines. They were allowed as much time as necessary to respond. An entire session took approximately 20 min to complete.

Results and Discussion

Center-of-mass estimates were measured as distance in centimeters from the center of the larger dot. We adopted this measure initially because it corresponds to the center-of-mass equation. In this and the succeeding experiments, responses greater than 3 standard deviations (*SDs*) above the mean were considered outliers and were deleted. Responses were analyzed using a (3 × 3) factorial analysis of variance (ANOVA) with size ratio and distance as factors. Effects of size ratio [$F(2,81) = 807.5, p < .01$], distance [$F(2,81) = 4,563.6, p < .01$], and their interaction [$F(4,81) = 224.7, p < .01$] were significant. A Scheffé post hoc comparison, with alpha set at .01, showed that estimates for size ratios of 1.0 ($M = 2.90, SE = 0.06$) were significantly larger than for ratios of 2.0 ($M = 2.10, SE = 0.04$), which were in turn larger than for ratios of

4.0 ($M = 1.64, SE = 0.03$). Estimates were significantly lower for center-to-center distances of 2.0 cm ($M = 0.80, SE = 0.06$) than for distances of 6.0 cm ($M = 2.10, SE = 0.02$), which in turn were lower than for distances of 10.0 cm ($M = 3.40, SE = 0.04$).

When size ratio is 1.0, the dots are the same size. These conditions were deliberately included to serve as controls. In this case, the center of mass is exactly halfway between dot centers, regardless of dimensionality, and the task corresponds to midpoint estimation. The data indicate that the participants were fairly accurate in estimating the center under these conditions.

Perceptual estimation of two-body center of mass was found to behave predictably in response to changes in relative size and distance. Estimates measured as distance from the center of the larger dot decreased as size ratio between the two dots increased. Estimates also increased with larger separation distances. These data conform generally to the physical center-of-mass formula. However, the data for unequal ratios show a consistent overestimation of the true center, for which the participants placed estimates in a direction away from the larger dot. We examined a possible explanation for this result in Experiment 2.

To empirically determine perceived dimensionality of the stimuli, the center-of-mass equation was reduced to $D/C = 1 + T^N$, where T was the ratio r_L/r_S . The proof of this derivation is given in the Appendix. If D/C is plotted versus T , a curve of the form $1 + T^N$, where N was the dimension, is expected. The data were plotted and fit to the curves $D/C = 1 + T^N$, where the dimensional value N was determined by the plotting routine. When this was done, a value of $N = .619$ is obtained, with an R^2 of .454. This value of N was less than 1.0, which does not have a physical correspondence. However, N was closest to a one-dimensional interpretation. Clearly, a three-dimensional representation was not seen. Figure 2 shows D/C as a function of size ratio for estimated center of mass. For comparison, the curve fit to the experimental data is included, along with the curve for $N = 1$.

The participants' self-ratings of the perceived dimensionality of the dots from the debrief did not correspond to the above analysis. There was a surprisingly even distribution of responses among the three possible dimensions. The majority (43%) of participants perceived the dots as two-dimensional, 29% perceived the dots as one-dimensional, and 28% perceived the dots as three-dimensional, with average perceived dimensionality centering at two ($M = 1.97, SE = 0.01$). A simple linear regression of center of mass by perceived dimensionality was not significant. Center-of-mass estimates did not decrease with an increase in perceived dimensionality, as might be expected. The fact that most participants perceived the dots in two dimensions but responded closer to a single dimension suggests that some factor other than perceived dimensionality was affecting the results.

One concern in this study was that individual dot pairs were viewed along with several others on each page. Although adjacent pairs were spaced apart vertically and

Figure 2. The ratio D/C is depicted as a function of dot size ratio. The data points are the medians for each condition in Experiment 1. The dashed line indicates the fit to the empirical data; the solid line shows the function for a dimensionality of one.

offset horizontally from each other, it is conceivable that they could have influenced the participants' judgments. To evaluate this, the mean estimates for each size ratio by distance combination (i.e., each unique stimulus pair) were compared across the different pages on which they occurred. A significant difference would indicate that a pair was judged differently in the context of one page compared with another. Out of 76 possible pairwise page comparisons, only 12 were statistically significant by Scheffé post hoc tests. The resulting .16 significance rate was low relative to the expected familywise error rate of .98 at an alpha level of .05 for that number of comparisons. It seems unlikely then that the surrounding dot pairs found on different pages affected estimates.

EXPERIMENT 2

One explanation for the overestimation seen in Experiment 1 was that the instructions could have biased the participants into using a see-saw or fulcrum analogy. If the stimuli were perceived in such a fashion, it may have been difficult to imagine a fulcrum placed inside the larger dot. This account seems unlikely, since in an everyday context a fulcrum can exist underneath (albeit not inside) a large body. This does occur on a see-saw. Another example is the position of a waiter's hand while holding a serving dish. To test this idea, additional debrief responses from Experiment 1 were analyzed. If a participant responded that he or she used a see-saw or fulcrum strategy or noted that the dots looked familiar as objects on a see-saw or balance, or if a participant stated that he or she used a see-saw or balance as a factor in solving the task, he or she was assigned to a fulcrum cat-

egory. All other responses were assigned as not using a fulcrum. There was no statistical difference between the two groups by a Scheffé post hoc test ($p = .38$). Fulcrum users ($M = 20.42$, $SE = 0.45$) did not overestimate in comparison to nonfulcrum users ($M = 20.87$, $SE = 0.25$).

The above analysis aside, it is still important to more fully elucidate any role a fulcrum interpretation may play in estimating center of mass. Estimates made without a fulcrum interpretation may show a different pattern of results for size ratio and distance. The consistent overestimation seen in Experiment 1 may have been due to a fulcrum interpretation. It could have biased the participants into not placing estimates inside or near the larger dot, since in an everyday context, such as a see-saw, a fulcrum is rarely positioned underneath the larger object. For this reason, Experiment 1 was replicated with any mention of *fulcrum* or *see-saw* removed from the instructions.

Method

Participants. Forty-five undergraduate Manhattan College students, 38 females and 7 males, participated for extra class credit. Their average age was 20 years. All data were collected in a classroom setting. All participants had normal or corrected-to-normal vision.

Apparatus and Procedure. The instructions used were identical to those in Experiment 1, with the exception that the two sentences referring to the see-saw analogy were removed. The identical sample dot pair used earlier followed to serve as an illustration. The ordering and position of the dot pairs on each page were the same as before. The participants again indicated their center-of-mass estimate by placing a short vertical line anywhere between the two dot centers.

This replication of Experiment 1 was performed in conjunction with a second control experiment, described later. Both were included in a single packet. In Form A, the participants responded to the Experiment 1 replication first, followed by the second control experiment. This order was reversed in Form B.

Results and Discussion

A meta-analysis was performed in which Experiment 1 and its replication were treated as a between-subjects variable. This new variable was called *instruction*. It consisted of two levels—fulcrum interpretation and no-fulcrum interpretation—and was included as an additional factor, along with size ratio and distance in a factorial ANOVA. The results showed that instruction did not significantly interact with either size ratio or distance. There was also no three-way interaction between these variables. There was a significant main effect of instruction [$F(1, 134) = 96.1$, $p < .01$]. Estimates in centimeters for the no-fulcrum interpretation were higher ($M = 2.35$, $SE = 0.03$) than were those for the fulcrum interpretation ($M = 2.10$, $SE = 0.02$).

An analysis of perceived dimensionality showed that the majority of participants perceived the dots as one-dimensional (50%), followed by a two- (30%) and three-dimensional (20%) interpretation. Average perceived dimensionality was, however, closest to two ($M = 1.70$, $SE = 0.02$).

The fact that instruction did not interact with either size ratio or distance demonstrates that a nonfulcrum interpretation of the task does not affect the influence of

either of these variables in perceiving center of mass. However, the significant main effect of instruction is an interesting finding. In every condition, the participants overestimated in the no-fulcrum case. This result is the opposite of what one would expect if a fulcrum interpretation were forcing estimates away from the larger dot. In this account, a fulcrum perception would produce comparative overestimation due to a reluctance to place a fulcrum inside or near the large dot.

One explanation is that, without a fulcrum/see-saw analogy, the participants were confused about how to perform the task. Rather than purely judging center of mass, they may have instead taken into account the midpoint between the dot pairs. If the participants were biased by the midpoint, they would overestimate, since the midpoint is always greater than the true center except when size ratio equals one, in which case they are identical. An estimate in this account may thus be the outcome of being pulled toward the larger dot for unequal size ratios by the real center of mass and simultaneously away from it by the midpoint. Support for this comes from the fact that, relative to Experiment 1, a greater portion of responses in the unequal ratio conditions were at or near the midpoint.

EXPERIMENT 3

The quantitative fit between the equation and the data in Experiment 1 may have been poor because there were too few conditions, particularly since the size ratio of 1.0 served as a control. Also, the ratios and distances employed were fairly small. It is conceivable that participants may adopt a different strategy when either the ratios or the distances are greater. Experiment 3 was therefore designed with more conditions, both to provide a better fit and to see whether the previous results generalize to larger ratios and distances.

Method

Participants. Sixty-two undergraduate Manhattan College students, 30 males and 32 females (average age = 19 years), voluntarily participated for extra class credit. All data were collected in a classroom setting. All participants had normal or corrected-to-normal vision.

Apparatus. The radius of the smaller dot was again 0.15 cm. Size ratios were 6.0, 7.0, 8.0, 9.0, and 10.0. A finer ratio spacing of 1.0 was chosen to provide a more accurate function. Center-to-center distances were 14.0, 18.0, and 22.0 cm. The range of distances varied in visual angle from approximately 19.0° to 29.0°. Each size ratio $\times$ distance combination was presented with the larger dot on the left and the right, yielding 30 unique dot pairs. Each of these pairs was repeated three times, for a total of 90 stimuli. The control condition of equal ratios was dropped because this scenario, which amounted to midpoint estimation or a bisection task, has previously been studied (Shuren, Jacobs, & Heilman, 1997). What is of significant theoretical interest here is the change in estimation for asymmetric ratios with distance.

Dot pairs were presented in random order on 21.5 $\times$ 28 cm sheets of white paper, aligned parallel to the long side of the page to accommodate the larger distances. The pairs were spaced apart vertically from dot centers by 12.0 cm and were displaced randomly to

the left or the right from the page center by 0.0, 0.5, and 1.0 cm. The edge of a dot was allowed to come no closer than 1.0 cm from the edge of a page. There were 2 pairs per page and 30 complete stimulus pages.

Procedure. The procedure was identical to that employed in Experiment 1. The participants received a packet containing the stimuli. The written instructions and example were the same as in Experiment 1. After reading the instructions, the participants estimated the center of mass for each pair by drawing a small vertical line. The participants were instructed to keep the page horizontal while viewing, and they were allowed sufficient time to respond. A debrief with the same questions as in Experiment 1 was completed afterward. An entire session lasted about 20 min.

Results and Discussion

A (3 $\times$ 3) factorial ANOVA again showed that effects of size ratio [$F(4,47) = 72.6, p < .01$], distance [$F(2,47) = 178.0, p < .01$], and their interaction [$F(8,47) = 4.5, p < .01$] were significant. Mean center-of-mass estimates decreased successively with increases in size ratio. Scheffé post hoc tests showed that estimates for each level of size ratio differed statistically at an alpha level of .05. Mean estimates for the center-to-center distance of 14.0 cm ($M = 3.90, SE = 0.51$) were significantly less than those for the 18.0-cm distance ($M = 4.70, SE = 0.60$), which were less than those for the 22.0-cm distance ($M = 5.44, SE = 0.66$), at an alpha level of .01.

The general results of Experiment 3 did not differ substantially from those of Experiment 1. Center-of-mass estimates decreased with both an increase in size ratio between the dots and a decrease in distance between their centers. The dimensionality was calculated in the same way as Experiment 1. We found that $N = .519$, with an R^2 of .755. We conclude that the participants' estimates were apparently made in the same way for these larger ratios and distances. Figure 3 depicts D/C as a function of size ratio. The curve fit for the empirical data is presented alongside the curve for $N = 1$. Also as in Experiment 1, the participants' perceived dimensionality as measured by the percentage of total responses was closer to two (43%) than one (28%) or three (29%). Mean estimated dimensionality was centered near two ($M = 2.03, SE = 0.01$).

As was the case in Experiment 1, other dot pairs simultaneously present on each page may have influenced estimates. To investigate this, judgments to each unique size ratio and distance combination were again compared across the different pages on which they occurred. Out of 150 possible pairwise comparisons (10 per unique combination), only 6 were statistically significant. This .04 probability of significance was quite low relative to the experimentwise error rate of .99, indicating that center-of-mass judgments to a particular dot pair were apparently unaffected by surrounding pairs. In fact, context defined this way had much less of an effect than in Experiment 1.

If the true center of mass is far removed from either dot—say, near the middle of the pair—one might expect a relatively normal distribution of responses about it. In this situation, estimates should fall equally to either side, and the skew of the distribution should approximate zero.

Figure 3. The ratio D/C plotted against dot size ratio. Data points are the medians for each condition of Experiment 3. The dashed line is the curve fit to this data; the solid line is the function when dimensionality is one.

This would be the case if estimates were not biased toward one dot or the other. If estimates were biased toward the larger dot, more cases would fall in the lower range, and skew would be positive. Bias away from the larger dot conversely places the bulk of cases in the upper range, producing a negatively skewed distribution. This was the case in our data, whether the larger dot was on the left or the right, since estimates were determined bidirectionally.

Table 1 depicts skew values for each size ratio and distance combination. The data clearly reveal a trend toward positive skew as the true center moved toward the large dot's center. As ratio increased and distance decreased, estimates did get closer to the larger dot, as reflected in the mean values already discussed. However, the response distribution also became more positively skewed, indicating that more and more of the bulk of the distribution's responses were on the lower end. This "piling up" of responses in the direction of the larger dot seems to reflect a hesitancy to place estimates near its edge and could explain the low obtained dimensionality.

EXPERIMENT 4

The data from the early experiments in preliminary analyses correspond best to a one-dimensional fit. This is a surprising result considering the inherent two-dimensionality of the stimuli. One possibility, supported by the skew data of Experiment 3, is that the participants avoided placing responses near the larger dot's edge. Although instructions allowed for placement anywhere between centers, the participants may have assumed that center of mass could exist only in the space between the

two edges and not inside the dots themselves. Since responses based on a one-dimensional interpretation in most conditions also fall outside the edge, it is unclear whether the previous results were due to a low-dimensional interpretation or an assumption that responses cannot be placed near the edge. Experiment 4 was designed to test these two explanations. Conditions were created in which the true center of mass fell on or near the edge for one and two dimensions. If participants perceive the dot pairs in a low dimensionality, they will place estimates near the edge. If instead they assume estimates cannot go inside of the edge, they should place them farther away.

Method

Participants. Fifty-five students, 22 males and 33 females, attending summer school classes at Manhattan College and the College of Mount Saint Vincent volunteered in exchange for extra course credit. Their average age was 27 years. All participants had normal or corrected-to-normal vision.

Apparatus and Procedure. The radius of the smaller dot was fixed at 0.50 cm and subtended 1.5° of visual angle. Size ratios were 2.0, 2.5, and 3.0. Distances were selected so that center of mass was on the edge of the larger dot or one fourth of its radius to the inside and outside. In this experiment, distance was not an independent variable of fundamental interest. It was employed only to a priori determine center-of-mass locations, and it varied from 3.75 to 18.75 cm (5.0° to 25.0° of visual angle). For each size ratio, distances were determined so that the center was thus 0.75, 1.00, or 1.25 the radius of the larger dot. This variable was called *center position* and was done for a one- and two-dimensional interpretation. The large dot was presented on the left and the right. The resulting 36 dot pairs were presented twice, totaling 72 stimuli.

The dot pairs were presented in random order, aligned parallel to the long side of a 21.5×28.0 cm sheet. They were positioned randomly 0.0, ± 0.50 , and ± 1.0 cm from page centers with a 7.0-cm vertical separation. Minimum separation distance to the page border was 2.0 cm. There were 3 dot pairs per page and 24 complete stimulus pages. Instructions and procedure were identical to those of Experiments 1 and 3.

Results and Discussion

The dependent variable for this experiment was accuracy, determined by subtracting the true center of mass, measured as distance from the center of the larger dot, from empirical estimates. This dependent measure was used to assess the difference between the participants' estimates and the actual center of mass as determined by a one- and two-dimensional solution. This measure was named *center-of-mass accuracy*. Positive values indicate overestimation, placing estimates farther away from the true center in a direction away from the edge. Negative values indicate underestimation, placing estimates away

Table 1
Skewness of Center-of-Mass Estimates
by Size Ratio and Distance in Experiment 2

Distance (cm)	Size Ratio				
	6.0	7.0	8.0	9.0	10.0
14.0	0.70	1.37	1.83	2.10	2.36
18.0	0.83	1.04	1.52	1.97	2.23
22.0	0.24	0.42	0.84	1.11	1.54

Figure 4. Mean center-of-mass accuracy measured as the difference between estimate and actual values. Data are for dimensionality = 2 in Experiment 4. Center position is true location of the center, expressed as a ratio of the larger dot. Distance is between dot centers. Bars are ± 1 SEM.

from the true center closer to the edge. Values at or near zero indicate accurate estimation. Overestimation was considered an indicator of any “edge effect.”

Factorial ANOVAs were conducted, with size ratio and center position as variables. These were done separately for each dimensionality. For the one-dimensional case, there was a significant effect of size ratio [$F(2,46) = 55.70, p < .01$] and center position [$F(2,46) = 4.84, p < .05$]. In the two-dimensional case, size ratio [$F(2,46) = 253.90, p < .01$], center position [$F(2,46) = 19.78, p < .01$], and their interaction [$F(4,46) = 6.24, p < .01$] were significant. Figure 4 shows means with standard errors for center position and size ratio in the two-dimensional case.

If the participants perceived the dots one-dimensionally or even two-dimensionally, responses would have corresponded to the actual centers, even if they were near the edge. Instead, estimates were consistently overestimated. Only 9.3% of all responses were at the true center of mass or closer to the larger dot. Clearly, this indicates a reluctance to place estimates near the larger dot, supporting an edge effect rather than a low-dimensionality explanation. In terms of perceived dimensionality, the majority (50%) of participants reported seeing the dots as three-dimensional, 31% reported seeing the dots as one-dimensional, and 19% reported seeing the dots as two-dimensional, with average perceived dimensionality near two ($M = 2.20, SE = 0.01$).

Overestimates in this experiment increased with size ratio for both dimensionalities. Larger dots, because of their prominent contours, may induce responding farther

from the edge. Overestimates also increased as the center of mass moved from inside to the edge and then away from it. This occurred only for large distances, corresponding to size ratios of 2.5 and 3.0 for two dimensions. For all other conditions, the participants appeared to be simply tracking the true center as it moved outward, as indicated by relatively unchanging overestimation values. The results for greater distances suggest that the edge effect peaks somewhere outside the edge, possibly farther out than 0.25 of the larger dot's radius.

Finally, to ensure that no context effect of other dot pairs was present, we again compared responses to unique pairs across different pages. A unique pair was defined not as a size ratio distance combination but instead as a size ratio edge (location of center of mass relative to edge of the larger dot) and dimension combination, because, in this case, these three variables together determined a unique stimulus condition. Only 7 of 108 possible pairwise comparisons (6%) were statistically different for an experimentwise error rate of .99. We conclude that context in this case, as in the previous experiments, did not bias estimates.

GENERAL DISCUSSION

In Experiment 1, we showed that the participants' estimates of the center between two dots were influenced by their relative size and distance. Estimates were made closer to the larger dot as size ratio between the dots increased. Similarly, estimates moved away from the larger dot as the distance between them increased. We predicted these effects on the basis of the physical center-of-mass equation. The findings were generalized and replicated in Experiment 3, with larger ratios and distances. In both experiments, the participants consistently overestimated the true center. In Experiment 4, we demonstrated that this was due—at least, in part—to a reluctance to place estimates near the larger dot's edge.

It was an interesting discovery that, in all the experiments, estimates were infrequently placed at the location of the true center when it was near the edge. This happened even though the instructions stated that the center could occur anywhere between the two dots' centers. One explanation for this—namely, that a fulcrum may be difficult to imagine inside of the larger body—is not supported by the results of Experiment 2. An alternative explanation comes from the Gestalt idea of “fixedness” of the sort encountered in the nine-dot problem (Dunker, 1945). In this problem, most participants are unable to connect nine dots in a square array with four straight lines because of a self-imposed restraint on staying within the boundary determined by the array. Similarly, the participants in our task may have imposed the false constraint on themselves that the center could occur only in the space between the dots and not inside of them.

The overestimation discovered in this study was expressed in the center-of-mass equation as a low perceived dimensionality. Our curve-fit data yielded a dimension-

ality less than one. However, mean self-report measures of perceived dimensionality were closer to two. It is not entirely clear why the empirically derived and self-report measures of dimensionality differed. The converging evidence from our experiments strongly implies an edge effect as responsible for the low-dimensional data obtained via the curve-fitting procedures. We believe that if this effect were attenuated (perhaps by making edge information less apparent), then any curve-fitting estimations of perceived dimensionality would increase to around two, in agreement with the self-reports. Another possibility is that the dimension self-reports made after center judgments did not accurately assess perceived dimensionality during the estimation process itself. These are all legitimate questions for further investigation.

Although the empirical data did not perfectly fit the equation, it is clear that estimates were influenced by size and distance much the same way that the true center of mass in a physical system is. Since this estimation appears useful in tasks such as judging relative location and grasping and manipulation, it may be an instance of the perceptual internalization of the laws of physics (Shepard, 1984, 1987). According to this idea, perceptual interaction with the environment promotes subjective perceptual "laws" similar to those governing the objective physical world. The ecological importance of center-of-mass estimation makes it a likely candidate for such a process.

It is worth mentioning that accuracy for the asymmetric two-object conditions that we employ here was fairly poor relative to accuracy for some of the asymmetric single-object conditions studied by other investigators. It is unclear whether this was due solely to the previously noted lack of experience that the participants had in manipulating mass distributions of this sort. Simple lack of experience would predict a uniform distribution of error, with relatively equal amounts of underestimation and overestimation. Our data consistently show overestimation and seem better accounted for by explanations such as the edge effect.

A final note concerns models of center-of-mass estimation. Our results support the notion that both luminance (manipulated here as size ratio) and boundary information affect judgments in a two-body system. Models such as those proposed by Bocheva and Mitrani (1993), which exclude contour information, may be adequate to describe estimation among multiple small elements but

cannot account for perception of large-scale objects, for which boundary information is more salient.

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APPENDIX

Derivation of Center-of-Mass Formula

For two masses, m_1 and m_2 , the center of mass (COM) along a particular axis is given by

$$\text{COM} = (m_1 X_1 + m_2 X_2) / (m_1 + m_2),$$

where X_1 and X_2 are the distances from an arbitrary reference point for masses m_1 and m_2 , respectively (Haliday, Resnick, & Walker, 1997). Let m_1 be the larger mass and m_2 be the smaller mass. Replace the subscripts with L and S to get

$$\text{COM} = (m_L X_L + m_S X_S) / (m_L + m_S).$$

Since the reference point is arbitrary, let it coincide with the center of mass of the larger mass, m_L . X_L is then zero, and X_S becomes D , the distance between the center of mass of the two objects:

$$\text{COM} = (m_S D) / (m_L + m_S).$$

If the two masses have the same uniform density, the equation can be rewritten in terms of their volumes. The density of an object is its mass divided by its volume:

$$\rho = m / V,$$

so we can express the mass as

$$m = \rho V.$$

Substitution yields the following:

$$\text{COM} = (\rho V_S D) / (\rho V_L + \rho V_S).$$

The density terms cancel, leaving

$$\text{COM} = (V_S D) / (V_L + V_S).$$

We have assumed that the masses are spheres. In this case the equation for volume is

$$V = (4/3) \rho R^3.$$

Substituting this equation to find the center of mass, after canceling the coefficients, yields

$$\text{COM} = (R_S^3 D) / (R_L^3 + R_S^3).$$

Factor out the R_S^3 term in both the numerator and the denominator to obtain

$$\text{COM} = (D) / (T^3 + 1),$$

where $T = R_L / R_S$ is the ratio of the radius of the larger mass to that of the smaller. In a similar fashion, the following equation can be obtained for the one- and two-dimensional cases. In general, the center of mass can be found by

$$\text{COM} = (D) / (1 + T^N),$$

where N is the dimension of the object. $N = 1$ is the one-dimensional result (diameter), $N = 2$ is the two-dimensional result (area), and $N = 3$ is the three-dimensional result (volume).