Notes and Comment

A Thurstonian model for the dual pair (4IAX) discrimination method

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In the dual pair method, the subject is presented with two stimuli in two pairs: One pair is composed of two samples of the same stimulus; the other pair is composed of two samples of different stimuli, one being the same as that in the identical pair. The task of the judge is to select the most different pair. The psychometric function for the dual pair method is derived and expressed in terms of a singly noncentral beta distribution. A table is provided that connects a measure of the degree of difference, d , to the probability of a correct response. This table assumes an unbiased observer and differencing decision rule. A table is provided to give an estimate of the variance of d , the experimental estimate of d . The power of the dual pair method is also investigated, and a formula to determine the sample size required to meet Type I and Type II error specifications is given. The dual pair method appears to be slightly less powerful than the duo-trio and the triangular methods. Experimental investigation is needed to explore the dual pair in applied research work.

A variety of sensory discrimination methods are available for investigating the difference or similarity of sensory stimuli. Some of those traditionally used are the triangular, duo-trio, same-different, 2-AFC (2-alternative forced choice), and 3-AFC methods. In the triangle test, also called 3-interval oddity, three stimuli are presented, two being the same, one being different. The task of the subject is to select the most different stimulus. In the duo-trio method, the subject is first presented with a reference stimulus. Two alternative stimuli are then presented, one being the same as the reference. The subject is instructed to select the stimulus that is most similar to the reference. This is the *reversed* design to that of the ABX method for which the subject is asked to match a single stimulus to one of two references. In the same-different test, pairs of stimuli are presented, sometimes identical, sometimes different. The subject is asked whether

the stimuli are the same or different. Lastly, for the 2-AFC and 3-AFC tests, directional instructions are given. For the 2-AFC test, the subject is required to select the stronger of two stimuli, whereas in the 3-AFC, which has the same presentation design as the triangle test, the strongest or weakest of the three stimuli, depending on the triad presented, needs to be chosen.

Using an approach proposed by Thurstone (1927) and developed in *signal detection theory* (Green & Swets, 1966), it is possible to obtain an estimate of the degree of difference between the stimuli. The population parameter is usually called δ , and its estimate is labeled d' . The 2-AFC and 3-AFC methods are statistically more powerful than the duo-trio and triangular methods. This is due to differences among the decision rules for the methods, due in part to the nature of the information available to the subject. Tables relating the subject's responses for these protocols and δ have been published (see for instance Elliot, 1964; D. M. Ennis, 1993; Frijters, 1982; Frijters, Kooistra, & Vereijken, 1980; Hacker & Ratcliff, 1979; Kaplan, Macmillan, & Creelman, 1978; Macmillan & Creelman, 1991). The variance of the estimate of δ for the triangle, duo-trio, 2-AFC and 3-AFC protocols can also be obtained using a set of tables (Bi, Ennis, & O'Mahony, 1997). The variance of d' can then be used to determine whether the two stimuli are perceived significantly different.

The duo-trio, triangular, and same-different methods have an advantage over the m -AFC methods because they do not require the nature of the sensory difference between the two products to be specified in the instructions. This might be particularly useful when consumers are subjects, since it might be difficult for them to effectively focus on a specific aspect of the product difference. On the basis of experimental evidence, it appears that the same-different method is a more powerful alternative to the triangular and duo-trio methods (Rousseau, Meyer, & O'Mahony, 1998; Rousseau, Rogeaux, & O'Mahony, 1999; Stillman & Irwin, 1995).

The dual pair is a modified version of the same-different method. In this protocol, two pairs of samples are presented simultaneously to the subject. One pair is composed of samples of the same stimulus, whereas the other is composed of samples of different stimuli, one of the stimuli being the same as that of the identical pair. The task of the subject is to select the most different pair of the two. This protocol has been called the 4-interval AX or 4IAX method (Creelman & Macmillan, 1979; Kaplan et al., 1978; Macmillan, Kaplan, & Creelman, 1977; Noreen, 1981).

One issue regarding these discrimination paradigms is that of response bias. Tables developed by Kaplan et al.

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(1978) and Macmillan and Creelman (1991) take into account response bias for the same-different, ABX, and dual pair (4IAX) methods. Another model exists to incorporate this cognitive factor into the 2-AFC data analysis. These models involve the proportions of hit and false alarm rates calculated from the experiment. Such proportions allow for the determination of the unbiased measure of sensitivity d' . Other models, like that developed by Versfeld, Dai, and Green (1996) for the triangle/3-interval oddity task, cannot take response bias into account and make the assumption that the observer is unbiased. This is also the assumption made in the tables developed by D. M. Ennis (1993) and J. P. Ennis, D. M. Ennis, Yip, and O'Mahony (1998). For these models, only the overall proportion of correct responses is considered in order to calculate d' .

In the present paper, a table for the dual pair test relating proportion of correct responses and δ , thus assuming an unbiased observer, is given. If the reader desires to incorporate response bias in d' calculation, the tables provided by Kaplan et al. (1978) should be consulted. The table presented here is derived from the dual pair psychometric function and is an addendum to those involving the proportion of correct responses described previously. Although it could be argued that, since no response bias is assumed, the proportion of correct responses appears to be a good alternative to δ , it would not permit comparison of results across various protocols. Furthermore, considering the S-shaped psychometric function, variations in proportion of correct answers would not precisely relate to the variation in the underlying perceived difference between stimuli. This difference is better described by δ .

Another particularity of the same-different, triangle, duo-trio, and dual pair protocols described above is that at least two alternate cognitive strategies can be assumed. When using the first strategy, the subject compares the relative perceptions of each stimulus, whereas for the second strategy, the subject considers the absolute perceptions of the stimuli, which involves the categorization of the sensations. These decision rules have been called the *sensory difference decision rule* and *optimal decision rule* (Noreen, 1981), the *differencing model* and the *independent observation model* (Macmillan & Creelman, 1991) and the *differencing model* and the *optimal model* (Stillman & Irwin, 1995). The differencing strategy has been described as nonoptimal compared with the alternative strategy (Macmillan et al., 1977), since the subject's performance would be lower. The conditions for using the two strategies have been reported to depend on the complexity of the stimuli (Irwin & Francis, 1995) and on the roving or fixed design of stimulus presentation (Dai, Versfeld, & Green, 1996; Macmillan & Creelman, 1991; Versfeld et al., 1996). For further discussion about this issue, see Rousseau (2001).

The model developed in the present paper assumes the differencing strategy for two reasons: First, this model would be of particular use in the food and personal care industry, where the nature of the difference between the stimuli being compared is infrequently known, preventing the categorization of the sensations, thus the optimal deci-

sion rule. If the nature of the difference was known and could be easily labeled, a 2-AFC or 3-AFC protocol would then be preferred since they are statistically more powerful (D. M. Ennis, 1993). Second, as noted previously, the information presented here is an extension to other published material using the same approach (D. M. Ennis, 1993; J. M. Ennis et al., 1998). Therefore, it is recommended to refer to Kaplan et al. (1978) and Macmillan and Creelman (1991) for further information about the independent-observations strategy in the dual pair method and other protocols.

As outlined earlier, in this paper we provide the psychometric function for the dual pair method and a table relating δ to the proportion of correct responses. A table of B parameters to facilitate the computation of the variance of d' , similar to those published by Bi et al. (1997) for other protocols, is also provided. Knowing the variance of d' allows hypothesis testing. Lastly, an equation permitting the calculation of the sample size necessary to detect a given d' , depending on the Type I and Type II errors, is given.

PSYCHOMETRIC FUNCTION FOR THE DUAL PAIR TEST

The psychometric function relates δ to the probability of a correct response, P_c .

$$P_c = 1 - G\left(1; \frac{\delta^2}{2}\right) \quad (1)$$

where G is a singly noncentral beta distribution with noncentrality parameter

$$\frac{\delta^2}{2}$$

evaluated from $-\infty$ to 1. A proof of Equation 1 is given in Appendix A.

Several expressions for a singly noncentral beta distribution have been proposed. A general expression for chi square, beta, and F distributions was provided by D. M. Ennis and Johnson (1993). This form describes the beta distribution as a particular case of an indefinite quadratic form. The psychometric function can then be expressed as

$$P_c = 0.5 + \frac{1}{\pi} \int_0^{\infty} \frac{\sin \theta(x)}{x \rho(x)} dx. \quad (2)$$

With

$$\theta(x) = 0.5 \left(\frac{\delta}{\sqrt{2}} \right)^2 \frac{x}{1+x^2} \quad (3)$$

and

$$\rho(x) = (1+x^2)^{0.5} \exp \left(\frac{0.5 \left(\frac{\delta}{\sqrt{2}} x \right)^2}{1+x^2} \right). \quad (4)$$

Equation 2 is then evaluated numerically to construct Table 1.

Table 1
Probability of a Correct Response ($\times 10^4$) as a Function of δ for the Dual Pair Method

δ	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	5000	5000	5000	5000	5001	5001	5002	5003	5005	5006
0.1	5007	5009	5011	5013	5015	5017	5020	5022	5025	5028
0.2	5031	5034	5038	5041	5045	5049	5053	5057	5061	5066
0.3	5071	5075	5080	5085	5091	5096	5102	5107	5113	5119
0.4	5125	5131	5138	5144	5151	5158	5165	5172	5179	5187
0.5	5194	5202	5210	5218	5226	5234	5243	5251	5260	5269
0.6	5278	5287	5296	5305	5315	5324	5334	5344	5354	5364
0.7	5374	5384	5395	5405	5416	5427	5438	5449	5460	5471
0.8	5483	5494	5506	5517	5529	5541	5553	5565	5578	5590
0.9	5603	5615	5628	5641	5653	5666	5679	5693	5706	5719
1.0	5733	5746	5760	5774	5787	5801	5815	5829	5843	5858
1.1	5872	5886	5901	5915	5930	5944	5959	5974	5989	6004
1.2	6019	6034	6049	6064	6079	6095	6110	6126	6141	6157
1.3	6172	6188	6204	6219	6235	6251	6267	6283	6299	6315
1.4	6331	6347	6363	6380	6396	6412	6429	6445	6461	6478
1.5	6494	6511	6527	6544	6560	6577	6593	6610	6627	6643
1.6	6660	6677	6693	6710	6727	6744	6760	6777	6794	6811
1.7	6828	6844	6861	6878	6895	6912	6929	6945	6962	6979
1.8	6996	7013	7029	7046	7063	7080	7097	7113	7130	7147
1.9	7164	7180	7197	7214	7230	7247	7264	7280	7297	7313
2.0	7330	7346	7363	7379	7396	7412	7428	7445	7461	7477
2.1	7494	7510	7526	7542	7558	7574	7590	7606	7622	7638
2.2	7654	7670	7686	7702	7717	7733	7749	7764	7780	7795
2.3	7811	7826	7842	7857	7872	7888	7903	7918	7933	7948
2.4	7963	7978	7993	8008	8022	8037	8052	8066	8081	8095
2.5	8110	8124	8138	8153	8167	8181	8195	8209	8223	8237
2.6	8251	8265	8278	8292	8306	8319	8333	8346	8359	8373
2.7	8386	8399	8412	8425	8438	8451	8464	8477	8490	8502
2.8	8515	8527	8540	8552	8564	8577	8589	8601	8613	8625
2.9	8637	8649	8661	8672	8684	8696	8707	8719	8730	8741
3.0	8753	8764	8775	8786	8797	8808	8819	8830	8840	8851
3.1	8861	8872	8882	8893	8903	8913	8924	8934	8944	8954
3.2	8964	8973	8983	8993	9003	9012	9022	9031	9040	9050
3.3	9059	9068	9077	9086	9095	9104	9113	9122	9131	9139
3.4	9148	9156	9165	9173	9182	9190	9198	9206	9214	9222
3.5	9230	9238	9246	9254	9262	9269	9277	9284	9292	9299
3.6	9307	9314	9321	9328	9336	9343	9350	9357	9363	9370
3.7	9377	9384	9390	9397	9404	9410	9416	9423	9429	9435
3.8	9442	9448	9454	9460	9466	9472	9478	9484	9489	9495
3.9	9501	9506	9512	9518	9523	9528	9534	9539	9544	9550
4.0	9555	9560	9565	9570	9575	9580	9585	9590	9595	9599
4.1	9604	9609	9613	9618	9622	9627	9631	9636	9640	9644
4.2	9649	9653	9657	9661	9665	9669	9673	9677	9681	9685
4.3	9689	9693	9697	9700	9704	9708	9711	9715	9718	9722
4.4	9725	9729	9732	9735	9739	9742	9745	9749	9752	9755
4.5	9758	9761	9764	9767	9770	9773	9776	9779	9782	9785
4.6	9787	9790	9793	9795	9798	9801	9803	9806	9809	9811
4.7	9814	9816	9818	9821	9823	9826	9828	9830	9832	9835
4.8	9837	9839	9841	9843	9845	9848	9850	9852	9854	9856
4.9	9858	9860	9862	9863	9865	9867	9869	9871	9873	9874
5.0	9876	9878	9879	9881	9883	9884	9886	9888	9889	9891
5.1	9892	9894	9895	9897	9898	9900	9901	9903	9904	9905
5.2	9907	9908	9909	9911	9912	9913	9914	9916	9917	9918
5.3	9919	9921	9922	9923	9924	9925	9926	9927	9928	9929
5.4	9930	9931	9932	9933	9934	9935	9936	9937	9938	9939
5.5	9940	9941	9942	9943	9944	9944	9945	9946	9947	9948
5.6	9949	9949	9950	9951	9952	9952	9953	9954	9954	9955
5.7	9956	9957	9957	9958	9959	9959	9960	9960	9961	9962
5.8	9962	9963	9963	9964	9965	9965	9966	9966	9967	9967
5.9	9968	9968	9969	9969	9970	9970	9971	9971	9972	9972

Table 1 gives the relationship between δ and P_c for the dual pair test. For instance, if $P_c = .6429$, then $\delta = 1.46$.

The model presented here is the same as that of Macmillan et al. (1977). They expressed the proportion of correct responses as a function of δ as

$$p_c = [\Phi(\delta/2)]^2 + [1 - \Phi(\delta/2)]^2, \quad (5)$$

where Φ is the normal integral. However, we are usually interested in the opposite relationship: p_c is known and δ needs to be estimated. By completing the square, we find that

Figure 1. The power of five difference testing methods: 3-AFC, 2-AFC, triangle, duo-trio, and dual pair as a function of δ at $\alpha = .05$ and for a sample $N = 50$.

$$\delta = 2\Phi^{-1}\left[\sqrt{\frac{p_c}{2} - \frac{1}{4} + \frac{1}{2}}\right]. \quad (6)$$

Equation 6 is easy to use to estimate δ using normal tables. However, Table 1 of the present paper permits the determination of δ without any calculations. It corresponds to the values given in Table 3 of Kaplan et al. (1978) for an unbiased observer using the differencing strategy. Nevertheless, Table 1 is given in order to complement similar tables available elsewhere for the triangle, duo-trio, 2-AFC, 3-AFC, and tetrad tests. It is interesting to note that it is also the equation given by Noreen (1981) to describe the proportion of correct answers for an observer using the optimal strategy in the same-different task.

THE POWER OF THE DUAL PAIR TEST

Ennis gave a general power equation for sensory discrimination methods for a given Type I error α , a sample size n , and a degree of difference δ . The sample size n represents the total number of tests performed by a given subject or combined over all subjects.

$$\text{Power} = 1 - \Phi\left[\frac{\sqrt{n} - \sqrt{n}(\delta)}{\sqrt{f(\delta)(1 - f(\delta))}}\right] \quad (7)$$

where

$$u = \frac{1}{m} + \frac{\Phi^{-1}(1 - \alpha)\sqrt{m - 1}}{m\sqrt{n}},$$

m is the number of decision alternatives (2 for the dual pair test), and $f(\delta)$ is the method's psychometric function evaluated at δ .

EXAMPLE. Let $n = 100$, $m = 2$, $\alpha = .05$, and $\delta = 0.74$. From Table 1 and Equation 1, $f(\delta) = 0.542$.

$$u = \frac{1}{2} + \frac{\Phi^{-1}(0.95)}{20} = 0.5822.$$

From Equation 2,

$$\text{Power} = 1 - \Phi\left[\frac{10 \times 0.5822 - 10 \times 0.542}{\sqrt{0.542 \times 0.458}}\right] = 0.21.$$

This value can be compared to a power of 0.24 for the duo-trio test, 0.26 for the triangle test, 0.995 for the 2-AFC, and of 0.998 for the 3-AFC test with the same parameters. Thus, at $\delta = 0.74$, the dual pair test is slightly less powerful than the duo-trio and triangle procedures.

Figure 1 illustrates the power of the five procedures for a sample size of $N = 50$. It can be seen that the dual pair test is the least powerful of all, even though the difference in power between the dual pair and the duo-trio is minimal compared with the difference in power between these two protocols and the 2-AFC method.

A general equation to calculate the sample size required for a given δ , and Type I and Type II errors is given in Appendix B.

Table 2
***B* Values for Estimation of the Variance of d' Obtained From the Dual Pair Method**

δ	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0		105684	105684	105684	7606	7606	3887	2607	1570	1310
0.1	1123	874.7	716.3	606.5	526.0	464.4	395.2	359.5	316.6	283.0
0.2	255.8	233.5	209.2	194.0	177.0	162.8	150.7	140.3	131.2	121.2
0.3	112.9	107.1	100.6	94.81	88.74	84.26	79.47	75.88	71.99	68.49
0.4	65.33	62.46	59.42	57.05	54.53	52.23	50.12	48.19	46.40	44.53
0.5	43.02	41.42	39.94	38.57	37.30	36.11	34.87	33.85	32.77	31.76
0.6	30.82	29.93	29.10	28.33	27.51	26.82	26.10	25.42	24.78	24.17
0.7	23.59	23.05	22.49	22.00	21.49	21.01	20.55	20.12	19.71	19.31
0.8	18.90	18.55	18.17	17.85	17.51	17.18	16.87	16.58	16.27	16.00
0.9	15.71	15.47	15.21	14.96	14.74	14.51	14.29	14.06	13.86	13.66
1.0	13.46	13.28	13.09	12.91	12.75	12.58	12.42	12.26	12.11	11.96
1.1	11.82	11.68	11.54	11.41	11.28	11.16	11.04	10.92	10.81	10.70
1.2	10.59	10.48	10.38	10.28	10.19	10.09	10.00	9.910	9.826	9.739
1.3	9.660	9.578	9.499	9.427	9.352	9.280	9.210	9.141	9.074	9.010
1.4	8.947	8.886	8.827	8.766	8.710	8.656	8.600	8.548	8.499	8.447
1.5	8.400	8.352	8.307	8.261	8.219	8.176	8.136	8.095	8.056	8.019
1.6	7.982	7.945	7.912	7.878	7.844	7.812	7.782	7.752	7.722	7.693
1.7	7.666	7.640	7.614	7.589	7.564	7.540	7.517	7.497	7.475	7.454
1.8	7.434	7.415	7.397	7.379	7.362	7.345	7.329	7.315	7.300	7.286
1.9	7.273	7.260	7.248	7.236	7.226	7.215	7.205	7.196	7.187	7.179
2.0	7.172	7.165	7.158	7.152	7.146	7.142	7.137	7.133	7.129	7.126
2.1	7.124	7.122	7.120	7.119	7.118	7.118	7.118	7.119	7.120	7.122
2.2	7.124	7.127	7.130	7.133	7.137	7.141	7.146	7.151	7.157	7.162
2.3	7.169	7.176	7.183	7.191	7.199	7.208	7.217	7.226	7.236	7.246
2.4	7.256	7.267	7.279	7.291	7.303	7.316	7.329	7.342	7.356	7.370
2.5	7.385	7.400	7.415	7.432	7.448	7.465	7.482	7.499	7.517	7.536
2.6	7.555	7.574	7.593	7.614	7.635	7.655	7.677	7.698	7.720	7.743
2.7	7.766	7.789	7.813	7.837	7.862	7.887	7.913	7.940	7.967	7.992
2.8	8.021	8.047	8.077	8.105	8.133	8.165	8.194	8.225	8.255	8.287
2.9	8.319	8.352	8.386	8.417	8.452	8.488	8.521	8.558	8.593	8.628
3.0	8.667	8.704	8.742	8.780	8.819	8.859	8.899	8.941	8.979	9.022
3.1	9.062	9.107	9.148	9.195	9.238	9.282	9.331	9.377	9.424	9.471
3.2	9.520	9.564	9.614	9.666	9.718	9.766	9.820	9.870	9.921	9.979
3.3	10.03	10.09	10.14	10.20	10.25	10.31	10.37	10.43	10.49	10.55
3.4	10.61	10.67	10.74	10.80	10.87	10.93	10.99	11.06	11.13	11.19
3.5	11.26	11.33	11.41	11.48	11.55	11.62	11.70	11.77	11.85	11.92
3.6	12.01	12.08	12.16	12.24	12.33	12.41	12.49	12.58	12.65	12.74
3.7	12.83	12.92	13.00	13.10	13.20	13.28	13.37	13.47	13.56	13.66
3.8	13.77	13.86	13.96	14.06	14.17	14.27	14.38	14.49	14.58	14.70
3.9	14.81	14.91	15.03	15.16	15.26	15.37	15.50	15.61	15.73	15.87
4.0	15.99	16.11	16.24	16.36	16.49	16.63	16.76	16.90	17.05	17.16
4.1	17.31	17.46	17.59	17.74	17.87	18.04	18.17	18.35	18.49	18.64
4.2	18.82	18.97	19.13	19.28	19.45	19.61	19.78	19.95	20.13	20.31
4.3	20.49	20.68	20.88	21.02	21.23	21.43	21.59	21.81	21.97	22.20
4.4	22.37	22.61	22.79	22.98	23.23	23.43	23.63	23.90	24.11	24.32
4.5	24.54	24.77	25.00	25.23	25.47	25.72	25.97	26.23	26.49	26.76
4.6	26.95	27.23	27.52	27.72	28.02	28.34	28.55	28.88	29.21	29.44
4.7	29.80	30.04	30.28	30.67	30.93	31.33	31.60	31.88	32.17	32.61
4.8	32.91	33.22	33.53	33.85	34.18	34.69	35.04	35.40	35.77	36.15
4.9	36.54	36.93	37.34	37.55	37.97	38.41	38.86	39.32	39.79	40.04
5.0	40.53	41.04	41.30	41.84	42.39	42.67	43.25	43.85	44.15	44.78
5.1	45.10	45.76	46.10	46.80	47.15	47.89	48.27	49.04	49.44	49.85
5.2	50.68	51.12	51.56	52.46	52.93	53.41	53.90	54.91	55.43	55.96
5.3	56.51	57.64	58.22	58.82	59.44	60.07	60.71	61.37	62.05	62.75
5.4	63.47	64.20	64.96	65.74	66.46	67.28	68.11	68.98	69.86	70.78
5.5	71.72	72.70	73.70	74.74	75.81	75.81	76.91	78.06	79.24	80.47
5.6	81.74	81.74	83.06	84.43	85.85	85.85	87.33	88.87	88.87	90.48

THE VARIANCE OF d'

Table 2 gives B values that can be used to calculate the variance of d' . This table is similar to those published by Bi et al. (1997). The table was generated by calculating the large sample ($N = 10,000$) variance of d' using the inverse of the second derivative of the maximum likelihood func-

tion with respect to δ . Then the B values were obtained on the basis of the central limit theorem (Bi et al., 1997), $B = Ns^2$, where N is the sample size and s^2 is the variance of d' .

EXAMPLE. From 50 judgments, a total of 37 correct answers were obtained. From Table 1, we get $d' = 2.04$. From Table 2 we get $B = 7.146$. Therefore the variance of d' is: $s^2 = 7.146/50 = 0.143$.

CONCLUSION

In this paper we showed that the psychometric function for the dual pair method can be expressed in terms of a noncentral beta distribution and provided tables to determine d' and its variance. We also compared the power of the method with other commonly used binary and triadic choice methods.

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APPENDIX A

Psychometric Function for the Dual Pair Method

Let P_c represent the probability of a correct response for the dual pair method. X and Y represent the two stimuli to be compared. Let x_1, x_2, x_3 , and y be the percepts generated during one trial of the dual pair method: x_1 and x_2 are the percepts of the identical pair, and x_3 and y are the percepts of the different pair. Assume that x_1, x_2 , and x_3 are randomly drawn from a normal distribution of mean 0 and variance 1 and that y is randomly drawn from a normal distribution of mean δ and variance 1.

A correct response will occur in a trial if the perceptual distance between x_3 and y is larger than the perceptual distance between x_1 and x_2 . The probability of a correct response can thus be written as

$$\begin{aligned} P_c &= P(|x_1 - x_2| < |y - x_3|) \\ &= P((x_1 - x_2)^2 < (y - x_3)^2). \end{aligned}$$

Let $z_1 = x_1 - x_2$ and $z_2 = y - x_3$. z_1 is distributed normally with mean 0 and variance 2, and z_2 is distributed normally with mean δ and variance 2.

Hence

$$\begin{aligned} P_c &= P(z_1^2 - z_2^2 < 0), \\ &= P(\mathbf{z}'\mathbf{A}\mathbf{z} < 0), \end{aligned} \tag{A1}$$

where

$$\mathbf{z}' = (z_1, z_2), \mathbf{z} = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}, \text{ and } \mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

APPENDIX A (Continued)

$\mathbf{z}$ has a bivariate normal distribution with mean vector

$$\mu = \begin{pmatrix} 0 \\ \delta \end{pmatrix}$$

and variance-covariance matrix

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}.$$

Standardizing $\mathbf{z}$,

$$\begin{aligned} \mathbf{w} &= \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}^{-1} (\mathbf{z} - \mu) \\ &= L^{-1}(\mathbf{z} - \mu) \end{aligned}$$

Hence $\mathbf{z} = L\mathbf{w} + \mu$

Substituting for $\mathbf{z}$ in Equation 1

$$\begin{aligned} P_c &= P((L\mathbf{w} + \mu)' A (L\mathbf{w} + \mu) < 0) \\ &= P\left(\left[\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \delta \end{pmatrix}\right]' \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \left[\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \delta \end{pmatrix}\right] < 0\right) \\ &= P\left(\begin{bmatrix} \sqrt{2}w_1 \\ \sqrt{2}w_2 + \delta \end{bmatrix}' \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \sqrt{2}w_1 \\ \sqrt{2}w_2 + \delta \end{bmatrix} < 0\right) \\ &= P\left(\begin{pmatrix} \sqrt{2}w_1, -\sqrt{2}w_2 + \delta \end{pmatrix} \begin{pmatrix} \sqrt{2}w_1 \\ \sqrt{2}w_2 + \delta \end{pmatrix} < 0\right) \\ &= P\left(2w_1^2 - (\sqrt{2}w_2 + \delta)^2 < 0\right) \\ &= P\left(2\left(w_1^2 - \left(w_2 + \frac{\delta}{\sqrt{2}}\right)^2\right) < 0\right) \\ &= P\left(\frac{w_1^2}{\left(w_2 + \frac{\delta}{\sqrt{2}}\right)^2} < 1\right) \\ &= 1 - P\left(\frac{\left(w_2 + \frac{\delta}{\sqrt{2}}\right)^2}{w_1^2} < 1\right) \end{aligned} \tag{A2}$$

The term

$$\frac{\left(w_2 + \frac{\delta}{\sqrt{2}}\right)^2}{w_1^2}$$

follows a singly noncentral beta distribution.

From Ennis and Johnson (1993)

$$P\left(\frac{\left(w_2 + \frac{\delta}{\sqrt{2}}\right)^2}{w_1^2} < 1\right) = 0.5 - \frac{1}{\pi} \int_0^\infty \frac{\sin \theta(x)}{x \rho(x)} dx,$$

with

$$\theta(x) = 0.5 * \left[\tan^{-1}(x) + \left(\frac{\delta}{\sqrt{2}}\right)^2 \frac{x}{1+x^2} - \tan^{-1}(x) \right],$$

APPENDIX A (Continued)

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and

$$\rho(x) = (1+x^2)^{0.5} \exp \left(\frac{0.5 \left(\frac{\delta}{\sqrt{2}} x \right)^2}{1+x^2} \right).$$

Thus,

$$P_c = 0.5 + \frac{1}{\pi} \int_0^{\infty} \frac{\sin \theta(x)}{x \rho(x)} dx.$$

APPENDIX B
Sample Size Calculation

A general equation to estimate the sample size needed for a method with psychometric function f , given δ , and Type I error and Type II error is (D. M. Ennis, 1993)

$$n = \left[\frac{\Phi^{-1}(\beta) [f(\delta)(1-f(\delta))]^{1/2} - y}{1/m - f(\delta)} \right]^2$$

where

$$y = \frac{\Phi^{-1}(1-\alpha) \sqrt{m-1}}{m},$$

and m is the number of decision alternatives.

This equation is derived by solving power Equation 2A for n .

Example:

Let $m = 2$, $\alpha = .05$, $\beta = 0.20$, and $\delta = 1.0$. From Table 1, $f(\delta) = 0.5733$.

$$y = \frac{\Phi^{-1}(0.95)}{2} = 0.8225$$

$$n = \left[\frac{\Phi^{-1}(0.20) [0.573 * 0.427]^{1/2} - 0.8225}{0.5 - 0.573} \right]^2 = 288.$$

This value can be compared to a required sample size of 228 for the duo-trio method, 198 for the triangular method, 22 for the 2-AFC method, and 15 for the 3-AFC method with the same parameters. Thus at $\delta = 1.0$, as shown on Figure 1, the dual pair method is slightly less powerful than the duo-trio and triangle procedures.

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