Rote memory and arithmetic fact processing

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The goal of the study was to examine the part played by skill in memorizing arbitrary sequences in the efficiency with which normal young adults perform simple arithmetic fact problems. The first experiment showed a clear independent role for sequence memory in all arithmetic fact processing, but a lesser role for semantic retrieval. This result was particularly true for large-answer multiplication problems and subtraction and division problems with large first operands. In a second experiment, which included a visuomotor processing control task, sequence memory predicted processing of all arithmetic problems apart from small additions independently of semantic retrieval, with the most robust independent contribution being to large-answer multiplication problems. The results, which are compatible with Dehaene and colleagues' triple-code model, suggest that rote learning may be a successful way for some people to process arithmetic facts efficiently.

Knowledge of simple arithmetic facts, such as $7 \times 3 = 21$, forms a fundamental component of mathematical competence because of its involvement in successful solution of many higher level arithmetic tasks, such as multidigit problems, computational sequences, and fractions. In the present study, we wished to investigate the nature of the verbal skills that are associated with mature adults' ability to retrieve such simple arithmetic facts rapidly and accurately. The importance of verbal skill in mathematical thinking has been stressed in recent theorizing by Dehaene and colleagues. Dehaene (1992) and Dehaene and Cohen (1995) put forward a "triple-code" model of numerical cognition, proposing that there are three independent ways of coding numbers, with links between the systems allowing one form of coding to be translated into either of the others. One code is analogical, according to which numbers are seen as having semantic representations along a "mental number line." Computations on the mental number line can be performed nonverbally when the magnitudes are very small, as in the enumeration or comparison of very small quantities, and when the distance between the number of elements to be compared is large, as in approximation. Two other codes are available to mature learners, one for processing Arabic digits, and one for coding numbers verbally. The verbal code is employed in learning and using number names, determining all but very small quantities precisely, and storing and retrieving simple arithmetic facts.

According to the triple-code model, many simple arithmetic facts are learned in the same way as other classes of verbal sequences, such as the alphabet or months of the year—namely, by a process of rote memorization. This claim has been evaluated by classifying arithmetic operations into those that are usually learned as verbal sequences (multiplication) and those that are thought to rely largely on meaningful processing of quantities (sub-

traction and division). Addition is regarded as being performed either by retrieving rote-learned facts or by mental manipulations of quantities. Providing evidence for this view, Dehaene and Cohen (1997) described a patient with a left subcortical lesion who had impaired memory for meaningless and meaningful rote-learned sequences. She had much more trouble with simple multiplication than with addition and subtraction. Another patient with a dominant inferior parietal lesion had spared long-term memory for sequences. She encountered more difficulty with both subtraction and division than with addition and multiplication. Cohen, Dehaene, Chochon, Lehéricy, and Naccache (2000) investigated a patient with damage to the left perisylvian area that had left her aphasic and severely impaired in processing numbers in a verbal format. With an Arabic format, she made very few errors on the retrieval of simple addition and subtraction facts, although she was extremely poor at multiplication. Functional MRI showed that a right parietal network was activated during the subtraction task, whereas multiplication activated the area near the left angular gyrus. Investigating two patients with brain lesions in different areas, Lemer, Dehaene, Spelke, and Cohen (2003) also recently showed a double dissociation between multiplication and subtraction. This dissociation generalized to other forms of numerical processing—such as exact calculation, as compared with approximation, and the enumeration of small numbers of objects, as compared with the counting of large numbers of objects—suggesting that the dissociations were based on separate nonverbal and verbal processing systems.

There is thus good evidence from brain-injured adults that skill at committing arbitrary sequences to memory contributes to the ability to retrieve some arithmetic facts rapidly and accurately. However, work on arithmetic fact representation and retrieval in normal, skilled adults has

not paid much attention to a role for this type of memory. Rather, prominent models, such as those of Campbell (1987) and Ashcraft (1992), have proposed an associative network model for the storage of arithmetic facts in memory, in which individual facts—particularly those of addition and multiplication—are thought to be accessed and retrieved via a process of spreading activation. This conceptualization is supported by numerous findings, such as the fact that when people make errors, they are more likely to give an answer to another problem containing the same operand than to give an answer to one that is completely different—for example, giving 56 as an answer to 7×9 (Campbell, 1994). Even if the facts are laid down in the form of a network, it is still possible that skill at memorizing sequences might assist in the coding and retrieval of the stored information. The goal of the present study was thus to evaluate the extent to which skill in memorizing arbitrary sequences plays a role in normal adults' abilities in remembering simple arithmetical facts. In line with Dehaene and colleagues' terminology, this memory skill will be called *rote memory*. In order to evaluate any independent role of rote memory, it is important to distinguish it from skill in retrieving information that has not been transferred to memory in this deliberate way. Accordingly, we contrasted the retrieval of elements from rote-learned nonnumerical sequences with the retrieval of semantic properties of familiar nonnumerical verbal concepts. We expected, nevertheless, that there would be many common processes involved in the two types of memory tasks.

In the two experiments presently reported, we assessed skill in solving simple facts using the four operations of addition, multiplication, subtraction, and division. As noted earlier, Dehaene and colleagues' model assumes that solving multiplication problems will be best achieved by retrieval from memory of a verbally coded proposition, whereas addition is regarded as being soluble either by such a procedure or by quantitative manipulations. This assertion is compatible with the finding that adults, such as introductory university students, report using strategies other than direct memory retrieval much more often for addition than for multiplication (Hecht, 1999; Smith-Chant & LeFevre, 2003). Moreover, subtraction is thought to be performed not by retrieving facts directly from memory, but by various analogical or counting procedures. In the face of brain damage, simple subtraction can still be performed even when multiplication is severely impaired (Cohen et al., 2000; Lee, 2000; Sandrini, Miozzo, Cotelli, & Cappa, 2003). In support of this finding, Lee and Kang (2002) found that normal young Korean adults were impaired in performing subtractions when they had to perform a concurrent visuospatial working memory task, whereas they were impaired in performing multiplications when they had to perform a concurrent spoken nonword task. Because the responses to both types of arithmetic problem were made in the same way, the researchers argued that the selective effects were operating at the "core stage" of number processing and that different processing strategies were being used. Dehaene, Piazza, Pinel, and Cohen (2003) suggested that the contrast between multiplication and subtraction provides a good test of the

distinct-system model, at least in relation to neuropsychological dissociations.

The operation of division has rarely been studied in neuropsychological cases, so we know little about how division patterns with the other arithmetic operations as a function of brain damage. As far as we are aware, children do not usually acquire knowledge of division problems as arbitrary sequences, the way they often learn multiplication facts. Instead, they may learn that the best way to solve divisions is by focusing on the divisor and iteratively going through the multiplication facts for that divisor until the answer is the same as the target dividend. Investigating strategies used for division by normal young adults, LeFevre and Morris (1999) found that participants did indeed report recasting large division problems as multiplications, testing the validity of their multiplication answers against the dividend. Thus, although division facts themselves might not have been rote learned as whole facts, they might still be solved more effectively by someone who can perform multiplications efficiently; thus, good rote sequence memory might enhance division ability. Alternatively, sequence memory might be of reduced importance, if the dominant process involves deciding which multiplication facts to activate during the iterative testing procedure.

For addition and multiplication, we divided the problems into those having a small answer and those having a large answer, expecting to find the well-known problem-size effect, with bigger problems taking longer to solve and causing more errors (see Zbrodoff & Logan, 2005). We also expected rote sequence memory to be of greater assistance for large-answer than for small-answer problems. For addition, adults claim to retrieve answers directly from memory over 80% of the time for small-answer problems (when the answer is 10 or less), but they report direct retrieval much less often as answers increase in size, going down to about 50% of trials when there is no time deadline (Campbell & Austin, 2002; LeFevre, Sadesky, & Bisanz, 1996). Answers obtained by direct retrieval were supplied more quickly than answers requiring a decompositional process (Campbell & Austin, 2002). Given the greater difficulty of the large-answer problems, being able to retrieve an intact verbal proposition from memory, rather than using some form of decompositional strategy, should enhance performance more for these problems than for those with a small answer.

EXPERIMENT 1

In Experiment 1, we assessed skill in solving simple addition, multiplication, subtraction, and division problems presented in Arabic numerals. For comparability, visual presentation was employed for all tasks. We used a verification format, with responses being made with buttonpresses so that any differences between participants in phonological retrieval skill and speech rate would not come into play. Although research has indicated that there might be some differences between producing and verifying answers to fact problems (Dagenbach & McCloskey, 1992; Zbrodoff & Logan, 1990), it is still the case that the participant has to know the answer in order to respond correctly in a verification task, so long as the response cannot be determined

by approximation. In order to encourage the retrieval of the answer given the arithmetic problem, the problem was presented temporally prior to the answer. The verbal tasks were therefore also presented in two temporal phases. For rote memory, participants saw an element from a well-learned sequence and then had to verify whether a target element followed it in the sequence. For semantic retrieval, the participant first viewed a category word and then had to verify whether a target word was a member of the category. By varying how typical the instances were of the category word, we expected to demonstrate a typicality effect, with typical instances being categorized more quickly and accurately than atypical instances. This manipulation was included simply to demonstrate that participants were performing the task in an attentive and predictable manner. The more automatically retrieved typical items were the focus of interest in predicting arithmetic retrieval skill. For all tasks, a self-paced presentation mode was adopted so that each participant had a chance to process the stimuli to the depth that they felt they required.

Method

Participants

A sample of 72 undergraduate and graduate students at the University of Melbourne volunteered to take part in the experiment. They either fulfilled requirements for the Psychology 1 Research Participation Programme or were paid a small sum of money. There were 29 males and 43 females, and their ages ranged from 18 to 34 years, with a mean of 21 years, 11 months. All participants were native speakers of English.

Materials and Tasks

Rote sequence memory. Inspection stimuli were the 26 individual letters of the alphabet, the 12 months of the year, and the 7 days of the week. On positive trials, each was followed by the letter, month, or day that normally follows in the particular sequence. On negative trials, each was followed by the element that normally precedes it in the sequence. There were 44 positive and 45 negative trials, with elements from the three different classes randomly interspersed. In order not to give participants practice on a subset of the pairs that would follow, they were told that the practice items would be slightly different but that they would still have to decide whether they were in the correct sequence. The 10 practice items contained pairs such as *Queen Elizabeth* and *States United*.

Semantic retrieval. Eight category words were presented as inspection stimuli: *fruit*, *vegetable*, *animal*, *body part*, *furniture*, *vehicle*, *weapon*, and *clothing*. Sixteen instances of each of these categories were selected, half of which were highly prototypical (e.g., *vegetable-carrot*), and the other half of which were less prototypical (e.g., *vegetable-mushroom*). For simplicity, the terms *typical* and *atypical* will be used for these conditions. The target words were then allocated to the appropriate category or to an inappropriate category, keeping the inappropriate cases within the general class of living or artifactual categories. Thus, examples of negative trials are *animal-broccoli* and *weapon-wardrobe*. The length of target words was matched across the conditions, with an average of 5.4 letters, and frequency of occurrence was matched across conditions within typicality (Kučera & Francis, 1967). Typical words averaged 52 occurrences/million, and atypical words averaged 18 occurrences/million. Practice items provided one positive and one negative instance of each of the categories that would follow, but used target words that were not on the test list.

Addition. All 45 addition facts involving the numbers 1–9 were presented in Arabic format—for example, $7 + 3$. The larger number was always presented on the left. The problems were classified into the 25 whose solutions came to 10 or lower and the 20 whose solu-

tions came to more than 10. The 45 problems were presented once with the correct answer and once with an answer that differed by one from the correct solution—for example, 11 for $7 + 3$. Whether the answer was 1 more or 1 less than the correct solution was pseudorandomized throughout the problems. Prior to the 90 test items, 10 practice trials containing problems different from the test problems were presented. Participants were instructed to respond “yes” if the answer was correct and “no” if it was incorrect.

Multiplication. All 45 multiplication facts involving the numbers 1–9 were presented in Arabic format—for example, 3×7 . The smaller number was always presented on the left. Twenty-three problems whose solution came to 20 or lower were classified as having small answers, and the remaining 22 with answers above 20 were classified as having large answers. The 45 problems were presented once with the correct answer and once with an incorrect answer that was different by 1 in the set of appropriate “table” answers—for example, 18 for 3×7 . Whether the answer was larger or smaller than the correct solution was pseudorandomized throughout the problems. Prior to the presentation of the 90 test items, 10 practice trials containing different problems from those on the test were presented. Participants were instructed to respond “yes” if the answer was correct and “no” if it was incorrect.

Subtraction. The 45 addition problems were transformed into subtraction problems—for example, $7 + 3$ became $10 - 3$. The problems were presented either with the correct answer or with an answer that differed from the correct answer by 1 (either 1 higher or 1 lower). Correct answers thus ranged from 1 to 9, and incorrect answers ranged from 1 to 10. Problems that had been classified as small-answer addition problems were classified as small subtraction problems, and problems that had been classified as large-answer addition problems were classified as large subtraction problems.

Division. The 45 multiplication problems were transformed into division problems—for example, 5×8 became $40 \div 8$. The problems were presented with either the correct answer or an answer that differed from the correct answer by 1 (either 1 higher or 1 lower). Correct and incorrect answers ranged from 1 to 9. Problems that had been classified as small-answer multiplication problems were classified as small division problems, and problems that had been classified as large-answer multiplication problems were classified as large division problems.

General Procedure

Each task was presented on a Pentium personal computer, with responses being made with a box containing two buttons marked “yes” and “no.” The tasks were controlled by the DMDX program (Forster & Forster, 2003). Lowercase letters, words, or Arabic-digit arithmetic problems were presented in the center of a computer screen. The program ensured that each participant received a different pseudorandom order of presentation, with items from each condition distributed throughout an experimental task. Participants were instructed that as soon as they had processed the inspection letter or word for the rote memory task, the category word for the semantic retrieval task, or the arithmetic problem, they were to press the “yes” button. Doing so would cause the first stimulus to disappear and bring on the next letter or word for the rote-memory task, the instance word for the semantic-retrieval task, or the answer for the arithmetic problems. They had to judge the appropriateness of the second stimulus by pressing a “yes” or a “no” button. The “yes” button was always pressed with the right hand, and the “no” button with the left hand. For all tasks, participants were asked to respond as quickly and as accurately as they could, and a new trial began 1 sec after they responded.

Participants completed the tasks in the following order: semantic retrieval, addition, multiplication, rote sequence memory, subtraction, and, finally, division. The data of 1 participant whose error rates approached chance on the subtraction and division tasks and whose inspection times in the addition and multiplication tasks were excessively long were discarded, leaving 71 participants. Two participants appeared to not understand the instructions of the semantic retrieval task, causing error rates higher than 50%, so their data for this task were eliminated.

Data Analysis

Three dependent variables were recorded. Inspection time was the time from the appearance of inspection stimulus on the screen until the press of the “yes” button; answer time was the time taken on correct trials to process the target stimulus, make the relevant decision, and press either the “yes” or the “no” button; error rate was the percentage of trials on which the response was incorrect. To minimize the influence of any excessively long or short times, participants’ response times (RTs) were truncated to a value corresponding to two *SDs* from the mean if they exceeded that value for a particular analysis. To determine whether participants were approaching each task in a typical way, condition means for the dependent variables were determined and compared using an ANOVA. Since it was not crucial to demonstrate that any effects were general for all items within a condition, only participant analyses were conducted.

To obtain a single index of skill in carrying out each task, inspection times and answer times were first added to create a measure of processing time. To give credit for accurate as well as prompt responding, processing times were divided by the proportion of correct responses. This procedure has been advocated by Townsend and Ashby (1983) as a way of overcoming any tendency for speed–accuracy trade-off and has recently been used in the numerical processing domain by Trick, Enns, and Brodeur (1996). If a participant were to make no errors, then their RT would stay the same, but any reduction in accuracy would augment the time score. Inspection of the distributions of the adjusted RTs revealed that 7 of the 781 scores lay three *SDs* or more beyond the mean, so they were classed as outliers and trimmed to this value. Finally, linear regression analyses were conducted to determine the extent to which rote memory efficiency contributed to variance in arithmetic-processing efficiency independently of semantic retrieval efficiency.

Results

Average Performance

Table 1 presents the average performance for all tasks. For rote memory, it was not meaningful to compare the times to process the completely different types of inspection stimuli for each class of stimuli. People classified sequences containing letters more slowly and less accurately (648 msec and 8.2%) than sequences containing months and days (563 msec and 3.3%) [$F(1,70) = 74.78, p < .001$, and $F(1,70) = 46.26, p < .001$, respectively]. Since there was no theoretical reason to keep the two stimulus types apart, they were combined in further analyses. For the semantic retrieval task, category words were the same

regardless of what kind of exemplar was to follow, so they were not analyzed further. As foreshadowed, atypical trials were not considered in further analyses, so the values given in the tables are for prototypical trials only. Typical instances were classified faster than atypical instances, with an atypical mean of 776 msec [$F(1,68) = 121.35, p < .001$], and with fewer errors, with an atypical mean of 11.1% errors [$F(1,68) = 145.80, p < .001$].

Addition problems with small answers were inspected for less time than were problems with large answers [$F(1,70) = 114.62, p < .001$]. Small addition problems also took less time to answer [$F(1,70) = 74.32, p < .001$] and resulted in fewer errors [$F(1,70) = 28.00, p < .001$]. For multiplication, in comparison with problems with large answers, those with small answers were inspected for less time [$F(1,70) = 94.67, p < .001$], took less time to answer [$F(1,70) = 92.73, p < .001$], and led to fewer errors [$F(1,70) = 67.02, p < .001$]. As compared with large problems, small subtraction problems were also viewed for less time [$F(1,70) = 79.58, p < .001$], took less time to answer [$F(1,70) = 66.38, p < .001$], and resulted in fewer errors [$F(1,70) = 24.42, p < .001$]. Similarly, as compared with large problems, small division problems were inspected for less time [$F(1,70) = 92.45, p < .001$] and were answered more quickly [$F(1,70) = 33.80, p < .001$], although they did not lead to more errors [$F(1,70) = 1.04, p > .05$].

Correlations Between Skills

The correlations between the adjusted processing times are presented in Table 2. Semantic retrieval correlated substantially with rote memory skill, and all the arithmetic fact tasks were highly intercorrelated. The critical correlations were those between the verbal and the arithmetic tasks. Rote memory correlated substantially with all the arithmetic-fact-processing tasks. Semantic retrieval correlated strongly with addition and with small multiplication, subtraction, and division, but more modestly with large multiplication, subtraction, and division.

Regression Analyses

The results of the regression analyses predicting arithmetic-fact-processing efficiency from rote memory and semantic retrieval skill are presented in Table 3. Rote memory predicted unique variance in each analysis, whereas semantic retrieval predicted independent variance in fewer conditions: small and large addition, small multiplication, and small subtraction. The percentages of unique variance accounted for by rote memory and typical retrieval were significantly different for large multiplication [$t(67) = 5.27, p < .001$], large subtraction [$t(67) = 2.40, p < .05$], and large division [$t(67) = 3.20, p < .001$].

Discussion

It is clear from the correlation between rote memory and semantic retrieval skill that the two tasks did evoke common processes. Nevertheless, rote memory predicted arithmetic fact processing independently of semantic retrieval for each of the arithmetic-fact-processing tasks. Both rote memory and semantic retrieval supplied independent variance in the prediction of addition, small multiplication, and

Table 1
Experiment 1: Mean Processing Time (in Milliseconds) and Mean Percentage Error for All Tasks, With Standard Deviations

Task	Response Measure					
	Inspection Time		Answer Time		Error Rate	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Verbal Processing						
Rote memory	554	141	606	157	5.7	3.7
Semantic retrieval	510	141	714	133	4.8	3.6
Arithmetic Fact Processing						
Small addition	604	165	479	100	2.3	3.0
Large addition	761	261	558	139	5.3	4.9
Small multiplication	639	187	492	93	2.1	2.1
Large multiplication	875	369	663	215	7.7	5.9
Small subtraction	659	244	503	115	3.8	4.1
Large subtraction	886	421	590	173	7.7	6.3
Small division	708	225	535	129	6.7	4.9
Large division	903	356	620	207	7.6	7.0

Table 2
Experiment 1: Correlations Between Adjusted Processing Times

	Semantic Retrieval	Small Addition	Large Addition	Small Multiplication	Large Multiplication	Small Subtraction	Large Subtraction	Small Division	Large Division
Rote memory	.44***	.55***	.56***	.62***	.58***	.57***	.49***	.58***	.53***
Semantic retrieval	—	.58***	.58***	.52***	.25*	.45***	.28*	.44***	.32**
Small addition		—	.90***	.87***	.73***	.82***	.69***	.85***	.72***
Large addition			—	.86***	.80***	.90***	.85***	.92***	.85***
Small multiplication				—	.88***	.84***	.74***	.84***	.79***
Large multiplication					—	.79***	.75***	.83***	.87***
Small subtraction						—	.93***	.90***	.85***
Large subtraction							—	.82***	.82***
Small division								—	.89***

* $p < .05$. ** $p < .01$. *** $p < .001$.

small subtraction, but only rote memory independently predicted large multiplication, large subtraction, and both small and large division. The percentage of unique variance was also significantly higher for rote memory than for semantic retrieval for large multiplication, subtraction, and division, with the difference being the greatest for large multiplication. On the face of it, these results support the expectation that efficient retrieval of some arithmetic facts is associated with efficient rote memory for arbitrarily connected sequences. The substantial effect for large multiplication is consistent with Dehaene and colleagues' idea that multiplication facts are the most likely to be helped by being committed to memory as a verbal sequence. The contribution of rote memory to large division would be seen as a consequence of this, since rapid and accurate division may be performed by iteratively applying multiplication facts. Large subtractions might also be aided by successful retrieval of the component addition facts, even if direct retrieval were less important for large addition itself.

Before accepting these conclusions, however, we have to consider several potential objections to the conclusions on the basis of methodological grounds. One is that the different pattern of results for rote memory and semantic retrieval might have come about because the tasks differed in the sequence of the processing steps involved. In the arithmetic fact tasks, participants viewed a problem from which they could generate a possible answer. In the rote memory task, participants were presented with an element from which they could also generate an answer, whereas

in the semantic retrieval task, participants saw the category word before the instance; thus, they were unlikely to generate a single plausible instance, given the category. The larger role for rote memory than for semantic retrieval might thus have occurred because its processing requirements were more similar to those of the arithmetic tasks than was the case for semantic retrieval. Another concern is that both types of verbal memory might have seemed important in arithmetic processing because all tasks involved short-term memory. In the verbal tasks, regardless of whether participants had already generated a possible answer, they had to keep a single element in short-term memory while the target element was evaluated. For the arithmetic tasks, if an answer had not been generated, the whole arithmetic problem would have to be retained in short-term memory while the answer was determined. If short-term memory skill was linking performance on all three tasks in some way, then it would be preferable to lessen the short-term memory demands and make them comparable across tasks. A final methodological question is whether the part played by both verbal memory skills in arithmetic performance might have been overestimated, since all tasks might involve a more fundamental visuo-motor skill factor that could be supplying considerable shared variance. Accordingly, we thought that it was important to see whether the same pattern of results could be found with these three concerns addressed.

EXPERIMENT 2

The aim of this experiment was to try to replicate the findings of the previous experiment with an improved methodology. The same simple arithmetic problems involving addition, multiplication, subtraction, and division were presented. Although the rote memory task was the same, the semantic retrieval task was changed so that the instance word appeared first and the participant had to judge whether the following category word was appropriate to the instance or not. Only prototypical instances of categories were included. In order to reduce short-term memory requirements and make them the same for each task, the participant no longer controlled the presentation time of the initial stimulus but viewed it for a constant period of time, depending on the task. The initial stimulus remained on the screen while the target element appeared below, not disappearing until the participant responded.

Table 3
Experiment 1: Summary of Linear Regression Analyses Predicting Arithmetic Fact Processing From Rote Memory and Semantic Retrieval

Operation	R^2	Rote Memory		Semantic Retrieval	
		Beta	% Unique Variance	Beta	% Unique Variance
Small addition	.44***	.37	11***	.42	14***
Large addition	.41***	.42	14***	.34	9***
Small multiplication	.48***	.51	21***	.30	7**
Large multiplication	.38***	.62	31***	-.02	0
Small subtraction	.38***	.47	18***	.24	5*
Large subtraction	.24***	.45	16***	.09	0
Small division	.39***	.49	20***	.22	4
Large division	.33***	.53	23***	.17	1

* $p < .05$. ** $p < .01$. *** $p < .001$.

Finally, a nonverbal visuomotor processing task was administered to allow variance in this fundamental skill to be partialled out from verbal and arithmetic performance. A verification format was again used, and all tasks were presented in the visual modality. In all other respects, the method was the same as that for Experiment 1.

Method

Participants

Participants were 75 native speakers of English who were undergraduate and graduate students at the University of Melbourne. They volunteered to take part in the experiment in order to either fulfill requirements for the Psychology 1 Research Participation Programme or receive a small sum for their participation. Their ages ranged from 18 to 31 years, with a mean of 20 years, 3 months. There were 18 males and 57 females.

Materials and Tasks

Rote sequence memory. The items for this task were the same as those before. The requirements of the task were also exactly the same, except that the first element appeared for a fixed duration of 600 msec prior to the onset of the target element.

Semantic retrieval. A total of 88 items were presented for this task. Eleven living categories (*animal, bird, color, fish, flower, fruit, gemstone, herb, insect, tree, vegetable*) and 11 nonliving categories (*clothing, crime, drug, fabric, furniture, metal, profession, sport, tool, vehicle, weapon*) were selected. Four prototypical instances of each category were chosen, two of which were allocated to their appropriate category and two of which were paired with an inappropriate category within the living and nonliving domains. Examples of items are *parsley-herb, cocaine-drug, trout-flower, cricket-furniture*. Target category words in each of the four conditions were matched on average letter length (at 5.4 letters), and the average frequencies of occurrence were as follows: for living instances, 20 for “yes” items and 24 for “no” items; for nonliving instances, 47 for “yes” items and 45 for “no” items. Instance words were presented for 600 msec prior to the onset of the category name.

Arithmetic facts. These tasks were the same as those before. However, the problems were presented for slightly different durations prior to the onset of the target answers, depending upon their approximate difficulty, as was demonstrated in Experiment 1. Addition problems were presented for 650 msec, multiplication and subtraction problems for 700 msec, and division problems for 750 msec.

Symbol matching. A set of 45 unfamiliar symbols that were not easy to give names to were selected for presentation. They were letters from the Devanagari alphabet, such as, ए, च, ऐ. A total of 54 sequences were presented, with 18 each containing three, four, or five such symbols. The symbols appeared beside each other in a straight line with spaces in between. Symbols were randomly selected from the total set, with a given symbol appearing from two to six times across all sequences. Three-symbol sequences appeared for 450 msec, four-symbol sequences appeared for 600 msec, and five-symbol sequences appeared for 750 msec, followed by a target stimulus. If the target was in the sequence, then the participant was to press the “yes” key, and if the target was not in the initial sequence, the participant was to press the “no” key. Half of the trials required a positive response, and the other half required a negative response. As with the other tasks, both initial and target stimuli remained on the screen until the participant responded. Items appeared in a different pseudorandom order for each participant, who was again asked to respond as quickly and as accurately as he or she could.

Procedure and Data Analysis

The tasks were run by computer in the same way as before, under the control of the DMDX system. Participants attended a single session, completing the tasks in the order of semantic retrieval, rote memory, addition, multiplication, subtraction, division, and, finally, visual symbol matching. The data of 4 participants whose error rates

were at chance on one or more of the tasks were discarded, leaving 71 participants. With these participants excluded, the highest error rate in any task was 37.5%. Because of experimenter error, 1 participant was not administered the multiplication task, and a different participant was not administered the symbol-matching task. Dependent variables were the time taken to make a decision, measured from the onset of the target stimulus to the buttonpress on correct trials, and the percentage of incorrect responses. The same Winsorization procedure was applied to the RTs for a given participant, and adjusted answer times were derived by dividing the RTs by the proportion of correct responses. Inspection of the distributions of the adjusted RTs revealed that 8 of the 923 scores lay beyond three SDs from the mean, and so they were trimmed to this value.

Results

Average Performance

The average performance for all tasks can be seen in Table 4. As before, participants classified sequences containing letters more slowly and slightly less accurately (937 msec and 7.7%) than sequences containing months and days (774 msec and 5.1%) [$F(1,69) = 135.70, p < .001$, and $F(1,69) = 14.38, p < .001$, respectively]. For the symbol-processing task, average classification time increased from 1,129 msec for three-symbol sequences, to 1,206 msec for four-symbol sequences, to 1,302 msec for five-symbol sequences [$F(2,69) = 85.48, p < .001$]. For some reason, presumably having to do with chance similarities of the symbols selected for the sequences, average error rates were slightly higher for four-symbol sequences (8.3%) than for sequences with either three symbols (6.0%) or five symbols (5.5%) [$F(2,69) = 5.06, p < .05$].

Addition problems with small answers took less time to answer than problems with large answers [$F(1,70) = 139.08, p < .001$] and resulted in fewer errors [$F(1,70) = 58.04, p < .001$]. For multiplication, problems with small answers took less time to answer than large problems [$F(1,69) = 129.12, p < .001$] and led to fewer errors [$F(1,69) = 91.03, p < .001$]. As compared with large problems, small subtraction problems also took less time to answer [$F(1,70) = 150.51, p < .001$] and resulted in fewer errors [$F(1,70) = 63.63, p < .001$]. Similarly, as compared with large problems, small division problems

Table 4
Experiment 2: Mean Response Time (RT, in Milliseconds)
and Mean Percentage Error for Verbal and
Arithmetic Fact Tasks, With Standard Deviations

Task	Response Measure			
	RT		Error Rate	
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
Verbal Processing				
Rote memory	855	260	6.4	4.5
Semantic retrieval	652	146	4.4	3.1
Arithmetic Fact Processing				
Small addition	530	140	2.7	2.7
Large addition	777	287	7.3	5.8
Small multiplication	601	168	2.6	3.1
Large multiplication	910	350	10.5	7.4
Small subtraction	647	115	3.8	4.1
Large subtraction	903	348	9.9	8.6
Small division	724	248	5.5	4.0
Large division	967	410	11.7	9.0

Table 5
Experiment 2: Correlations Between Adjusted Processing Times

	Rote Memory	Semantic Retrieval	Small Add.	Large Add.	Small Mult.	Large Mult.	Small Sub.	Large Sub.	Small Div.	Large Div.
Symbol processing	.36**	.52***	.43***	.27*	.35**	.20	.34**	.23	.34**	.21
Rote memory	—	.54***	.44***	.46***	.50***	.47***	.48***	.43***	.55***	.44***
Semantic retrieval		—	.56***	.41***	.48***	.32**	.38**	.32**	.46***	.33**
Small addition			—	.88***	.78***	.64***	.83***	.74***	.76***	.61***
Large addition				—	.73***	.70***	.79***	.77***	.75***	.67***
Small multiplication					—	.84***	.79***	.75***	.84***	.78***
Large multiplication						—	.66***	.66***	.81***	.87***
Small subtraction							—	.92***	.77***	.62***
Large subtraction								—	.75***	.66***
Small division									—	.88***

Note—Add., addition; Mult., multiplication; Sub., subtraction; Div., division. * $p < .05$. ** $p < .01$. *** $p < .001$.

were answered more quickly [$F(1,70) = 88.26, p < .001$] and led to more errors [$F(1,70) = 37.59, p < .001$]. These results are highly similar to those found in Experiment 1.

Correlations Between Skills

The correlations between the adjusted processing times are given in Table 5. As would be expected, the visuomotor control task correlated positively with all of the verbal and arithmetic tasks, although to varying degrees. It correlated substantially with semantic retrieval and less so with rote memory. Although it also correlated with the processing of small arithmetic problems, it did not correlate significantly with the processing of large arithmetic problems, with the exception of a just-significant correlation for large addition. As was the case in Experiment 1, semantic retrieval skill correlated moderately with rote memory skill. However, partialling out symbol processing did little to reduce the correlation [$r(66) = .47, p < .001$]. Again, all of the arithmetic fact tasks were highly intercorrelated. Rote memory correlated less strongly than did semantic retrieval with small addition, about the same as semantic retrieval with large addition and small multiplication, but more strongly with performance in the remaining arithmetic conditions.

Regression Analyses

For comparison with the results of Experiment 1, the first regression analyses compared the independent contribution of semantic retrieval and rote memory to arithmetic processing for each operation, without taking into account symbol processing. Table 6 gives the results of these analyses. In contrast with what was found in Experiment 1, rote memory no longer contributed independently to small addition processing and now shared predictive power with semantic retrieval for small multiplication as well as for large addition processing. Aside from these changes, the pattern of results for this experiment was similar to that shown in Experiment 1. Rote memory independently predicted all problems except for small addition, whereas semantic retrieval independently predicted only small and large addition and small multiplication problems. The minor independent contribution of semantic retrieval to small subtraction obtained in Experiment 1 was not replicated. Note, however, that although the pattern of results was similar, the percentage of unique variance accounted for by rote memory was substantially smaller in

this experiment than that found in Experiment 1. Finally, the percentage of unique variance was significantly higher for rote memory than for semantic retrieval for large multiplication [$t(66) = 2.02, p < .05$], and there was a significantly greater percentage of unique variance for semantic retrieval than for rote memory for small addition [$t(66) = 2.01, p < .05$].

To see whether the effects of rote memory diminished when visuospatial skill was controlled, regression analyses were performed with the variance that was due to symbol processing removed. The results of these analyses are given in Table 7. One can see that symbol memory did not account for variance in any of the analyses. The major effect of its inclusion was to lower the amount of independent variance accounted for by semantic retrieval, particularly in addition. The pattern of results for all other operations remained virtually the same. The percentage of unique variance was still significantly higher for rote memory than for semantic retrieval for large multiplication [$t(66) = 2.21, p < .05$], but the greater unique variance for semantic retrieval than for rote memory for small addition was no longer significant ($t < 1$).

Comparison of Average Performance Across Experiments

In order to try to explain the apparently smaller role of rote memory in Experiment 2 than in Experiment 1, we decided to compare the average performance of the two

Table 6
Experiment 2: Summary of Linear Regression Analyses Predicting Arithmetic Fact Processing From Rote Memory and Semantic Retrieval

Operation	R^2	Rote Memory		Semantic Retrieval	
		Beta	% Unique Variance	Beta	% Unique Variance
Small addition	.35***	.19	2	.47	16***
Large addition	.26***	.32	7*	.26	5*
Small multiplication	.32***	.34	8**	.30	6*
Large multiplication	.23***	.41	12**	.11	1
Small subtraction	.25***	.39	11**	.17	2
Large subtraction	.20***	.35	9**	.14	1
Small division	.35***	.42	12***	.24	4
Large division	.22***	.36	9**	.16	2

* $p < .05$. ** $p < .01$. *** $p < .001$.

Table 7
Experiment 2: Summary of Linear Regression Analyses Predicting Arithmetic Fact Processing From
Symbol Processing, Rote Memory, and Semantic Retrieval

Operation	Problem Size	R^2	Symbol Processing		Rote Memory		Semantic Retrieval	
			Beta	% Unique Variance	Beta	% Unique Variance	Beta	% Unique Variance
Addition	Small	.36***	.19	3	.14	2	.37	8**
	Large	.25***	.05	0	.32	7*	.22	3
Multiplication	Small	.33***	.10	1	.31	7*	.27	4*
	Large	.22***	.03	0	.42	12**	.06	0
Subtraction	Small	.28***	.14	2	.35	8**	.15	1
	Large	.21***	.04	0	.33	7*	.16	2
Division	Small	.36***	.10	1	.41	11**	.19	2
	Large	.21***	.28	0	.36	9**	.13	1

* $p < .05$. ** $p < .01$. *** $p < .001$.

groups of participants. For the rote memory and arithmetic fact tasks, even though presentation times were self-paced in Experiment 1 and under experimenter control in Experiment 2, it seemed reasonable to compare average error rates, since the same items were used in the two experiments. Since the semantic retrieval tasks used different items as well as different presentation conditions, the participants were not compared on this task. Only effects involving the factor experimental group will be reported. Experiment 1 participants made significantly fewer errors than did Experiment 2 participants in the rote memory task [$t(137) = 2.44, p < .05$]. For the arithmetic tasks, Experiment 1 participants also outperformed Experiment 2 participants, with a significant main effect of experiment for addition [$F(1,137) = 5.02, p < .05$], multiplication [$F(1,137) = 8.37, p < .01$], and division [$F(1,137) = 8.37, p < .05$], although not for subtraction [$F(1,137) = 1.69, p > .05$]. Additionally, there was a significant interaction between experiment and problem size for each operation, so that the difference in skill between the two groups of participants was more marked for large than for small problems [for addition, $F(1,137) = 4.36, p < .05$; for multiplication, $F(1,137) = 5.72, p < .05$; for subtraction $F(1,137) = 5.47, p < .05$; and for division $F(1,137) = 4.59, p < .05$]. Experiment 1 participants were significantly older, by 1.5 years on average, than participants in Experiment 2 [$t(140) = 2.66, p < .001$]. This age difference was an unintentional consequence of the time of year at which the students were recruited. Experiment 1 used more higher level students than did Experiment 2, because it was run later in the year when the Psychology 1 research participation pool had become depleted.

Discussion

Exchanging the order of the instance and category words in the semantic retrieval task did not make this task a better predictor of arithmetic processing than it was in Experiment 1. In fact, whereas typical retrieval contributed unique variance to both small and large addition, small multiplication, and small subtraction in Experiment 1, it now contributed uniquely only to small addition and small multiplication. In comparison, rote memory continued to predict unique variance in all arithmetic conditions, except for small addition. Thus, the results lend little support to the possibility that rote memory predicted arithmetic perfor-

mance better than did semantic retrieval in Experiment 1, because its processing demands were closer to those of the arithmetic tasks. Similarly, it seemed that reducing short-term memory demands and making them comparable across tasks did not greatly alter the basic contrasting pattern of results for the two verbal tasks. Moreover, the inclusion of the baseline symbol-matching task barely reduced the predictive ability of rote memory, with semantic retrieval being more affected than rote memory by the partialling out of visuomotor skill. However, on the strictest test, rote memory predicted significantly more variance than did semantic retrieval only for large multiplication problems. One possible reason for the reduced influence of rote memory in this experiment was the different overall skill level of the participants. Even though the differences were very small, the older participants of Experiment 1 were better at the rote memory task as well as at all the arithmetic tasks, especially the more difficult ones.

GENERAL DISCUSSION

The results of both experiments have demonstrated that the unimpaired young adults in our sample responded to the conditions of the tasks we administered in a standard way. Rote memory for sequences of elements was better for small sets of items, such as days and months, than for a large set of items, such as the letters of the alphabet, and retrieving category information about verbal concepts was easier when the instance was prototypical rather than atypical of its class. Simple addition and multiplication problems were easier when the answer was small than when it was large, and an analogous effect was found when the problems were converted to subtraction and division problems. Despite these average condition effects, there were substantial individual differences in the efficiency with which the participants could perform the tasks. To seek effects on arithmetic performance that were specific to skill at memorizing arbitrary sequences, we also assessed skill in retrieving information that would not have been laid down in long-term memory using deliberate rote strategies—namely, skill at retrieving semantic properties of common verbal concepts. Although it is clear from the substantial correlation of the two skills that they share many processing and retrieval operations, we nevertheless found that sequence memory contributed variance to

the arithmetic-processing tasks independently of semantic retrieval.

The most robust finding of the experiments was that people who were particularly efficient at retrieving elements from rote-learned sequences were also the most efficient at performing large multiplications, independently of their semantic retrieval skills. This finding is in line with the triple-code model of Dehaene (1992) and Dehaene and Cohen (1995), which assumes that multiplication facts are typically coded and stored as verbal propositions. It is also consistent with the case study evidence discussed in the introduction. Regarding problem size effects, we know that children use a repeated-addition strategy for small-answer multiplications when they have not yet automated the result (e.g., *two times three is—three, four, five, six*) (Lemaire & Siegler, 1995). Even adults report using strategies for multiplication, such as repeated addition, as well as counting by the number denoted by one of the operands (e.g., *four times two is—two, four, six, eight*) (Hecht, 1999). Intuitively, it seems that these strategies would be more available for the small than for the large multiplication problems, and in both experiments, there were smaller amounts of unique variance accounted for by rote memory in small-answer than in large-answer multiplication problems. Of course, our results do not demonstrate that all individuals store large multiplication problems in phonological form in long-term memory. Just one example that this is unlikely to be the case comes from a report by Whalen, McCloskey, Lindemann, and Bouton (2002), who demonstrated that two brain-damaged individuals who frequently read arithmetic problems out loud incorrectly could usually give correct answers to them. However, the present findings do suggest that those people who are good at this form of coding perform the most efficiently on large multiplication problems, regardless of their semantic retrieval skill. A plausible interpretation is that their skill at rote memory is used to aid memorization and storage of the problems in long-term memory.

The part played by rote memory was anticipated to be weaker in solving addition than multiplication problems, and this is what we found. Rote memory shared an independent predictive role with semantic retrieval for small addition problems in Experiment 1, and it played no role in their solution in Experiment 2. In both experiments, the two verbal skills independently predicted large addition performance. The triple-code model would be able to account for this finding by the idea that people can approach addition problems with either a verbal or a nonverbal strategy. Addition problems can more easily be performed using strategies other than recall of a rote-learned proposition, such as imagining quantities analogically, counting on from the first operand, or transforming the problem into simpler subcomponents (LeFevre et al., 1996). Adults' intuitions are also in line with this possibility, in that they report using such strategies more often than direct memory retrieval when performing addition, although much less so when performing multiplication (Hecht, 1999; Smith-Chant & LeFevre, 2003). Note, however, that it would be misleading to make a simple equation between self-reports of direct retrieval and retrieval of

a rote-learned proposition. As has been mentioned, people report retrieving answers directly from memory most of the time for small-answer addition problems, but much less often for large-answer problems (Campbell & Austin, 2002; LeFevre et al., 1996). Our results showed that efficient rote memory played less of a role than did semantic retrieval in small-answer than in large-answer addition problems. What this result indicates is that participants' report of direct retrieval does not reveal why the memory response felt immediate and direct. Retrieval could have either a semantic associative basis, as suggested by Campbell (1987) and Ashcraft (1992), or an asemantic rote-learned basis, without participants' being able to differentiate the underlying mechanisms.

The operations of subtraction and division have been examined much less often than addition and multiplication. The results of Experiment 1 indicated that rote sequence memory made a significant unique contribution to small and large problems in subtraction and division, whereas semantic retrieval did not, apart from a slight contribution to small subtraction processing. Rote memory made a significantly higher contribution than did semantic memory to large problems. The results of Experiment 2 confirmed the unique predictive power of rote memory in the solution of subtraction and division problems and demonstrated no significant predictive power for semantic retrieval. Although contrasting the independent contributions of rote memory and semantic retrieval directly did not produce significant differences, these results can be seen at the minimum as indicating a trend for a greater predictive role for rote memory than for semantic retrieval in subtraction and division. We found some post hoc evidence that this effect may be a function of the age or skill level of the participant, with older, more adept participants more likely to use rote learning as a strategy. It would be worthwhile to test this hypothesis directly.

Finding an influence of rote memory in subtraction is not necessarily in conflict with the triple-code model. Subtraction can be performed analogically, by imagining values on the mental number line, or by various other procedures, such as counting backward from the minuend or by transforming the problem into an addition. Moreover, Lee and Kang's (2002) finding that people solving simple subtraction problems were selectively disrupted by irrelevant visuospatial demands suggests that some kind of analogical processing is often used in subtraction. Thus, anyone who had impaired rote memory would be able to rely on such procedures if necessary. However, it still may be more efficient for people to have stored the answers in long-term memory in phonological form. Regarding division problems, even if they are reframed as multiplication problems, as indicated by LeFevre and Morris (1999), rapid retrieval of the relevant component multiplication facts could still contribute to fast and accurate solution of these problems.

Our conclusions should be considered in the light of the particular task conditions that we used. For example, by assessing efficiency in recognizing adjacent elements in the rote memory task, we did not evaluate skill in recognizing whether one element followed another further

away in the sequence. However, there is little reason to think that the ability to recognize distant order relationships might be any better a measure than recognizing adjacent links in the sequence. Additionally, it could be that in the rote memory task, some people may remember the sequence in chunks larger than adjacent pairs of elements (see, e.g., Klahr, Chase, & Lovelace, 1983) and may need to go back to the beginning of a chunk to verify a target pair. We do not deny that some people might use rehearsal of chunks in our task, but we maintain that someone who can minimize such rehearsal will have more proficient sequence memory than will someone who has to resort to it frequently. We have contended that it is through their more efficient rote memory that people can become more adept at arithmetic processing. An alternative possibility is that rote memory and arithmetic processing are related largely due to similarly inefficient processing. For example, it might be that people who use chaining and repetition of elements in the rote memory task also use cumbersome analogical procedures to perform subtraction processing. This view implies that there should be a closer association between rote memory and arithmetic processing at the unskilled than at the skilled end of the spectrum. However, inspection of the scatterplots revealed the opposite effect: The two variables were much more closely associated among the more efficient than among the less efficient rote memorizers, particularly in Experiment 2. Thus, although we cannot rule out this option definitively, we do not see it as a particularly persuasive alternative.

Some might also contend that although memorizing the alphabet, the days of the week, or the months of the year involves learning associations between verbally encoded elements, it is not the verbal nature of the learning that is crucial, but the learning of elemental associations. Although it would be interesting to assess the hypothesis that associative learning competence is the more critical factor, it is not easy to envisage an analogous learning task in which participants cannot use verbal coding strategies. In defense of the view that the sequences used in both the present study and previous work are verbally coded, we point out that the first attempt to remember a sequence of unrelated elements by rote is exactly the same as a standard short-term memory trial, where there is little contention about the phonological nature of the coding involved. If there were a semantic basis for *a* to appear before *b* rather than the other way around, then undoubtedly the form of coding would be different.

Another possible limitation stems from our use of verification rather than production tasks, which we did to avoid individual differences in phonological retrieval and articulation skill. Dagenbach and McCloskey (1992) have argued that people can verify a target answer accurately without generating an answer to the problem internally. They reported a case study of a brain-damaged individual who performed poorly on subtraction problems, as compared with the other arithmetic operations, when he had to produce the answers but was no longer impaired when he had to verify an entire equation. Zbrodoff and Logan (1990) have also claimed that people do not perform veri-

fication tasks by producing an answer internally and then comparing it with the target answer. When delays were imposed between the presentation of the problem and the answer, university students still showed effects of problem size in the answer times, suggesting that they may not have completed processing by the time the answer arrived and that they may have compared the equation as a whole against memory, rather than first generating an answer. However, it is clear that participants in the first experiment reported in the present article were not simply waiting for the target answer before performing any processing, since we found substantial problem size effects in the problem inspection times for all operations, before the target answer had been presented. There is no reason to think that participants in Experiment 2 would inhibit processing the problem in the period of time that it was available for inspection before the answer appeared. Moreover, even if it were the whole equation that was checked against information stored in memory, and not just the problem itself, this circumstance does not alter the fact that the individual would have to know the appropriate answer and be able to retrieve this information rapidly. The question of whether diverging results would be found with a production format remains to be determined.

In sum, the present findings have shown that superior long-term memory for arbitrary sequences is typically a better predictor of ability to retrieve simple arithmetic facts than is skill in retrieving information about common semantic concepts, at least for multiplication, subtraction, and division. The strongest unique contribution of rote memory was to the solution of large multiplication problems, which cannot be performed by quantitative, analogical procedures in a realistic amount of time. Although it is of vital importance for people to understand the basic conceptual principles underlying each of the arithmetic operations, the present results suggest that rote learning the facts may be a successful way for some people to obtain correct answers rapidly. The validity of this claim is bolstered by our finding that the relative importance of rote memory over semantic memory in predicting arithmetic performance was not simply an artifact of shared visuo-motor processing demands. However, a valuable next step in consolidating our conclusions would be to show that rote memory does not play a significant part in certain other kinds of numerical processing over and above any common lower level processes. For example, rote memory skill would be expected to be irrelevant to the ability to perform nonverbal numerical judgments, such as making approximate comparisons on dot arrays. This important issue awaits further research.

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