ERRATA

Erratum to: ““Weak” Measurements and Supraluminal Communication” [Moscow University Physics Bulletin 71, 482 (2016)]

A. V. Belinsky* and A. K. Zhukovskiy**

Department of Physics, Moscow State University, Moscow, 119991 Russia

*e-mail: belinsky@inbox.ru

**e-mail: andrez@rambler.ru

Received July 11, 2017; in final form, August 3, 2017

Abstract—We present the calculations which show that the measurements performed according to the scheme proposed in the commented paper do not differ from each other before and after the collapse of the state vector.

Keywords: non-demolition measurements, entanglement, quantum state vector collapse.

DOI: 10.3103/S0027134917060066

In the cited paper, a method for supraluminal communication using quantum non-demolition measurements was proposed (see Fig. 4 of the cited paper for the schematic diagram of the method). Let us perform simple calculations in order to verify these considerations. Let us take the single-photon Fock state $|1\rangle^p$ as the probe mode. After its splitting by the 50% light beam splitter the following superposition is obtained:

$|\psi_p\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1^p|0\rangle_2^p + |0\rangle_1^p|1\rangle_2^p)$. The quantum state of the system as a whole after generation of the pair of entangled photons and their separation by polarization prisms of observers A and B is described by the pure state with the vector

$$|\psi_{abp}\rangle = \frac{1}{2}((|1\rangle_1^a|1\rangle_1^p|0\rangle_2^a|0\rangle_2^p + |1\rangle_1^a|0\rangle_1^p|0\rangle_2^a|1\rangle_2^p)|1\rangle_x^b|0\rangle_y^b + (|0\rangle_1^a|1\rangle_1^p|1\rangle_2^a|0\rangle_2^p + |0\rangle_1^a|0\rangle_1^p|1\rangle_2^a|1\rangle_2^p)|0\rangle_x^b|1\rangle_y^b). \quad (1)$$

The influence of nonlinearity described by the operator $\hat{U} = e^{-i\bar{\chi}_{ap}\hat{n}_a\hat{n}_p/2}$ yields

$$|\psi'_{abp}\rangle = \frac{1}{2}((|1\rangle_1^a|1\rangle_1^p|0\rangle_2^a|0\rangle_2^p e^{-i\bar{\chi}_{ap1}/2} + |1\rangle_1^a|0\rangle_1^p|0\rangle_2^a|1\rangle_2^p)|1\rangle_x^b|0\rangle_y^b + (|0\rangle_1^a|1\rangle_1^p|1\rangle_2^a|0\rangle_2^p + |0\rangle_1^a|0\rangle_1^p|1\rangle_2^a|1\rangle_2^p e^{-i\bar{\chi}_{ap2}/2})|0\rangle_x^b|1\rangle_y^b). \quad (2)$$

In the Heisenberg representation the action of the light beam splitter situated in front of the detectors is described as $\hat{a}'_p = \frac{\hat{a}_p \pm \hat{a}_2}{\sqrt{2}}$. The plus sign corresponds to one detector, and the minus sign, to the other detector.

Then we obtain that the average number of photon counts of one of the detectors is equal to $\frac{1}{4}\left(2 + \cos\frac{\bar{\chi}_{ap1}}{2} + \cos\frac{\bar{\chi}_{ap2}}{2}\right)$, and the average number of photon counts of the other detector is equal to $\frac{1}{4}\left(2 - \cos\frac{\bar{\chi}_{ap1}}{2} - \cos\frac{\bar{\chi}_{ap2}}{2}\right)$.

In the Schrödinger representation the quantum state of the system at the output of the Mach–Zehnder interferometer after the output light beam splitter is described by the vector

$$|\psi_{abp}^n\rangle = \frac{1}{2\sqrt{2}}((|1\rangle_1^a|0\rangle_2^a|1\rangle_x^b|0\rangle_y^b(e^{-i\bar{\chi}_{ap1}/2} + 1) + |0\rangle_1^a|1\rangle_2^a|0\rangle_x^b|1\rangle_y^b(1 + e^{-i\bar{\chi}_{ap2}/2}))|1\rangle_1^d|0\rangle_2^d + (|1\rangle_1^a|0\rangle_2^a|1\rangle_x^b|0\rangle_y^b(e^{-i\bar{\chi}_{ap1}/2} - 1) + |0\rangle_1^a|1\rangle_2^a|0\rangle_x^b|1\rangle_y^b(1 - e^{-i\bar{\chi}_{ap2}/2}))|0\rangle_1^d|1\rangle_2^d). \quad (3)$$

Here, $|1\rangle_1^d|0\rangle_2^d$, $|0\rangle_1^d|1\rangle_2^d$ are the states at the detector inputs. It can be seen that if one of the detectors is activated, the superposition $|\psi_b\rangle = \frac{1}{\sqrt{2}}(|1\rangle_x^b|0\rangle_y^b + |0\rangle_x^b|1\rangle_y^b)$ is not reduced to one of the components of this state, i.e., this measurement is a truly non-demolition measurement. This expression also yields the previous results containing the cosine functions.

What happens in the case of “strong” measurement of polarization by observer B? The $|\psi_{abp}^n\rangle$ state is reduced either to the first and the third or the second

and the fourth terms of (3). The detector activation probabilities are equal, their value is either $\frac{1}{2}\left(1 \pm \cos \frac{\bar{\chi}_{ap1}}{2}\right)$ or $\frac{1}{2}\left(1 \pm \cos \frac{\bar{\chi}_{ap2}}{2}\right)$, where, similar to the above said, the plus and minus signs correspond to the first and the second detectors, respectively, i.e., the upper sign corresponds to the first detector, and the lower sign, to the second detector. Thus, the pure state is transformed to the mixed state with equal probabilities (1/2) of both results. This means that the “strong” measurement performed by observer B does not allow one to distinguish between the pure state $|\psi_{abp}^n\rangle$ and the mixed state with the probability $\frac{1}{2}\left(1 \pm \cos \frac{\bar{\chi}_{ap1}}{2}\right)$ or $\frac{1}{2}\left(1 \pm \cos \frac{\bar{\chi}_{ap2}}{2}\right)$.

Thus, supraluminal communication cannot be obtained in the case of probing by the single-photon state. What happens in the case of an arbitrary state? Let us write it in the form

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\rangle_1 |m\rangle_x^b |n\rangle_y^b + |\rangle_2 |n\rangle_x^b |m\rangle_y^b). \quad (4)$$

Let us average the results of measurement of the phase difference cosine which is described by the operator $\hat{C}$,

$$\langle \psi | \hat{C} | \psi \rangle = \frac{1}{\sqrt{2}} (\langle_y^b | n \rangle_x^b \langle m |_1 | \hat{C} | \rangle_1 | m \rangle_x^b | n \rangle_y^b + \langle_y^b | m \rangle_x^b \langle n |_1 | \hat{C} | \rangle_2 | n \rangle_x^b | m \rangle_y^b). \quad (5)$$

Thus, the results of measurements are the sum of two probabilities of finding the particles in channels x and y for observer B, which completely coincides with the probabilities after reduction as a result of “strong” measurement performed by this observer, similar to the case with single-photon states considered above.

However, if two additional detectors identifying the interferometer channel within which a photon from an entangled pair is found after the probe photon detection are introduced for observer A, the variation of the probabilistic laws of triggering of detectors of observer A allows one to determine whether observer B has performed a collapsing measurement.