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EDITORIAL

FCAA RELATED NEWS, EVENTS AND BOOKS (FCAA–VOLUME 17–4–2014)

Virginia Kiryakova

Dear readers,

in the Editorial Notes we announce news for our journal, anniversaries, information on international meetings, events, new books, etc. related to the FCAA ("Fractional Calculus and Applied Analysis") areas.

This is a Special Issue partly dedicated
to the 80th Anniversary of Professor Ivan Dimovski
with invited papers and some selected papers from TMSF '14

1. Great success for the FCAA Journal

In the end of July 2014, Web of Knowledge of Thomson Reuters and Scopus of Elsevier announced the scientific impact metrics for 2013. FCAA have been indexed there since 2011 and it is the first time it receives these metrics officially.

According to **Thomson Reuters' 2013 Journal Citation Reports (JCR)** Science Edition, Release July 2014,

http://admin-apps.webofknowledge.com/JCR/JCR,

the Journal Title **"FRACT CALC APPL ANAL"**

has Impact Factor JIF (2013) = 2.974.

This launches our journal at **rank 4/95** in category Mathematics, Interdisciplinary Applications and **rank 5/250** in category Mathematics, Applied – immediately after "SIAM Rev", "ACM T Math Software" "Commun Pur Appl Math". Although FCAA is not ranked in "Mathematics" (pure, where it would be most suitable classification), the data from JCR show that it can be there also at 4th place (of 300), after "Commun Pur Appl Math", "J AM Math Soc", "Acta Math (Djusholm)", and before the next journals in the list as "Ann Math", "Fixed Point Theory", "Found Comput Math", "Invent Math", "Mem AM Math Soc", "Duke Math J".

According to **Journal Metrics Values, Scopus Release 2014**,

http://www.journalmetrics.com,

the title "Fractional Calculus and Applied Analysis" obtained the following scientific metrics, including SJR (SCImago Journal Rang):

2012: SNIP=0.934, IPP=0.909, SJR=1.069

2013: SNIP=2.079, IPP=2.333, SJR=2.106

that confirm the excellent performance of FCAA also at Scopus database.

pp. 907–935 , DOI: 10.2478/s13540-014-0205-1

Springer

2. Reports on International Events Related to FC

"International Conference on Fractional Differentiation and Its Applications' 14" (ICFDA' 14) June 23-25, 2014, Catania - Sicily, Italy Website http://www.icfda14.dieei.unict.it/

The conference was attended by 170 participants from 40 countries, with altogether 210 submitted/ presented papers.

The ICFDA14 audience enjoyed the following invited talks:

Plenary talks, by:

- Igor Podlubny, The Evolution of Generalized Differentiation
- Yury Luchko, Selected Topics in Fractional Transport Equations
- Rudolf Hilfer, Time Flow and the Foundations of Nonequilibrium Statistical Mechanics
- Enzo Orsingher, Random Flights Governed by Fractional D'Alembert Operators
- Jose Antonio Tenreiro Machado, The Persistence of Memory

Semiplenary talks:

- Mark M. Meerschaert, Tempered Fractional Calculus
- Todd Freeborn, Fractional-Order Circuits: State-of-the Art Design and Applications
- Teodor Atanackovic, Linear Viscoelasticity of Real and Complex Fractional Order
- Giuseppe Nunnari, Evidences of Self-Organized Criticality in Volcanology

The following awards have been assigned to conference participants:

Mittag-Leffler Award: FDA Achievement Award

- Winner: Francesco Mainardi

Riemann-Liouville Awards: Best FDA Papers (theory, application)

- Winner (theory): Mark M. Meerschaert;
- Winner (application): Igor Podlubny;
- Winners (application): Jean-Claude Trigeassou, Nezha Maamri, Alain Oustaloup

Grunwald-Letnikov Awards: Best Student Papers (theory, application)

- Winner (theory): Roberto Garra (other finalists: Gioacchino Alotta, Le Ha Vy Nguyen, Alberto Di Matteo)
- Winner (application): Iftikhar Ali (other finalists: Boris Jakovljevic, Norelys Aguila-Camacho, Xiaomin Tian)

FDA Dissemination Award
– Winner: J. A. Tenreiro Machado

Anatoly A. Kilbas Award(s), diplomas presented to 4 winners (groups):
– Yuri Luchko;
– Juan J. Trujillo;
– Boris B. Jakovljevic, Zoran D. Jelicic,
Milan R. Rapaic, Tomislav B. Sekara;
– Rafail K. Gazizov, Alexey A. Kasatkin, Stanislav Yu. Lukashchuck.

The participants in ICFDA14 had the opportunity to publish their papers in *Conference IEEE Xplore Proceedings* (110 such papers), and in special / topical issues in the journals "*Fractional Calculus and Applied Analysis*", "*Journal of Applied Nonlinear Dynamics*", "*Discontinuity, Nonlinearity, and Complexity*".

On behalf of participants, we like to thank the **local organizers – Riccardo Caponetto and his collaborators** for the great efforts to organize the conference, the IEEE support and to ensure fruitful and pleasant stay in Catania.

The Steering Committee for FDA events elected **YangQuan Chen as its new Chairman**, to inherit on that position Francesco Mainardi. It was approved that the **next international conference FDA will take place in 2016, hosted by University of Novi Sad – Serbia**, to be organized by **Teodor Atanackovic, Dragan Spasic** and others from their group.

Chair of Program Committee, *Virginia Kiryakova*

7th Minisymposium
"Transform Methods and Special Functions" (TMSF '14)
in frames of "Mathematics Days in Sofia" (MDS 2014)
July 6 – 10, 2014, Sofia, Bulgaria
Website http://www.math.bas.bg/∼tmsf/2014/

This was the 7th in the series of the *international meetings TMSF* organized periodically in Bulgaria, details at http://www.math.bas.bg/∼tmsf.

It was dedicated to *the 80th anniversary of Professor Ivan Dimovski* (http://degruyteropen.com/people/dimovski/), member of our Editorial Board, and **this special FCAA issue contains selected papers presented at TMSF '14 as well as some invited papers dedicated to his anniversary**.

TMSF '14, with 30 participants and 25 talks, was a part of the international conference "*Mathematics Days in Sofia*", organized by the Institute of Mathematics and Informatics – Bulgarian Academy of Sciences (BAS), that was attended by more than 280 participants from all over the World, see all details at http://www.math.bas.bg/mds2014/.

By tradition, the topics of TMSF '14 included: – Classical and Generalized Integral Transforms; – Fractional Calculus; Fractional Differential and Integral Equations; – Operational and Convolutional Calculus; – Special Functions, Classical Orthogonal Polynomials; – Generalized Functions; – Geometric Function Theory, Functions of One Complex Variable; – Related Topics of Analysis, Differential Equations, Applications, etc.

Invited Speakers were:

– Ivan Dimovski (Bulgaria), Nonclassical convolutions and their uses
– Andrzej Kaminski (Poland), On associativity of the convolution of ultradistributions
– Gradimir Milovanovic (Serbia), Nonstandard quadratures of Gauss-Lobatto type and applications in the fractional calculus
– Blagovest Sendov (Bulgaria), Stronger Rolle's theorem for complex polynomials
– Sergei Sitnik (Russia), Buschman-Erdélyi transmutations: classification, analytical properties and applications to differential equations and integral transforms.

The scientific program, booklet of abstracts, list of participants and some selected photos for TMSF'14 can be found at
http://www.math.bas.bg/∼tmsf Chair, *Virginia Kiryakova*

International Symposium on Fractional Signals and Systems (FSS 2015), October 1 – 3, 2015, Cluj-Napoca, Romania http://www.fss-conference.com

The organizing committee has the pleasure of inviting you to participate at the International Symposium on Fractional Signals and Systems, FSS 2015, hosted by the Technical University of Cluj-Napoca, Romania.

Topics: – Fractional order control (tuning, implementation issues, new algorithms); – Signal analysis and filtering with fractional tools (restoration, reconstruction, analysis of fractal noises), – Fractional modeling; – Fractional system identification (linear, nonlinear, multivariable methods).

Important deadlines: – Submission opens: 1 May 2015; – Initial submission: 1 July 2015; – Author notification: 10 August 2015; – Final submission: 1 September 2015; – Registration deadline: 10 September 2015.

Debate sessions moderators:
YangQuan Chen, Valerio Duarte, Clara Ionescu, Manuel Ortigueira

Local Organizing Committee: Cristina Muresan, Eva H. Dulf, Roxana Both, Bogdan Pop, Ioan Nascu

International Program Committee: see at
http://fss-conference.com/organizing-commitee.php

For queries and contacts, at http://fss-conference.com/contact.php, or by e-mail: cristina.pop@aut.utcluj.ro

On behalf of Organizers, *Cristina Muresan*

3. New Books

Richard Herrmann, *Fractional Calculus. An Introduction For Physicists*. World Scientific, N. Jersey, London etc., 2nd Ed., 2014, 479 p., ISBN : 978-981-4551-07-6.

Details on 1st Ed.:
http://www.worldscientific.com/worldscibooks/10.1142/8072.

Details on 2nd Ed.:
http://www.worldscientific.com/worldscibooks/10.1142/8934,

Download sample chapter: Introduction,
http://www.worldscientific.com/doi/suppl/10.1142/8934/suppl_file/8934_chap01.pdf.

Information on the 1st Edition and a review on it (by Ralf Metzler) we have published in Editorial Note of *FCAA*, Vol. **15**, No 1 (2012), p. 7, http://link.springer.com/article/10.2478/s13540-012-0001-8,

and in No 2 (2012), pp. 166–167,
http://link.springer.com/article/10.2478/s13540-012-0011-6.

Review on 2nd Edition of the book:

This is the second edition of a popular book on fractional calculus, which has proven useful to many new researchers in the field. The second edition includes new chapters describing applications of fractional calculus to image processing, folded potentials in cluster physics, infrared spectroscopy, and local fractional calculus. A very welcome new feature in the second edition is the inclusion of exercises at the end of every chapter, with detailed solutions in the back of the book.

The book contains of 27 chapters. The first few chapters develop and explain the main ideas of fractional calculus on one dimension, from the point of view of the author. The remaining chapters include extensions to multiple dimensions, and a wide array of applications to physics.

By this time there are many good books on fractional calculus and its applications. This book is specifically aimed at physicists, although many of my colleagues outside physics have also found it useful. This is particularly true of graduate students and beginning researchers, or those new to the subject of fractional calculus. The book takes a practical approach, which will be especially appealing to those accustomed to thinking about modeling in terms of differential equations and transforms. The author takes some care to explain how the fractional models extend the traditional approach using integer order derivatives.

I agree with the comments by R. Metzler, in his review of the first edition, that the book lacks any discussion of statistical physics (e.g., random walks), which some readers might find helpful. For example, the discussion of the Feller derivative seems hard to grasp without some discussion of the stable laws (Lévy flights) that motivated Feller in his work on this subject. For those interested in the probabilistic approach, one might also consider the recent book, M. Meerschaert, A. Sikorskii, *Stochastic Models for Fractional Calculus*, which was reviewed in the Editorial Note of a previous *FCAA* issue, Vol. **15**, No 3 (2012), pp. 348–349: http://link.springer.com/article/10.2478/s13540-012-0025-0.

Readership: Students and researchers in physics and related areas.

Reviewer: Mark Meerschaert, *Dept. of Statistics and Probability Michigan State University, MI 48824–1027, USA*

Alain Oustaloup, *Diversity and Non-Integer Differentiation for System Dynamics*. J. Wiley & Sons, and ISTE, 2014, 384 p., ISBN (Print): 978 1848214750, ISBN (Online): 9781118760864.

Details: http://onlinelibrary.wiley.com/book/10.1002/9781118760864

Book Description: Based on a diversity of structured approach which is notably inspired by various natural forms of diversity (biological among others), this book unquestionably offers a framework, on the one hand, to the introduction of non-integer derivative as a modeling tool and, on the other hand, to the use of such a modeling form to highlight dynamic performances (and notably of damping) unsuspected in an "integer" approach of mechanics and automatic control. The "non-integer" approach indeed enables us to overcome the mass-damping dilemma in mechanics and consequently the stability-precision dilemma in automatic control.

This book has been written so that it can be read on two different levels: the first chapter achieves a first level of presentation which goes through the main results while limiting their mathematical development; the five remaining chapters constitute a second level of presentation in which the theoretical passages, deliberately avoided in the first chapter, are then developed at the mathematical level, but with the same goal of simplicity which aspires to make this book an example of pedagogy.

Contents:

A. Kh. Gilmutdinov and P.A. Ushakov, ***Fractal Elements***. Publ. House of Kazan State Tehnical University, 2013, 306 pp., ISBN 987-5-7579-1922-5, UDK 621.372.54, In Russian.

Foreword: Today one can say with certainty that two kinds of researches, namely scientists and engineers, have widely recognized the necessity to use the fractals theory along with the theory of fractional integral-differential operators and fractal treatment of different signals for solution of variety of problems that emerge in various fields of modern science and technology. The terms "fractals" and "fractal" reflect the modern view of the physical nature of real objects and processes; this view was firmly established after publication of the pioneering works of B. Mandelbrot related to the fractal geometry of the Nature.

The term "fractional operators" reflects the modern approach to mathematical description and identification of different fractal objects and processes associated with the properties of the geometry and dynamics of the object studied. They were described by integer order differential equations only approximately with a stretch of imagination. Today we are able to describe the dynamics of these objects and processes using the non-integer (fractional) order equations that fill the gaps between equations of the first, the second and other integer orders.

Despite the fact that the concept of fractional derivatives was known as early as at the end of the XVII century, the systematic use of fractional calculus in practical science and technology actually can be attributed to the time of appearance of the pioneering works by Rashid Shakirovich Nigmatullin, the founder and the scientific director of the Kazan scientific school of investigation and application of electrochemical converters of information (ECCI), and his disciples. R. Sh. Nigmatullin was one of the first who realized physical implementation of fractional integration and differentiation (FID) operations on the basis of real elements (in particular, electrochemical ones). He also was the first to develop methods for synthesis of ladder-type resistive-capacitive and resistive-inductive circuits that implement these operations in time-domain. He proposed a number of certain useful applications of such elements, in particular, to increase the resolution of osillopolarographic spectrum. It was further widely used to develop the corresponding devices both in Russia and abroad. R. Sh. Nigmatullin has also offered the block diagram of a computer meant for solving equations of linear, spherical and cylindrical diffusion. The device is based on operational amplifiers with special RC-bipoles (fractional order integrators and differentiators). He showed the possibility to use the semi-infinite RC-cable in order to create some special functions out of the trigonometric. All these results were obtained and published within the period from 1962 to 1968.

The appearance of the following works has ultimately established interest to the fractional calculus: the well-known works of K.B. Oldham (e.g., K.B. Oldham, J. Spanier, *The Fractional Calculus*, Academic Press, N. York, 1974, 234 p.), of B.B. Mandelbrot (B.B. Mandelbrot, *Les Objects Fractals: Forme, Hasard et Dimension*, Flammarion, Paris, 1975, 187 p.; B.B. Mandelbrot, *Fractals: Forme, Chance and Dimension*, Freeman, San Francisco, 1977, 365 p.; B.B. Mandelbrot, *The Fractals Geometry of Nature*, Freeman, N. York, 1982, 468 p.) and the fundamental book of S.G. Samko, A.A. Kilbas and O.I. Marichev (S.G. Samko, A.A. Kilbas, O.I. Marichev, *Fractional Order Integrals and Derivatives and Some of Their Applications*, Nauka i Tekhnika, Minsk, 1987, 688 p. (in Russian); and its transl. and enlarged ed. in EN, *Fractional Integrals and Derivatives. Theory and Applications*, Gordon and Breach, Switzerland etc., 1993).

For a short historical period the pure mathematics tools of the fractional calculus have found their applications in various fields of science, such as classical and quantum physics, field theory, electrodynamics, solid state physics, fluid dynamics, turbulence, general chemistry, biology and medicine, stochastic analysis, nonlinear control theory, image processing, seismology, geology, etc. Nowadays, the necessity of applications of the fractals theory together with the fractional calculus and interpretations in fractals'sense for various problems arising in different domains of contemporary science and technics achieved wide recognition not only among the scientific but also in the engineering collegia. The numerous scientific publications and monographs confirm this fact. Here we remind only three of them, published in Russian. The fist is the monograph by A.A. Potapov (*Fractals in Radiophysics and Radar: Sample Topology*, Univ. Kniga, Moscow, 2005, 848 p.). The second is the monograph by V.V. Uchaikin (*The Method of Fractional Derivatives*, Artichoke Publ., Ulyanovsk, 2008, 512 p.). Each of these monographs contains more than one thousand references. And the third one is the following fundamental book: A.A. Potapov, Y.V. Gulyaev, S.A. Nikitov, A.A. Pakhomov, V.A. German / Ed. by A.A. Potapov, *The Modern Image Processing Techniques*, FizMatLit, Moscow, 2008, 496 p.

Another indicator for the great interest to the fractional analysis and its applications is the fact that a variety of international conferences on these matters are conducted annually. For example, the representative conferences "Fractional Differentiation and its Applications" (FDA '02, FDA '04, FDA '06, FDA '08, FDA '12, etc.) were organized and their papers published in specialized scientific journals, such as *Chaos, Solutions and Fractals*, *Nelineyny Mir* (*Nonlinear World*), *Fractional Calculus and Applied Analysis*, etc.

However, the Russian science, including applied sciences and industry, demonstrates absolutely insufficient use of these concepts and of emerging opportunities to understand the Nature and to acquire new knowledge, to create new methods and measurement tools, and better models of technical equipment. One reason for this is that there is not enough scientific-technical and especially educational literature that would reflect both theoretical understanding of fractional differentiation and integration operations and their hardware implementation together with the practical use. That is why scientists and engineers do not have the required knowledge associated with the fractional analysis as well in the field of design of fractional order elements ("fractal elements") that would make it possible to physically implement fractional operators and other devices for information and signal processing.

This manual book (*tutorial*) has appeared as a result of the systematized outcomes of theoretical and experimental researches of the authors. The book partly fills the mentioned gap. The tutorial can be used to develop general engineering and special education courses along with the corresponding teaching materials. The aim is to actively introduce the concepts of fractal geometry and fractional analysis into the minds of the future engineering professionals and scientists who would be able to work at the production industry and in research laboratories and would be able to embody these ideas into new instruments, devices and systems.

Chapter 1 summarizes the fundamentals of the fractals theory, fractal dimension and scaling. The concept of fractal signals and some methods for their processing is explained.

Chapter 2 provides the essential information from the fractional analysis theory. This information will be used below for description of the fractional order systems and to perform frequency domain analysis of circuits containing fractal elements (FE). In this chapter some examples of problems from electrical engineering and electrochemistry that cause fractional order differential equations are considered also.

Chapter 3 introduces the concept of FEs and gives their mathematical de-scription. Versions of the known devices and electrical circuits where the input impedance frequency dependence is expressed in the power-law exponents is analyzed. The multilayer RC structure is substantiated as the base for creating FEs.

Chapters 4 - 6 describe design, schematic and technological fundamentals for implementation of different FEs based on multilayer resistive-capacitive medium. Powerful capabilities to obtain the required parameters and characteristics of FEs by means of static and dynamic inhomogeneities of the medium are shown.

Chapter 7 opens and explains physical effects that are used to create controlled resistors and capacitors. We can assume that application of these effects to resistive and dielectric materials in multilayer resistive-capacitive structures will make it possible to create parametric and non-linear FEs; and the latter will significantly expand the capabilities of these structures.

Chapter 8 provides an overview of FE applications for modeling, signal processing, creation of control systems, hybrid computers, etc. Breadth of applications demands also a wide range of FE characteristics and parameters that can be implemented based on the multilayer RC medium. Therefore, the authors proposed a universal structural framework suitable for implementation of FEs in various application areas. This structural framework contains seven alternating layers of resistive, dielectric and conductive materials. The whole of these layers constitute a generalized virtual element.

Chapter 9 describes the technique of forming a system of partial differential equations for potential distribution in the resistive layers of the proposed virtual element. An example of FEs classification by resistive layers potential distribution is shown for the particular case of the fractal element with "resistor-insulator-resistor" layers structure.

Chapters 10 - 12 describe in details how to calculate the so-called y-parameters of bipolar and (in the general case) multipole elements formed on the basis of the RC multilayer medium which contains static and dynamic heterogeneities. These chapters are of particular importance for the practical implementation of FEs as long as this kind of problems had no satisfactory solutions till now. The authors used their proposed method of finite distributed elements to show that the external parameters of FEs can be calculated regardless of the complexity of the structure, the heterogeneities nature and distribution in the RC medium. Several variants of algorithms for calculating the y-parameters for different design implementations of FEs are proposed to the readers.

Practical exercises and different tests are given at the end of every chapter with the aim to consolidate the given material and to provide self-studying.

The authors suppose that this tutorial sufficiently fulfills educational and innovative objectives in application of ideas of fractal geometry and fractional analysis purposed to create fractal radioelectronics devices, communication systems, systems for identification and control the distributed and fractional objects and processes.

The authors dedicated this book to the blessed memory of Rashid Shakirovich Nigmatullin, the founder of the Kazan scientific school of fractional order operators.

Contact with authors can be via Prof. Anis Kh. Gilmutdinov,
e-mail: agilmutdinov@rambler.ru

Written by:
Deputy Director of the Kotel'nikov Institute of Radioengineering and Electronics of the R.A.S. (Russian Academy of Sciences), D.Sci. in Physics and Mathematics, Corresponding member of the R.A.S., *Prof. S.A. Nikitov*

4. Prof. Mourad E.H. Ismail's 70th Anniversary

Personal data: Born April 27, 1944, in Cairo, Egypt.
Canadian and Egyptian citizen, permanent resident in the United States.
Education:
Ph.D. 1974 (Alberta), M.Sc. 1969 (Alberta), B.Sc. 1964 (Cairo).
Affiliation: Department of Mathematics, University of Central Florida, Orlando, FL 32816 - USA.

Web-page: http://math.cos.ucf.edu//∼ismail/

Research Interests: approximation theory, asymptotics, combinatorics, integral transforms and operational calculus, mathematical physics, orthogonal polynomials and special functions.

Honors and Awards:

– Undergraduate Merit Scholarship, Cairo University 1960-1964;
– Dissertation Fellowship, University of Alberta, 1973-1974;
– Theodore and Venette Askounes-Ashford Distinguished Scholar Award University of South Florida, 1992-1993;
– Leverhulme research fellow, Imperial College, London, 1996;
– University visiting research professorship, City University of Hong Kong, 2000-2001;
– USF Presidential Excellence Award (= 10 % raise), 2003;
– Listed among the highly cited: www.isihighlycited.com ;
– Elected fellow of the Institute of Physics, December 2004;
– Elected fellow of the European Society of Computational Mathematics in Science and Engineering.

Editorial Boards:

1. *Constructive Approximation*, Springer-Verlag, 1988 - present.
2. *Encyclopedia of Mathematics,* Cambridge Univ. Press, 1992 - present.
3. *Journal of Approximation Theory*, formerly published by Academic Press and now by Elsevier, 2000 - present.
4. *Journal of Physics A: Mathematical and General*, 2001-2004.
5. *The Ramanujan Journal,* formerly published by Kluwer and now by Springer-Verlag , 1996 - present.
6. *Methods and Applications of Analysis*, International Press, 1992 - 1999.
7. *International J. of Mathematics and Mathematical Sciences*, 1993 - 2008.
8. *J. of Computational Analysis and Applications*, Plennum, 1998 - 2008.
9. *The Indian Journal of Mathematics*, 1997 - present.
10. *Fractional Calculus and Applied Analysis*, 1998 - present.
11. *The Egyptian Journal of Mathematics*, 2003 - present.
12. *Arab J. of Science and Engineering*, Math. Section.
13. *Journal of Advanced Research* (the official Journal of Cairo University), 2009 - present.
14. *Jordan Journal of Mathematics and Statistics.*
15. Collaborating Problem Editor, *American Math. Monthly*, 1992 - 1997.

Visiting Positions:

2009	Visiting member, The Isaac Newton Institute, University of Cambridge, UK
2008	Von Neumann Professor, the Technical University of Munich, Munich, Germany, June & July
2008	Visiting Scholar, Hong Kong U of Sc. and Tech., May & June
2006	Visiting Scholar, City University of Hong Kong, May & June
2002	Visiting Scholar, Hong Kong U of Sc. and Tech., May & June
2000-2001	Visiting university professor, City University of Hong Kong
1999	Visiting member, Mat. Sc. Res. Inst., Berkeley, three months
1996	Visiting Professor and Leverhulme research fellow, Imperial College, London
1990-1991	Adjunct Professor, University of Toronto
1990	Visiting Professor, University of Paris VII (10 weeks in the summer)
1988	Visiting Professor, National University of Colombia (1 month)
1987-1990	Adjunct Professor, York University
1987	Visiting Professor, University of Alberta (1 month in the summer)
1986	Visiting Professor, University of Paris VII (10 weeks in the summer)

1984-1985	Visiting Professor, University of Minnesota, Minneapolis
1982	Visiting Professor, Kuwait University (winter and summer semesters)
1976	Visiting Scholar, Mathematics Research Center University of Wisconsin, Madison
1975-1976	Visiting Lecturer and Research Associate, University of Toronto
1974-1975	Assistant Scientist, Department of Mathematics Research Center, University of Wisconsin, Madison

Master's Students:

(1) Richard Ruedemann, Arizona State University, August 1987.
Thesis title: *"Positivity Results in Combinatorics"*.

(2) Ruiming Zhang, Arizona State University, August 1987.
Thesis title: *"The Hellmann-Feynman Theorem and Zeros of Special Functions"*.

(3) David Milligan, University of South Florida, December 1997,
Thesis title: *"How Mathematics Aids Engineering and Engineering stimulates Mathematics and an Example Involving Fuel Spray"*.

(4) David Wallace, University of Central Florida, July 2005,
Thesis title: *"The Hellmann-Feynman Theorem"*.

Doctoral Students:

(1) Edward Bank, Arizona State University, April 1984.
Dissertation title: *"Pollaczek Polynomials and Functions"*.

(2) Jairo Charris, Arizona State University, August 1984.
Dissert. title: *"Sieved Pollaczek and Random Walk Polynomials"*.

(3) Li-Chen Chen, University of South Florida, August 1989.
Dissertation title: *"On Asymptotics of Certain Hypergeometric Functions and $6-j$ Symbols"*.

(4) Richard Ruedemann, University of South Florida, August 1992.
Dissertation title: *"Relation Between Polynomials Orthogonal on the Unit Circle With Respect to Different Weights"*.

(5) Ruiming Zhang, University of South Florida, April 1993.
Dissertation title: *"Some Formulas of W. Gosper and Spectral Properties of Certain Operators in Weighted Spaces"*.

(6) Jifeng Ma, University of South Florida, May 1997.
Dissertation title: *"Spectrum of Some Integral Operators"*

(7) Zeinab Mansour, Co-supervisor with Mahmoud Annaby, Cairo University, January 2006.
Dissertation title: *"q-Difference Equations"*.

(8) Jemal Gishe, University of South Florida, July 2006.
Dissertation title: *"A Finite Family of q-Orthogonal Polynomials and Resultants of Chebyshev Polynomials"*.

(9) Daniel J. Gallifa, University of Central Florida, May 2009.
Diss. title: *"The Sheffer B-Type 1 Orthogonal Polynomial Sequences"*.

Publications:

Prof. MEH Ismail is author of a long list of books, more than 250 papers, also editor in many special issues in journals, author of prefaces to books, solutions to problems, books'reviews, etc. Of them we give here only the books'set and for the others, pls. look at

http://math.cos.ucf.edu//~ismail/ismail_files/cv7.pdf.

Books:

(1) *Mathematical Analysis, Wavelets, and Signal Processing*, Proc. of an International Conf. on Mathematical Analysis and Signal Processing, coeditor with M.Z. Nashed, A.I. Zayed and A.F. Ghaleb, Contemporary Mathematics, Vol. 190, AMS, Providence, 1995.
(2) *Special Functions, q-Series and Related Topics*, coeditor with D. Masson and M. Rahman, Fields Institute Communications, volume 14, AMS, Providence, 1997.
(3) *Q-Series from a Contemporary Perspective*, coeditor with D. Stanton, Contemporary Mathematics, volume 254, AMS, Providence, 2000.
(4) *Special Functions*, coeditor with C. F. Dunkl and R. Wong, World Scientific, Singapore, 2000.
(5) *Special Functions 2000, Current Perspectives and Future Directions*, coeditor with J. Bustoz, and S. K. Suslov, Kluwer, Dorchester, 2001.
(6) *Symbolic Computation, Number Theory, Special Functions, Physics and Combinatorics 2001*, Coeditor with F. G. Garvan, Developments in Mathematics, Volume 4, Kluwer, Dorchester, 2001.
(7) *Theory and Applications of Special Functions: A Volume Dedicated to Mizan Rahman*, Coeditor with H. Koelink, Developments in Mathematics, Springer+Business Media, New York, 2005.
(8) *Classical and Quantum Orthogonal Polynomials in one Variable*, Cambridge University Press, 2005.
(9) *Classical and Quantum Orthogonal Polynomials in one Variable*, paperback edition, Cambridge University Press, 2009.

5. Prof. Yury Brychkov's 70th Anniversary

Professor Yury Aleksandrovich Brychkov, born on February 29 of 1944 in Moscow, the capital of Russia, is a well-known authority in the field of analysis. He started his mathematical career in Moscow State University and graduated its Faculty of Mechanics and Mathematics in 1966. In 1966-1969 he took a Post-Graduate course in Department of Quantum Field Theory in Steklov Mathematical Institute of USSR Academy of Sciences under the guidance of Prof. Yu.M. Shirokov. In 1971 he successfully defended Ph.D. Thesis. Since 1969 till nowadays, Y.A. Brychkov has been working in the Computer Center of USSR (now Russian) Academy of Sciences (currently at Dept. of Special Functions), and meanwhile in the period 2000-2002, he has been Researching Professor, in the Symbolic Computation Group, in Fac. of Mathematics - University of Waterloo, Canada.

The first area of Brychkov's mathematical research was devoted to generalized functions. He obtained new results in this area, basically connected with asymptotic properties of generalized functions, and his results, published in a series of papers, were presented in the book *"Integral Transforms of Generalized Functions"* written together with A.P. Prudnikov.

The second area of Brychkov's mathematical research deals with investigation of integral transforms with generalized and special functions in the kernels. In this field he (with co-authors) has proved factorization properties of general integral transforms of convolution type and mapping properties of integral transforms with the Appel hypergeometric function F_3 in the kernels. Yu.A. Brychkov together with H.-J. Glaeske, A.P. Prudnikov and Vu Kim Tuan have published the monograph *"Multidimensional Integral Transformations"*. This book was the first publication devoted to properties not only of the classical multi-dimensional integral transforms by Fourier, Laplace and Mellin but also multi-dimensional integral transforms with special functions in the kernels.

Under the influence of Prof. A.P. Prudnikov, the next area of Brychkov's mathematical research was devoted to the evaluation of integrals and series. He and his coauthors A.P.Prudnikov and O.I. Marichev have obtained modern results in this field published in three volumes of the bestseller of A.P. Prudnikov, Yu.A. Brychkov and O.I. Marichev *"Integrals and Series"*, Vols. 1-3, Nauka, 1981, 1983, 1986. These notebooks, translated in English, present many new results in this field. The continuations of these books were two volumes in this series devoted to the properties of the direct and inverse Laplace transforms: A.P. Prudnikov, Yu.A. Brychkov and O.I. Marichev *"Integrals and Series.* Vol. 4. *Direct Laplace transforms"*, Gordon and Breach Science Publishers, 1992, and *"Integrals and Series.* Vol. 5. *Inverse Laplace Transforms"*, Gordon and Breach Science Publishers, 1992.

In the recent years, Professor Brychkov has a very productive research work, especially on special functions. He derived a lot of new formulas for elementary and special functions of Bessel, Anger, Weber and other type, implemented into the internet page http://functions.wolfram.com/ containing more than 300 000 formulas. He published also the monograph *"Handbook of Special Functions. Derivatives, Integrals, Series and Other Formulas"*, CRC Press, 2008, and takes active part in other handbooks' recent projects.

Professor Brychkov is in close contacts with mathematicians from many countries. He was invited to give talks at many international conferences, seminars and workshops in Bulgaria, Germany, Japan, Poland, Spain, USA and other countries, was a Visiting Professor in Volfram Research Corporation.

Professor Brychkov gives a lot of his energy to publishing and editorial activities. He published many notebooks in Russian, republished in English. He is a member of the Editorial Boards of *"Integral Transforms and Special Functions"* (where he took an active part as a Secretary of this journal) and *"Fractional Calculus and Applied Analysis"*, and others.

Prof. Brychkov is an author of numerous scientific papers. Therefore here is a "selected" list of the the books authored / coauthored by him.

Books

1. Brychkov, Yu.A.; Prudnikov, A.P. *Integral Transforms of Generalized Functions*. Mathematical Reference Library (Russian), Nauka, Moskva, 1977, 287 p.

2. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series. Elementary Functions* (Russian), Nauka, Moskva, 1981, 799 p.

3. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series. Special Functions* (Russian), Nauka, Moskva, 1983, 752 p.

4. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series. Complementary Chapters* (Russian), Nauka, Moskva, 1986, 800 p.

5. Brychkov, Yu.A.; Marichev, O.I.; Prudnikov, A.P. *Tables of Indefinite Integrals*. Handbook (Russian), Nauka, Moskva, 1986. 192 p. **Zbl.0676.00032**.

6. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series*. Vol. 1: *Elementary Functions* (English), Gordon and Breach Science Publishers, New York, etc., 1986. 798 p. **Zbl.0733.00004**.

7. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series*. Vol. 2: *Special Functions* (English), Gordon and Breach Science Publishers, New York, etc., 1988,. 750 p.

8. Brychkov, Yu.A.; Marichev, O.I.; Prudnikov, A.P. *Tables of Indefinite Integrals* (Englsh), Gordon and Breach Science Publishers, New York, etc., 1989, 192 p.

9. Brychkov, Yu.A.; Prudnikov, A.P. *Integral Transforms of Generalized Functions* (English), Gordon and Breach Science Publishers, New York, etc., 1989, 343 p.

10. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series*. Vol. 3: *More Special Functions* (English), Gordon and Breach Science Publishers, New York, etc., 1990, 800 p.

11. Brychkov, Yu.A.; Marichev, O.I.; Prudnikov, A.P. *Tables of Indefinite Integrals* (German), Harry Deutsch, Thun, 1992, 200 p.

12. Brychkov, Yu.A.; Glaeske, H.-J.; Prudnikov, A.P.; Nuan, Vu Kim. *Multidimensional Integral Transformations* (English), Gordon and Breach Science Publishers, Philadelphia, PA, 1992, 386 p.

13. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series*. Vol. 4: *Direct Laplace Transforms* (English), Gordon and Breach Science Publishers, New York, 1992, 619 p.

14. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series*. Vol. 5: *Inverse Laplace Transforms* (English), Gordon and Breach Science Publishers, New York, 1992, 595 p.

15. Brychkov, Yu.A. *Mellin Transforms: I* (Russian), Vychislitel'nyi Tsentr RAN, Moscow, 1997, 64 p.

16. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series.* Vol. 1: *Elementary Functions*, 2nd revised ed. (Russian), Fiziko-Matematicheskaya Literatura, Moskva, 2003, 632 p.

17. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series*. Vol. 2: *Special Functions*, 2nd revised ed. (Russian), Fiziko-Matematicheskaya Literatura, Moskva, 2003, 664 p.

18. Prudnikov, A.P.; Brychkov, Yu.A.; Marichev, O.I. *Integrals and Series.* Vol. 3: *Complementary Chapters*, 2nd revised ed. (Russian), Fiziko-Matematicheskaya Literatura, Moskva, 2003, 688 p.

19. Brychkov, Yu.A. *Handbook of Special Functions. Derivatives, Integrals, Series and Other Formulas* (English), Chapman and Hall/CRC, Boca Raton, FL, 2009, xix+780 p.

6. Prof. Ivan Dimovski's 80th Anniversary

Professor Dr.Sc. Ivan Dimovski, Institute of Mathematics and Informatics - Bulgarian Academy of Sciences (B.A.S.), Corresponding Member of B.A.S., and a *Member of Editorial Board of our "FCAA" Journal*, is born on July 7, 1934 and has this year a 80'th jubilee, to which the international symposium "*Transform Methods and Special Functions '14*" was dedicated (see separate information in this Editorial Note).

Prof. Dimovski started his successful mathematical carrier yet as a pupil in the secondary school in the town of Troyan, near to his born villa-

-ge, when he received a prize as a winner in the first national mathematical olympiad in Bulgaria. Then he graduated at the Mathematics Dept. at Sofia University, and another reason to be proud, was his active participation in the famous seminar of Prof. Jaroslav Tagamlitski on topics of real analysis, where many other students took part, later becoming known scholars. There is a period in his studies and research interests dedicated also to Mechanics.

His carrier as a teacher and assistant professor started in the town of Rousse. Then it continued in Sofia in the Institute of Mathematics of Bulgarian Academy Sciences, till nowadays as an Emeritus Prof. and Honorary member of Institute - starting from a young mathematician (1959); through full professor (1982); a long-year chief of "Complex Analysis" department (1986-2004). He had been also a lecturer in many Bulgarian universities on large variety of courses (calculus, history of mathematics, operational calculus, potential theory, theory of elasticity and continuum mechanics); and President of Scientific Council on "Applied Mathematics and Mechanics"(2005-2009). In 1997, Prof. Dimovski was elected as a corresponding member of Bulgarian Academy of Sciences.

Prof. Dimovski is author of more than 100 scientific papers, and of the *monograph* [2] *"Convolutional Calculi"* (1990), Kluwer, referred to by other authors more than 700 times. Many papers, theses and monographs of his Bulgarian collaborators and of foreign authors are inspired by his ideas and results and are using them essentially, some of them even containing his name in the titles or in the names of new mathematical notions.

He is an invited speaker at many international conferences and visiting professor in foreign universities, in: Russia, Germany, Yugoslavia, Poland, Serbia, Spain, Venezuela, Kuwait, Macedonia, etc.; member of the Editorial Boards of the international journals *"Integral Transforms and Special Functions"*, *"Fractional Calculus and Applied Analysis"* and *"Special Functions and Inequalities"*; reviewer for many other international journals and publishers; organizer of a series of international mathematical conferences.

Among his contributions is the creation of a *Bulgarian mathematical school* (consisting of 11 Ph.D. students, many M.Sc. students and research collaborators), as well as a group of foreign collaborators and followers, in the field of Operational Calculus (extended by him as Convolutional Calculus) and Integral Transforms. *His PhD students were*: Prof. Dr.Sc. Nikolai Bozhinov; Prof. Dr.Sc. Sava Grozdev; Asso.Prof. Dr. Radka Petrova; Asso.Prof. Dr. Stefan Koprinski; Dr. Dimiter Mineff; Prof. Dr.Sc. Virginia Kiryakova; Assist. Prof. Mladen Vassilev; Asso.Prof. Dr. Emilia Bazhlekova; Asso.Prof. Dr. Svetlana Mincheva-Kaminska; Asso.Prof. Dr. Margarita Spiridonova; Dr. Yulian Tsankov.

In his young years, Prof. Dimovski *translated more than 50 famous mathematical books* from foreign languages (English, German, Russian and French) and thus he made many brilliant mathematical masterpieces available to the audience of Bulgarian mathematicians. Vice versa, he translated from Bulgarian to English or German some works of famous Bulgarian mathematicians. Another trend of his activities is the field of *"School Mathematics"* - as a lecturer for pupils prepared for mathematical olympiads, and as author of many classroom books on elementary mathematics, agendas and school curricula.

For his scientific achievements and contributions to Bulgarian science, Prof. Ivan Dimovski has been awarded the academic price on the name of Bulgarian famous mathematician "Nikola Obrechkoff" (1979), the honor medal "Marin Drinov" of Bulgarian Academy of Sciences (2004), and now he is elected Honorary member of Institute and nominated for the Honorary Sign of Academy's President (2014).

Short annotation
of Prof. Dimovski's scientific contributions

The key role in Prof. Dimovski's studies, from his very first publication [6] to the recent ones, is played by the term *"convolution of a linear operator"*. A new notion in mathematics usually deserves a long-standing role, only if it helps to solve problems that can be formulated without its use, but their solution requires an essential use of it. Such a notion happens to be the one introduced by Dimovski, as the base of his *"convolutional approach"*. By means of this approach, he has built new operational calculi for local and nonlocal boundary value problems, extending the area of applicability of the multipliers theory and relating it to the theory of commuting linear operators. Based on this convolutional approach, a *new variant of the Duhamel principle* has been developed, for a large variety of important nonlocal BVPs for equations of mathematical physics.

The classical operational calculus of Jan Mikusinski is based on the well known convolution of Duhamel

$$f * g(t) = \int_0^t f(t-\tau)g(\tau)d\tau. \qquad (1)$$

Yet in his first scientific paper [6] of 1962, Dimovski proved that putting on the base of the operational calculus any other continuous convolution of the integration operator, this leads to another operational calculus, isomorphic to Mikusinski's one. The idea to generalize the direct algebraic approach of Mikusinski for building operational calculi for other operators different from the integration operator, has encountered both *conceptual* *and technical problems*, yet in the first attempts made by some Russian, German and Hungarian mathematicians. Dimovski's notion *"convolution of a linear operator"* opened the way to such generalizations, allowing to speak about "operational calcul*i*" (in *plural* form). Its origin is hidden yet in paper [6] and formulated for some particular operators and spaces in [28], with the most important examples for convolutions proposed in [11], [12], [21], to justify its general character. Since in each particular case of an operator, one needs to solve the problem of constructing a convolution in explicit form, the happy hint for Dimovski has been to start (1966-1974) with a very general class of operators of Bessel type or arbitrary order $m \geq 2$ (nowadays called *"hyper-Bessel operators"*) and then to continue with the general linear differential operators of first and second order. The whole theory with the known applications (by then) can be found in his monograph [2]: Ivan Dimovski, *Convolutional Calculus*, Kluwer Academic Publishers, Dodrecht - Boston - London, 1990 (its first edition being by Printing House of Bulg. Acad. Sci. in 1982).

Dimovski's basic definition is the following: *A bilinear, commutative and associative operation* $* : X \times X \mapsto X$ *is called a convolution of the linear operator* $L : X \mapsto X$*, mapping a given linear space* X *into itself, when* $L(f * g) = (Lf) * g$ *for all* $f, g \in X$.

To find a convolution of a given linear operator in explicit form, as a rule, is a difficult and nontrivial task. However, once found, a convolution operation allows easily to build an operational calculus for the corresponding operator, and thus to solve the basic spectral problems, related to it: finding various spectral functions of this operator, and also of its commutant. In this respect, the paper [4] of 1978 is important, including a fact unknown by then: if such an operator has a cyclic element, then the rings of the multipliers of a nontrivial convolution of it, and of the operators commuting with it, *coincide*! This allowed Dimovski to find an elegant explicit characterization of the linear operators $M : C[0,1] \mapsto C[0,1]$, commuting with the classical integration operator $Lf(t) = \int\limits_0^t f(\tau) d\tau$. In his monograph [2] it is proven that for the commutation of M and l it is necessary and sufficient that M has an integral representation of the form $Mf(t) = \dfrac{d}{dt} \int\limits_0^t f(t-\tau)\alpha(\tau) d\tau$, where $\alpha(\tau)$ is a function simultaneously continuous and with bounded variation. This seems to be a result of wide mathematical importance.

The main approach in finding new convolutions, using some already known basic ones, happened to be the *"similarity method"*, called also a *"method of transmutations"*. For the first time, Dimovski formulated this idea in [3] in 1974, and with good justifying examples, in [5]. The method of similarity is based on the following simple fact: If $T : X \mapsto \widehat{X}$ and $\widehat{*}$ is a convolution of the linear operator $\widehat{L} : \widehat{X} \mapsto \widehat{X}$, then the operation $f{*}g := T^{-1}[(Tf)\widehat{*}(Tg)]$ is a convolution for the linear operator $L = T^{-1}\widehat{L}T$, mapping X into X. Usually, T is called a transmutation or similarity operator, and L and $\widehat{L}$ are similar operators.

Dimovski used essentially this approach, for the first time, in the case of the *Bessel-type operators*, [42], [31], [32], [43]. The corresponding similarity operator T found by him, is a generalization of the classical transformations of Poisson and Sonine (nowadays, we use to call it a *"Poisson-Sonine-Dimovski" transformation*). Next, by means of the similarity operators of Delsarte-Povzner, he proved the possibility for building operational calculi not only for initial value problems for the general 2nd order differential operator ([22]) but also for a wide class of nonlocal boundary value problems, including as a very special case the famous Sturm-Liouville problem ([16], [17], [26], [27]).

From a point of view of wide-range mathematics, Dimovski himself considers as most valuable the *contributions related to the spectral theory of the classical (1st order) differential operator*. In the 30's of the 20th century, the French mathematician Jean Delsarte (the founder of "Bourbaki" group) made several unsuccessful attempts to find a convolution, related to the most general spectral problem for the differentiation operator. His only achievement then was the generalization of the classical Taylor formula for this operator. The general spectral problem for the differentiation operator (in a corresponding space) consists in studying the resolvent operator L_λ, the result of its action, $y = L_\lambda f$ being considered as a solution of the nonlocal boundary value problem (BVP) $y' - \lambda y = f$, $\Phi(y) = 0$, where Φ is an arbitrary nonzero linear functional.

Dimovski solved the problem of Delsarte in 1974 (papers [7], [8]) when he found and proved that the operation

$$(f * g)(t) = \Phi_\tau \left\{ \int_\tau^t f(t + \tau - \sigma) g(\sigma) d\sigma \right\} \tag{2}$$

is a convolution of the resolvent L_λ, and built the respective operational calculus ([9]). The same convolution was rediscovered independently, by the German mathematician Lothar Berg (member of German Academy of Sciences) in 1976, who made reference to Dimovski's paper of 1974, thus acknowledging his priority. This convolution (called now as *Dimovski-Berg convolution*) allowed to obtain a complete solution of *the problem for multipliers of the Leontiev extensions in exponentials in the complex domain* (papers [10], [18]).

Interesting analogues of the differentiation operator are *the operator for backward shift translation* $\Delta f(t) = (f(t) - g(t))/t$ (called also the Pommiez operator) and *the finite-difference operator in the space of sequences*. It was confirmed that Dimovski's general scheme works successfully also in these two cases (see [17], [14], [15]). The experts acknowledged strongly also the results on *finding commutant of the Gelfond-Leontiev integration operator*, from joint papers (with Kiryakova) as [19], [20], referred to in the books by M.K. Fage and N.I. Nagnibida, "Problem of Equivalency of Ordinary Differential Operators", Nauka, Novosibirsk, 1987 (In Russian) and by S.G. Samko, A.A. Kilbas and O.I. Marichev, "Fractional Integrals and Derivatives", 1987 (Nauka) and 1993 (Gordon and Breach).

From the point of view of applied mathematics, the most important Dimovski's results concern the *convolutions for nonlocal BVP for 2nd order linear differential operators*, since these operators play basic role in the *problems of mathematical physics*. In the mathematical literature, there has been a lack of a well-developed theory of nonlocal BV problems. Each of the few authors studying such BVPs, has considered some very special cases, without any general view on these kind of problems. As to the local BVPs of mathematical physics, the most popular method for their solving is the Fourier method. Unfortunately, this method hardly allows a computer realization. Concerning the widely known difference methods, the experts confess that "taking into account the subsequent principle of calculation of the contemporary computers, such an approach requires a .... expense of time" (e.g. V.Z. Alad'ev, M.L. Shishakov, "Automated Working Space of a Mathematician", Moscow, 2000 (in Russian), p. 644). Anyway, by recently it seems nobody has tried to solve, by means of commonly used PCs, serious local and nonlocal BV problems related to equations of mathematical physics. Dimovski has proved, at least at principle, the possibility of using the *"convolutional method"* for such a task. The essence of his approach consists in *combining the Fourier method with the Duhamel principle*. The weakness of the Fourier method is in the necessity to use expansions of the boundary functions in series of eigenfunctions. This procedure requires calculating of number of integrals (from dozens to hundreds) and afterwards, summing the series obtained as solution in many points. In his works [21], [27], [23], [2], Dimovski proposes convolutions for BV problems for Sturm-Liouville operators with one local, and another - in general - nonlocal boundary value conditions, and thus opened the way to extend the Duhamel principle from a time-variable to space-variables in linear problems of mathematical physics. Generally speaking, the Duhamel principle consists in finding all solutions of a BVP by means of one particular solution of same problem. Such a particular solution is defined in terms of simple boundary functions and does not require numerical computation of definite integrals. Combining the Fourier method with the Duhamel principle allows to avoid two hard stages, from computing point of view, in the realization of Fourier method: expansion of the boundary functions in series of eigen- and associated functions, and the summation in many points of the obtained solution which is a rule, a slow convergent series. The numerical experiments ([48]) show high efficiency of such a convolutional approach. A natural question arises about "If the combination of these two methods is so efficient both in theoretical and computational aspects, why nobody had the hint before, to use it?" Possibly, the answer is that nobody had expected the existence of a simple explicit expression for the corresponding convolutions of [21] ...

Another application of Dimovski's convolutions for nonlocal BVPs for 2nd order linear ordinary differential operators, is the *possibility to generalize the notion of finite integral transformation for each of the corresponding BV problems* ([26], [27]). This solves automatically also the problem of obtaining of explicit convolutions for these finite integral transforms. As a special case, it is obtained a solution of the problem posed in 1972 by Churchill, to find convolutions of the finite Sturm-Liouville transformations (see Dimovski's monograph [2]). As a by-side product of the convolutions related to the general Bessel-type operators (paper [29]) it is found an explicit convolution of the classical Meijer transform (paper [24]). Another well appreciated Dimovski's result is the explicit convolution of the discrete Hermite convolution ([25]). A whole chapter is dedicated to this result in the book *"Integral Transforms and Their Applications"* by L. Debnath, one of the pioneers in searching for such a convolution.

From the point of view of Bulgarian mathematics and its traditions, the most important Dimovski's contribution is the identification, studying and giving an international popularity to the so-called *"Obrechoff integral transform"*, nowadays being a widely popular generalization of the Laplace transform ([8], [9], [24], etc). For his achievements on the subject, Dimovski was awarded in 1979 with the "Nikola Obrechkoff" Prize of Bulgarian Academy of Sciences. The Bulgarian mathematician Nikola Obrechkoff (1896-1963) himself never claimed for an authorship of a new integral transform but only for a formula for integral representation of functions on the real half-axis, *as an extension of a result of S. Bernstein.* In 60's-70's Dimovski studied the so-called general Bessel type differential operators, i.e. the singular differential operators of arbitrary order $m \geq 2$ naturally extending the 2nd order Bessel operator,

$$B = t^{\alpha_0} \frac{d}{dt} t^{\alpha_1} \frac{d}{dt} t^{\alpha_2} \dots \frac{d}{dt} t^{\alpha_m} = t^{-\beta} \left(t\frac{d}{dt} + \beta\gamma_1 \right) \dots \left(t\frac{d}{dt} + \beta\gamma_m \right)$$
$$= t^{-\beta} \left[t^m \frac{d^m}{dt^m} + a_1 t^{m-1} \frac{d^{m-1}}{dt^{m-1}} + \dots + a_{m-1} t \frac{d}{dt} + a_m \right], \quad 0 < t < \infty, \quad (3)$$

nowadays known as *"hyper-Bessel differential operators"*. Developing operational calculi for the corresponding integral operators L, initial right inverse to B $(BL = I)$, he had the idea that the Obrechkoff integral transform of 1958 (a slight modification of it) can be used successfully as a transform basis, in the same way as the Laplace transform is used in the classical operational calculus for the usual differentiation/ integration operators. Later on, the studies on the Obrechkoff transform and on the hyper-Bessel operators have been prolonged in some joint papers of Dimovski and Kiryakova, e.g. [33]–[37], [39] and extended to a theory of generalized fractional calculus and related to important classes of special functions in the monograph: V. Kiryakova, *"Generalized Fractional Calculus and Applications"*, Longman - J. Wiley, 1994. The keys of these further developments were given by: considering the *fractional powers of the hyper-Bessel operators* as the simplest "generalized operators of fractional integration and differentiation"; and by the role of some Meijer G-functions as kernel-functions of the Obrechkoff transform and of the hyper-Bessel integral operator, as well as solutions of classes of hyper-Bessel differential equations. For the details on this matter, see the survey paper by V. Kiryakova, From the hyper-Bessel operators of Dimovski to the generalized fractional calculus, published in this same issue of the journal *FCAA*, Vol. **17**, No 4 (2014).

Another important result, related to the Bessel type operators of arbitrary order, is the wide *generalization of the classical transmutation operators of Sonine and Poisson* (Dimovski's papers [40]–[42], [31], [38]), by means of which the equivalency of every two Bessel type operators of one and same order is proven. For the applications, the generalized Sonine operator is important, since it transforms an arbitrary Bessel-type differential operator of order m into the m-tuple differentiation $(d/dt)^m$. It is worth mentioning that in each particular case, the corresponding Dimovski's formula gives the best possible result. *This is a solution of the Delsarte problem*, posed by him in a manuscript (about 80 p.) published only posthumously in 1970 in the 2nd volume of his collected papers.

There exists a field of mathematics, which would look like quite different today if there were not the contributions of Dimovski, the *"Operational Calculus"*, nowadays extended to so-called *"Convolutional Calculus"*. In "Mathematics Subject Classification" it is classified as a section A44. It leads its origin from the studies of O. Heaviside in the end of 19th century, and remained without a strong mathematical formulation by the 50's of 20th century, when the Polish mathematician Jan Mikusinski proposed his direct algebraical approach based on the classical Duhamel convolution (1), thus justifying the Heaviside calculus. Predecessors of Mikusinski were Volterra and Pérèz (1924, 1943) to whom belonged the idea of using convolutional fractions for the same purpose. In 1957 the Russian mathematician V.A. Ditkin gave an example of operational calculus, different from Mikusinski's one. Namely, while Mikusinski's calculus is concerned with the Cauchy problem for the differentiation operator d/dt, Ditkin's calculus concerns the same problem but for the simplest operator of Bessel type, $(d/dt)t(d/dt)$. New examples of operational calculi of Bessel type of rather particular forms appeared in the 60's, by different authors. In the papers [13], [28]–[31], Dimovski established the applicability of Mikusinski's approach to the most general Bessel-type operator (3). In [30] and [42] he proved, for the first time, that the operational calculi for all Bessel-type operators in the Mukusinski scheme, are isomorphic. Especially, they are isomorphic to the Mikusinski operational calculus, since the classical differential operator is also a Bessel-type operator. In a paper [22] joint with N. Bozhinov, Dimovski proved that the operational calculi for initial value problems for the general 2nd order linear differential operator are also isomorphic to Mikusinski's calculus.

Conceptually new are *Dimovski's contributions for building operational calculi for boundary value problems* (BVPs) *and* especially, *for nonlocal BVPs for linear differential operators of 1st and 2nd order.* In contrary to the "Algebraic Analysis" of D. Przeworska-Rolewicz and to the approach of P. Bittner, in which two algebraic systems are considered - a ring of operators and a linear space, *in Dimovski's scheme it is considered a single algebraic system* - the ring of the multiplier quotients. This approach was described first in the Dr.Sc. thesis [1] of Dimovski and then, in his monograph [2]. *The advantage of using multipliers quotients instead of convolutional ones*, as it is in Mukisinski's approach, can be seen in the operational calculi of functions of several variables (papers e.g. [44], [45], [46]). The more, there exist cases (*resonance cases*) when the convolutional quotients' approach is not applicable (see [12], [13], [51]), while the multipliers' one works successfully.

To summarize, Dimovski's basic contributions can be classified in the following domains of mathematical analysis: operational calculus, integral transforms, theory of multipliers of convolutional algebras, expansions in eigenfunctions and associated functions, explicit characterization of commutants and automorphisms in them, linear nonlocal boundary value problems for equations of mathematical physics.

The recent papers of Dimovski and his collaborators show the continuation and further developments and applications of his approach and ideas, some few of them see at http://degruyteropen.com/people/dimovski/.

Some publications of Ivan Dimovski

[1] *Convolutional Method in Operational Calculus*. Summary of D.Sc. Thesis, Sofia, 1977.

[2] *Convolutional Calculus*. Kluwer Academic Publishers, Dordrecht-Boston-London, 1990; First publ.: Publ. House of Bulg. Acad. Sci., Sofia, 1982.

Scientific Articles

1. Some Popular Papers and Surveys

[3] On the bases of operational calculus. In: *"Mathematics and Math. Education. Talks of 2nd Spring Conf. UBM"* (1974); 103-112 (In Bulgarian).

[4] Convolutions of right inverse operators and representation of their multipliers. *Compt. rend. Acad. bulg. Sci.*, **31**, No 11 (1978), 1377-1380.

[5] The convolutional method in operational calculus. In: *Generalized Functions and Operational Calculus. Proc. Varna Conf.'1975* (1979), 69-68.

2. Convolutions, Operational Calculi, Multipliers and Commutants Related to Differentiation Operator

[6] Uniqueness of the Mikusinski field. *Fiz.-matemat. spisanie*, **5**, No 1 (1962), 56-59 (In Bulgarian).

[7] On an operational calculus for vector-valued functions. *Math. Balkanica*, **4** (1974), 129-135.

[8] Convolutions for the right-inverse operators of the general linear differential operator of the first order. *Serdica*, **2** (1976), 82-86.

[9] Nonlocal operational calculi. In: *Trudy Matemat. Instituta im. V.A.. Steklova*, **203** (1993), 58-73 (In Russian); Engl. Transl.: Nonlocal operational calculi. *Proc. Steklov Inst. Math.* (1995), Issue 3, 53-65.

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