Erratum to: “Subnormal, permutable, and embedded subgroups in finite groups”

Erratum

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Abstract: The original version of the article was published in Central European Journal of Mathematics, 2011, 9(4), 915–921, DOI: 10.2478/s11533-011-0029-8. Unfortunately, the original version of this article contains a mistake: Lemma 2.1 (2) is not true. We correct Lemma 2.2 (2) and Theorem 1.1 in our paper where this lemma was used.

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A subgroup H of a group G is said to be *S-semipermutable* if H permutes with every Sylow subgroup X of G such that $(|H|, |X|) = 1$. A group G is called an *SPS-group* if every subnormal subgroup of G is *S-semipermutable* in G . In [2], the authors introduce the concept of *τ -semipermutability*: A subgroup H is called *τ -semipermutable* in G if H permutes with every Sylow subgroup X of G such that $(|H|, |X|) = 1$ but $(|H|, |X^G|) \neq 1$, where X^G is the normal closure of X in G . Hence an *S-semipermutable* subgroup of a group is *τ -semipermutable*; however, the converse is not true, as seen in [2, Example 1].

[1, Lemma 2.1 (2)] states that if H is *S-semipermutable* in a group G and N is a normal p -subgroup of G for some prime p , then HN/N is an *S-semipermutable* subgroup of G/N .

This lemma was used in two critical places in [1], namely the proofs of Lemma 2.2 (2) and Theorem 1.1. Unfortunately, Lemma 2.1 (2) is in general false, as the following example shows:

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Example 1.

Let $G = A_4 \times C_3$, where A_4 is the alternating group on four letters and C_3 is the cyclic group of order 3. Let x be an element of order 2 in A_4 and let $H = \langle x \rangle \times C_3$. Since $\pi(H) = \pi(G)$, H is an S -semipermutable subgroup of G . Put $N = 1 \times C_3$. Then G/N is isomorphic to A_4 and H/N is isomorphic to $\langle x \rangle$. But $\langle x \rangle$ does not permute with any Sylow 3-subgroup of A_4 since A_4 has no subgroup of order 6. Hence H/N is not S -semipermutable in G/N .

[2, Theorem 1.2] states that a group G is a solvable PST -group if every subgroup of the generalized Fitting subgroup $F^*(G)$ is τ -semipermutable. In a solvable group, $F^*(G) = F(G)$, the Fitting subgroup of G . Now if G is solvable, every subgroup of the nilpotent subgroup $F(G)$ of G is subnormal in the normal subgroup $F(G)$ of G , and is therefore subnormal in G . Hence if G is a solvable SPS -group, every subgroup of $F(G)$ is S -semipermutable, and hence τ -semipermutable, in G . Thus by [2, Theorem 1.2], G is a PST -group. Thus a solvable SPS -group is a PST -group; clearly a PST -group is an SPS -group, so a solvable group is a PST -group if and only if it is an SPS -group. This establishes [1, Theorem 1.1]. [1, Lemma 2.2 (2)] also follows from [2, Theorem 1.2], but it is no longer required.

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References

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