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A realistic resistance deterioration model for time-dependent reliability analysis of aging bridges*

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Abstract: Bridge resistance and reliability may deteriorate with time due to aggressive environmental conditions and increasing road freight volumes, resulting in an increase of potential economic loss. This is thus a great concern to decision-makers managing the bridges' continued future service. Reasonable models of bridge resistance and applied loads are the fundamentals of accurate estimation/prediction of a bridge's serviceability. In this paper, a new model for resistance deterioration is proposed, which enables the non-increasing property and auto-correlation in the stochastic deterioration process to be incorporated. To facilitate the practical application of the model, methods to determine its parameters using obtained data on structural resistance are developed and illustrated through simple numerical examples. Time-dependent reliability analysis is conducted using the proposed resistance deterioration model based on Monte Carlo simulation, and the effect of auto-correlation in the deterioration process on structural time-dependent reliability is investigated.

Key words: Time-dependent reliability, Aging bridges, Resistance deterioration, Auto-correlation, Deterioration model

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1 Introduction

Bridges may suffer from severe environmental conditions and ever increasing traffic loads in service. Such environmental and loading conditions may cause changes in structural strength and stiffness, impairing the bridge's safety and serviceability (Mori and Ellingwood, 1993; OBrien *et al.*, 2014; Li *et al.*, 2015). As a result, it is often necessary to perform a serviceability assessment and/or maintenance optimization regarding the safety of these bridges, because the service reliability may eventually fall below the acceptable level which would be specified for new ones. In practice, uncertainties are associated with structural properties and loading conditions. In such cases, reliability analysis is a widely-used tool providing a rational criterion for the comparison of the possible results of decisions taken considering those uncertainties (Stewart and Val, 1999).

Many researchers have focused on the safety assessment and damage assessment of aging bridges. The methodology developed by Mori and Ellingwood (1993) was one of the first attempts to assess time-dependent reliability of aging structures considering both the randomness of resistance and the stochastic nature of load. This method was further used by Ellingwood and Mori (1997) to help optimize maintenance measures for aging structures, and by Enright and Frangopol (1998) to predict the service life of deteriorating bridges. Akiyama *et al.* (2010) analyzed the time-dependent structural reliability of port structures taking the hazard associated with airborne chlorides into account, in which random variables related to observed information were updated with the sequential Monte Carlo simulation. Li *et al.* (2015) proposed an improved method for

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time-dependent reliability analysis of aging structures, which enables the non-stationarity in the loading process to be incorporated. However, in these researches, the resistance deterioration process has been treated as fully-correlated, and is unable to account for the effects of auto-correlation. In reality, the resistance deterioration process, without rehabilitation or other types of strengthening, is a non-increasing process, and is neither statistically independent nor fully correlated. The challenge then arises in modeling the deterioration process realistically, consistent with the physics of resistance deterioration.

This paper proposes a new model for the realistic resistance deterioration process, whose parameters can be determined by regression analysis of observed data of structural resistance. Methods to calibrate these parameters are developed separately for the resistance deterioration of an individual bridge and a type of bridges in similar service conditions. The calibration methods are illustrated through simple numerical examples. Time-dependent reliability analyses of an illustrative aging bridge, whose deterioration behavior is described by the proposed deterioration model, are conducted by Monte Carlo simulation, and the effects of auto-correlation in the deterioration process on the estimate of time-dependent reliability are investigated.

2 Resistance deterioration model

2.1 A review

The manner in which material properties of components and systems vary with time essentially affects the structural service life. Aging mechanisms causing deterioration of concrete structures include chemical or physical attack of either the cement-paste matrix or aggregates (Clifton and Knab, 1989; Ellingwood, 2005).

A frequently used model to describe the resistance deterioration of aging structures with time is

$$R(t) = R_0 \cdot g(t), \tag{1}$$

where $R(t)$ is the resistance at time t, R_0 is the initial resistance (a random variable), and $g(t)$ is the degradation function (a stochastic process).

The stochastic process model of deterioration described by $g(t)$ is non-stationary in nature. Most models of structural deterioration that have been used in previous time-dependent reliability analysis of aging infrastructures rely on experimental laboratory data (Sun and Hong, 2002; Mohd *et al.*, 2014; Sharifi and Paik, 2014). Reliability analyses requiring extrapolation beyond the range of limited experimental data are actually problematic because the service conditions might differ significantly from the laboratory conditions (Ellingwood, 2005; Melchers, 2008; Saad-Eldeen *et al.*, 2013). For example, a critical review of corrosion propagation models showed that the existing models have significant discrepancy in predicting the deteriorations of concrete structures and have not been validated in many cases (Tarighat and Jalalifar, 2014). Reasonable deterioration models should incorporate both the deterioration mechanisms and observed information, because the mechanism determines the overall shape of the deterioration function while the *in-situ* or experimental data may reduce the epistemic uncertainties associated with the mechanism-based models. However, note that the deterioration of structures is a multifarious process, which may include multiple deterioration mechanisms. In practice, the observed data associated with structural resistance deterioration are often limited, with which it is difficult or impossible to calibrate all the deterioration parameters associated with the multifarious deterioration processes. The objective of this study is to develop a realistic deterioration model for bridges' resistance deterioration focusing on the overall trend of the deterioration reflected by the observed information, which is capable of incorporating the observed information once it becomes available.

For a fully correlated deterioration process, a general model for the deterioration function, $g(t)$, takes the form of (Mori and Ellingwood, 1993)

$$g(t) = 1 - a \cdot t^{\alpha}, \tag{2}$$

where a and α are parameters determining the deterioration shape. Eq. (2), which represents a generic deterioration process capturing the overall trend of the deterioration mechanism, can be applied to the cases of environmental attacks. The value of α is determined according to the dominant deterioration mechanism of interest, such as corrosion ($\alpha = 1$), sulfate attack ($\alpha = 2$), and diffusion-controlled aging ($\alpha = 0.5$). However, if the inspection data are sufficient, the value of α should be determined by the observed data. Moreover, the deterioration model in Eq. (2) may appear more realistic in some cases if it includes an additive noise, $\epsilon(t)$, incorporating the uncertainty associated with the deterioration process, i.e.,

$$g(t) = 1 - (a \cdot t^{\alpha} + \epsilon(t)), \tag{3}$$

where $\epsilon(t)$ is an independent, stationary, and zero-mean normal sequence. However, such a deterioration process in Eq. (3) implies that the resistance may increase with time since $\epsilon(t)$ is possibly negative. Bhattacharya *et al.* (2008) proposed a model for $g(t)$ which incorporates a multiplicative noise term ensuring that the resistance deterioration function is non-decreasing in time:

$$\frac{\mathrm{d}g(t)}{\mathrm{d}t} = \begin{cases} 0, & t \leq t_{\text{initial}}, \\ \beta(t - t_{\text{initial}})^{\gamma}\exp(\eta(t)), & t > t_{\text{initial}}, \end{cases} \tag{4}$$

$$\frac{\mathrm{d}\eta(t)}{\mathrm{d}t} = -k\eta(t) + \sqrt{D}\xi(t), \tag{5}$$

where t_{initial} is the time to activate the deterioration process; $\eta(t)$ is the exponentiated noise item; $\xi(t)$ is the white noise; β and γ are random parameters independent of time, while k and D are constants. This model allows the non-stationarity and the auto-correlation in resistance deterioration process involved. It is not difficult to generate a realization for such a deteriorating process utilizing Monte Carlo simulations when the parameters are determined. However, the calibration of the parameters using observed data is quite challenging due to the non-explicit form of the model. To give a realistic and simple description for resistance deterioration, the Gamma process has been considered by previous studies (Dieulle *et al.*, 2003; Saassouh *et al.*, 2007; van Noortwijk *et al.*, 2007; Li *et al.*, 2015), because such a process describes a continuous stochastic process with non-decreasing trajectories. However, these researches mainly focus on the parametric studies for the deterioration process; no effort has been made on the calibration of the deterioration model parameters using obtained data. In this paper, a resistance deterioration model for aging structures is proposed in Section 2.2, which satisfies the physics of deterioration and provides convenience for the calibration of parameters, as will be seen in Section 3.

2.2 Proposed deterioration model

The resistance deterioration process, $X(t)$ in time, defined as $R_0 - R(t)$, is a non-decreasing process with independent increments assumed. With this, a qualitative realization of $R(t)$ is illustrated in Fig. 1.

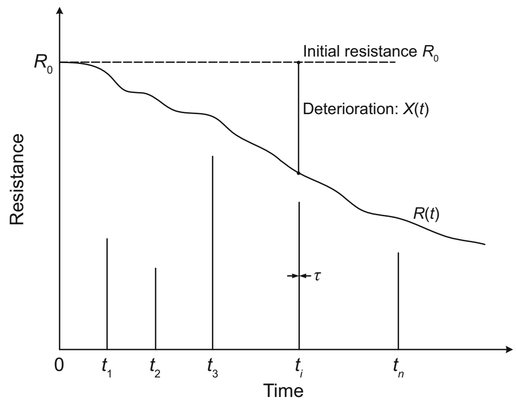

Fig. 1 Stochastic processes of deterioration and loads. Reprinted from (van Noortwijk *et al.*, 2007) with modification, Copyright 2006, with permission from Elsevier

For the service period of interest, $(0, T]$, suppose that a sequence of load effects, $S_1, S_2, \ldots, S_n$, occurs at times $t_1, t_2, \ldots, t_n$, and the duration of each load event (τ) is short enough compared with T (Fig. 1). The time sequence, $\{t_1, t_2, \ldots, t_n\}$ in ascending order, divides $(0, T]$ into $n+1$ sections. With the resistance deterioration function, $g(t)$, modeled as a Gamma process (an introduction to the Gamma distribution and Gamma process is found in Appendix A), the increment of deterioration during time interval $(t_{i-1}, t_i]$, G_i, is assumed statistically independent (Appendix A). Suppose that G_i is Gamma distributed with scale parameter b and shape parameter a_i, $G_i \sim \mathrm{Ga}(a_i, b)$, $i = 1, 2, \ldots, n$, then the resistance at time t_k, $R(t_k)$, is

$$R(t_k) = R_0 \cdot g(t_k) = R_0 \cdot \left(1 - \sum_{i=1}^{k} G_i\right). \tag{6}$$

Thus,

$$g(t_k) = 1 - \sum_{i=1}^{k} G_i, \quad k = 1, 2, \ldots, n. \tag{7}$$

Eq. (6) generates a non-increasing process since each $G_i > 0$. Note that $G_i \sim \mathrm{Ga}(a_i, b)$, $\sum\limits_{i=1}^{k} G_i \sim$

$\mathrm{Ga}(\sum_{i=1}^{k} a_i, b)$, the mean value and variance of $g(t_k)$ are determined respectively by

$$\mathrm{Mean}[g(t_k)] = 1 - b \cdot \sum_{i=1}^{k} a_i, \tag{8}$$

$$\mathrm{Var}[g(t_k)] = b^2 \cdot \sum_{i=1}^{k} a_i, \tag{9}$$

for $k = 1, 2, \ldots, n$. It is seen from Eqs. (8) and (9) that the mean value of $g(t_k)$ decreases with time while the variance of $g(t_k)$ increases in time due to the involvement of more uncertainties. In this paper, a_i and b are called deterioration parameters, and it is assumed that the deterioration process is independent of the initial resistance R_0.

The deterioration parameters, a_i and b, will be determined using observed resistance data, as will be discussed in Section 3. The effects of auto-correlation in deterioration process will be discussed in Section 5.

According to Eqs. (7) and (9), the coefficient of correlation between $g(t_i)$ and $g(t_j)$, $\rho_{i,j}$, is given as

$$\rho_{i,j} = \sqrt{\frac{\mathrm{Var}[g(t_i)]}{\mathrm{Var}[g(t_j)]}} = \sqrt{\frac{\sum_{l=1}^{i} a_l}{\sum_{l=1}^{j} a_l}}, \quad t_i < t_j. \tag{10}$$

Eq. (10) demonstrates that the proposed deterioration model accounts for the auto-correlation in the realistic deterioration process, and $\rho_{i,j}$ decreases as the time lag $t_j - t_i$ increases and reaches zero when the time lag is sufficiently long, which is consistent with the physics of deterioration.

3 Determination of deterioration parameters

3.1 Condition inspection and resistance evaluation of existing bridges

Practically, deterioration conditions of existing bridges are assessed by *in-situ* inspection following established standards or manuals (e.g., AASHTO, 2008; MOT, 2011). The inspection results provide useful information to the resistance evaluation of the bridge. For example, according to the Chinese code "Specification for inspection and evaluation of load-bearing capacity of highway bridges" (MOT, 2011), the residential resistance of existing reinforced concrete girders, R_c, is estimated by

$$R_\mathrm{c} = R(f_\mathrm{d}, \xi_\mathrm{c} a_\mathrm{dc}, \xi_\mathrm{s} a_\mathrm{ds}) \cdot Z_1 \cdot (1 - \xi_\mathrm{e}), \tag{11}$$

where f_d is the design strength of materials; ξ_c and ξ_s are the section reduction coefficients of concrete and steel bars; a_dc and a_ds are the geometry dimensions of concrete and steel, respectively; Z_1 is the comprehensive modification coefficient of bridge load-bearing capacity; and ξ_e is the deterioration coefficient of load-bearing capacity. With R_c obtained, the deterioration function at time t_c, $g(t_\mathrm{c})$, is calculated as

$$g(t_\mathrm{c}) = R_\mathrm{c}/R_0, \tag{12}$$

where t_c is the survival age of the inspected bridge. The initial resistance of the girder, R_0, in Eq. (12) may be calculated following the manuals or national codes (e.g., MOT, 2004). For example, according to the stress-block model, if the neutral axis is within the flange, the initial bending moment capacity, R_0, can be expressed as

$$R_0 = A_\mathrm{s} f_\mathrm{y} \left(d - 0.5 \frac{A_\mathrm{s} f_\mathrm{y}}{0.85 f'_\mathrm{c} \cdot w} \right), \tag{13}$$

where A_s is the area of reinforcing steels, f_y is the yielding strength of reinforcing steel, d is the depth of girder, f'_c is the compressive strength of concrete, and w is the width of girder flange.

In practice, the bridge resistance is seldom evaluated during the service life partially because of the relatively high cost required; as a result, available resistance information for an existing bridge is limited. Recognizing this, two methods to calibrate the deterioration parameters using limited resistance information are developed in this section. Method 1 is for an individual existing bridge whose resistance is evaluated currently. Method 2 is for a type of bridges in similar service conditions; the resistance of these bridges follows a common deterioration function; some individual bridges of the type were evaluated at different ages and these data are available.

3.2 Method 1

In Eq. (7), as indicated in Appendix A, $\sum_{i=1}^{k} G_i \sim \mathrm{Ga}(\sum_{i=1}^{k} a_i, b)$, where a_i and b are unknown. The choice of a_i is mechanism-related, which determines the shape of the deterioration function referring to Eq. (8). For example, if the deterioration is mainly controlled by corrosion of reinforcement, the mean value of the deterioration function is usually modeled as linear with time (Mori and Ellingwood, 1993). In this case, a_i takes the form of

$$a_i = \kappa_{\mathrm{l}}(t_i - t_{i-1}), \quad i = 1, 2, \ldots, n, \tag{14}$$

where κ_{l} is a parameter indicating the deterioration rate. Substituting Eq. (14) into Eq. (8), we have

$$\mathrm{Mean}[g(t_k)] = 1 - b \cdot \kappa_{\mathrm{l}} \cdot t_k. \tag{15}$$

Eq. (15) implies that the mean value of resistance deteriorates linearly with time. Similarly, if the square root model in Eq. (2) ($\alpha = 0.5$) due to the mechanism of diffusion-controlled aging, or the parabolic model ($\alpha = 2$) due to the mechanism of sulfate attack is used, a_i is in the form of

$$a_i = \kappa_{\mathrm{s}}(\sqrt{t_i} - \sqrt{t_{i-1}}), \tag{16}$$

$$a_i = \kappa_{\mathrm{p}}({t_i}^2 - t_{i-1}^2), \quad i = 1, 2, \ldots, n, \tag{17}$$

where κ_{s} and κ_{p} are deterioration rate parameters similar to κ_{l} in Eq. (14).

A general form for Eqs. (14), (16), and (17) is

$$a_i = \kappa({t_i}^{\alpha} - t_{i-1}^{\alpha}), \quad i = 1, 2, \ldots, n, \tag{18}$$

where κ is a parameter indicating the deterioration rate; $\alpha = 0.5$, 1, or 2 corresponding to different deterioration mechanisms.

Now if the mean value and variance of $g(t^*)$ at time t^* are known as m^* and v^*, according to Eqs. (8) and (9),

$$m^* = 1 - b \cdot \kappa \cdot (t^*)^{\alpha}, \tag{19}$$

$$v^* = b^2 \cdot \kappa \cdot (t^*)^{\alpha}. \tag{20}$$

Thus, the estimates of parameters b and κ, $\hat{b}$ and $\hat{\kappa}$, are determined as

$$\hat{b} = \frac{v^*}{1 - m^*}, \tag{21}$$

$$\hat{\kappa} = \frac{(1 - m^*)^2}{v^* \cdot (t^*)^{\alpha}}. \tag{22}$$

An illustrative application of Method 1 can be found in Section 5.

3.3 Method 2

Different from Method 1 which aims at determining the deterioration function for an individual bridge, Method 2 aims at determining a common deterioration function for a specific type of bridges in similar service conditions.

Now suppose that the deterioration functions (i.e., the ratio of residential resistance to initial resistance) of p existing bridges in similar service conditions, whose survival ages are $t_1^*, t_2^*, \ldots, t_p^*$ (in ascending order and in years) respectively, are obtained as $m_1, m_2, \ldots, m_p$. With Eqs. (8) and (18), we have

$$1 - m_i = b \cdot \kappa \cdot (t_i^*)^{\alpha}. \tag{23}$$

Taking the logarithmic form for both sides of Eq. (23),

$$\ln(1 - m_i) = \ln(b \cdot \kappa) + \alpha \ln(t_i^*), \tag{24}$$

where $i = 1, 2, \ldots, p$. The estimates of α and $b \cdot \kappa$, $\hat{\alpha}$ and $\hat{b} \cdot \hat{\kappa}$, can be obtained graphically in the coordinates with the abscissa of $\ln(t_i^*)$ and the ordinate of $\ln(1 - m_i)$. Using the "least squares method" (Spiegel *et al.*, 2009), $\hat{\alpha}$ equals the slope of the linearly fitted curve $\ln(1 - m_i)$ with respect to $\ln(t_i^*)$, while $\hat{b} \cdot \hat{\kappa}$ equals the intercept of the fitted line accordingly.

With $\hat{\alpha}$ and $\hat{b} \cdot \hat{\kappa}$, the estimated mean values of the deterioration function at time instants $t_1^*, t_2^*, \ldots, t_p^*$ can be obtained as $\hat{m}_1, \hat{m}_2, \ldots, \hat{m}_p$. Note that Eq. (9) implies that the variance of the deterioration function increases with time in proportion to t^{α}. However, the limited data can not reflect the change of variance of the deterioration function with time since the number of samples corresponding to each time instant is very few. Recognizing this, this paper uses the average variance of the deterioration function, which is $\frac{1}{p}\sum_{i=1}^{p}(m_i - \hat{m}_i)^2$, to determine the unknown parameters. With this, we have

$$\hat{b}^2 \cdot \hat{\kappa} \cdot \sum_{k=1}^{p}(t_k^*)^{\alpha} = \sum_{i=1}^{p}(m_i - \hat{m}_i)^2. \tag{25}$$

Thus, $\hat{b}$ and $\hat{\kappa}$ are determined as

$$\hat{b} = \frac{\sum_{i=1}^{p}(m_i - \hat{m}_i)^2}{\hat{b} \cdot \hat{\kappa} \cdot \sum_{k=1}^{p} t_k^{\alpha}}, \quad \hat{\kappa} = \frac{\hat{b} \cdot \hat{\kappa}}{\hat{b}}. \tag{26}$$

As a widely-used indicator of goodness of fit estimate (Rousson and Goşoniu, 2007; Tellinghuisen and Bolster, 2011), R^2 is employed in this paper to assess the fitting of mean deterioration value upon the bridge's survival age (t_i^*, $i=1,2,\ldots,p$), which is defined as

$$R^2 = 1 - \frac{\sum_{i=1}^{p}(m_i - \hat{m}_i)^2}{\sum_{i=1}^{p}(m_i - \overline{m})^2}, \tag{27}$$

where $\overline{m} = \frac{1}{p}\sum_{i=1}^{p} m_i$. A perfect fit yields $R^2 = 1$.

For illustration purposes, suppose we obtained the resistance data of some bridges at different ages, i.e., 11 years, 12 years, ..., 25 years, and the values of the deterioration function at these ages were calculated, as presented in Table 1.

Table 1 Observed values of deterioration function in the illustrative example

t^* (year)	Mean[$g(t^*)$]	t^* (year)	Mean[$g(t^*)$]
11	0.817	19	0.737
12	0.897	20	0.719
13	0.788	21	0.726
14	0.802	22	0.630
15	0.723	23	0.707
16	0.739	24	0.673
17	0.835	25	0.562
18	0.804		

With the data obtained, first, $\hat{\alpha}$ and $\hat{b}\cdot\hat{\kappa}$ are determined using the graphical method mentioned above: $\hat{\alpha}=1.0847$ and $\hat{b}\cdot\hat{\kappa}=\exp(-4.5216)=0.0109$, as shown in Fig. 2. $\hat{b}$ and $\hat{\kappa}$ are further determined as 0.008645 and 1.2575, respectively, using Eq. (26). With the deterioration parameters, α, b, and κ determined as $\hat{\alpha}$, $\hat{b}$, and $\hat{\kappa}$, respectively, the observed and estimated values of the deterioration process are shown in Fig. 3. The variances associated with the calibrated deterioration function can also be calculated by Eqs. (9) and (18) and are also plotted in Fig. 3, where the coefficient of variance (COV) is simply the ratio of standard deviation to mean value. The value of R^2 is calculated as 0.68 according to Eq. (27) for Fig. 3, which is an indicator of the model uncertainty, related to the uncertainties associated with $\hat{\alpha}$ and $\hat{b}\cdot\hat{\kappa}$.

The coefficients of correlation $\rho_{i,j}$ between $g(t_i)$ and $g(t_j)$ are calculated according to Eq. (10) and are presented in Fig. 4. $\rho_{i,j}$ equals 1 when $t_i=t_j$ and decreases as the time lag, t_j-t_i, increases. Moreover, $\rho_{i,j}$ depends on both t_i and t_j, rather than the time lag only.

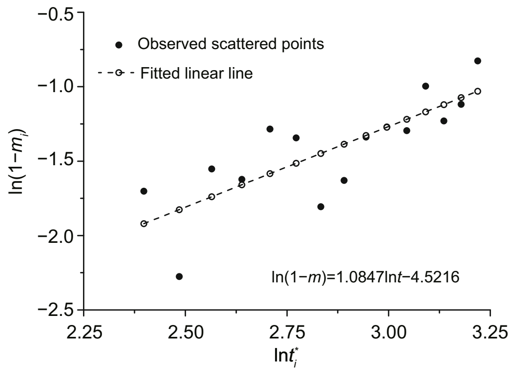

Fig. 2 Determination of $\hat{\alpha}$ and $\hat{b}\cdot\hat{\kappa}$ using the graphical method

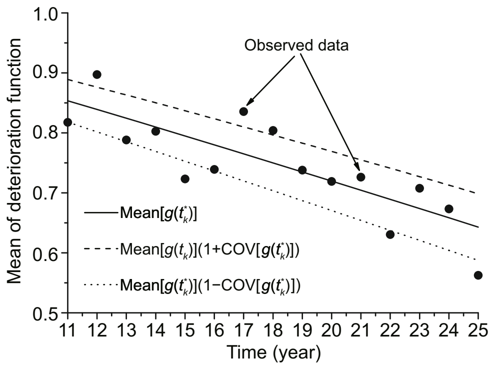

Fig. 3 Estimated mean values and variances of the deterioration function

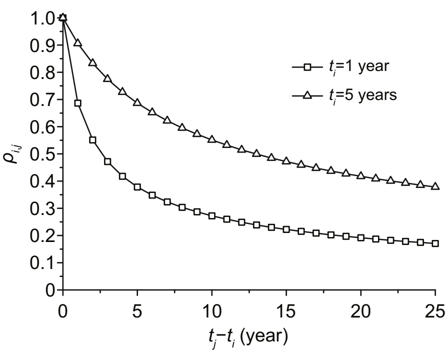

Fig. 4 Coefficients of correlation associated with the deterioration process

4 Time-dependent reliability analysis

Structural resistance, R, and applied load, S, are random variables, and the margin of safety is

$$Z = R - S. \tag{28}$$

The structural failure probability, P_f, is defined as the possibility that $Z < 0$. Correspondingly, the reliability, Rel, equals $1 - P_\mathrm{f}$.

Taking into account the time-variant properties of R and S, let $R(t)$ and $S(t)$ denote the resistance and applied load at time t ($R(t)$ has been defined in Eq. (1)), and the margin of safety at any time, t, is then

$$Z(t) = R(t) - S(t). \tag{29}$$

Under the assumption that R and S are statistically independent, the instantaneous probability of failure, $P_\mathrm{f}(t)$, is

$$P_\mathrm{f}(t) = \Pr[Z(t) < 0] = \int_0^\infty F_R(x,t) f_S(x,t)\mathrm{d}x, \tag{30}$$

where Pr[ ] is the probability of the event in the bracket, $F_R(x,t)$ is the cumulative density function (CDF) of $R(t)$, and $f_S(x,t)$ is the probability density function (PDF) of $S(t)$.

For the purpose of service life prediction and reliability analysis, the probability of structural expected performance over the period of interest, say, $(0,T]$, is of more importance than the snapshot of the reliability (or the failure probability) expressed in Eq. (30). The probability that a structure survives over $(0,T]$, $\mathrm{Rel}(0,T]$, is

$$\mathrm{Rel}(0,T] = \Pr\{R(\tau) \geq S(\tau), \forall \tau \in (0,T]\}. \tag{31}$$

Significant load events occur randomly in time with random intensities. Suppose that the load intensity varies negligibly or slowly during the interval in which it occurs, and there is no dynamic response, its effect on the structure may be considered as static, and for the purpose of reliability analysis, the load intensity may be treated as constant during the load event (Fig. 1). With these assumptions, suppose that during interval $(0,T]$ n discrete statistically independent loads $S_1, S_2, \ldots, S_n$ occur at times $t_1, t_2, \ldots, t_n$ (The load events may be eventually correlated (Li and Wang, 2014). In this study, the correlation in load events is ignored, i.e., the loads are treated as statistically independent), then $\mathrm{Rel}(0,T]$ becomes

$$\mathrm{Rel}(0,T] = \Pr\{R(t_1) > S_1 \cap \ldots \cap R(t_n) > S_n\}. \tag{32}$$

The discrete loading process can be modeled as a stationary Poisson process, within which the occurring time of each load event is uniformly distributed in $(0,T]$ independently.

With Eq. (6), the time-dependent reliability in Eq. (32) becomes

$$\mathrm{Rel}(0,T] = \Pr\left\{\bigcap_{k=1}^{n}\left[R_0 \cdot \left(1 - \sum_{i=1}^{k} G_i\right) > S(t_k)\right]\right\}. \tag{33}$$

Because of the difficulty of solving Eq. (33) in close form, the Monte Carlo simulation is performed herein for structural time-dependent reliability analysis. To estimate $\mathrm{Rel}(0,T]$, the procedure of each simulation run is summarized as follows.

(1) Generate a sample of initial resistance, r_0, according to the PDF of R_0, $f_{R_0}(r)$.

(2) Since the number of load events during $(0,T]$, N, is Poisson distributed, simulate a sample of N, n, and then generate n random variables uniformly in $(0,T]$, $t_1 < t_2 < \ldots < t_n$.

(3) Knowing that $G_i \sim \mathrm{Ga}(a_i, b)$, generate G_1, $G_2, \ldots, G_n$ independently for time intervals $(0,t_1]$, $(t_1,t_2]$, ..., $(t_{n-1},t_n]$.

(4) Corresponding to $t_1, t_2, \ldots, t_n$, simulate n independent load effects, $S(t_1), S(t_2), \ldots, S(t_n)$.

(5) If $S(t_i)$ does not exceed $R(t_i)$ for all $i = 1, 2, \ldots, n$, then the structure is deemed to survive during $(0,T]$.

Performing the above procedures for M times ($M = 1\,000\,000$), if the structure survives for m times, the time-dependent reliability, $\mathrm{Rel}(0,T]$, is then m/M.

5 Illustrative examples

With the proposed resistance deterioration model, the effect of auto-correlation in the deterioration process on time-dependent reliability is investigated in this section using several parametric representations of resistance deterioration and load process.

5.1 Models of resistance and loads

The probabilistic models of initial resistance and load history of an existing bridge used in this parametric study are summarized in Table 2. The initial resistance, R_0, is assumed to be lognormally distributed with mean value of $1.15R_\mathrm{n}$ (R_n is the nominal or code-specified resistance) and COV of 0.15. This assumption is consistent with many observations in structural reliability analysis where uncertainties in time-varying loads contribute a major

Table 2 Statistical descriptions for structural resistance and loads

Resistance and load	Mean	COV	Distribution	Occurrence rate
Initial resistance	$1.15R_n$	0.15	Lognormal	–
Dead load	$1.05D_n$	0	–	–
Live load 1 (LL1)	$0.5L_n$	0.40	Extreme type I	1.0/year
Live load 2 (LL2)	$0.6L_n$	0.40	Extreme type I	1.0/year

portion of the overall uncertainties (Nowak, 1995; Enright and Frangopol, 1998; Faber *et al.*, 2000). The dead load is equal to $1.05D_n$ (D_n is the nominal value), and its variability is assumed to have a negligible impact on reliability in comparison with the uncertainty in the time-varying live load. The time-dependent reliabilities for the bridge's service periods up to 20 years are considered.

The loading process is assumed to be a stationary Poisson process with occurrence rate $\lambda = 1.0$/year (on average 1 load event occurs per year). To illustrate the effect of load magnitude on bridge reliability, two live load models are considered (Table 2). Each live load follows the extreme type I distribution with COV = 0.40.

The mean value of initial resistance is evaluated according to the design requirement for concrete structures in flexure (AASHTO, 2007):

$$1.0R_n = 1.25D_n + 1.75L_n. \tag{34}$$

For the purpose of simplicity, it is assumed that $D_n = L_n = 1.0$, from which R_n is obtained as 3.0 according to Eq. (34).

5.2 Deterioration model

For the existing bridge, suppose the resistance deterioration function at the end of 20 years, $g(20)$, is estimated to have a mean value of 0.7 and COV of 0.30. Parameters b and κ in the deterioration function (Eqs. (7) and (18)) can be determined using Method 1 as follows.

For linear deterioration due to the mechanism of corrosion ($\alpha = 1$), parameters b and κ can be solved from Eqs. (21) and (22) as $\hat{b} = 0.147$, $\hat{\kappa} = 0.102$. For the cases of square root deterioration ($\alpha = 0.5$), $\hat{b} = 0.147$, $\hat{\kappa} = 0.456$, and for parabolic deterioration ($\alpha = 2$), $\hat{b} = 0.147$, $\hat{\kappa} = 0.0051$. Some illustrative realizations of linear deterioration processes are plotted in Fig. 5; the coefficients of correlation $\rho_{i,j}$ between $g(t_i)$ and $g(t_j)$ for the linear deterioration are plotted in Fig. 6.

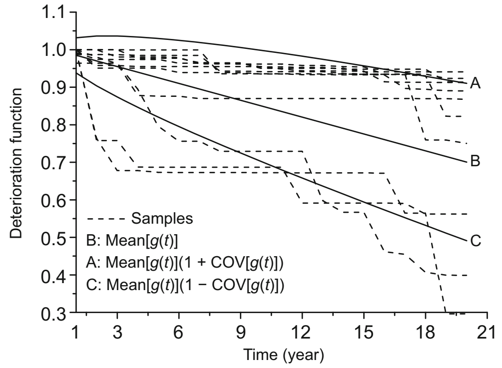

Fig. 5 Samples of linear deterioration processes generated by Eq. (9)

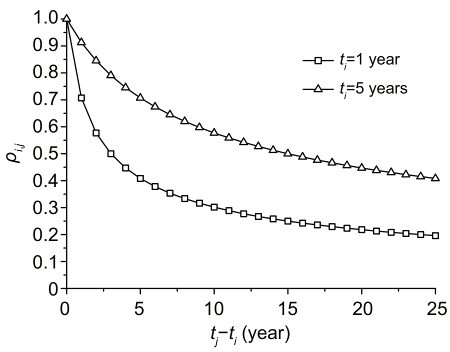

Fig. 6 Coefficients of correlation associated with the resistance deterioration process for the illustrative example

To investigate the effects of auto-correlation in the deterioration process on the time-dependent reliability of structures, the case of fully correlated deterioration process is considered with the same mean value and variation as the proposed one. Note that for the proposed model, the mean value of the deterioration function decreases with time in proportion to t^α and the variance increases with time in proportion to t^α, as seen from Eqs. (8), (9), and (18). Thus, the mean value and variance of the fully correlated deterioration function should also decrease/increase linearly with t^α. With this, the fabrication and simulation of the fully correlated deterioration process can be found in Appendix B.

5.3 Results

With the realistic resistance deterioration process (the proposed one), the time-dependent reliabilities for periods up to 20 years are obtained using Monte Carlo simulation (Section 4) and are plotted in Fig. 7. The ordinate of Fig. 7 is the probability of failure, $P_f(0,T]$, defined as $1-\mathrm{Rel}(0,T]$. For comparison purposes, time-dependent reliabilities with fully correlated deterioration processes are also obtained by simulation (Appendix B) and are plotted in Fig. 7.

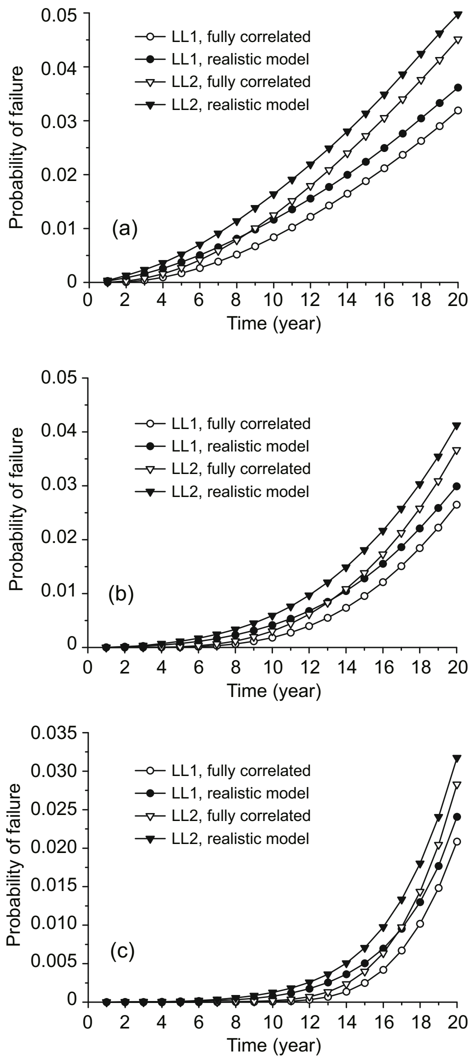

Fig. 7 Time-dependent reliabilities for periods up to 20 years: (a) square root deterioration; (b) linear deterioration; (c) parabolic deterioration

The effect of auto-correlation in the resistance deterioration on time-dependent reliability is seen through Fig. 7. In all cases, the failure probability increases with time, which is characteristic of structural aging; $P_f(0,T]$ associated with live load 2 is greater than that associated with live load 1 as expected. The probability of failure associated with the fully correlated deterioration process is smaller than that associated with the Gamma deterioration process (the realistic one), indicating that structural reliability may be overestimated when the deterioration process is modeled as fully correlated (e.g., Eq. (2)). For example, for the parabolic deterioration process, when live load 1 is considered, the failure probability is 0.0209 for fully correlated resistance, about 0.87 times that associated with the realistic deterioration model. This observation is consistent with that by Li *et al.* (2015). Further, it is observed from Fig. 7 that at the early stages of the service life where the failure probability is small, the failure probability for the case of "LL1, realistic model" is greater than that associated with the case of "LL2, fully correlated model", although the intensity of LL2 is greater than LL1, indicating that $P_f(0,T]$ is more sensitive to the auto-correlation in the resistance deterioration process than the intensity of live loads. However, as the service life becomes longer, the failure probability for the case of "LL2, fully correlated model" becomes close to and even exceeds the failure probability for the case of "LL1, realistic model", implying that $P_f(0,T]$ becomes gradually more sensitive to the live load intensity.

Moreover, the difference between the failure probabilities associated with the fully correlated deterioration process and those associated with the realistic one becomes smaller as the failure probability increases. This can also been seen through Table 3, where the ratios of failure probabilities associated with the realistic deterioration model to those associated with the fully correlated one are presented for the case of linear deterioration. Therefore, it is concluded that when the realistic deterioration model is treated as fully correlated, it may result in a significant overestimate for the time-dependent reliability if the failure probability is small. This observation can be explained conceptually as follows. For simplicity, consider two extreme cases: (1) the resistance $R(t)$ is fully correlated and (2) the resistance $R(t)$ is statistically independent for different time instants. Suppose that no resistance deterioration occurs (i.e., $R(t)=R$) during the considered time period and two loads, S_1 and S_2, occur. With this, the failure probabilities for the two cases are determined respectively as

$$P_{\mathrm{f},1} = F_R(\max(S_1, S_2)), \tag{35}$$

$$P_{\mathrm{f},2} = 1 - (1 - F_R(S_1))(1 - F_R(S_2)), \tag{36}$$

where $F_R(r)$ is the CDF of the resistance, R. Now if $S_1 = S_2 = S$, then

$$\frac{P_{\mathrm{f},2}}{P_{\mathrm{f},1}} = 2 - F_R(S). \tag{37}$$

Table 3 Ratios of failure probabilities associated with the realistic deterioration model to those associated with the fully correlated one (the unit of T is year)

Load	Ratio of failure probabilities				
	$T=4$	$T=8$	$T=12$	$T=16$	$T=20$
LL1	54.250	3.694	1.690	1.303	1.127
LL2	10.727	3.005	1.560	1.254	1.124

It is seen from Eq. (37) that $P_{\mathrm{f},2}$ is greater than $P_{\mathrm{f},1}$ since $0 < F_R(S) < 1$, indicating that the fully-correlated deterioration model results in an underestimate of failure probability. In addition, the ratio of $P_{\mathrm{f},2}$ to $P_{\mathrm{f},1}$ decreases as $F_R(S)$ increases, implying that the effect of auto-correlation in the deterioration process on structural reliability becomes smaller as the failure probability increases.

The comparison among Figs. 7a–7c shows that the square root deterioration model generates the greatest failure probability, followed by that associated with the linear model and the parabolic model. This is because for the same $g(20)$, the square root model generates the lowest resistance for the considered periods up to 20 years while the parabolic model generates the highest resistance. In addition, the failure probability is sensitive to the choice of deterioration type for the early stage of service life. For example, for LL1, if a service period of 10 years is considered, the failure probabilities, $P_f(0, 10]$, are 0.0118, 0.0041, and 0.0008 for square root, linear, and parabolic deterioration types, respectively, using the realistic deterioration model. However, the failure probability becomes less sensitive to the deterioration type as the service life becomes longer. For example, $P_{\mathrm{f}}(0, 20]$ are found as 0.0361, 0.0299, and 0.0241 respectively for the three deterioration types mentioned above.

6 Conclusions

A new model for a bridge's resistance deterioration is presented in this paper for evaluating time-dependent reliability of aging bridges, which enables non-decreasing properties and auto-correlation in deterioration process to be incorporated. For the practical application of the proposed deterioration model, calibration methods are developed to determine the deterioration parameters for two cases: the deterioration function of an individual bridge and the common deterioration function of similar bridges in similar service conditions. Numerical examples are presented to illustrate the application of the realistic model and the calibration methods. The proposed deterioration model is then used in time-dependent reliability analyses of an illustrative bridge, and the effects of auto-correlation in the resistance deterioration process are investigated. It is found that if the deterioration process is treated as fully correlated, the failure probability will be underestimated, and this underestimate may be significant if the failure probability is small. In addition, the square root deterioration process generates the lowest reliability compared with that associated with the linear model and the parabolic model. Time-dependent reliability is sensitive to the choice of deterioration type at the early stage of service life and becomes less sensitive to the deterioration type as the service period becomes longer.

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中文概要

题 目：用于桥梁时变可靠度分析的承载力衰减新模型

目 的：提出能够反映劣化桥梁承载力的非增特性以及自相关性的承载力衰减新模型，并利用该模型进行结构可靠度分析。

创新点：提出描述劣化桥梁承载力衰减的新的随机过程模型，给出利用实测数据进行模型参数拟合的方法，并利用提出的衰减模型对劣化桥梁进行时变可靠度分析。

方 法：采用Gamma过程描述承载力衰减的随机过程（公式7），并基于Monte Carlo模拟的方法研究承载力衰减过程的自相关性对可靠度结果的影响。如果将承载力衰减过程假定为完全相关的过程，则会低估结构的失效概率。

关键词：时变可靠度；劣化桥梁；衰减模型；自相关性

Appendix A

A random variable, X, has a Gamma distribution with shape parameter $a > 0$ and scale parameter $b > 0$ if the PDF of X takes the form of

$$f_X(x) = \frac{(x/b)^{a-1}}{b\Gamma(a)}\exp(-x/b), \quad x \geq 0, \tag{A1}$$

where $\Gamma()$ is the Gamma function. Such an X is usually written as $X \sim \mathrm{Ga}(a, b)$. The mean value and variance of X are ab and ab^2, respectively.

A very useful property for Gamma distribution is that for two parameters $X_1 \sim \mathrm{Ga}(a_1, b)$ and $X_2 \sim \mathrm{Ga}(a_2, b)$, the sum of X_1 and X_2 also follows Gamma distribution, i.e., $X_1 + X_2 \sim \mathrm{Ga}(a_1 + a_2, b)$.

The Gamma process with shape function $a(t) > 0$ and scale parameter $b > 0$ is a continuous stochastic process $\{X(t), t > 0\}$ if satisfying the following three properties:

(1) The probability that $X(0) = 0$ is 1;

(2) $X(t)$ has independent increments;

(3) The increments in (2) also follow Gamma distribution with the same scale parameter b.

With these, the mean value and variance of $X(t)$ are respectively:

$$\mathrm{Mean}[X(t)] = a(t) \cdot b, \quad \mathrm{Var}[X(t)] = a(t) \cdot b^2. \tag{A2}$$

Appendix B

For comparison purposes, the mean value of the fully correlated deterioration function, $g(t)$, should decrease linearly with t^α while the variance of $g(t)$ should increase in proportion to t^α. With this, $g(t)$ takes the form of

$$g(t) = 1 - \xi \cdot t^{\alpha/2} + \mathrm{Mean}[\xi] \cdot (t^{\alpha/2} - t^\alpha), \tag{B1}$$

where the random variable ξ indicates the deterioration rate. The mean value and variance of $g(t)$ in Eq. (B1) are

$$\mathrm{Mean}[g(t)] = 1 - \mathrm{Mean}[\xi] \cdot t^\alpha, \tag{B2}$$

$$\mathrm{Var}[g(t)] = \mathrm{Var}[\xi] \cdot t^\alpha. \tag{B3}$$

It is easy to see that the mean value and variance of $g(t)$ in Eq. (B1) varies linearly with t^α as expected.

The Monte Carlo simulation is employed in this paper to analyze time-dependent reliabilities with fully correlated deterioration processes. For each simulation run, the procedure is summarized as follows:

(1) Generate a sample of initial resistance, r_0, and then generate a Gamma distributed random variable X with mean value of 0.3 and COV of 0.7; set $g(20) = 1 - X$;

(2) Determine the value of ξ in Eq. (B1) with sampled $g(20)$. To do this, Mean[ξ] is firstly determined with Eq. (B2), and then ξ is obtained as

$$\xi = \frac{1 + \mathrm{Mean}[\xi] \cdot (20^{\alpha/2} - 20^\alpha) - g(20)}{20^{\alpha/2}}; \tag{B4}$$

(3) Simulate a sample of number of load events N, n, and then generate n random variables uniformly in $(0, T]$, $t_1 < t_2 < \ldots < t_n$;

(4) The resistance at time t_i, $R(t_i)$, is calculated according to Eqs. (1) and (B1) for $i = 1, 2, \ldots, n$;

(5) Corresponding to $t_1, t_2, \ldots, t_n$, simulate n independent load effects, $S(t_1), S(t_2), \ldots, S(t_n)$;

(6) If $S(t_i)$ does not exceed $R(t_i)$ for all $i = 1, 2, \ldots, n$, then the structure is deemed to survive during $(0, T]$.