

CARTOMETRIC ANALYSIS OF THE CZECHOSLOVAK VERSION OF 1:75 000 SCALE SHEETS OF THE THIRD MILITARY SURVEY (1918–1956)

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Open Access of this paper is sponsored by the Hungarian
Scientific Research Fund under the grant No. T47104 OTKA
(for online version of this paper see www.akkrt.hu/journals/ageod)

The 1:75 000 scale Special Map (SM75) series of the Third Military Survey is analysed in this work. Geographic co-ordinates of the sheet corners are computed from the sheet number codes, as well as their grid co-ordinates in the modern Křovák Projection and Czech and Slovak national grid called S-JTSK. Distortions caused by paper drying and shrinkage were analysed and a mathematical algorithm is given to compute their effects. Statistical analysis shows that the shrinkage is direction-dependent at the studied 125 map sheets throughout the Czech Republic. This analysis also verified the usefulness and practical adaptability of the shrinkage correction method to obtain more precisely rectified map sheets for GIS applications.

Keywords: Czechoslovakia; Křovák projection; map projections; map shrinkage;
polyedric projection system; S-JTSK; Third Military Survey

1. The Third Military Survey in the Czech Lands

The historical Third Military Survey, concerning the territory of the Czech Crown, was carried out in the Austria-Hungarian Empire in the period of 1870–1883, particularly in Bohemia 1877–1879, in Moravia 1876–1877 and in Silesia 1876. The survey was carried out in decimal scale of 1:25 000 and resulted in topographic sections. Numeric base for the survey was a network of trigonometric stations in the co-ordinate systems of Gusterberg and St. Stephen transformed (Timár et al. 2006) into the polyedric projection. The graphical base consists of cadastral maps reduced into scale of 1:25 000. Four topographic sections (TS25) represented one sheet of Special Map (scale of 1:75 000; SM75) with dimensions of 30' in longitude and 15' in latitude. The sheets of SM75 were labelled by the number of row

(Arabic figures), the number of column (Roman figures) and the name of the most significant settlement, e.g. 5-X Kladno. The layers were numbered starting from parallel $51^{\circ}15'$ southwards and the columns starting from meridian 27° from Ferro eastwards (Biszak et al. 2007). After 1917 a new labelling was introduced that is valid until nowadays.

The maps of the Third Military Survey survived the end of the Austro-Hungarian Empire in 1918 thanks to their importance and became the state map series in Austria, Hungary and Czechoslovakia.

The new Czechoslovak Military Geographic Institute of Prague (founded in 1919) took over these map sheets from the Austrian Military Geographic Institute for the territory of Bohemia, Moravia, Silesia, Slovakia and Subcarpathian Ruthenia. Altogether 699 topographic sections, 189 sheets of the special map and 33 sheets of the general map all of them in the form of printing documents, i.e. copper printing plates.

The German and Hungarian settlement and other geographical names on the maps were replaced by the Czech and Slovak versions. The contents of the SM75 was gradually upgraded by green filling of forest areas and by apply a kilometer grid of the co-ordinate system S-JTSK in the Křovák projection (see below), used in Czechoslovakia since 1927 until today.

Especially the SM75 series enjoyed a great popularity and was produced in several thematic series. Even today, the SM75 maps is very popular in the Czech Republic. In 1956 the maps of the Third Military Survey were obsolete by a unified military map series used in the Warsaw Pact Countries. These maps have been produced in Pulkovo (S-42) geodetic datum and in Gauss-Krüger projection. Their validity expired by the 31st December 2005, when the Czech Topographic Series adopted the NATO standards, i.e. the WGS84 datum and the UTM/WGS84 projection. Importance of the Third Military Survey maps for studies of countryside development and introduction of new technologies in cartography is enormous and a great attention should be therefore dedicated to their study.

2. The Czech/Slovak S-JTSK co-ordinate system and the theory of the Křovák projection

S-JTSK is a geodetic co-ordinate system called in Czech as "System Jednotné Trigonometrické Site Katastrální — The System of the Unified Czech/Slovak Trigonometric Cadastral Net". Scale, location and orientation of the S-JTSK on the surface of the Bessel ellipsoid were derived from the results of the historical Austro/Hungarian military surveys in the years 1862–1898 (see Molnár and Timár 2009). There are 42 identical points on the Czech territory used for transformation computations. Astronomical orientation was measured only on the Hermannskogel, a trigonometric point in Austria, near Vienna. The Křovák projection and the national grid S-JTSK were adopted on the territory of the Czech and Slovak republics (former Czechoslovakia) in 1927. The using of this system for all geodetic surveying and cartographic activities (state mapping) is in context with the Czech State Law Nr. 200/94.

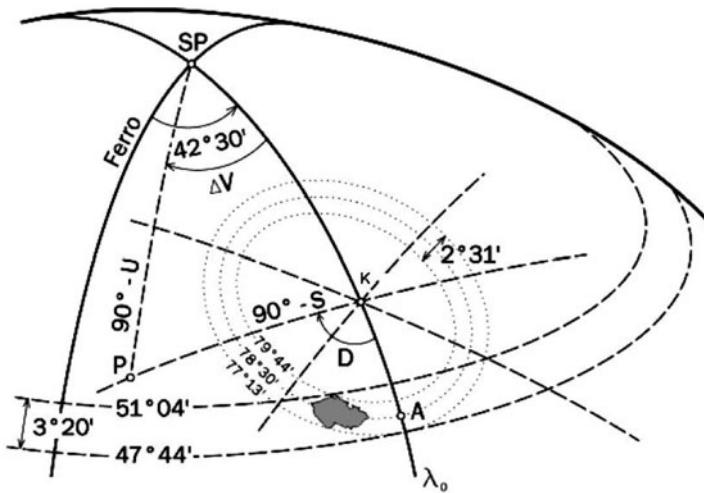


Fig. 1. Křovák projection – location of the Czech Republic on the Bessel Ellipsoid (Talhofer 2007)

The Conformal Oblique Conic Projection of Czechoslovakia was prepared by Josef Křovák in the year 1922 for compilation of cadastral and topographical maps with low distortion for the civil geodetic service of Czechoslovakia. The Bessel 1841 ellipsoid ($a = 6\ 377\ 397.155$ m; $1/f = 299.152812853$), which was widely adopted in Central Europe, is used. Longitude values refer to the virtual Ferro meridian (20 degrees west of Paris in round numbers; Timár 2007), not the Greenwich one round value of $17^{\circ}40'$ is used for cadastral and topographic mapping in the Czech Republic/Slovakia and Austria as the longitude difference between Ferro and Greenwich. The projection is conformal, therefore the meridians and parallels intersect at right angles in the maps. The projection cone is in oblique position, the pseudo pole is in the Bay of Finland, near Tallinn, Estonia (Fig. 1). The X-axis normally coincides with the meridian of $42^{\circ}30'$ from Ferro, increasing to south. The Y-axis is perpendicular to the X-axis and increasing to west.

This computation of this projection consists of the following steps: conformal projection of Bessel ellipsoid to Gauss sphere; a projection from the sphere to the oblique cone and a computation of the planar co-ordinates in the S-JTSK (Fig. 2). For more detailed information see (Kuska 1960, Talhofer 2007).

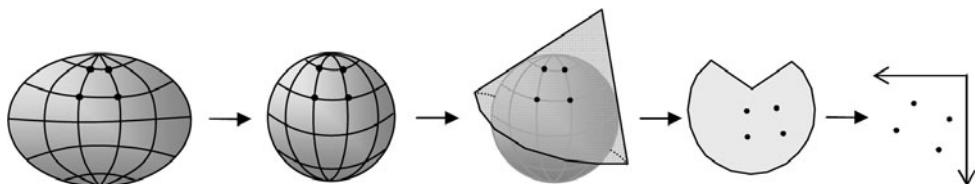


Fig. 2. Computing of S-JTSK co-ordinates

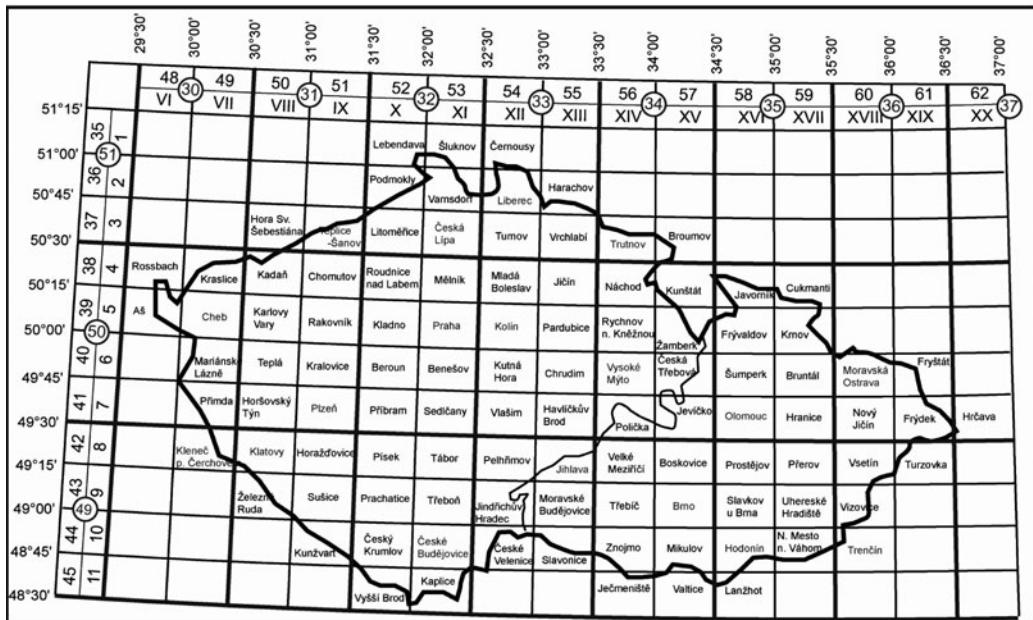


Fig. 3. Index of SM75 map on the territory of the Czech Republic

3. Elimination of distortions in the SM75 map by Helmert transformation

We made an experiment to substitute of the original polyedric projection of the SM75 maps by the Křovák Projection within the Czech Republic. According to the results, this is possible and advantageous. The meridians and parallels will be theoretically represented as common curves. As far as meridians are concerned they will be represented as concave curves with respect to meridian $42^{\circ}30'$ which also represents X axis in the S-JTSK system. Considering the dimensions of the map sheets we can presume that the west and east section lines represented by meridians are straight lines without any loss in map accuracy. A bit more complicated is the case of the north and south section lines represented by parallels. These should be taken for circles of a large radius. This circle can be calculated from three known points, where two of them are the corner points of the map and the mid point is obtained by averaging of their geographic co-ordinates and recalculating into S-JTSK plane by MATKART program. For more detail information see Veverka and Čechurová (2003).

It has to be underlined that not only the S-JTSK could be used as target system of this analysis. The method has been checked using the sinusoid projection suggested by Molnár and Timár (2009, this volume) and it provides almost the same level of errors, with a difference of a few meters. This indicates that any projection, which provides low distortion in the mapped area, can be used as target system of the described study.

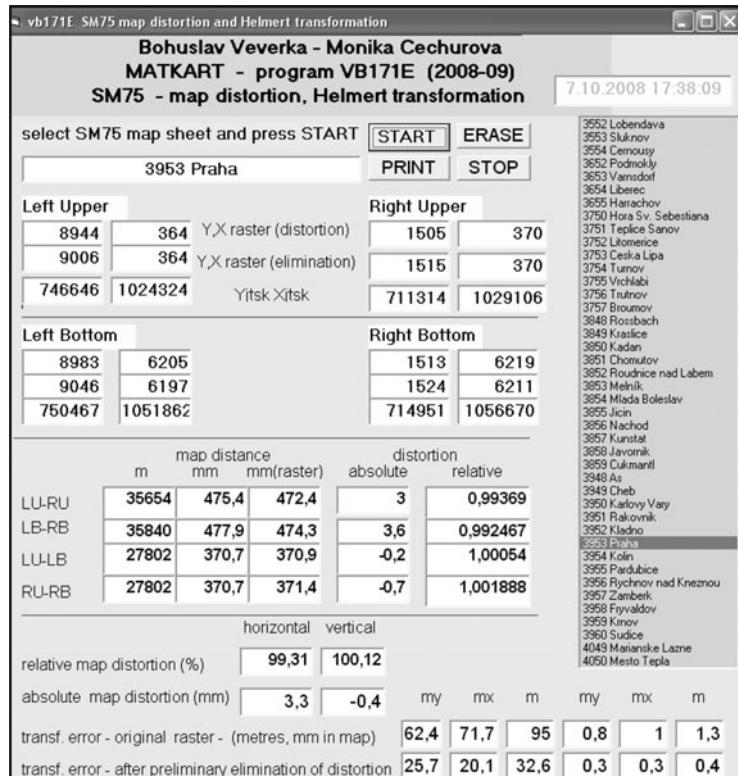


Fig. 4. Program VB171E – computation of the SM75 size, map distortion (shrinkage) and transformation errors

According to Chapter 1, the SM75 maps from the Czech territory (Fig. 3) dates several decades back. It is therefore important to take into account their distortion, that means a shrinkage of each map sheet. The map paper subjected to a long-term influence of its environment. Namely to a changing temperature and air humidity changes in the course of time its dimensions these changes are quite regular, the axis of the deformation symmetry are central transversals of the map frame that are at as a rule parallel or perpendicular to the direction of paper production. The shrinkage of the map has character of a systematic error and influences the map scale. As it is continuously changing its determination should be precede each individual cartometric investigation in order to eliminate its influence.

We consider a scanned image of a SM75 map sheet. Without knowing the raster co-ordinates of corners of these maps, presume to know the ideal co-ordinates of individual map corners. We can derive also the geographic co-ordinates of their corners ($\varphi, \lambda_{\text{Ferro}}$) from the labels of individual map sheets and calculate their values in S-JTSK. It is possible to use a suitable transformation for recalculation of co-ordinates obtained by raster digitisation. For this purpose Program VB171E were used the user screen of which is shown at Fig. 4. A total number of 125 SM75 type maps have been evaluated with this software.

4. Main steps of the algorithm when calculating the shrinkage and its elimination

- a) By applying a suitable software we obtain raster co-ordinates of corner points in a local co-ordinate system of the scanned map sheet where index LU = upper left, RU = upper right, LB = left bottom, RB = right bottom (see Fig. 4).
- b) Then the real lengths of the map sheet sides, influenced by the shrinkage are computed (in metres).

$$\begin{aligned}
 d_{LU-RU} &= \sqrt{(x_{LU} - x_{RU})^2 + (y_{LU} - y_{RU})^2} \cdot \frac{25.4}{DPI}, \\
 d_{LU-LB} &= \sqrt{(x_{LU} - x_{LB})^2 + (y_{LU} - y_{LB})^2} \cdot \frac{25.4}{DPI}, \\
 d_{RU-RB} &= \sqrt{(x_{RU} - x_{RB})^2 + (y_{RU} - y_{RB})^2} \cdot \frac{25.4}{DPI}, \\
 d_{LB-RB} &= \sqrt{(x_{LB} - x_{RB})^2 + (y_{LB} - y_{RB})^2} \cdot \frac{25.4}{DPI}.
 \end{aligned} \tag{1}$$

- c) Ideal positions of corner points of map sheet in Křovák Projection and co-ordinate system S-JTSK are computed from the geographic co-ordinates of the sheet corners given on the Bessel ellipsoid (cf. Fig. 2).

$$\begin{aligned}
 [\varphi, \lambda_{\text{Ferro}}]_{LU} &\rightarrow [U, V]_{LU} \rightarrow [S, D]_{LU} \rightarrow [\varrho, \delta]_{LU} \rightarrow [X, Y]_{LU} \\
 [\varphi, \lambda_{\text{Ferro}}]_{RU} &\rightarrow [U, V]_{RU} \rightarrow [S, D]_{RU} \rightarrow [\varrho, \delta]_{RU} \rightarrow [X, Y]_{RU} \\
 [\varphi, \lambda_{\text{Ferro}}]_{LB} &\rightarrow [U, V]_{LB} \rightarrow [S, D]_{LB} \rightarrow [\varrho, \delta]_{LB} \rightarrow [X, Y]_{LB} \\
 [\varphi, \lambda_{\text{Ferro}}]_{RB} &\rightarrow [U, V]_{RB} \rightarrow [S, D]_{RB} \rightarrow [\varrho, \delta]_{RB} \rightarrow [X, Y]_{RB}
 \end{aligned} \tag{2}$$

- d) Ideal dimensions of the map sheet in Křovák projection in co-ordinate system S-JTSK (in metres) is calculated.

$$\begin{aligned}
 d_{LU-RU}^{JTSK} &= \frac{\sqrt{(X_{LU} - X_{RU})^2 + (Y_{LU} - Y_{RU})^2}}{75}, \\
 d_{LU-LB}^{JTSK} &= \frac{\sqrt{(X_{LU} - X_{LB})^2 + (Y_{LU} - Y_{LB})^2}}{75}, \\
 d_{RU-RB}^{JTSK} &= \frac{\sqrt{(X_{RU} - X_{RB})^2 + (Y_{RU} - Y_{RB})^2}}{75}, \\
 d_{LB-RB}^{JTSK} &= \frac{\sqrt{(X_{LB} - X_{RB})^2 + (Y_{LB} - Y_{RB})^2}}{75}
 \end{aligned} \tag{3}$$

- e) The absolute shrinkage s^a of sides is calculated from the length differences between the real and ideal map sheet sizes.

$$\begin{aligned}
 s_{LU-RU}^a &= d_{LU-RU}^{JTSK} - d_{LU-RU}, \quad s_{LU-LB}^a = d_{LU-LB}^{JTSK} - d_{LU-LB} \\
 s_{RU-RB}^a &= d_{RU-RB}^{JTSK} - d_{RU-RB}, \quad s_{LB-RB}^a = d_{LB-RB}^{JTSK} - d_{LB-RB}
 \end{aligned} \tag{4}$$

- f) The average relative shrinkage s^r is the ratio of the real and ideal values of individual map sheet side lengths.

$$\begin{aligned} s_{LU-RU}^r &= \frac{d_{LU-RU}^{JTSK}}{d_{LU-RU}}, & s_{LU-LB}^r &= \frac{d_{LU-LB}^{JTSK}}{d_{LU-LB}}, \\ s_{RU-RB}^r &= \frac{d_{RU-RB}^{JTSK}}{d_{RU-RB}}, & s_{LB-RB}^r &= \frac{d_{LB-RB}^{JTSK}}{d_{LB-RB}} \end{aligned} . \quad (5)$$

- g) The average relative shrinkage s_h in horizontal direction or s_v in vertical direction is computed as the arithmetic mean of relative shrinkages of upper and lower, or left and right sides of the map sheet.

$$s_h = \frac{s_{LU-RU}^r + s_{LB-RB}^r}{2}, \quad s_v = \frac{s_{LU-LB}^r + s_{RU-RB}^r}{2}. \quad (6)$$

- h) Positional corrections of real corner co-ordinates are introduced by dividing them by the average relative shrinkage in the respective direction.

$$\begin{aligned} y'_{LU} &= \frac{y_{LU}}{s_h}, & y'_{RU} &= \frac{y_{RU}}{s_h}, & y'_{LB} &= \frac{y_{LB}}{s_h}, & y'_{RB} &= \frac{y_{RB}}{s_h} \\ x'_{LU} &= \frac{x_{LU}}{s_v}, & x'_{RU} &= \frac{x_{RU}}{s_v}, & x'_{LB} &= \frac{x_{LB}}{s_v}, & x'_{RB} &= \frac{x_{RB}}{s_v} \end{aligned} . \quad (7)$$

- i) Thus we obtain two sets of entry co-ordinates for Helmert transformation. The first set is represented by the original co-ordinates measured in the raster influenced by the shrinkage $[x, y]_{LU}$, $[x, y]_{RU}$, $[x, y]_{LB}$, $[x, y]_{RB}$. The second set is represented by co-ordinates corrected by the influence of the shrinkage $[x, y]_{LU}'$, $[x, y]_{RU}'$, $[x, y]_{LB}'$, $[x, y]_{RB}'$. The equations of Helmert transformation are published in any textbooks of geodesy (Helmert 1868). This linear conform transformation is suitable especially because it does not cause any further deformations of the grid of points.

5. Final analysis of map distortion for the State Map Series SM75 of the Czech Republic

The distortion (shrinkage) of the selected map sheet 3953 (Prague) (Fig. 4) is 3.3 mm in the horizontal direction and on the contrary, in the vertical direction the map got longer by 0.4 mm. Helmert transformation used on the raster co-ordinates influenced directly by the shrinkage determined a mean square error of the set of identical points — in this instant map corners by values of 62.4 m in Y-axis and 71.7 m in X-axis, i.e. altogether 95 m. This represents at the map scale to 0.8 mm for Y-axis, 1 mm for X-axis and 1.3 mm on the whole. This leads to a conclusion that the Helmert transformation is not a very appropriate conform procedure to eliminate the irregular shrinkage. If we preliminary eliminate the shrinkage by means of a calculation method, then the results can be significantly improved, the deviations decrease to 25.7 m, 20.1 m in the directions of the axes and 32.6 m as

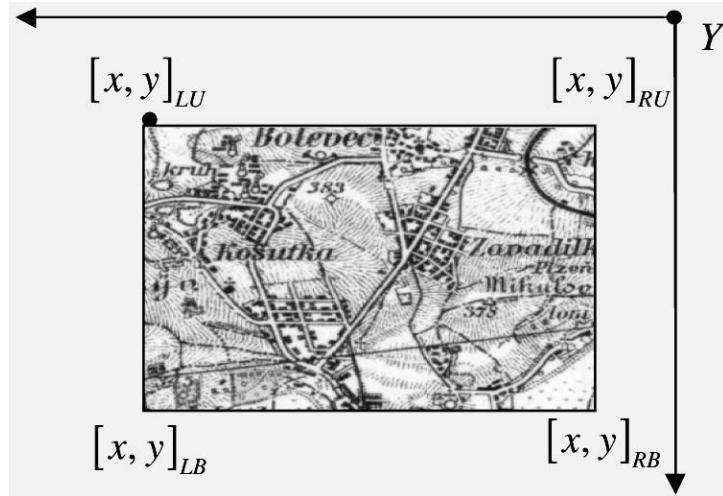


Fig. 5. Map sheet corners

the whole. This represents at the map scale 0.3 mm, 0.3 mm and 0.4 mm, which corresponds to the limit of graphic accuracy of the map drawing.

The goal of this activity has been to find out extreme values of distortion, it means shrinkage of map sheets and namely suitability of Helmert transformation for recalculation of raster co-ordinates of a map influenced by distortion into the S-JTSK system. The analysis has been carried out in an EXCEL environment with interesting results (see Figs 6 and 7). These figures show that among elements of root mean square errors m_y and m_x that characterize the accuracy of the Helmert transformation for each individual map sheet influenced by its shrinkage a strong statistical dependency has been found. This can be described by a correlation field with a clear linear dependency (see Fig. 6) represented by regression line given by equation $y = 1.19 + 0.81x$ and by correlation coefficient $R = 0.97326$. The results in Fig. 6, however, shows that the shrinkage of the map sheets are anisotropic; the example figures and their ratio taken from the sheet 3953 (Prague) is representative for the other investigated sheets. Figure 7 demonstrates the effectiveness of the shrinkage elimination.

If the shrinkage of the map is eliminated beforehand by calculation by calculating the absolute or relative shrinkage value of the map sheets then after a repeated application of Helmert transformation, the correlation field of m_y and m_x deviations becomes flatly spread (see Fig. 7). The regression line is represented by equation $y = 2.55 + 0.95x$ and correlation coefficient $R = 0.40983$ proves independency of root mean square errors m_y and m_x after previous shrinkage elimination. In case of preliminary shrinkage elimination the values of m_y and m_x decrease to about one third of their original value and as an average they do not reach over 0.3 mm for a map sheet.

The results in Fig. 6, however, shows that the shrinkage of the map sheet is anisotropic. The example figures taken from the sheet 3953 (Prague) is representa-

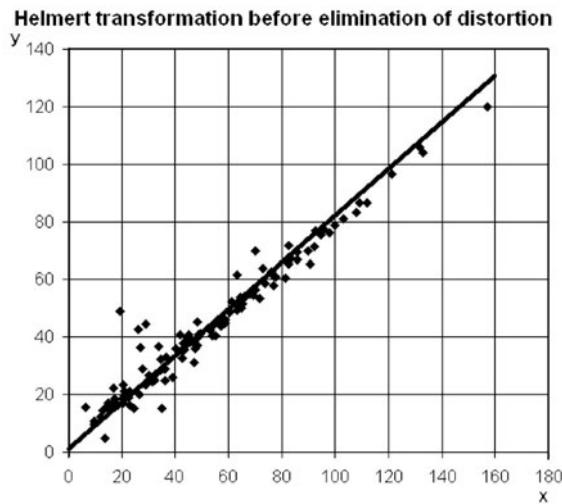


Fig. 6. Original shrinkage of the map sheets in two directions, in metres

tive for the other investigated sheets. Figure 7 demonstrates the effectivity of the shrinkage elimination.

This conclusion proves that even the relatively old maps as the SM75 sheets are, printed mostly on paper of an inferior quality, can be successfully converted into a scanned and rectified raster image by removing the total influence of the scanning errors. Errors of manual interactive digitization of map sheet corners and errors due to non-homogenous shrinkage may be eliminated by procedure described in this contribution.

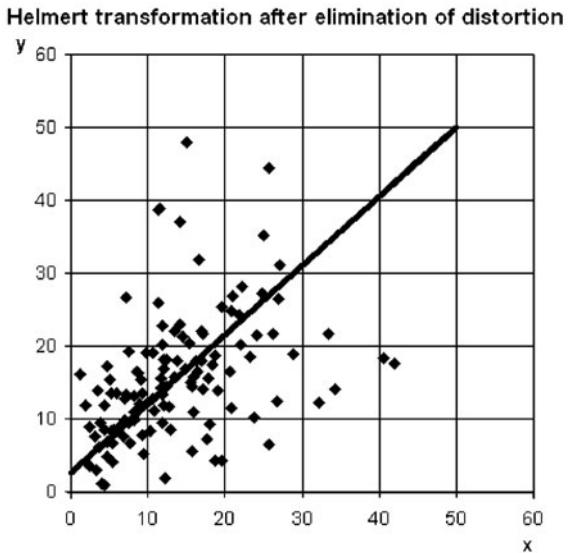


Fig. 7. Corrected shrinkage of the map sheets in two directions, in metres

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