



ERRATUM

ERRATUM TO: INVARIANT SUBSPACE METHOD: A TOOL FOR SOLVING FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS, IN: FCAA-20-2-2017

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This is erratum to our paper [1] in: *Fract. Calc. Appl. Anal.* **20**, No 2 (2017), pp. 477-493, DOI: 10.1515/fca-2017-0024. Here we point out that Theorem 3.1 and Theorem 3.2 of [1] hold true more generally when fractional partial time derivatives $\frac{\partial^{\alpha+i}}{\partial t^{\alpha+i}}$ and $\frac{\partial^{i\alpha}}{\partial t^{i\alpha}}$ in FPDE (3.1) and (3.7) are Caputo as well as **Riemann-Liouville (R-L)** time derivatives.

Further, note that $\frac{d^\alpha(t^{-\alpha})}{dt^\alpha}$ is not defined in case of Caputo derivative if $\alpha \in (0, 1) \setminus \{\frac{1}{2}\}$. In the literature many authors have mistakenly ignored the fact that $\frac{d^\alpha(t^{-\alpha})}{dt^\alpha} = \frac{\Gamma(1-\alpha)t^{-2\alpha}}{\Gamma(1-2\alpha)}$ is not valid in case of Caputo derivative, though it holds correct for R-L derivative.

In view of these facts, Example 4.2 and Example 4.7 are wrong if $\frac{d^\alpha f}{dt^\alpha}$ is taken as Caputo time derivative, but correct if we take fractional time derivative as R-L time derivative, i.e., $\frac{{}^{RL}d^\alpha f}{dt^\alpha}$ in both these examples.

Hence Example 4.2 given in [1] should read as follows:

$$\frac{{}^{RL}\partial^\alpha f}{\partial t^\alpha} + f \frac{\partial^\beta f}{\partial x^\beta} - d \frac{\partial^\beta}{\partial x^\beta} \left(\frac{\partial^\beta f}{\partial x^\beta} \right) = 0, \quad \alpha \in (0, 1), \quad \alpha \neq \frac{1}{2}, \quad \beta \in (0, 1], \quad (4.6)$$

where the solution continues to be the same, i.e.,

$$f(x, t) = at^{-\alpha} - \left[\frac{\Gamma(1 - \alpha)t^{-\alpha}}{\Gamma(1 + \beta)\Gamma(1 - 2\alpha)} \right] x^\beta.$$

Proceeding on similar lines, Example 4.7 from [1] should be considered as:

$$\frac{{}^{RL}\partial^\alpha f}{\partial t^\alpha} - 6f \frac{\partial^\beta f}{\partial x^\beta} + \frac{\partial^{2\beta}}{\partial x^{2\beta}} \left(\frac{\partial^\beta f}{\partial x^\beta} \right) = 0, \quad \alpha \in (0, 1) \setminus \{1/2\}, \quad \beta \in (0, 1]. \quad (4.38)$$

The solution continues to be the same, i.e.,

$$f(x, t) = at^{-\alpha} + \left[\frac{\Gamma(1 - \alpha)t^{-\alpha}}{6\Gamma(1 + \beta)\Gamma(1 - 2\alpha)} \right] x^\beta.$$

References

- [1] S. Choudhary and V. Daftardar-Gejji, Invariant subspace method: A tool for solving fractional partial differential equations. *Fract. Calc. Appl. Anal.* **20**, No 2 (2017), 477–493, DOI: 10.1515/fca-2017-0024; <https://www.degruyter.com/view/j/fca.2017.20.issue-2/issue-files/fca.2017.20.issue-2.xml>.

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