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CORRIGENDUM

CORRIGENDUM: FRACTIONAL INTEGRAL ON MARTINGALE HARDY SPACES WITH VARIABLE EXPONENTS

Yong Jiao [1], Dejian Zhou [1,*], Ferenc Weisz [2], Zhiwei Hao [1]

Abstract

This is an erratum to the paper "Fractional integral on martingale Hardy spaces with variable exponents", *Fract. Calc. Appl. Anal.* **18**, No 5 (2015), 1128–1145; DOI: 10.1515/fca-2015-0065.

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In [2], the authors introduced the following condition ([2, Condition (1.1)]): there exists a constant $K_{p(\cdot)} \geq 1$ depending only on $p(\cdot)$ such that

$$\mathbb{P}(A)^{p_-(A)-p_+(A)} \leq K_{p(\cdot)}, \quad \forall A \in \mathcal{F}. \tag{1}$$

Indeed, if $\mathcal{F}$ is a non-atomic algebra and $p(\cdot)$ satisfies (1), then $p(\cdot)$ is a constant. Assume that $p(\cdot)$ is not a constant. Then $0 < p_- < p_+$. Take $x_1, x_2 \in \Omega$ such that $p_- \leq p(x_1) < p(x_2) < p_+$. Since $\mathcal{F}$ is non-atomic, there exist $A_1^n, A_2^n \in \mathcal{F}$, so that $x_1 \in A_1^n$, $x_2 \in A_2^n$, $A_1^n \cap A_2^n = \emptyset$ and $\mathbb{P}(A_1^n) = \mathbb{P}(A_2^n) = 2^{-n}$. Set $A = A_1^n \cup A_2^n$. Then $\mathbb{P}(A) = 2^{-n+1}$. By (1), we have

$$2^{(-n+1)(p(x_1)-p(x_2))} = \mathbb{P}(A)^{p(x_1)-p(x_2)} \leq \mathbb{P}(A)^{p_-(A)-p_+(A)} \leq K_{p(\cdot)}.$$

Taking $n \to \infty$, we find that the above inequality does not hold if $p(\cdot)$ is not a constant.

In order to avoid this problem, we introduce the following condition: there exists a constant $K_{p(\cdot)} \geq 1$ depending only on $p(\cdot)$ such that

$$\mathbb{P}(A)^{p_-(A)-p_+(A)} \leq K_{p(\cdot)}, \quad \forall A \in \cup_n A(\mathcal{F}_n), \tag{2}$$

where, for each $n \geq 0$, $\mathcal{F}_n$ is generated by countable atoms and $A(\mathcal{F}_n)$ denotes the set of atoms in $\mathcal{F}_n$.

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Condition (1.1) in [2] should be replaced by the condition (2). Furthermore, if we replace "for any set $A \in \mathcal{F}$" in [2, Lemma 4.1] by "for any atom $A \in \cup_{n\geq 0} A(\mathcal{F}_n)$", then the lemma is still correct. This means that only Lemma 4.5 and Theorem 4.1 of [2] should be modified.

Note that for each stopping time ν, there are disjoint atoms $I_{j,i} \in \cup_{n\geq 0} A(\mathcal{F}_n)$ such that $\{\nu < \infty\} = \cup_{j=0}^{\infty}\{\nu = j\} = \cup_{j=0}^{\infty} \cup_i I_{j,i}$. This, together with [2, Theorem 3.1], allows us to get, for every $f \in \mathcal{Q}_{p(\cdot)}$, $f = \sum_{k\in\mathbb{Z}} \mu_k a^k = \sum_{k\in\mathbb{Z}} \sum_{j=0}^{\infty} \sum_i 3 \cdot 2^k \|\chi_{\{\nu_k<\infty\}}\|_{p(\cdot)} a^k \chi_{I_{k,j,i}}$. Repeating the proof of [2, Lemma 4.5], for a $p(\cdot)$-atom a associated with ν, we obtain

$$\|I_\alpha(a\chi_{I_{j,i}})\|_{\mathcal{Q}_{q(\cdot)}} \lesssim C_\alpha \|\chi_{\{\nu<\infty\}}\|_{p(\cdot)}^{-1} \|\chi_{I_{j,i}}\|_{p(\cdot)}. \tag{3}$$

We use (3) instead of [2, Lemma 4.5]. With the help of (3), repeating the proof of [2, Theorem 4.1], we can reprove [2, Theorem 4.1].

We also find that [1, 3, 4] have applied condition (1). The corresponding condition used in these papers should be replaced by (2).

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[1] *School of Mathematics and Statistics, Central South University*
410083 Changsha, CHINA

e-mail: jiaoyong@csu.edu.cn , zw_hao8@163.com
* *zhoudejian@csu.edu.cn*
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[2] *Department of Numerical Analysis, Eötvös L. University*
H–1117 Budapest, Pázmány P. sétány 1/C., HUNGARY

e-mail: weisz@inf.elte.hu

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