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Comparison of Selected Geopotential Models in Terms of the GOCE Orbit Determination Using Simulated GPS Observations

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Abstract

This work contains a comparative study of the performance of six geopotential models in an orbit estimation process of the satellite of the Gravity Field and Steady-State Ocean Circulation Explorer (GOCE) mission. For testing, such models as ULUX_CHAMP2013S, ITG-GRACE 2010S, EIGEN-51C, EIGEN5S, EGM2008, EGM96, were adopted. Different sets of pseudo-range simulations along reference GOCE satellite orbital arcs were obtained using real orbits of the Global Positioning System satellites. These sets were the basic observation data used in the adjustment. The centimeter-accuracy Precise Science Orbit (PSO) for the GOCE satellite provided by the European Space Agency (ESA) was adopted as the GOCE reference orbit. Comparing various variants of the orbital solutions, the relative accuracy of geopotential models in an orbital aspect is determined. Full geopotential models were used in the adjustment process. The solutions were also determined taking into account truncated geopotential models. In such case, an accuracy of the solutions was slightly enhanced. Different arc lengths were taken for the computation.

Key words: geopotential models, GOCE satellite orbit.

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1. INTRODUCTION

Over the past few decades, numerous models describing the gravity field have been determined. Some of them were based on both terrestrial and satellite data, as, for example, EGM96 (Lemoine *et al.* 1998) or EGM2008 (Pavlis *et al.* 2012). Others have been obtained using solely satellite data, for example GOGRA02S (Yi 2012) or JYY_GOCE02S (Yi *et al.* 2013). The high quality of gravity field models is essential for modeling, for example, satellite orbits and geoid. The accuracy of these models can be described by internal and external quality parameters. Many models contain quality information expressed by variance-covariance matrices connected with a least squares solution or a Monte-Carlo approach (Gruber *et al.* 2011). However, this internal error characteristic requires verification by obtaining external quality parameters to estimate the performance of these models in several aspects. Generally, an evaluation of gravity field models may refer to two types of tests. The first one concerns satellite orbit determination, where RMS of observation residuals and orbit predictions are used for gravity model evaluations (Sośnica 2014). The second test type is connected with comparisons of geoid and derivative quantities (Gruber *et al.* 2011). Both test types are complementary because the first one investigates the quality of long wavelength part of gravity field models (in the orbit determination procedure), whereas the second one allows a model performance to be assessed in the spatial domain (in geoid comparisons) (Gruber *et al.* 2011).

Many different works refer to considering the issue of a gravity model evaluation. For example, Lejba *et al.* (2007) and Sośnica *et al.* (2012) compared the impact of selected gravity field models on the estimation of Laser Geodynamics Satellites (LAGEOS) orbits, taking into account the RMS values of satellite laser ranging (SLR) residuals. Sośnica (2014) also presents the results of different gravity model validation, where the estimated LAGEOS satellite orbits were compared with the predicted orbits. The obtained orbits estimated using the tested gravity models were also compared directly with each other. In this work, the strong dependence of orbital solutions obtained on the quality of the C_{20} coefficient for the tested gravity models is emphasized. This can also be observed in the estimated sine term of the once-per-revolution cross track acceleration (Sośnica 2014). In turn, Gruber *et al.* (2011) evaluated three geopotential models derived from the Gravity Field and Steady State Ocean Circulation Explorer (GOCE) mission taking into account orbital residuals for the very precise reduced-dynamic orbit of a satellite of the Challenging Mini-satellite Payload mission (CHAMP) (Reigber *et al.* 2005) and of the satellites of the Gravity Recovery and Climate Experiment (GRACE) mission (Tapley *et al.* 2004). These orbits were converted into the inertial Earth-centered reference frame. The

Cartesian coordinates *X*, *Y*, *Z* for these orbits were then treated as observations in a fully dynamic orbit determination (Gruber *et al.* 2011). On the other hand, a comparison of geoid heights computed from validated models with corresponding values obtained from GPS-leveling points was performed in an area of medium-to-higher spatial scales of the gravity field of the Earth (Gruber *et al.* 2011). The different truncation of gravity models allowed to recognize to which degree the models give significant results. This can be described equivalently as the degree from which the tested models start to lose signal. It is caused by the attenuation of a gravity signal with satellite height (Gruber *et al.* 2011).

In a similar way, Förste *et al.* (2014) tested the combined model EIGEN-6C4p. They investigated the performance of this model by fitting estimated orbital arcs of the GOCE satellite into the positions originating from the precise science orbit (PSO orbit) (Bock *et al.* 2011). These positions were treated as observations (Förste *et al.* 2014). Papanikolaou and Tsoulis (2014) validated GOCE gravity models using an adapted dynamic orbit determination algorithm. They compared selected models of the gravity field by the band-limited performance in the orbit determination and by generated orbit perturbations. Cheng and Ries (2015) compared the performance of selected GOCE gravity field models for satellite orbit determination based on SLR observations. Comparisons showed a similar performance of all recent GOCE and GRACE based models in terms of the RMS fit of the SLR observations. They also showed that the estimate of C_{20} is a dominant factor in the long-wavelength error of gravity field models (Cheng and Ries 2015).

GOCE-derived geopotential models were also locally tested in certain areas, *e.g.*, in Norway (Šprlák *et al.* 2015), Germany (Voigt and Denker 2015), in the Mediterranean area (Carrion *et al.* 2015) and in South America (de Matos *et al.* 2015). Usually, such quantities as gravity anomalies and height anomalies computed based on tested gravity field models are compared with corresponding quantities obtained independently using local terrestrial data. Hirt *et al.* (2015) compared gravity from the GOCE mission and from RET2014 topography. The degree of similarity between both signals was taken as an indicator of quality for the GOCE gravity field models. It was found that the 5th-generation GOCE models describe parts of the gravity field down to about 70 km spatial scales (Hirt *et al.* 2015).

The GOCE satellite was a key component of the aforementioned the Gravity Field and Steady State Ocean Circulation Explorer mission, which was the first Earth explorer core mission of the European Space Agency (ESA). This satellite was launched on 17 March 2009 into a sun-synchronous dusk-dawn orbit with a very low initial altitude of about 280 km (Bock *et al.* 2011). The core onboard three-axis gradiometer performed measurements of gravity gradients whereas the 12-channel dual-frequency Global

Positioning System (GPS) receiver delivered phase-code observations of GPS satellites. The extremely low altitude of this satellite was necessary to ensure a proper level of gravity signal. Such altitude was realized by a drag-free flight, which was maintained using the drag-free and attitude control system (DFACS). This means that mainly the atmospheric drag acting on the satellite in flight direction was compensated (Bock *et al.* 2011). However, direct and indirect solar radiation pressure also occurs. The collected time series of gravity gradient observations and GPS code-phase measurements were used for the realization of the main objective of the mission which was the estimation of the new generation of the Earth's static gravity field models (Pail *et al.* 2011). Code-phase GPS measurements were also used to estimate the GOCE satellite precise orbit as a reduced-dynamic orbit and a kinematic orbit. The precise orbit was needed for the geolocation of observations derived from the mission (Bock *et al.* 2011). The satellite positions taken from the kinematic orbit were used for the recovery of both the static and time-variable gravity fields (Jäggi *et al.* 2015). This also illustrates the sensitivity of the very low orbit of GOCE satellite to temporal variations of the gravity field. The GOCE mission ended with the re-entry of the satellite in dense layers of the atmosphere 13 November 2013 (ESA 2014).

This work contains various tests, in which the performance of six selected gravity field models in the process of determining the GOCE satellite orbit was compared. Since an average operational altitude of GOCE satellite was equal to about 255 km (Rummel *et al.* 2009), a comparison of the performance of selected gravity models therefore refers to the extremely low Earth orbit.

The aim of this work was to compare the quality of long-wavelength parts of selected gravity field models in the aspect of GOCE satellite orbit determination with an indication of the preferred models. The obtained results may be useful, for example, for a fully dynamic approach (Casotto *et al.* 2013) for the GOCE orbit determination by using the preferred gravity field models.

The tested models were based on the data coming from terrestrial sources and such space missions as CHAMP and GRACE. GOCE-derived gravity models were not included in this study. This is caused by the fact that the same data sets were partially used for the estimation of these models and the GOCE orbit.

2. RESEARCH

The basic tool used in this work is a software package called the Orbital Computation System (OCS), which is an extension of the Toruń Orbit Processor (TOP) package (Drożyner 1995). An important task realized by the OCS package is to determine a satellite orbit in the field of gravitational and non-gravitational perturbing forces. Taking into account the Cowell 8th order method, the equation motion of a satellite is numerically integrated in order to obtain a time series of position and velocity vectors. The right-hand side of the equation of motion contains the vector presenting the satellite's Keplerian motion in the central gravity field and the vector describing the effect of perturbing forces (Eshagh and Najafi-Alamdari 2007). In the framework of the OCS software, a mathematical model of the forces governing the satellite motion is created at the given epoch during the computation (Drożyner 1995). This model, for the GOCE satellite, includes the gravitational accelerations generated by: the geopotential (a given tested gravity field model), ocean tides and Earth tides, the third body effect and the relativity effects. The ocean and Earth tides were described by the MERIT (Monitoring Earth Rotation and Intercomparison of Techniques) standards model (Melbourne *et al.* 1983), whereas the acceleration due to the third body effect was computed using the planetary ephemerides DE200/LE200 (Standish *et al.* 1992). The relativistic acceleration was computed by means of the Painleve formulation, taking into account spherical symmetrical space-time with the Schwarzschild metric. This formulation was implemented in the Toruń Orbit Processor software (Drożyner 1995). Mathematical formulas describing the MERIT standards model and the relativistic acceleration are presented in detail by Bobojć and Drożyner (2011). The satellite orbit computation is a part of the orbit estimation process, which is the main task realized by the OCS package.

The simulated observations of pseudo-ranges between the GOCE satellite and maximum twelve Global Positioning System (GPS) satellites at a given epoch are used in the orbit determination. These simulations were obtained taking into account the reduced-dynamic PSO orbit of GOCE satellite and the orbits of GPS satellites provided by the ESA as an L2 product of the GOCE mission (ESA 2010, Bock *et al.* 2011). The computation of sets of the pseudo-ranges was based on the Cartesian coordinates of GOCE and of GPS satellites with respect to the inertial reference frame (IRF) of standard epoch J2000.0 (ESA 2010). Additionally, the time of GPS signal travel between the GPS satellites and the GOCE satellite was taken into account in this computation. In order to express of the GOCE and GPS satellite coordinates with respect to IRF, the elements of orientation of ITRF2005 and IRF (ESA 2010) were used. These elements in terms of quaternions were obtained through the ESA and they were also used in the orbit determination.

The orbit estimation process is based on the following observation equation:

$$D_{jk}^{o} + v_{jk} = D_{jk}^{c} + \frac{\partial D_{jk}^{c}}{\partial \boldsymbol{r}} \frac{\partial \boldsymbol{r}}{\partial(\boldsymbol{r}_o, \dot{\boldsymbol{r}}_o)} [\Delta \boldsymbol{r}_o, \Delta \dot{\boldsymbol{r}}_o]^T . \tag{1}$$

In the above formula:

D_{jk}^{o}, D_{jk}^{c} – observed and computed pseudo-range between the GOCE satellite and the j-th GPS satellite at epoch k, respectively,

v_{jk} – correction to the observed pseudo-range D_{jk}^{o},

$\partial D_{jk}^{c} / \partial \boldsymbol{r}$ – partial derivative of the computed pseudo-range D_{jk}^{c} with respect to the position vector $\boldsymbol{r} = [x, y, z]^T$ at epoch k,

$\boldsymbol{r}_o, \dot{\boldsymbol{r}}_o$ – initial position vector and initial velocity vector, respectively, at initial epoch t_o, $\boldsymbol{r}_o = [x_o, y_o, z_o]^T$ and $\dot{\boldsymbol{r}}_o = [\dot{x}_o, \dot{y}_o, \dot{z}_o]^T$,

$\partial \boldsymbol{r} / \partial(\boldsymbol{r}_o, \dot{\boldsymbol{r}}_o)$ – partial derivative of the position vector with respect to the initial state vector $\boldsymbol{p}_o = [\boldsymbol{r}_o, \dot{\boldsymbol{r}}_o]^T$,

$\Delta \boldsymbol{r}_o, \Delta \dot{\boldsymbol{r}}_o$ – unknown correction vectors to the initial position and velocity vectors; $\Delta \boldsymbol{r}_o = [\Delta x_o, \Delta y_o, \Delta z_o]^T$, and $\Delta \dot{\boldsymbol{r}}_o = [\Delta \dot{x}_o, \Delta \dot{y}_o, \Delta \dot{z}_o]^T$,

In the orbit estimation process, the unknown corrections to the initial state vector are estimated in successive iterations using the classical least squares method until a convergence. Finally, the initial state vector, corrected in the last iteration, enables determining the satellite orbit.

In order to estimate the quality of obtained solutions, the root mean square (RMS) of the difference between the determined orbit and the reference one is used. This parameter expresses the accuracy of the given solution. It is determined by means of the following formula:

$$\mathrm{RMS} = \sqrt{\sum_{i=1}^{3} (\mathrm{RMS}_i)^2} . \tag{2}$$

The quantities of $\mathrm{RMS}_i \, (i = 1, 2, 3)$ are computed using the expression:

$$\mathrm{RMS}_i = \sqrt{\frac{\sum_{j=1}^{n} \left[(x_i)_j - (x_i)_{j\mathrm{REF}} \right]^2}{n}} , \tag{3}$$

where $(x_i)_j$, $(x_i)_{j\mathrm{REF}}$ ($i = 1, 2, 3$; $x_1 = x$, $x_2 = y$, $x_3 = z$) are the satellite's Cartesian coordinates at epoch j w.r.t. IRF, in the estimated orbit and in the reference orbit, respectively, and n is the total number of epochs – the same for both orbits. The RMS parameter can be explained as the mean distance between corresponding points of both orbits (for the same epoch) or as a measure of the fit of the orbit determined to the reference orbit. In the case of GOCE orbit computed directly without adjustment, the RMS value determines the threshold of the orbital solution effectiveness, *i.e.*, solutions with RMS values less than the corresponding threshold values can be treated as effectively estimated orbital variants.

The reference orbit – the reduced-dynamic PSO GOCE orbit, is acquired through ESA as a Level 2 GOCE mission product and was estimated relying on the GPS observations. The gravity field model and the remaining dynamical models were also used in the estimation process. The generation of this orbit was a multi-step process. In the first step, an approximate orbit using pseudo-range measurements was determined. In the next step, this orbit was improved in an iterative procedure using zero-difference phase observations. After an appropriate number of iterations, this orbit was determined by six initial osculating elements, three constant empirical accelerations and a set of pseudo-stochastic piecewise constant accelerations in 6-minute intervals (Bock *et al.* 2007). These pseudo-stochastic accelerations absorb the errors of dynamic models used and the accelerations induced by non-gravitational forces such as the direct solar radiation pressure and the Earth albedo. The effect of the dynamic models on this orbit is limited by estimated pseudo-stochastic parameters. Hence, this orbit is called a reduced-dynamic orbit. An approach applied to the estimation of reduced-dynamic orbit has also been successfully used to determine, for example, the satellite orbit of the CHAllenging Minisatellite Payload (CHAMP) mission with an accuracy of about 3 cm (Jäggi *et al.* 2006). In turn, the accuracy of the PSO orbit of GOCE satellite is at a level of 2 cm, which is based on the SLR validation (Bock *et al.* 2011). The precise orbit of GOCE satellite was also determined as a kinematic solution. This kinematic orbit was estimated solely based on the GPS phase measurements, *i.e.*, no dynamic models were taken into account. Thus, this geometrical solution is more sensitive to changes in the quality of GPS measurements. The accuracy of the kinematic orbit is at a similar centimeter level as in the case of a reduced-dynamic orbit (Bock *et al.* 2011).

The estimated GOCE orbits were integrated and compared with the reference orbit (the reduced-dynamic PSO orbit) with respect to IRF, whose origin is located at the center of mass of the Earth. The reference orbit was previously transformed from ITRF2005 to IRF using the instantaneous rotation matrices generated on the basis of a given set of ESA-delivered quaternions.

Ten orbital arcs were selected for the orbit estimation process in which the corrections to the corresponding initial state vectors were estimated. These initial state vectors were taken from the reduced-dynamic PSO orbit of the GOCE satellite (Bock *et al.* 2011) at the following epochs [UTC]:

6 November 2009, 23 h 59 m 45.00 s,
19 November 2009, 23 h 59 m 45.00 s,
2 December 2009, 23 h 59 m 45.00 s,
18 December 2009, 23 h 59 m 45.00 s,
29 December 2009, 23 h 59 m 45.00 s,
6 January 2010, 23 h 59 m 45.00 s,
16 January 2010, 23 h 59 m 45.00 s,
26 January 2010, 23 h 59 m 45.00 s,
5 February 2010, 23 h 59 m 45.00 s,
10 February 2010, 23 h 59 m 45.00 s.

In order to obtain the computed (approximated) orbital arcs, the same set of initial state vectors was used.

Taking into account the mentioned variants of orbital arcs and six selected geopotential models, the different solutions of orbit estimation process were determined. The geopotential models are expressed in terms of spherical harmonic coefficients according to the following formula (Heiskanen and Moritz 1967):

$$V(r,\theta,\lambda) = \frac{\mathrm{GM}}{r}\sum_{n=0}^{N_{\max}}\left(\frac{a}{r}\right)^{n}\sum_{m=0}^{n}\left(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right)\bar{P}_{nm}(\cos\theta). \tag{4}$$

In this equation: V is the potential of the gravity field; r, θ, λ – geocentric coordinates of a given point: r – distance from Earth's center, $\theta = 90° - \varphi$ – colatitude, and φ – geocentric latitude, λ – geocentric longitude; a – equatorial radius of the Earth ellipsoid; $\bar{C}_{nm}, \bar{S}_{nm}$ – spherical harmonic coefficients (Stokes' coefficients) of degree n ($n = 0, 1, \ldots, N_{\max}$, $N_{\max}$ – the maximum degree of the spherical harmonic expansion) and order m ($m = 0, 1, \ldots, n$), and $\bar{P}_{nm}(\cos\theta)$ – normalized associated Legendre function of degree n and order m.

All geopotential models used in this work are listed in Table 1. They were obtained through International Center for Global Earth Models (ICGEM) at Deutsches GeoForschungsZentrum Potsdam (Drewes 2012). The ICGEM is one of six centers of the International Gravity Field Service of the International Association of Geodesy. The library of available models is constantly updated by the ICGEM.

All of the parameters constituting the tested models were adopted for computation without any changes. The exception was the ITG-GRACE 2010S model, where the non-zero coefficients of first degree were replaced by zero values. It was done to remove the effect of geocenter shift.

Table 1

List of the gravity field models used in this work

Gravity field model	Reference
EGM2008	Pavlis *et al.* 2012
EIGEN-5S	Förste *et al.* 2008
EIGEN-51C	Bruinsma *et al.* 2010
ITG-GRACE2010S	Mayer-Gürr *et al.* 2011
ULux_CHAMP2013S	Weigelt *et al.* 2013
EGM96	Lemoine *et al.* 1998

3. NUMERICAL TEST RESULTS

Using selected six gravity field models and the corresponding sets of pseudo-ranges, the different variants of GOCE orbital arcs were determined using the OCS software package taking into account the remaining dynamical models. The accuracies of estimated orbital arcs were expressed by the RMS difference between these arcs and the corresponding reference ones (reduced-dynamic PSO orbital arcs). The variants of estimated orbital arcs were obtained taking into account the first three initial epochs and all ten initial epochs listed in the previous section. Thus, the three and the ten solutions of the orbit determination process were estimated for each geopotential model taking into account a given orbital arc length. Subsequently, the RMS parameters computed for obtained solution variants for a given gravity field model were averaged.

Table 2 shows the mean values (for three orbital arcs) of the RMS parameter for particular geopotential models for the 1-day orbital arcs. Nominally, the reference orbit (the GOCE reduced-dynamic PSO orbit) was just obtained in the form of the 1-day arcs (Bock *et al.* 2011).

There are two basic options in Table 2. Both assume, of course, the use of the given gravity field model in the orbit determination. However, the first one additionally implies the use of the additional dynamical models with or without the observations (G-DM and G-DM-O modes in Table 2), whereas the second one uses the geopotential model only with or without the observations or additionally the truncated geopotential model (G, G-O, and G-O-Gtr modes in Table 2). In both options, the addition of observations (simulated pseudo-ranges) is connected with the orbit improvement with respect to the corresponding approximated orbit (computed without using the observations). An effectiveness threshold of improvement is defined by the $\overline{\mathrm{RMS}}$ values, which are obtained without using the measurements.

Table 2

Mean RMS (for three arcs) of differences between the estimated 1-day orbital arcs and the corresponding reference arcs (reduced-dynamic PSO arcs), depending on the applied geopotential model

Gravity field model	$\overline{\mathrm{RMS}}$ [m]				
	G-DM[1]	G-DM-O[2]	G[3]	G-O[4]	G-O-Gtr[5]
EIGEN-51C	15.97	2.52	30.90	8.01	7.72 / ~92
EGM2008_360×360	15.76	2.72	31.68	8.08	7.79 / ~104
EIGEN-5S	15.61	2.55	31.45	8.05	7.84 / ~92
ITG-GRACE2010S	10.03	2.47	29.99	8.13	7.84 / ~92
ULux_CHAMP2013S	9.21	2.56	30.69	8.14	7.88 / ~93
EGM96	43.46	4.48	47.90	8.42	8.36 / ~124

Explanations: Data set used in the orbit determination for the following modes:
1) geopotential model (G), dynamical models (DM) – ocean tides, solid Earth tides, third body effect, relativity;
2) geopotential model, dynamical models, observations (O),
3) geopotential model,
4) geopotential model and observations,
5) geopotential model, observations and the truncation of geopotential model (Gtr) for the improved initial state vector resulting from the G-O variant.
The results are given in the form: $\overline{\mathrm{RMS}}$ / mean degree and order of the truncation.

It is clearly seen that all solutions in Table 2 are effective because they have the $\overline{\mathrm{RMS}}$ values after using the observations several times less than the ones obtained for the approximated orbit. As can be seen, the older model, *i.e.*, EGM96 generates clearly worse solutions (higher $\overline{\mathrm{RMS}}$ values) than the newer ones. In the framework of the first option (using the additional dynamical models in orbit determination), the best results are achieved for such models as ITG-GRACE2010S, EIGEN-51C, EIGEN-5S, and ULux_CHAMP2013S where the corresponding $\overline{\mathrm{RMS}}$ values are equal to 2.47, 2.52, 2.55, and 2.56 m, respectively. These values were determined taking into account the G-DM-O mode.

In order to isolate and emphasize the impact of the gravity field models and to obtain an independence of the results of the orbit determination from the specified set of dynamic models used, the second aforementioned option is selected which comprises the use of only the geopotential models and observations in the orbit improvement process. The smallest $\overline{\mathrm{RMS}}$ values in this option are given by the application of gravity field models truncated at the same degree and order (G-O-Gtr mode). This degree and order is obtained by an analysis of the RMS values of the difference between the orbit computed for the estimated state vector (taken from the estimation based on the full geopotential model and observations) and the reference orbit (reduced-dynamic PSO orbit). Due to the changing of geopotential model truncation, the RMS values change, reaching the minimum at the specific value of degree and order of truncation. The values presented in Table 2 (last column) are the averages taking into account the RMS values and the degree and order truncation values for the three orbital arcs, which were the subject of estimation. For the degree and order of truncation, it is the mean value rounded to the nearest integer. For all tested geopotential models, the truncation to the specified degree and order causes the increase of determined arc accuracy of a few dm. This indicates the well-known fact that a usable signal for the given gravity field model in an orbital aspect primarily covers the long wavelength part. On the other hand, the obtained degree and order of the gravity models truncation may be also connected with the different ways of regularizations, which were used in the process of the model estimation. Unlike in the previous option (the use of remaining dynamical models) the best result is obtained for the EIGEN-51C model – $\overline{\mathrm{RMS}}$ equals 7.72 m, but the successive places are occupied by the geopotential models, such as EGM2008, EIGEN-5S, ITG-GRACE2010S with the $\overline{\mathrm{RMS}}$ values from 7.79 to 7.84 m (G-O-Gtr mode). These models are involved with the GRACE mission data. Slightly worse results are obtained by the use of ULux_ CHAMP2013S model ($\overline{\mathrm{RMS}}$ at a level of 7.88 m) which is based on the CHAMP mission data. Clearly inferior values of $\overline{\mathrm{RMS}}$ were obtained for the older EGM96 model (G-O and G-O-Gtr mode).

Table 3 contains similar results as in Table 2. In order to significantly reduce of errors due to the disabling of the remaining dynamic models in the G-O and G-O-Gtr modes, a much shorter orbital arc, which is equal to about 90 minutes, is selected. This is approximately the period of GOCE satellite revolution. Compared to Table 2, $\overline{\mathrm{RMS}}$ values for the best solutions decreased from meters to a level of decimeters. It is caused by the aforementioned decrease of errors, which are connected mainly with the use of only geopotential model with the observations (G-O and G-O-Gtr mode) and a simplified parameterization of the orbit determination process. This parameterization only includes the estimation of six corrections to the initial state vector of satellite. The orbital errors increase with increasing length of the determined arc. As for the results in Table 2, slightly better results were achieved using the truncated geopotential models in the orbit estimation. The decrease of $\overline{\mathrm{RMS}}$, obtained as a difference between the G-O mode and the

G-O-Gtr mode in Table 3, is included in the range of 0.7 cm for the ULux_CHAMP2013S model to 10.0 cm for the EGM96 model.

Table 3

Mean RMS of differences between the estimated 90-minute orbital arcs and the corresponding reference ones (reduced-dynamic PSO orbital arcs) depending on the applied geopotential model. Mean RMS values in the columns 2, 3, 4 are obtained for the three arcs. The results in the column 5 refer to the ten arcs

Gravity field model	$\overline{\mathrm{RMS}}$ [cm]			
	G[1]	G-O[2]	G-O-Gtr[3]	G-DM-O[4]
EGM2008_360×360	218.6	56.3	55.0 / ~ 126	14.0
EIGEN-5S	220.4	56.6	55.1 / ~ 116	13.8
EIGEN-51C	217.7	56.4	55.2 / ~ 124	13.7
ITG-GRACE2010S	210.3	58.3	57.2 / ~ 124	13.9
ULux_CHAMP2013S	216.5	59.6	58.9 / ~ 110	15.3
EGM96	564.6	123.1	113.1 / ~ 93	120.4

Explanations: Data set used in the orbit determination for the following modes:
1) gravity field model only,
2) gravity field model and observations,
3) geopotential model, observations and the truncation of geopotential model (Gtr) for the corrected initial state vector resulting from the G-O variant,
4) geopotential, dynamical models (ocean tides, solid Earth tides, third body effect, relativity), observations.
The results are given in the form: $\overline{\mathrm{RMS}}$ / mean degree and order of the truncation.

Unlike for the longer arcs, this time the best result in the G-O-Gtr mode was obtained for the EGM2008 gravity field model ($\overline{\mathrm{RMS}}$ = 55.0 cm – the G-O-Gtr mode). The next best solutions were obtained using the EIGEN-5S and EIGEN-51C models with the $\overline{\mathrm{RMS}}$ value equal to 55.1 and 55.2 cm, respectively (Table 3). The use of the ITG-GRACE2010S and ULux_CHAMP 2013S generates slightly worse results (57.2 and 58.9 cm, respectively). The result for the EGM96 model (113.1 cm) clearly differs from the other ones.

The values of $\overline{\mathrm{RMS}}$ in the G-DM-O mode (Table 3) were determined taking into account the ten orbital arcs with the initial epochs listed in the previous section. In this mode, the obtained results are about four times better than in the G-O mode due to the addition of dynamic models. The result for the EGM96 model is worse than the results for the remaining models by almost one order of magnitude. For this model, a relatively small increase in the accuracy of about 1.02 times in the G-DM-O mode w.r.t. the G-O mode occurs. The best results are achieved for such models as EIGEN-51C ($\overline{RMS}$ = 13.7 cm) and EIGEN-5S ($\overline{RMS}$ = 13.8 cm). Slightly worse result can be seen for the ULux_CHAMP2013S model ($\overline{RMS}$ = 15.3 cm).

In turn, each of the values in Table 4 is computed using ten RMS values obtained for ten 45.0-minute orbital arcs with the initial epochs listen in the Section 2. Comparing the results in the G-O mode in Tables 2-4, a similar order of the gravity models can be noted, except the EGM2008 model, which changes position from third (Table 2, $\overline{RMS}$ = 8.08 m) through the first (Table 3, $\overline{RMS}$ = 56.3 cm) to fourth (Table 4, $\overline{RMS}$ = 23.43 cm). The relative order (G-O mode for increasing values) of the remaining models is the same in all three cases (Tables 2-4): EIGEN-51C, EIGEN-5S, ITG-GRACE2010S, ULux_CHAMP2013S, EGM96.

Just as in Table 3, the best results were achieved using the G-DM-O mode, taking into account the dynamic models (Table 4). As for the 90-minute arcs, the accuracy of solutions increases about four times w.r.t. the G-O mode with the exception of solutions for the older EGM96 model, where the increase in accuracy is characteristically low – about 1.02 times. The considered results confirm the dominance of the EIGEN-5S model ($\overline{RMS}$ = 4.9 cm) and the EIGEN-51C model ($\overline{RMS}$ = 5.1 cm). On the other hand, the slightly worse results for the ULux_CHAMP2013S model and the much worse results for the EGM96 model are also confirmed.

Table 4

Mean RMS (for ten arcs) of differences between the estimated 45.0-minute orbital arcs and the corresponding reference ones (reduced-dynamic PSO orbital arcs) depending on the applied geopotential model for the G-O mode and the G-DM-O mode

Gravity field model	$\overline{RMS}$ [cm]	
	G-O[1]	G-DM-O[2]
EGM2008_360x360	23.43	5.3
EIGEN-5S	23.29	4.9
EIGEN-51C	23.13	5.1
ITG-GRACE2010S	23.34	5.2
ULux_CHAMP2013S	23.52	5.8
EGM96	44.15	41.1

Explanations: Data set used in the orbit determination for the following modes:
[1)] gravity field model and observations,
[2)] geopotential, dynamical models (ocean tides, solid Earth tides, third body effect, relativity), observations.

Direct comparison between the performance of all gravity models is given in Table 5 (the G-O mode). This comparison is done for the 45-minute arcs and the 1-day arcs. The presented results are the absolute values of differences of $\overline{\mathrm{RMS}}$ (hereafter referred to briefly as differences) of the fit between the tested models. For the 45-minute arcs, these differences are below one centimeter in the group of newer models (EIGEN-51C, EIGEN-5S, ITG-GRACE2010S, EGM2008, ULux_CHAMP2013S) and they do not exceed 0.4 cm − the largest of them, between EIGEN51-C and ULux_CHAMP 2013S, is equal to 0.39 cm. The EGM96 model clearly differs from the others, showing above a fifty times greater difference with the EIGEN51C model (21.02 cm). In the case of longer, 1-day arcs (Table 5 – differences in bold) differences remain at the centimeter level, reaching 13 cm for EIGEN51-C and ULux_CHAMP2013S (for the newer models). This time, the greatest difference with the EGM96 model is equal to 41 cm. This value is approximately three times greater than the largest difference in the group of newer models. It follows that the disproportion between the performance of newer models and the EGM96 model decreased significantly with the increasing length of the orbital arcs (from 45-min to 1-day), because the aforementioned ratio of differences decreased from about fifty to almost three.

In order to obtain a measure of the disproportion between the performance of newer models, two other ratios were computed for the 45-minute and 1-day orbital arcs by dividing the greatest difference in the group of newer models (Table 5) by the mean value of difference in this group of models. The ratios are equal to about 2.12 and 1.91 for the 45-minute and 1-day orbital arcs, respectively, which means that the disproportion between the performance of newer models is almost the same for both the 45-minute and 1-day orbital arcs. Strictly speaking, the disproportion slightly decreased for the longer orbital arc.

Table 5

Absolute values of differences between mean RMS of fit for all compared models, taking into account the estimated 45.0-minute orbital arcs and 1-day arcs (in bold) for the G-O mode (using the gravity field model and observations)

$\lvert\Delta\overline{\mathrm{RMS}}\rvert$ [cm]	EIGEN-51C	EIGEN-5S	ITG-GRACE 2010S	EGM2008	ULux_CHAMP 2013S	EGM96
EIGEN-51C	–	0.16 / **4**	0.21 / **12**	0.30 / **7**	0.39 / **13**	21.02 / **41**
EIGEN-5S	–	–	0.05 / **8**	0.14 / **3**	0.23 / **9**	20.86 / **37**
ITG-GRACE2010S	–	–	–	0.09 / **5**	0.18 / **1**	20.81 / **29**
EGM2008	–	–	–	–	0.09 / **6**	20.72 / **34**
ULux_CHAMP2013S	–	–	–	–	–	20.63 / **28**

4. SUMMARY AND CONCLUSIONS

Six selected gravity field models were used in the process of GOCE satellite orbit determination. These models are involved in this process directly in the satellite motion model. For the computations, the ten GOCE initial state vectors from the reference orbit (reduced-dynamic PSO orbit for the GOCE satellite) were adopted. The sets of GPS pseudo-ranges simulated along the reduced-dynamic PSO orbit were the observations used in the adjustment. Different solution variants of the GOCE orbit have been computed, taking into account the data processed in the following modes: the geopotential model only, the geopotential model and the remaining dynamical models, the geopotential and the observations, the geopotential and the remaining dynamical models with the observations.

In order to compare the quality of determined solutions using the selected geopotential models, the RMS differences between the estimated orbits and the corresponding reference orbits were averaged in the frame of each aforementioned solution mode. Thus, the $\overline{\mathrm{RMS}}$ value for the given geopotential model is based on the RMS values computed for the three and ten orbit estimations with the same arc lengths and the different initial state vectors. Three groups of $\overline{\mathrm{RMS}}$ values were obtained. They refer to the length of the estimated orbital arcs of about 45 minutes, 90 minutes, and 1 day.

For tested arc lengths, the best results (the smallest $\overline{\mathrm{RMS}}$ values) were achieved especially for the EIGEN-51C model (for 45-minute arcs – in the G-O mode and for 1-day arcs – in the G-O and G-O-Gtr mode). The results in the G-DM-O mode depend to some extent on the orbital arc length. In the case of shorter, 45- and 90-minute arcs, the best solutions were obtained for the EIGEN-51C model and the EIGEN-5S model, whereas the ITG-GRACE2010S model is preferred for the 1-day arcs. However, the EIGEN-51C model was the second preferred model for these orbital arcs. The EIGEN-51C is a combined solution based on the GRACE and CHAMP mission data and terrestrial gravimetric data (Bruinsma *et al.* 2010). Similarly, the EIGEN-5S is based on the GRACE mission data and additionally on LAGEOS data (Förste *et al.* 2008). The results obtained in the G-DM-O mode also depend on the taken set of remaining dynamical models. Although these results both confirm good performance of the aforementioned EIGEN-51C model and the EIGEN-5S model and slightly worse performance the ULux_CHAMP2013S model, based on the CHAMP-only data (Weigelt *et al.* 2013). These results show clearly worse solutions for the older EGM96 model, which incorporates gravity anomalies derived from altimeter data, surface gravity data and, among others, such data as Doppler observations, SLR and optical observations (Lemoine *et al.* 1998).

After truncation of the geopotential models at the determined degree and order, the obtained results are slightly improved, which indicates a useful long and medium wavelength part of the assessed models in terms of determination of the GOCE orbit. In other words, the obtained degree and order values of truncation may indicate a sensitivity limit of GOCE orbit in terms of modeling geopotential. The mentioned part of the assessed models reaches the degree and order from 92 to 124 for the longer, 1-day arcs and from 93 to 126 for the shorter, 90-minute arcs. It seems that, for the shorter, 90-minute arcs, the useful part of the geopotential models is generally larger than for the longer, 1-day arcs. The mean degree and order of truncation is equal to about 99 for the longer, 1-day arcs whereas the corresponding mean value for the shorter, 90-minute arcs equals about 115. This might be related to smaller errors in the process of shorter arc determination. Additionally, in the case of the shorter, 90-minute arcs, the useful part of models is larger for the gravity models with the better results – smaller $\overline{RMS}$ values (EIGEN-51C, EGM2008, EIGEN-5S, ITG-GRACE2010S) than for the models with worse results – greater mean RMS values (ULux_CHAMP2013S, EGM96). The gravity field models lost the signal above determined values of degree and order, which means that a certain range of spherical harmonic coefficients is not useful in the considered orbital aspect. On the other hand, the presented truncation effect is not only due to the signal loss, but also due to the aforementioned regularization of the gravity field solutions.

An example increase of the time range of research and an example decrease of the length of orbital arc determined does not lead to significant changes in relative results – the order of five of six tested models in terms of performance in the GOCE orbit estimation did not change. The following sequential pattern is visible for all three cases of orbital arc lengths used (in the G-O mode, Tables 2-4): EIGEN-51C, EIGEN-5S, ITG-GRACE2010S, ULux_CHAMP 2013S, EGM96.

In comparing the results for the 45-minute and 1-day orbital arcs (Table 5), a significant decrease of disproportion of the performance between the newer models (EIGEN-51C, EIGEN-5S, ITG-GRACE2010S, EGM2008, ULux_CHAMP2013S) and the EGM96 model can be observed. It seems that larger errors which arise in the process of generating longer, 1-day orbital arcs can lead to a reduction of this disproportion with respect to the disproportion for the shorter, 45-minute arcs. In the case of the disproportion of the performance between the newer models, it is almost the same for both the 45-minute and 1-day orbital arcs (only slightly decreases for the longer arcs). The determined ratios measuring this disproportion are similar for both orbital arc lengths – 2.12 for the 45-minute arcs and 1.91 for the 1-day arcs.

Taking into account all obtained results (Tables 2-4), the generated GOCE orbital solutions particularly prefer such models as: EIGEN-51C and EIGEN-5S, EGM2008, ITG-GRACE2010S (especially this model in the G-DM-O mode for the 1-day arcs). All of these models are based on the GRACE mission data.

It is worth noting that the first model in the above list, *i.e.*, EIGEN-51C, is a combined solution that uses both satellite data and (especially) terrestrial data, which may have some importance for an orbit with such an extremely low altitude as the GOCE satellite orbit.

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