On Data Interpolation at Three Croatian Repeat Stations by Using the Spherical Elementary Currents Systems Method

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Abstract

A general mathematical tool for expanding vector systems on a sphere into basis functions, spherical elementary current system (SECS) method, was applied for separation of the geomagnetic field variations into external and internal parts, over a limited region of central and southeastern Europe. The registered variations at three Croatian repeat stations were compared to the variations estimated by the SECS method using the variations from the different sets of observatories. The results of the SECS method were also compared to a simple assumption that the variations at repeat station are equal to those at particular observatory. The relevance of this comparison was to get an insight about the possibility of using the SECS method for estimating the geomagnetic field variations over Croatia. The guidelines for the application of the SECS method for the purpose of reducing repeat station data were also given.

Key words: geomagnetic repeat station, geomagnetic observatory, geomagnetic data interpolation, spherical elementary currents system.

1. INTRODUCTION

The geomagnetic field measured at the Earth's surface is a superposition of several contributions. The main field originates from the electrically conductive fluid motions in the outer core, and the lesser part of the geomagnetic

field originates from the magnetized rocks in the crust and upper mantle (the lithospheric field). There are also contributions of the ionospheric and magnetospheric currents (*i.e.*, the external field) and their induced effects in the lithosphere. The geomagnetic field is continuously monitored at geomagnetic observatories and periodically on the repeat stations network (Mandea and Purucker 2005).

The external field variations (and associated induced effects) at some repeat stations and at surrounding observatories can be prominently different, and this occurs, *e.g.*, when these sites are quite far away (especially in latitude), and/or the lithospheric electrical conductivity differs under these sites (Korte and Thébault 2007). The time variations at some point can be interpolated by using the data recorded at relatively nearby magnetometers, up to few hundreds of kilometers, by using the simple assumption that these variations (about some arbitrary baselines) are the same at those sites. Such a method is commonly used in repeat stations data reduction procedure (Newitt *et al.* 1996).

Several methods, based on spectral decomposition of magnetic potential in spherical or rectangular geometry (Amm and Viljanen 1999 and references therein), were developed in order to model the spatial distribution of geomagnetic time variations at the Earth's surface. In this paper the attention is focused on spherical elementary current systems (SECS) method, introduced by Amm (1997). This method for field continuation from ground to the ionosphere is not based on spectral decomposition. It expands the measured ground geomagnetic time variations into a sum of the magnetic field contributions of spherical elementary current systems placed in the ionosphere (Amm 1997, Amm and Viljanen 1999, Pulkkinen *et al.* 2003b), or in the ionosphere and at some depth inside the Earth (Pulkkinen *et al.* 2003a). The centers of these elementary current systems (*i.e.*, "poles") can be placed freely, such that their locations are most suitable with respect to the type of magnetic variations to be analysed or to the density of measurement sites (Amm and Viljanen 1999). The amplitudes of elementary systems (scaling factors) are represented by the scaling factor matrix (Pulkkinen *et al.* 2003a), and it is the solution to be determined by the data from measurement sites.

Some examples of using the SECS method are: modelling of Cowling effect (Amm 1997, Amm and Viljanen 1999, Amm *et al.* 2013), modelling of some ionospheric events and their electrical current configurations, the magnetic field effects of which were recorded by BEAR and/or IMAGE magnetometer arrays (Pulkkinen *et al.* 2003a, b; Vanhamäki *et al.* 2003, Apatenkov *et al.* 2004), determining of ionospheric currents from CHAMP satellite measurements (Juusola *et al.* 2006), data interpolation at some observatories in Europe and North America (McLay and Beggan 2010), and deriving the

geomagnetically induced electric field at the Earth's surface by using the data from BEAR and IMAGE magnetometer arrays (Vanhamäki *et al.* 2013).

The prime aim of this work was to inspect an implementation of the SECS method for the estimation of the centered geomagnetic field time series over Croatia. The usage of centered data means that the variations' baselines are the mean values of geomagnetic components over some interval. The other variations with different baselines could also be used for testing the SECS method, and some of those baselines are the quiet night values and the annual mean values. Their accuracies at Croatian repeat stations were not satisfactory for this case study. The estimations derived by the SECS method were also compared to the corresponding estimations derived by a simple method, which assumes that the centered data at a repeat station are equal to the centered data registered at the relatively close observatory. Implicitly, the relevance of this comparison was to inspect an alternative (improved) method for the reduction of repeat station data.

2. METHODS AND DATA

2.1 Spherical elementary currents systems method

Two types of spherical elementary sheet currents have been defined by Amm (1997), one being divergence-free and the other curl-free (Amm and Viljanen 1999). In the case of ground disturbance continuation, it is necessary to consider only the divergent-free elementary systems (Amm and Viljanen 1999, Juusola *et al.* 2006). Their poles are placed in the ionosphere of radius R_i and below the Earth's surface at radius R_g (Pulkkinen *et al.* 2003a), respectively, both in geocentric reference frame. The definition of one elementary sheet currents system, in a spherical coordinate system (r', θ', φ') in which the pole of the elementary system is at $\theta' = 0$, is (Amm and Viljanen 1999, McLay and Beggan 2010):

$$J_c(\theta) = (I_{0,c}/4\pi R_c) \cdot \cot(\theta/2) \cdot \hat{\phi}', \quad (1)$$

where index $c = i, g$ (i stands for the ionosphere and g for the ground), $I_{0,c}$ is scaling factor of the elementary system, radii R_i and R_g are defined as 110 km above the Earth's surface and 100 km below it, respectively (Amm and Viljanen 1999, McLay and Beggan 2010). In fact, there are two infinitely thin horizontal current layers, the first in the ionosphere and the second inside the Earth, since any divergence-free current system can be composed by superposition of elementary current systems (Pulkkinen *et al.* 2003a). The magnetic effect of these two layers at some point on the Earth's surface is (in geocentric frame):

$$\begin{aligned} \mathbf{B}(r, \theta, \varphi, t) = & \sum_{m=1}^M I_{m(0,i)}(t) \cdot T_i(r, \theta, \varphi, R_i, \vartheta_m, \varphi_m) \\ & + \sum_{s=1}^S I_{s(0,g)}(t) \cdot T_g(r, \theta, \varphi, R_g, \vartheta_s, \varphi_s), \end{aligned} \quad (2)$$

where the scaling factors I are the functions of time t , $T_{i,g}$ are geometric parts related to the external and internal part ($R_g < r < R_i$) of magnetic field produced by each elementary current system located at $(R_i, \theta_m, \varphi_m)$ and $(R_g, \theta_s, \varphi_s)$, while M and S are the numbers of poles related to these two current layers, respectively. The expressions for $T_{i,g}$ are given by Amm and Viljanen (1999). The linear system of equations given by Eq. 2 can be written in a matrix form as (McLay and Beggan 2010):

$$\mathbf{B} = \mathbf{T} \cdot \mathbf{I}, \quad (3)$$

A procedure of deriving $\mathbf{I}$ can be taken, *e.g.*, under an assumption that the standard deviations of the errors in measurements are equal at each measurement point at the Earth's surface (Pulkkinen *et al.* 2003a), which can be a reasonable assumption for geomagnetic observatories. The matrix $\mathbf{T}$, of the order $3N$ by $(M+S)$, where N is a number of measurement points, is then of the form (Pulkkinen *et al.* 2003a):

$$\begin{bmatrix} \dots & & & \\ T_{i,\theta}^{k,1} & T_{i,\theta}^{k,M} & T_{g,\theta}^{k,1} & T_{g,\theta}^{k,S} \\ T_{i,\varphi}^{k,1} & \dots T_{i,\varphi}^{k,M} & T_{g,\varphi}^{k,1} & \dots T_{g,\varphi}^{k,S} \\ T_{i,r}^{k,1} & T_{i,r}^{k,M} & T_{g,r}^{k,1} & T_{g,r}^{k,S} \\ \dots & & & \end{bmatrix}, \quad (4)$$

where k refers to k th measurement point, and, *e.g.*, the element $T_{i,\theta}^{k,1}$ is the geometric part related to the magnetic field of internal (i) origin, produced at k th point in polar direction (θ) of geocentric frame, and by superscripts (1, ..., M) and (1, ..., S) are denoted the elementary systems, respectively. Further, the matrix $\mathbf{B}$, of the order $3N$ by 1, can be derived from the geomagnetic field variations (B_θ, B_φ, B_r) measured at the points (1, ..., N) on the Earth's surface (in geocentric frame), and in this case it is equal to a transpose of:

$$\left(B_\theta^1 \ B_\varphi^1 \ B_r^1 \dots B_\theta^k \ B_\varphi^k \ B_r^k \dots B_\theta^N \ B_\varphi^N \ B_r^N \right). \quad (5)$$

After deriving matrices **T** and **B**, matrix **I** can be calculated. This system of equations is highly underdetermined (Amm and Viljanen 1999, Pulkkinen *et al.* 2003a, McLay and Beggan 2010), since the number of unknowns ($M + S$) is in general much greater than the number of measurements at the Earth's surface (3N), at a particular time moment (one minute in this case). The inversion of matrix **T** can be performed by its singular value decomposition (Press *et al.* 2001), and after that the matrix **I** can be calculated at each minute. The usual procedure is that the stabilization in inversion of matrix **T** is done, by choosing the threshold ε for singular values related to different basis vectors of the decomposition, and if larger ε is chosen, the solutions for **I** will, in general, be smoother (Amm and Viljanen 1999, Pulkkinen *et al.* 2003a).

Matrix **I** can be derived, *e.g.*, by using two networks of poles of elementary current systems and field values from observatories, the latter being distributed over some region on the Earth's surface (McLay and Beggan 2010). Once matrix **I** is known, the magnetic field variation at some point on the Earth's surface can be calculated by using Eq. 3, but with matrix **T** calculated for that position and given elementary current systems networks.

These estimated variations can be compared to the measured one at the repeat station, in order to derive root-mean-square (rms) error, defined as $\text{rms}(\text{error}) = (\sum \text{error}^2 / n)^{0.5}$, where n is the length of one-minute time-series used for testing the interpolation by SECS method, and the maximal absolute error ($\max(|\text{error}|)$) of interpolation. These two parameters were obtained for the errors of estimated centered data, and the estimations were derived, respectively: by the SECS method and by a simple method (an assumption that the centered data at repeat station are equal to the simultaneous centered data at particular observatory).

Further, the SECS method could also be used for the purpose of reducing repeat station data to their annual mean values. Provided that $E(t)$ is the absolute measurement of geomagnetic component E at the instant t , $\text{var}(t)$ is the corresponding variation of that component with respect to its annual mean value, and if $\text{var}(t)$ can be estimated by the SECS method according to Eq. 3, then the annual mean value at repeat station can be determined from:

$$E^{\text{annual mean}} = E(t) - \text{var}(t) . \quad (6)$$

The simultaneous geomagnetic components' variations at the observatories, which are necessary as the input values in the SECS method (for deriving the matrix **I**), are derived as: $E_O(t) - E_O^{\text{annual mean}}$, where subscript O is for observatory. If a simple method is used for data reduction, one has (Newitt *et al.* 1996):

$$E^{\text{annual mean}} = E(t) - \left[E_O(t) - E_O^{\text{annual mean}} \right]. \quad (7)$$

2.2 Data used

During July 2010 the Overhauser effect dIdD (deflected Inclination deflected Declination) variometer was installed successively on three Croatian repeat stations (Fig. 1), Krbavsko Polje (KRBP), Sinjsko Polje (SINP), and Pala-gruža (PALA), in a frame of the joint Croatian-Hungarian project (Csontos *et al.* 2012). More on the Croatian geomagnetic networks can be found in Brkić (2013). The one-minute values of X (north), Y (east), and Z (vertical) components were calculated after the instrument calibration. Due to some unexpected artificial disturbances, several intervals of data for each station were dropped. In Table 1, the usable intervals larger than five hours are displayed, and these twelve time series (four per repeat station) are denoted with letters a, b, ..., l. The baselines of these variations were taken as the mean values of components over each time interval. The total field time series from KRBP, SINP, and PALA are displayed in Fig. 1, and twelve time series (a, b, ..., l) are also denoted.

In this case study, the one-minute values from ten European geomagnetic observatories (Fig. 1) were used for interpolation of the intervals given in Table 1: Budkov (BDV), Black Forest (BFO), Castello Tesino (CTS), Fürstenfeldbruck (FUR), Grocka (GCK), Hurbanovo (HRB), Nagycenk (NCK), Panagjurishte (PAG), Surlari (SUA), and Tihany (THY). The one-minute data were obtained from at Intermagnet Web Site (<http://www.intermagnet.org>). These observatories were selected because of their proxi-

Fig. 1. The total field time series from KRBP (top), SINP (middle), and PALA (bottom). The durations of twelve time series (a, b, ..., l) are also denoted.

Table 1

The durations of twelve (a, b, ..., l) time series used in this study which were recorded at three Croatian repeat stations

Repeat station	Time series	Time interval [UTC]	Duration [min]	Kp
KRBP	a	20 July, 02:03-10:28	503	1-2
	b	20 July, 15:57 – 21 July, 07:08	903	1-2
	c	21 July, 07:24 – 22 July, 08:28	1501	0-2
	d	22 July, 08:44-23:37	894	1
SINP	e	23 July, 19:26 – 24 July, 10:09	881	0-3
	f	24 July, 10:20 – 25 July, 02:20	949	0-2
	g	25 July, 15:36-21:04	326	1
	h	26 July, 16:34 – 27 July, 00:56	500	1-3
PALA	i	27 July, 18:35 – 28 July, 09:18	884	2-4
	j	28 July, 13:23 – 29 July, 01:38	725	2-3
	k	29 July, 11:06 – 30 July, 12:48	1540	1-2
	l	30 July, 23:28 – 31 July, 05:58	391	1-2

Note: The activity index Kp is also given (obtained from <http://swdcwww.kugi.kyoto-u.ac.jp>).

Table 2

The distances (in km) of three Croatian repeat stations (KRBP, SINP, and PALA) from ten geomagnetic observatories

km	BDV	BFO	CTS	FUR	GCK	HRB	NCK	PAG	SUA	THY
KRBP	505	691	347	512	407	406	340	728	839	304
PALA	764	905	548	751	441	628	584	650	844	518
SINP	637	828	478	653	344	483	443	622	771	374

mity to Croatian repeat stations, and a few of them (FUR, GCK, and THY) were used previously in data reduction of Croatian repeat stations surveys (Vujić *et al.* 2011). The distances of these observatories from Croatian repeat stations are given in Table 2. One can notice that there is no observatory for data interpolation southwest and south of repeat stations.

Considering the SECS method (*i.e.*, deriving of matrix **I**) and data interpolation at repeat stations, four sets of observatories were used, respectively: set A (CTS, GCK, HRB, NCK and THY), set B (BDV, CTS, GCK, PAG and THY), set C = set A + BDV + PAG (seven observatories), and set D consists of all ten observatories. Set A contains five closest observatories to the repeat stations, and set B consists of the observatories that constitute ap-

Fig. 2. The locations of ten geomagnetic observatories (black and red dots) used for data estimation at three Croatian repeat stations, and the locations of those repeat stations (red triangles) inside a spatial domain of the SECS's poles. The SECS pole grid is marked with blue dots (with 0.5° spacing). The observatories from set A (CTS, GCK, HRB, NCK, and THY) are marked by red dots, the observatories from set B (BDV, CTS, GCK, PAG, and THY) are marked with black circles, and the observatories from set C (set A + set B) are denoted by red quadrangles.

proximately equidistant spatial distribution of observatories around the repeat stations. The data of the Croatian stations themselves were not used for the SECS method. Beside those sets, two networks of poles of elementary currents systems were used, and their spatial domain is also shown in Fig. 2. These two networks were rectangular ones with spacing δ . Further, as none of the three Croatia stations is situated inside the hull made up by the observatories, one can talk about extrapolation, not interpolation, in a spatial sense. The term interpolation refers to the interpolation of the one-minute time series.

3. RESULTS AND DISCUSSIONS

The first step in this case study was to find the optimal values of parameters ε and δ . The values of rms(error) and (max(|error|), *i.e.*, error parameters, were derived for repeat stations time series by using the observatory sets (A, B, C, and D), with the values for ε and δ in the ranges $\varepsilon = 0.01$ - 0.1 , as suggested in Amm and Viljanen (1999), and $\delta = 0.2^\circ$ - 0.8° , respectively. The following conclusions arised from these analyses: (i) the errors parameters

Fig. 3a. The root-men-square errors of SECS method in geomagnetic components for given time intervals (a, b, ..., l), derived by using different observatory sets (A, B, C, and D) for data interpolation, with $\delta = 0.5^\circ$ and $\varepsilon = 0.1$.

Fig. 3b. Maximal absolute errors of SECS method in geomagnetic components for given time intervals (a, b, ..., l), derived by using different observatory sets (A, B, C, and D) for data interpolation, with $\delta = 0.5^\circ$ and $\varepsilon = 0.1$.

are almost not sensitive to δ , for a given set of observatories and ε , but slightly better values are for $\delta = 0.5^\circ$; (ii) the error parameters were sensitive to ε for Y , but were stable for X and Z , and the minimal values were for $\varepsilon = 0.1$; (iii) set C was the most suitable for KRBP, while set A for SINP and PALA, for geomagnetic variations interpolation; (iv) set D has given the worst results in errors parameters. The errors parameters derived with different observatory sets and values of $\delta = 0.5^\circ$ and $\varepsilon = 0.1$ are displayed in Figs. 3a and 3b, respectively. On average, they have the lowest values in X for KRBP, and in Y and Z for SINP, and in general they are the smallest in Z and highest in Y .

As mentioned, none of these repeat stations is situated inside the hull made up by the observatories. In Pulkkinen *et al.* (2003a) it was derived that at the edges of the magnetometer array, and at the regions where the two-dimensionality of the array is lost, errors of data interpolation may become larger. One can expect the results from the Croatian repeat stations to be different provided that the stations are situated inside the hull. The same procedure was performed also when the data from a particular station were included in the SECS method. In this case, the errors were much lower, as can be expected.

The sets A or C gave the best results for those three repeat stations, on average. If the sets of observatories are ordered with respect to their mean distances from KRBP and SINP stations, one obtains: A, C, B, and D, *i.e.*, A is the closest set, and D is the farthest one. The same order for station PALA is: A, B, C, and D. Further, a magnetic field produced by the elementary system decreases with the polar angle, as measured from its pole (Amm and Viljanen 1999). It can be presumed that the scaling factors of the elementary systems which are closer in latitude to the observatories will be more accurately determined by inversion process using the data from those observatories, than the scaling factors of the elementary systems that are farther from the observatories. On the other hand, a magnetic field at repeat station that is closer to those observatories or is situated inside the hull of those observatories, will be more accurately determined. These could be the reasons why a spatial interpolation is dominant over extrapolation. Based on these simple arguments, one can qualitatively explain why the set A could be the most suitable for SINP and PALA stations, and the set D the worst choice for all three stations.

The set C comprises two sets, A and B, so it could be expected that the results for the set C are at least the same as for either of them, since generally adding information should never worsen the output. It can be presumed that this is not the case for stations SINP and PALA due to a spatial extrapolation. The observatories BDV and PAG are probably relatively far away from them, and it could be possible that the elementary systems in the vicinity of those observatories produce some unfavorable total contribution (noise) at SINP and PALA. The total contribution from the elementary systems in the vicinity of those two observatories could be probably favorable at KRBP station, since it is possible that BDV is close enough and PAG is far enough from that station.

Also, it was checked whether the SECS method or a simple method (by particular observatory) better estimates the centered data (variations) at repeat stations. The results, in terms of the minimal values of errors parameters, are presented in Table 3. One can notice that the results are about the same for both methods of interpolation, and none of them is dominant. The

Table 3

The results of analysis whether the SECS method or some observatory (Obs.) better explains the centered data at a particular repeat station

Set or observatory	rms (error)			max (error)		
Time series	X	Y	Z	X	Y	Z
a	SECS	Obs.	SECS	Obs.	Obs.	SECS
b	Obs.	SECS	SECS	Obs.	SECS	SECS
c	Obs./SECS	Obs.	SECS	Obs./SECS	Obs.	SECS
d	Obs.	SECS	Obs.	SECS	SECS	Obs.
e	SECS	Obs.	Obs./SECS	SECS	Obs.	SECS
f	Obs.	Obs./SECS	SECS	Obs.	Obs.	SECS
g	Obs.	Obs./SECS	Obs./SECS	Obs.	Obs./SECS	Obs./SECS
h	Obs./SECS	SECS	Obs./SECS	Obs./SECS	SECS	Obs./SECS
i	Obs.	Obs.	SECS	Obs.	Obs.	SECS
j	Obs.	SECS	SECS	Obs.	SECS	SECS
k	Obs.	Obs.	Obs.	Obs.	Obs.	SECS
l	Obs./SECS	SECS	SECS	Obs.	Obs.	SECS

Note: The multiple results for some time series mean that the minimal values of errors' parameters were achieved in both cases.

SECS method was generally better for Z component estimations, and the method with particular observatory was generally better for X component, while for Y component they were about the same. In Table 4 there are displayed relatively high values of linear correlation coefficients between original and reconstructed (interpolated by SECS method) time variations of X , Y , and Z components at repeat stations. Table 5 gives error parameters for some characteristic variations (quiet-night and larger parts of diurnal variation). These results could be an indication that the quiet-night values could be relatively accurately estimated.

As the examples, the next two figures display the time variations of geomagnetic components for some particular time intervals, together with the simultaneous reconstructed series derived with the SECS method. In Fig. 4 are the results for major part of diurnal variation and a smaller part of quiet-night interval at KRBP station. The maximal absolute errors were 4.5, 3.9, and 5.0 nT in X , Y , and Z , respectively. In Fig. 5 are displayed time variations that include bay disturbance connected with geomagnetic pulsations that begun at 20:32 UTC on 27 July (Dr. H.J. Linthe, Niemegk Observatory, private communication). The maximal absolute errors for data in Fig. 5 were

Table 4

The linear correlation coefficients (CC) between original and reconstructed (interpolated by SECS method) time variations of X , Y , and Z components at repeat stations

Time series	CC_X	CC_Y	CC_Z
a	0.9886	0.9810	0.9801
b	0.9880	0.9965	0.9149
c	0.9844	0.9730	0.9936
d	0.9900	0.9910	0.9547
e	0.9934	0.9907	0.9797
f	0.9300	0.9936	0.9614
g	0.9396	0.9353	0.9870
h	0.9955	0.9876	0.8951
i	0.9861	0.9876	0.9412
j	0.9887	0.9972	0.7050
k	0.8828	0.9965	0.9898
l	0.9194	0.9831	0.9211

Note: In the case of KRBP variations, observatories set C was used, and in cases of SINP and PALA, set A was used.

Table 5

The root-mean-square and maximal absolute errors for quiet-night variations (first four rows), and for the parts of diurnal variations (last two rows), of geomagnetic components at repeat stations

Variation	Station	rms (error)			max (error)		
		X [nT]	Y [nT]	Z [nT]	X [nT]	Y [nT]	Z [nT]
20 July, 22:00 – 21 July, 01:00 UTC	KRBP	0.1	0.2	0.2	0.4	0.5	0.6
21 July, 22:00 – 22 July, 01:00 UTC	KRBP	0.2	0.3	0.2	0.5	0.7	0.6
24 July, 22:00 – 25 July, 01:00 UTC	SINP	0.5	0.3	0.2	1.1	0.9	0.5
29 July, 22:00 – 30 July, 01:00 UTC	PALA	0.3	0.3	0.2	1.0	0.9	0.4
21 July, 07:20-20:00 UTC	KRBP	1.0	4.5	1.1	2.4	8.2	3.1
22 July, 08:44-20:00 UTC	KRBP	2.3	1.5	2.2	4.4	3.4	4.9

Note: The used observatories sets are the same as for the results in previous table.

3.3, 1.9, and 1.3 nT in X , Y , and Z , respectively. One can notice that there is no systematic under-estimations or over-estimations of variations in those two cases, and that X and Y components time series are very highly correlated.

Fig. 4. The time variations of geomagnetic components at KRBP station during the time interval 22 July, 08:44-23:37 UTC. The observatories set C was used for data reconstruction.

Fig. 5. The time variations of geomagnetic components at PALA station during the time interval 27 July, 20:32-22:32 UTC. The observatories set A was used for data reconstruction.

4. CONCLUSIONS

The method of spherical elementary current systems was implemented for interpolation of geomagnetic time variations at three Croatian repeat stations for the first time. The data from different sets of geomagnetic observatories were used for interpolation of geomagnetic time variations at the positions of repeat stations. The time variations in this case study were the centered time series of geomagnetic components. It was possible to find which method parameters and sets of observatories describe the variations at stations in a best manner, *i.e.*, to have the smallest root-mean and maximal absolute errors. They were in general smallest in Z component and highest in Y component.

This method was compared to the results of a simple assumption that the time variations of the geomagnetic components are the same at the particular observatory and the repeat station. These two methods gave about the same results; on average, SECS method gave better results in Z component, and simple method in X component, while in Y the results were about the same. The time series from three stations were statistically relatively small samples, and for more consistent conclusions it is necessary to have longer time series of quiet as well as disturbed conditions. Also it could be interesting to see such results if variometers could be installed simultaneously in southern and southwestern part of spatial domain, since for determining current scale factors in SECS method there were no data from that region.

The relevance of the described comparison was to implicitly present an alternative method for the reduction of repeat station data. Since the annual mean values of geomagnetic components at Croatian repeat stations have quite lower accuracies than the corresponding values at observatories, they were not used as variations' baselines in this case study. However, the instructions for the usage of the SECS method for the purpose of reducing repeat station data were also given, in the case when an influence of the differences of secular variations between a repeat station and the observatories can be neglected. The application of this method for the purpose of reducing repeat station data from Croatia can be expected in future.

Considering overall message/recommendation to someone who may face a similar problem in a different region, one can presume that there are two approaches to determine the (optimal) SECS method parameters: without an *in situ* variometer, or to perform an empirical analysis by using a variometer data. The former approach is possible only if the time variations of geomagnetic components can be modeled over the region of interest, prior to an application of the SECS method. However, one can also determine those parameters by finding the minimum misfit between the SECS method predictions and known observations, which are recorded by a variometer. Those conclusions have to be based on an adequate amount of the time series of interest (e.g., the diurnal variations for data reduction).

A spatial distribution of the observatories around the points of interest is also important, as derived here and in the previous studies. Those observatories are necessary for deriving a scalar factors' matrix. It is advisable to inspect which combination of the observatories gives the best solution for the time series of interest. Furthermore, as derived in the cited previous studies, it will be more efficient if the points of interest are inside of the hull made up by the observatories, i.e., if the data at those points are interpolated in a spatial sense, rather than extrapolated (which was the case in this exercise).

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