New Relations of Water Saturation's Calculus from Well Logs

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Abstract

The saturation in water or hydrocarbon is an important petrophysical parameter used for the evaluation of oil and gas reservoirs. It represents the amount of hydrocarbons in a reservoir. There are many relations to determine the water saturation from well logs by using the physical properties of rocks. The accurate determination of the hydrocarbon formation or the water saturation is given by the accuracy of the parameters used for calculating and by the relations used.

This paper makes an analysis of the relations for water saturation calculation using the Walter H. Fertl classification. The four types of relations were modified using the MacLaurin series of the exponential function. It also analyses the cases when the water saturation is not to be found in the variation range $[0,1]$. The new relations give values that are always greater than 0 and we have established the conditions for the water saturation to be less than 1.

In the final part of the paper we make an application which confirms the theoretical aspects presented in the first part of the paper.

Key words: water saturation, well log, true resistivity, MacLaurin series.

1. INTRODUCTION

The water saturation is the fraction of the pore volume in the reservoir rock that contains water. It is generally assumed, unless otherwise known that the pore volume contains not only water but also gas/oil. Determining water and hydrocarbon saturation is one of the basic objectives of the interpretation of well logging.

To determine the water saturation in the rock reservoirs from well logs, we are using equations that have the following parameters: true resistivity, water resistivity, porosity, volume of shale, shale resistivity, saturation and cementation exponents (Eq. 1). The implicit equation for the calculus of water saturation is the following:

$$S_w = f(\Phi, R_t, R_w, R_{sh}, V_{sh}, a, C_j) \quad (1)$$

where S_w is the water saturation in the virgin zone, R_t the true resistivity, R_w the water resistivity, R_{sh} the shale resistivity, Φ the porosity, V_{sh} the volume of shale, C_j the coefficients which dependent on the equation used, and a the lithological coefficient.

Some of the explicit mathematic relations used to calculate the content of shale sedimentary down-tanks in water are presented by Fertl (1987). Fertl has classified these equations in four types as follows:

Type 1

$$\frac{1}{R_t} = \alpha S_w^n + \gamma \quad (2)$$

Type 2

$$\frac{1}{R_t} = \alpha S_w^n + \gamma S_w^s \quad (3)$$

Type 3

$$\frac{1}{R_t} = \alpha S_w^n + \beta S_w^r + \gamma \quad (4)$$

Type 4

$$\frac{1}{R_t} = \alpha S_w^n + \beta S_w^r + \gamma S_w^s \quad (5)$$

In Eqs. 2-5, α , β , γ are functions depending on $\Phi, R_w, R_{sh}, V_{sh}, C_j$, and n , r , s are saturation exponents with medium values between 1 and 2, (Hamada 2008).

The calculated values of S_w with these equations are not always within $[0,1]$. They are considered to be values outside of the variation field when $S_w < 0$ (more precisely S_w has very low values of the order of 10^{-3}), when $S_w > 1$, and when S_w is not a real number (these values do not have physical meaning).

The main purpose of this article is to establish new formulas for the calculus of water saturation, formulas that can also be applied in cases when the old formulas give negative values for water saturation.

We present an analysis of the four types of the calculus relations of water saturation and, at the same time, we modify them using the MacLaurin series of the exponential e^{-x} function (Nikolsky 1981). In this way we replace the expression $(1 - x)$ by the expression e^{-x} . It is easy to notice that $|e^{-x} - (1 - x)| \leq 1/e$, $(\forall)x \in [0,1]$, so the amount of truncation error of MacLaurin series is less than $1/e$.

At the same time we set out new conditions for the new relations to ensure the criterion: $S_w \leq 1$. In terms of resistivity these conditions have been obtained as follows: $R_t > R_{c,n}$, $n = 1, 2, 3, 4$ where the parameter $R_{c,n}$ is determined for each type of formula.

The structure of the paper is as follows: in Section 2 we analyze the four types of relations, we give an example for each type of relation, we modify them starting from the MacLaurin formula and finally we give the conditions for using these new formulas; in Section Three we make an application in which we compare the old and the new formulas; the conclusions of this co-operation are presented in the final section.

2. THE ANALYSIS OF THE WATER SATURATION FORMULAS

2.1 The Type 1 relations analysis

The Type 1 relations are the ones given by formula 2. According to them, water saturation is:

$$S_w^n = \frac{1}{\alpha \cdot R_t} (1 - \gamma \cdot R_t). \quad (6)$$

The fraction $1/(\alpha \cdot R_t)$ is positive but the expression $(1 - \gamma \cdot R_t)$ can take negative values, for $\gamma \cdot R_t > 1$. In this case the water saturation S_w takes complex values or real negative values (depending on the n exponent), values that are not compatible with the variation field of saturation in water. This restriction is eliminated, if we use the MacLaurin series for the exponential function e^{-x} .

The $(1 - \gamma \cdot R_t)$ expression, according to MacLaurin series can be approximated with $e^{-R_t \gamma}$, and the Type 1 relations may be written as follows:

$$S_w^n = \frac{1}{\alpha R_t} \cdot e^{-R_t \gamma} . \quad (7)$$

In Eq. 7, the $S_w > 0$ criterion is satisfied and the restriction for the $(1 - \gamma \cdot R_t)$ expression to be positive is eliminated.

In order to $S_w \leq 1$, the following criterion must be satisfied:

$$\frac{1}{\alpha \cdot R_t} \cdot e^{-\alpha \cdot R_t} \leq 1 . \quad (8)$$

This is written as:

$$e^{\gamma \cdot R_t} \geq \frac{1}{\alpha \cdot R_t} . \quad (9)$$

The inequality 9, using the development of MacLaurin series is:

$$1 + \gamma \cdot R_t \geq \frac{1}{\alpha \cdot R_t} \quad (10)$$

from which, one can deduce that:

$$R_t \geq \frac{2}{\alpha + \sqrt{\alpha^2 + 4 \cdot \alpha \cdot \gamma}} = R_{c1} . \quad (11)$$

It results that water saturation calculated using Type 1 relations will have values bigger than 0 if the relation 7 is used and lower than 1 if the criterion 11 is satisfied.

Example

Type 1 relations include: the relations that correspond to the laminar shale model (Schlumberger 1989), Pupon-Leveaux relations, Simandoux relation (Simandoux 1963), Hossin relation (Bassiouni 1994).

The solution to these equations is given by Eq. 6 and can be rewritten like 7. The relation that corresponds to the laminar shale (Schlumberger 1989, Iheanacho 2014) is:

$$\frac{1}{R_t} = \frac{\Phi^2 \cdot S_w^2}{a \cdot (1 - V_{sh}) \cdot R_w} + \frac{V_{sh}}{R_{sh}} \quad (12)$$

where

$$\alpha = \frac{\Phi^2}{a \cdot (1 - V_{sh}) R_w} \quad (13)$$

and

$$\gamma = \frac{V_{sh}}{R_{sh}}. \quad (14)$$

It results that the water saturation is given by:

$$S_w^2 = \frac{a \cdot (1 - V_{sh}) \cdot R_w}{\Phi^2 \cdot R_t} \cdot \left(1 - V_{sh} \frac{R_t}{R_{sh}} \right). \quad (15)$$

In relation 15, the expression:

$$\left(1 - V_{sh} \frac{R_t}{R_{sh}} \right) \quad (16)$$

can be approximated according to MacLaurin series with:

$$\left(1 - V_{sh} \frac{R_t}{R_{sh}} \right) \approx e^{-V_{sh} \frac{R_t}{R_{sh}}}. \quad (17)$$

With this approximation, relation 15 becomes:

$$S_w^2 = \frac{a \cdot (1 - V_{sh}) \cdot R_w}{\Phi^2 \cdot R_t} \cdot e^{-V_{sh} \frac{R_t}{R_{sh}}}. \quad (18)$$

The same method can be used for the other Type 1 equations (Hossin relation, Simandoux relation, *etc.*).

2.2 The Type 2 relations analysis

The Type 2 equations are written, in their general form, in Eq. 3. For $n = 2$ and $s = 1$, the Type 2 equations become:

$$\frac{1}{R_t} = \alpha \cdot S_w^2 + \gamma \cdot S_w \quad (19)$$

or

$$S_w^2 + \frac{\gamma}{\alpha} \cdot S_w - \frac{1}{\alpha \cdot R_t} = 0. \quad (20)$$

The equation's discriminant is:

$$\Delta = \left(\frac{\gamma}{\alpha} \right)^2 + \frac{4}{\alpha \cdot R_t} \quad (21)$$

and is always positive.

The solution of Eq. 20 is:

$$S_{w1,2} = -\frac{\gamma}{2 \cdot \alpha} \pm \sqrt{\left(\frac{\gamma}{2 \cdot \alpha} \right)^2 + \frac{1}{\alpha \cdot R_t}}. \quad (22)$$

For the minus sign (–), the water saturation values will always be negative, resulting in a non-physical solution. As a result the formula for water saturation calculus is:

$$S_w = -\frac{\gamma}{2 \cdot \alpha} + \sqrt{\left(\frac{\gamma}{2 \cdot \alpha} \right)^2 + \frac{1}{\alpha \cdot R_t}} \quad (23)$$

Equation 23 can be written as

$$S_w = \sqrt{\left(\frac{\gamma}{2 \cdot \alpha} \right)^2 + \frac{1}{\alpha \cdot R_t}} \cdot e^{-\sqrt{\frac{\gamma \cdot R_t}{\gamma^2 \cdot R_t + 4 \cdot \alpha}}}. \quad (24)$$

In Eq. 24 the two factors are positive and, thus, the value of water saturation is always positive $S_w > 0$.

From Eq. 23 it results that the criterion for the saturation value to be less than or equal to 1 is:

$$R_t \geq \frac{1}{\alpha + \gamma} = R_{c2}. \quad (25)$$

Relation 25 represents the criterion for the water saturation to take values smaller than 1.

Example

The modified Simandoux equation is:

$$\frac{1}{R_t} = \frac{S_w^2}{F \cdot R_w} + \frac{V_{sh} \cdot S_w}{R_{sh}} \quad (26)$$

and is written for $n = 2$ and $s = 1$, where

$$\alpha = \frac{1}{FR_w} \quad (27)$$

and

$$\gamma = \frac{V_{sh}}{R_{sh}}. \quad (28)$$

The solution of Eq. 26, according to Eq. 23, for $F = \frac{1}{\Phi^2}$ is:

$$S_w = -\frac{V_{sh} \cdot R_w}{2 \cdot R_{sh} \cdot \Phi^2} + \sqrt{\left(\frac{V_{sh} \cdot R_w}{2 \cdot R_{sh} \cdot \Phi^2}\right)^2 + \frac{R_w}{R_t \cdot \Phi^2}}. \quad (29)$$

Using the MacLaurin series, it can be written as:

$$S_w = \sqrt{\left(\frac{V_{sh} \cdot R_w}{2 \cdot \Phi^2 \cdot R_{sh}}\right)^2 + \frac{R_w}{\Phi^2 R_t}} \cdot e^{-\frac{R_t}{R_t + 4 \cdot \frac{\Phi^2 \cdot R_{sh}^2}{R_w \cdot V_{sh}^2}}}. \quad (30)$$

Similarly, other Type 2 equations (L. de Witte 1950, A.J. de Witte 1957, Patchett and Rausch 1967, Waxman and Smith 1968, Simandoux 1963, Schlumberger 1989) can be written like this. Criterion 25 can be used so that the saturation values will be smaller than 1.

2.3 The Type 3 relations analysis

The Type 3 equations are written, in their general form, like Eq. 4. For $n = 2$ and $r = 1$, Eq. 4 can be rewritten as:

$$\frac{1}{R_t} = \alpha S_w^2 + \beta S_w + \gamma \quad (31a)$$

or

$$S_w^2 + \frac{\beta}{\alpha} S_w + \frac{1}{\alpha} \left(\gamma - \frac{1}{R_t} \right) = 0. \quad (31b)$$

Equation 31b has real solutions if and only if $\Delta \geq 0$, where:

$$\Delta = \left(\frac{\beta}{\alpha} \right)^2 - \frac{4}{\alpha} \cdot \left(\gamma - \frac{1}{R_t} \right). \quad (32)$$

The solutions of Eq. 31b are:

$$S_{w_{1,2}} = -\frac{\beta}{2 \cdot \alpha} \pm \sqrt{\left(\frac{\beta}{2 \cdot \alpha} \right)^2 - \frac{1}{\alpha} \cdot \left(\gamma - \frac{1}{R_t} \right)}. \quad (33)$$

If $\gamma - 1/R_t < 0$, from the solutions given by Eq. 33, only the positive solution has a physical sense. It results that S_w can be calculated using only the relations:

$$S_w = -\frac{\beta}{2 \cdot \alpha} + \sqrt{\left(\frac{\beta}{2 \cdot \alpha}\right)^2 - \frac{1}{\alpha} \cdot \left(\gamma - \frac{1}{R_t}\right)}. \quad (34)$$

If $(\gamma - 1/R_t) > 0$ and $\Delta > 0$, then both solutions of Eq. 31b are negative and they are not physically valid. It results that the Type 3 equation given by 31b can be used in this form only if the following conditions are valid:

$$\Delta = \left(\frac{\beta}{\alpha}\right)^2 - \frac{4}{\alpha} \cdot \left(\gamma - \frac{1}{R_t}\right) > 0 \quad (35)$$

and

$$\left(\gamma - \frac{1}{R_t}\right) < 0. \quad (36)$$

In order to eliminate the restrictive criteria of the Type 3 equations, the solution 34 can be written as:

$$S_w = -\frac{\beta}{2 \cdot \alpha} + \sqrt{\left(\frac{\beta}{2 \cdot \alpha}\right)^2 + \frac{1}{\alpha \cdot R_t} \cdot (1 - \gamma \cdot R_t)}. \quad (37)$$

Using the approximation $1 - \gamma \cdot R_t \approx e^{-\gamma \cdot R_t}$, one can obtain:

$$S_w = -\frac{\beta}{2 \cdot \alpha} + \sqrt{\left(\frac{\beta}{2 \cdot \alpha}\right)^2 + \frac{1}{\alpha \cdot R_t} \cdot e^{-\gamma \cdot R_t}}. \quad (38)$$

Rearranging the terms in Eq. 38, we get:

$$S_w = \sqrt{\left(\frac{\beta}{2 \cdot \alpha}\right)^2 + \frac{1}{\alpha \cdot R_t} \cdot e^{-\gamma \cdot R_t}} - \frac{\beta}{2 \cdot \alpha} \quad (39)$$

and, using the MacLaurin series development, Eq. 39 becomes:

$$S_w = \sqrt{\left(\frac{\beta}{2 \cdot \alpha}\right)^2 + \frac{1}{\alpha \cdot R_t} \cdot e^{-\gamma \cdot R_t}} \cdot e^{-\frac{1}{1+4 \cdot \frac{\alpha \cdot e^{-\gamma \cdot R_t}}{\beta^2 \cdot R_t}}}. \quad (40)$$

The criterion that $S_w \leq 1$ results from relation 38:

$$\sqrt{\left(\frac{\beta}{2 \cdot \alpha}\right)^2 + \frac{1}{\alpha \cdot R_t} \cdot e^{-\gamma \cdot R_t}} \leq 1 + \frac{\beta}{2 \cdot \alpha} \quad (41)$$

from which one can deduce that:

$$e^{\gamma \cdot R_t} \geq \frac{1}{R_t (\alpha + \beta)}. \quad (42)$$

Taking into account that $e^{\gamma \cdot R_t} \geq 1 + \gamma \cdot R_t$, the inequality 42 is satisfied if:

$$1 + \gamma \cdot R_t \geq \frac{1}{R_t (\alpha + \beta)}. \quad (43)$$

From Eq. 43 it results that:

$$R_t \geq \frac{2}{\alpha + \beta + \sqrt{(\alpha + \beta)^2 + 4 \cdot \gamma \cdot (\alpha + \beta)}} = R_{c3}. \quad (44)$$

The Type 3 equations can be used under the form given by relation 40. The criterion $S_w \leq 1$ is given by relation 44.

Example

One of the Type 3 relations is Alger's relation (Alger 1966):

$$\frac{1}{R_t} = \frac{(1-q)^2}{F \cdot R_w} \cdot S_w^2 + \frac{q \cdot (1-q) \cdot (R_w + R_{sh})}{F \cdot R_w \cdot R_{sh}} \cdot S_w + \frac{q^2}{FR_{sh}} \quad (45)$$

where

$$\alpha = \frac{(1-q)^2}{F \cdot R_w} \quad (46)$$

$$\beta = \frac{q \cdot (1-q) \cdot (R_w + R_{sh})}{F \cdot R_w \cdot R_{sh}} \quad (47)$$

and

$$\gamma = \frac{q^2}{F \cdot R_{sh}}. \quad (48)$$

The solution of Eq. 45, according to Eq. 34, is:

$$S_w = \frac{q}{2 \cdot (1-q)} \cdot \left(\sqrt{A} - \frac{R_w + R_{sh}}{R_{sh}} \right) \quad (49)$$

where A is:

$$A = \frac{(R_w + R_{sh})^2}{R_{sh}^2} + \frac{4 \cdot F \cdot R_w}{q^2 \cdot R_t} \cdot \left(1 - \frac{q^2 \cdot R_t}{F \cdot R_{sh}}\right). \quad (50)$$

One can make a first approximation $\left(1 - \frac{q^2 \cdot R_t}{F \cdot R_{sh}}\right) \approx e^{-\frac{q^2 \cdot R_t}{F \cdot R_{sh}}}$ and formula 50 becomes:

$$A = \frac{(R_w + R_{sh})^2}{R_{sh}^2} + \frac{4 \cdot F \cdot R_w}{q^2 \cdot R_t} \cdot e^{-\frac{q^2 \cdot R_t}{F \cdot R_{sh}}}. \quad (51)$$

Formula 49 can be written as:

$$S_w = \frac{q \cdot \sqrt{A}}{2 \cdot (1-q)} \cdot \left(1 - \frac{R_w + R_{sh}}{R_{sh} \cdot \sqrt{A}}\right). \quad (52)$$

It results that the equation for water saturation calculus, using MacLaurin series development, is:

$$S_w = \frac{q \cdot \sqrt{A}}{2 \cdot (1-q)} \cdot e^{-\frac{R_w + R_{sh}}{R_{sh} \cdot \sqrt{A}}}. \quad (53)$$

2.4 The Type 4 relations analysis

The Type 4 relations can be divided into two categories:

- Type 4a, where the saturation exponents are equal, $n = r = s$ (the equations Poupon and Levaux (1971), “Indonesia Formula”, Woodhouse, *etc.*, correspond to the case $n = r = s = 2$)
- Type 4b, where the saturation exponents are different $n \neq r \neq s$ (“Dual-porosity model”) (Fertl 1987). For an equation of this type, one cannot give a general solution; each case will be analysed depending on the values of n , r , and s .

In the first case we have the relation:

$$\frac{1}{R_t} = \alpha S_w^n + \beta S_w^n + \gamma S_w^n \quad (54)$$

or

$$S_w^n = \frac{1}{R_t (\alpha + \beta + \gamma)}. \quad (55)$$

Example

Poupon and Leveaux relation (simplified “Indonesia Formula”) for $V_{sh} \leq 0.5$ is:

$$\frac{1}{R_t} = \frac{S_w^2}{F \cdot R_w} + 2 \cdot \sqrt{\frac{V_{sh}}{F \cdot R_w \cdot R_{sh}}} \cdot S_w^2 + \frac{V_{sh}^2}{R_{sh}} \cdot S_w^2 \quad (56)$$

where α , β , and γ are as follows

$$\alpha = \frac{1}{F \cdot R_w}, \quad (57)$$

$$\beta = 2 \cdot \sqrt{\frac{V_{sh}}{F \cdot R_w \cdot R_{sh}}} \quad (58)$$

and

$$\gamma = \frac{V_{sh}^2}{R_{sh}} \quad (59)$$

The solution of Eq. 56 is:

$$S_w = \left[\frac{F \cdot R_w \cdot R_{sh}}{R_t \cdot \left(R_{sh} + 2 \cdot \sqrt{F \cdot R_w \cdot R_{sh} \cdot V_{sh}} + V_{sh}^2 \right)} \right]^{\frac{1}{2}}. \quad (60)$$

In relation 60, all terms are positive and, consequently, there are no situations where the relation cannot be used. This relation covers the entire range of variation of the measures. The criterion for $S_w \leq 1$ is given by the relation:

$$R_t \geq \frac{F \cdot R_w \cdot R_{sh}}{\left(R_{sh} + 2 \cdot \sqrt{F \cdot R_w \cdot R_{sh} \cdot V_{sh}} + V_{sh}^2 \right)} = R_{c4}. \quad (61)$$

3. APPLICATION

The application was made on the oil field Barbuncesti. This oil field is part of the diapirs faults (Fig. 1; Beca and Prodan 1981). From the stratigraphical point of view there are Miocene and Pliocene deposits with a cumulative thickness of about 3500 m. They developed in detritic facies forming an alternance of pelitic and arenitic rocks as sandstones, sands, and shales.

Fig. 1. The distribution of the oil and gas fields in the Diapers Faults Area (Beca and Prodan 1981).

The oil field Barbuncesti is an asymmetric anticline oriented NE-SW affected by longitudinal and transversal faults which divides it into different tectonic sealed blocks forming distinct hydrodynamic units (Fig. 2). Its size is about 6 km in length and 2 km wide. The traps are of structural (fault seal) type.

The main petroleum accumulations are hosted by the upper Miocene reservoirs. These deposits (about 600-800 m thick) are separated in 8 productive complexes (Fig. 3).

Two wells, A and B, were selected. The following logs were recorded in these wells: caliper (CAL), natural gamma ray intensity (GR), spontaneous potential (SP), dual-laterolog with deep and shallow sensing modes (DLL-RD, RS) and microlog (RMLL), density log (DEN), neutron log (CN), and sonic log (DT). These logs are presented in Figs. 4 and 5.

We have selected an interval between 2715 and 2745 m of depth for well A and an interval between 2553 and 2583 m of depth for well B. For each of the two wells, the following values were used: saturation exponents $m = n = 2$, lithological coefficient $\alpha = 1$ and $q = 0.5$. The mean value for shale in the log is $R_{sh} = 2.5 \Omega m$. Formation water resistivity was taken from measurements of water samples. Its values are of $0.024 \Omega m$ for well A and $0.027 \Omega m$ for well B.

Fig. 2. Structural map, oil field Barbuncesti (Beca and Prodan 1981).

For these intervals we calculated the water saturation values using the initial relation and the modified one (according to MacLaurin series):

□ for Type 1 – relations 12 (the initial relation) and 18 (the modified one);

□ for Type 2 – relations 29 (the initial one) and 30 (the modified one);

□ for Type 3 – relations 52 (the initial one) and 53 (the modified one).

For Type 4, only relation 60 was used because, for Case 4a, modification of the relation with MacLaurin series is not necessary.

All the values of the water saturation calculated with the relations mentioned above were compared with the values obtained by Archie's relation.

The calculations were made using Interactive Petrophysics software (Interactive Petrophysics, v. 3.6; LR Senergy Ltd. 2015, licence).

Fig. 3. Geological section through oil field Barbuncesti (Beca and Prodan 1981). The symbols P1(upper), P2(middle), P3(lower), are subdivisions of the reservoir rock formations of Pontic age, The symbols 1s, 1i, 2s, 2i, etc., are the subdivisions of the reservoir rock formations of Meotian age; "s" is for upper and "i" for lower.

Fig. 4. Logs from well A – caliper (CAL), natural gamma ray intensity (GR), dual – laterolog with deep and shallow sensing modes (DLL-RD, RS) and microlog (RMLL), density log (DEN), neutron log (CN), and sonic log (DT).

The results are presented in Fig. 6 for well A and Fig. 7 for well B.

Analysing the results one can conclude as follows:

- Well A was drilled with saline mud; therefore, the RMLL resistivity curve shows values lower than RD and RS curves. Well B was drilled with fresh mud; therefore, the resistivities RMLL, RD, and RS have approximately the same values. The true resistivity calculated with “Interactive Petrophysics” shows similar values (see Fig. 8 for well A and Fig. 9 for well B).
- For the Type 2, 3 and 4 relations:
 - a) the water saturation values obtained using the initial relations, the modified relations and the Archie relation are almost similar.

Fig. 5. Logs from well B – caliper (CAL), natural gamma ray intensity (GR), dual – laterolog with deep and shallow sensing modes (DLL-RD, RS) and microlog (RMLL), density log (DEN), neutron log (CNC), sonic log (DT).

Fig. 6. Water saturations calculated, for well A, using the relations: Type 1 – relations 12, 18, and Archie's; Type 2 – relations 29, 30, and Archie's; Type 3 – relations 49, 53, and Archie's; Type 4 – relations 60 and Archie's.

Fig. 7. Water saturations calculated, for well B, using the relations: Type 1 – relations 12, 18, and Archie's; Type 2 – relations 29, 30, and Archie's; Type 3 – relations 49, 53, and Archie's; Type 4 – relations 60 and Archie's.

Fig. 8. The log of the conditions $R_t > R_{c,n}$, $n = 1, 2, 3, 4$, obtained using the criterion $S_w \leq 1$, for well A.

Fig. 9. The log of the conditions $R_t > R_{c,n}$, $n = 1, 2, 3, 4$, obtained using the criterion $S_w \leq 1$, for well B.

□ For the Type 1 relations:

- b) the values obtained using the initial relations and the modified one are sensibly smaller than the ones obtained using the Archie relation;
- c) for relatively larger intervals of depth the initial relation gives negative values of the water saturation, while the modified relation gives positive values for the whole interval.

□ It is important to stress out the fact that, as regards well B, the Type 3 initial relation too gives negative values of the water saturation.

□ All water saturation values are smaller than 1. In Fig. 8 (well A) and Fig. 9 (well B) we present the conditions $R_t > R_{c,1}$ (C11), $R_t > R_{c,2}$ (C25), $R_t > R_{c,3}$ (C44), and $R_t > R_{c,4}$ (C61) obtained using the criterion $S_w \leq 1$. We can see that these conditions are fulfilled. One can observe that this criterion is satisfied for the two wells being analysed. Therefore, the validity of these conditions is confirmed.

4. CONCLUSIONS

In this paper we proposed new relations for water saturation calculus. They were obtained by using MacLaurin series of exponential function.

The results obtained from well data enable us to say that the relations enlarge the applicability field of the initial relations without sensibly changing the calculated values for the saturation in water.

To support the above statements we could stress out the following result: in wells A and B the values of saturation in water could not be calculated with the initial relation of Type 1, the relation 12 (corresponds to the laminar shale model), since for certain intervals of depth the values are negative.

For the entire interval, using the modified relation 18 all the values of the saturation in water could be calculated (see Figs. 6 and 7) and the values are to be found in the range of variation.

The values obtained for the water saturation using the initial relations, the modified relations and the Archie relation, are similar which allow us to affirm that the new relations do not bring errors. The saturation values calculated using the initial relation and the modified one according to the MacLaurin series are similar. The correlation coefficients r (Mohan and Godofredo 2002) calculated on the basis of the saturation values, using relations 13 and 18 for Type 1 relations, 29 and 30 for Type 2 relations, and 49 and 51 for Type 3 relations, are given in Table 1.

Table 1

The correlation coefficients between water saturation estimates

No.	Relations	Well A r	Well B r
1	13-18	0.998	0.950
2	29-30	0.972	0.999
3	49-51	0.999	0.998

The correlation coefficients vary between 0.950 and 0.999. This shows that the water saturation values calculated using the formulas put forward in this article concord with the water saturation values obtained using the initial formulas, which shows that using the equations written with MacLaurin series does not modify the results of calculation.

For the two wells, the conditions $R_t > R_{c,n}$, $n = 1, 2, 3, 4$, were checked so that the saturation values in water should be less than 1. The results show that all these conditions have been satisfied (see Fig. 8 for well A and Fig. 9 for well B).

Using modified relations concomitantly with the established conditions leads to positive values of the saturation in water and lower than 1.

In conclusion we believe that the new relations obtained here can be used to calculate the saturation in water from well logs, with a satisfying level of reliability.

Nomenclature

- S_w – water saturation of the virgin zone;
 R_t – true resistivity;
 R_w – water resistivity;
 R_{cl} – shale resistivity;
 Φ – porosity;
 C_j – coefficients;
 α, β, γ – coefficients;
 n, r, s – saturation exponents;
 q – dispersed shale volume fraction of intermatrix porosity.

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