Spherical Elementary Current Systems Method Applied to Geomagnetic Field Modeling for the Adriatic

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Abstract

The aim of this work was to derive an accurate regional model of geomagnetic components on the Adriatic. Data of north, east and vertical geomagnetic components at repeat stations and ground survey sites enclosing the Adriatic Sea were used to obtain a geomagnetic model at 2010.5 epoch. The core field was estimated by use of the global Enhanced Magnetic Model, while the crustal field by a mathematical technique for expanding vector systems on a sphere into basis functions, known as spherical elementary current systems method. The results of this method were presented and compared to the crustal field estimations by the Enhanced Magnetic Model. The maps of isolines of the regional model are also presented.

Key words: geomagnetic field model, crustal field, spherical elementary current systems method.

1. INTRODUCTION

Several methods for modeling of a spatial distribution of geomagnetic field components over a limited region at the Earth's surface have been developed by now (Haines 1985, De Santis *et al.* 1990, Thébault *et al.* 2006). Haines

(1985) introduced Spherical Cap Harmonic Analysis (SCHA), where the solution of Laplace's equation over the spherical cap includes associated Legendre functions of non-integral degree and integral order. Further modifications of SCHA are given by Adjusted Spherical Cap Harmonic analysis (De Santis 1992) and by Translated Origin of Spherical Cap Harmonic Analysis (De Santis 1991). The Revised Spherical Cap Harmonic Analysis was introduced in order to combine ground, aeromagnetic, and satellite data, and the advantage of this method is improved altitude dependence (Thébault *et al.* 2004, 2006).

The primary objective of this study is testing new method for modelling the crustal field, as well as making a new crustal model over the Adriatic. In this work the regional model is a combination of the Enhanced Magnetic Model (EMM) for the core field, and a model of the crustal field which was derived by the spherical elementary current systems (SECS) method, introduced by Amm (1997). EMM is the high-degree global model up to degree and order of 720 based on spherical harmonic analysis (Maus 2010), and is derived from CHAMP Satellite Data, ØRSTED Satellite Data, INTERMAGNET Observatory Data and GEODAS magnetic data. The core field is described by degrees of 1-15. On the other hand, the SECS method is not based on spectral decomposition of a scalar magnetic potential (Amm and Viljanen 1999). It is based on two sets of basis functions (curl-free and divergence-free) in spherical coordinates, which can be used, *e.g.*, to expand any continuously differentiable vector field on a sphere (Amm 1997, Amm and Viljanen 1999, Juusola *et al.* 2006). The SECS method can expand the measured ground geomagnetic field into a sum of the magnetic field contributions of spherical elementary current systems, placed above the measurement sites and at some depth inside the Earth in geocentric frame (Pulkkinen *et al.* 2003).

2. METHOD AND DATA

2.1 The SECS method

Two types of spherical elementary sheet currents have been defined by Amm (1997): curl-free and divergence-free. In a case of the ground disturbances continuation, it is necessary to consider only the latter (Amm and Viljanen 1999). The centers of these elementary current systems (their poles) can be placed in the ionosphere and below the Earth's surface at fixed radii in geocentric reference frame (Pulkkinen *et al.* 2003). Within those two planes the poles can be placed freely, such that their locations are most suitable with respect to the type of magnetic variations to be analysed or to the density of measurement sites (Amm and Viljanen 1999).

Although the ground disturbances caused by ionospheric and/or magnetospheric currents were not analysed in this work, the same technique was applied for modeling of the crustal field at the Earth's surface. In this approach the spatial variations of the crustal field were modeled by using the two effective current layers placed in the Earth's interior and at height above the Earth's surface, respectively. For matching all three components of the measured field, field separation into internal and external parts have to be carried out by placing two equivalent current layers, one inside the ground and second above the ground (Vanhamäki *et al.* 2003). In this work a spatial distribution of the crustal field is modeled, not the real sources inside the Earth or the external contributions (and their induced counterparts). Thus, it does not matter where the sources are placed, only their total magnetic effects are modelled by those two current layers.

The definition of one of such divergence-free elementary sheet currents systems (Amm 1997, Amm and Viljanen 1999), in a spherical coordinate system (r', θ', φ') in which the pole of the elementary system is at $\theta' = 0$, is given by: $\mathbf{J}_c(\theta') = (I_c/4\pi R_c) \cdot \cot(\theta'/2) \cdot \hat{\varphi}'$, where $c = e, i$ (e stands for the external layer and i for the internal layer), I_c is a scaling factor of the elementary system, radii R_e and R_i are defined in geocentric frame as $R_e = R_E + H_e$ and $R_i = R_E - H_i$, and the mean Earth's radius is $R_E = 6371.2$ km.

Thus, there are two infinitely thin horizontal current layers, the first above at height H_e and the second inside the Earth at depth H_i , since any divergence-free current system can be composed by superposition of elementary current systems (Amm 1997, Pulkkinen *et al.* 2003). The magnetic effect of these two layers at some point given by radius vector $\mathbf{r}$ in geocentric frame on the Earth's surface is:

$$\mathbf{B}(\mathbf{r}) = \sum_{k=1}^K I_i^k \mathbf{G}_i(\mathbf{r}, R_i, \theta_k, \varphi_k) + \sum_{l=1}^L I_e^l \mathbf{G}_e(\mathbf{r}, R_e, \theta_l, \varphi_l), \quad (1)$$

where $\mathbf{G}_{i,e}$ are geometric terms related to the internal and the external part ($R_i < r < R_e$) of magnetic field produced by each elementary current system located at $(R_i, \theta_k, \varphi_k)$ and $(R_e, \theta_l, \varphi_l)$, while K and L are the numbers of poles related to the internal and the external layers, respectively. The expressions for $\mathbf{G}_{i,e}$ are given in Amm and Viljanen (1999) in detail. The above linear system of equations can be written in a matrix form as (Amm and Viljanen 1999, McLay and Beggan 2010):

$$\mathbf{B} = \mathbf{G} \mathbf{I},$$

where

$$\begin{pmatrix} \vdots \\ B_\theta^s \\ B_\phi^s \\ B_r^s \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ G_{i,\theta}^{s,1} & G_{i,\theta}^{s,K} & G_{e,\theta}^{s,1} & G_{e,\theta}^{s,L} \\ G_{i,\phi}^{s,1} & \dots & G_{i,\phi}^{s,K} & G_{e,\phi}^{s,1} & \dots & G_{e,\phi}^{s,L} \\ G_{i,r}^{s,1} & G_{i,r}^{s,K} & G_{e,r}^{s,1} & G_{e,r}^{s,L} \\ \vdots \end{pmatrix} \cdot \begin{pmatrix} I_i^1 \\ \vdots \\ I_i^K \\ I_e^1 \\ \vdots \\ I_e^L \end{pmatrix} \quad (2)$$

with s referring to s -th measurement site. The matrix $\mathbf{B}$ (of the order $3N$ by 1), where N is a number of measurement sites, comprises the crustal field (B_θ, B_ϕ, B_r) estimated at the sites $(1, \dots, N)$ on the Earth's surface (in geocentric frame). The matrix $\mathbf{G}$ is of the order $3N$ by $(K + L)$, where, for example, the term $G_{i,\theta}^{s,l}$ denotes θ -component (in polar direction of geocentric frame) of the magnetic effect of internal origin, of an elementary current system with a scaling factor of 1A and its pole denoted by 1, at the measurement site denoted by s .

Matrix $\mathbf{I}$ can be calculated after deriving the matrices $\mathbf{B}$ and $\mathbf{G}$. The system of equations is highly underdetermined (Amm and Viljanen 1999, Pulkkinen *et al.* 2003, McLay and Beggan 2010, Weygand *et al.* 2011), since the number of unknowns $(K + L)$ is in general much greater than the number of measurements at the Earth's surface $(3N)$. The inversion of matrix $\mathbf{G}$ can be performed by its singular value decomposition (Press *et al.* 2001), and after that the matrix $\mathbf{I}$ can be calculated. The procedure is that the stabilization in inversion of matrix $\mathbf{G}$ is done by choosing the threshold ε for singular values related to different basis vectors of the decomposition, and if larger ε is chosen, the smoother solutions for $\mathbf{I}$ will be in general (Amm and Viljanen 1999, Pulkkinen *et al.* 2003, Vanhamäki *et al.* 2003).

It is also possible to use only one (internal) layer for modeling the spatial variations of the crustal field, which is placed inside the Earth at depth $H = H_i$. In this case, the matrix $\mathbf{G}$ is of the order $3N$ by K , and the matrix $\mathbf{I}$ is of the order K by 1. The internal SECS produces a different set of scaling factors compared to a case of using the two current layers (McLay and Beggan 2010).

2.2 Data

For obtaining the geomagnetic field over the Adriatic Sea region, the values of north (X), east (Y), and vertical (Z) geomagnetic components from 6 Croatian repeat stations, 54 Croatian ground survey stations, 32 Italian repeat stations, and 10 Albanian repeat stations were used (Fig. 1). The distances between the first neighbours are in the range of 5-67 km, with the average of 31 km. The data from Italian and Albanian repeat stations were attained from

Fig. 1: Left: the positions of measurement sites (squares) used in deriving the regional geomagnetic model. The sites used for finding the optimal values of parameters of the SECS method are denoted with crosses (see Section 3); Right: the SECS poles grid (spacing 0.1°) over the Adriatic Sea region used in this work.

Dominici *et al.* (2012), and these data were reduced to 2010.0 epoch. The measurements of the Basic Geomagnetic Network of Republic of Croatia were performed in 2008 and 2009 (Brkić *et al.* 2013), and the data were reduced to 2008.5 and 2009.5 epochs, respectively, by the method described in Brkić *et al.* (2012).

3. RESULTS AND DISCUSSION

To find the optimal values of parameters of the SECS method, implemented for the crustal field modelling, the method was tested by using a known model of crustal magnetic field (degrees higher than 15 of the EMM model), as follows. The crustal field model values of X , Y , and Z components were calculated at the 102 survey sites (Fig. 1), on which a random noise up to 5.5 nT (the estimated uncertainties due to instrumental and external variations elimination errors) was added (now representing virtual measurements). These values were used as the input into the SECS method.

The SECS method spacing and threshold parameters were tested in the ranges $\delta = 0.05\text{--}0.3^\circ$ (due to the distances between survey sites) and $\varepsilon = 0.01\text{--}0.2$. In this work, due to simplicity, the radii of current layers in geocentric frame were taken as $R_i = R_E - H$ and $R_e = R_E + H$, with heights in the range $H = 5\text{--}30$ km (due to positions of survey sites in geocentric frame). The parameter of the SECS method misfit was the root-mean-square (rms) of residuals (rsd), where $\text{rsd} = \text{“real values”} - \text{“estimations”}$. The residuals of the SECS method were calculated at 151 sites at the sea level over the Adriatic (marked as crosses in Fig. 1, not overlapped with the survey sites). The “real values” at those 151 sites were the EMM crustal field values, while

“estimations” were the crustal field values obtained by the SECS method. It was found that the smallest root-mean-square of residuals were obtained with choice $\delta = 0.1^\circ$, $\varepsilon = 0.036$, and $H = 5$ km, and their values were 15.5 nT in X , 15.0 nT in Y , and 3.9 nT in Z . The regular SECS pole grid with spacing $\delta = 0.1^\circ$ is shown in Fig. 1.

After the process of finding the optimal values of parameters of the SECS method, the aim was to produce the regional crustal field model over the Adriatic. The input values into the SECS method were the annual mean values of X , Y , and Z components at 102 survey sites, subtracted by EMM up to degree of 15, for 2008.5, 2009.5, and 2010.0 epochs, respectively, depending on reduction epoch. Those values were finally corrected for the long-term external variations. This means that the crustal field estimations at survey sites were the input data into the SECS method.

The long-term external variations and their induced effects do not average out completely in observatory annual mean values, and the same is reasonable for the repeat station and ground survey annual mean values (Korte and Thébault 2007). An empirical method for estimation of the long-term external variations can be used, as suggested in detail by (Korte and Thébault 2007). If one can assume that the external influence is homogeneous over some region, it can be estimated by using the data of observatories inside or close to that region. In this study the data from observatories Grocka (20.8 °E, 44.6 °N) and Tihany (17.9 °E, 46.9 °N) were used. The same procedure was used for both observatories, by using observatory data and the corresponding EMM (up to degree 720) estimations. The averaged values of external influences for X , Y , and Z were taken as corrections for all the sites in this study. Their absolute values are smaller than 4 nT in X , 1.5 nT in Y , and 3.5 nT in Z .

The results for rms(rsd) of the SECS method calculated at survey sites are presented in Fig. 2, for all three geomagnetic components. The errors of model increase with the height H above (below) the mean Earth's radius, and they are smallest for Z component up to 14 km, and for X component from 14 km up to 30 km. The corresponding values obtained by EMM crustal field estimations (degrees 16-720) at survey sites were 68.7, 57.4, and 87.2 nT in X , Y , and Z components, respectively. For H up to 10 km, rms(rsd) are smaller than 23 nT in X , 29 nT in Y , and 17 nT in Z . The rms values of residuals at survey sites in the case when just one (internal) layer was used in the SECS method, with $\delta = 0.1^\circ$ and $\varepsilon = 0.036$, are also shown in Fig. 2. For the values of $H \leq 10$ km the results are better for X component in a case of internal layer, for Z in a case of two layers, while for Y the results are about the same for both cases.

Figures 3a-c display the isolines of X , Y , and Z components of the crustal field at the sea level over the Adriatic Sea, obtained with the SECS method

Fig. 2. The root-mean-square values of residuals of X , Y , and Z components of the crustal field, respectively, derived at survey sites with the SECS method ($\varepsilon = 0.036$, $\delta = 0.1^\circ$), as a function of the current layers height (H) above and below the sea level in geocentric frame. The rms values of residuals in the case of just one (internal) layer are also shown, when that layer was placed at depth H .

Fig. 3a. The regional model of the crustal field at the sea level of X component over the Adriatic Sea. The scale bar is in nT. It was obtained by the SECS method by using the two current layers (with $H = 5$ km, $\varepsilon = 0.036$, $\delta = 0.1^\circ$).

Fig. 3b. The same as Fig. 3a, but for *Y* component.

Fig. 3c. The same as Fig. 3a, but for *Z* component.

($H = 5$ km, $\varepsilon = 0.036$, $\delta = 0.1^\circ$). The maximal calculated values of X , Y , and Z are 158, 124, and 502 nT, respectively, while the minimal values are -300 , -146 , and -180 nT, respectively. Finally, the regional geomagnetic field model for 2010.5 epoch was equal to a sum of the core field for 2010.5 (obtained with EMM) and of the crustal field derived by the SECS method.

The regional geomagnetic model of X , Y , and Z components for 2010.5 epoch is presented in Figs. 4a-c, respectively.

Both the crustal and geomagnetic field model maps present an unprecedented view of the crustal and the geomagnetic field features on the Adriatic. However the densification of the data is needed to depict more reliable Adriatic crustal and geomagnetic field.

Fig. 4a. The regional model at the sea level of X component over the Adriatic Sea for 2010.5 epoch. The isolines are in nT. The main field was obtained from the EMM, and the crustal field with the SECS method by using the two current layers (with $H = 5$ km, $\varepsilon = 0.036$, $\delta = 0.1^\circ$).

Fig. 4b. The same as Fig. 4a, but for Y component.

Fig. 4c. The same as Fig. 4a, but for Z component.

4. CONCLUSION

An alternative approach of modeling the geomagnetic field was envisaged by use of spherical elementary current systems method and investigated on the Adriatic. The presented procedure, in which the main field is removed from the measurements, allows modelling of residuals, or in other words, the crustal field. For a favorable set of parameters (spacing, threshold, and the current layers height), the SECS method gives relatively small modelling errors. The root-mean-square of residuals of the crustal field components were found to be 4-22 times smaller (with $H = 5$ km, $\varepsilon = 0.036$, $\delta = 0.1^\circ$) in respect to the Enhanced Magnetic Model crustal field estimations. Finally, the geomagnetic field was restored by adding back the main field to obtained residuals. The presented crustal and geomagnetic field maps depict features with unprecedented informativeness, but limited by availability and distribution of the input data; thus the maps are expected to be more reliable in the measured areas, close to the Adriatic Sea coast. A new and more accurate crustal and geomagnetic field maps are generally needed for scientific research, *e.g.*, to build tectonic models of oceanic and continental crust, to estimate the crustal thermal states, to contribute to geodynamic models, *etc.* (Thébault *et al.* 2010), while in practical purposes for the natural resources assessment and the navigation, and for some of these reasons could also be used in our future work.

The input data came from the Adriatic coast and islands, while the open sea was without measurements (for both simulated and real cases), and this is a clear drawback of the final model. It would be contrasted by using data in the sea. In a case of “perfect data” there would not be any problem because of the validity of Laplace equation, but here we are quite far from having “perfect data”. Furthermore, the satellite data from recent missions should also be used in the future work, in order to fill the gap across the sea, and to understand the real potentiality of the SECS method in case of data at different altitudes as well.

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