Local Recovery of Sub-Crustal Stress Due to Mantle Convection from Satellite-to-Satellite Tracking Data

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A b s t r a c t

Two integral transformations between the stress function, differentiation of which gives the meridian and prime vertical components of the sub-crustal stress due to mantle convection, and the satellite-to-satellite tracking (SST) data are presented in this article. In the first one, the SST data are the disturbing potential differences between twin-satellites and in the second one the line-of-sight (LOS) gravity disturbances. It is shown that the corresponding integral kernels are well-behaving and therefore suitable for inversion and recovery of the stress function from the SST data. Recovery of the stress function and the stress components is also tested in numerical experiments using simulated SST data. Numerical studies over the Himalayas show that inverting the disturbing potential differences leads to a smoother stress function than from inverting LOS gravity disturbances. Application of the presented integral formulae allows for recovery of the stress from the satellite mission GRACE and its planned successor.

Key words: integral equation, upward/downward continuation, integral kernel, truncation error, GRACE satellite mission, stress function.

1. INTRODUCTION

The sub-crustal stress due to mantle convection is one of the important geophysical phenomena. It allows for interpretation of seismicity, volcanicity, kimberlite magmatism, ore formation, tectonic and magnetic features (Liu 1977). Runcorn (1964, 1967) was one of the pioneers who proposed modelling the sub-crustal shear stress components in terms of spherical harmonic coefficients of gravity field. His formula was successfully applied, *e.g.*, by Liu *et al.* (1976), Liu (1977, 1978, 1980a, b), and Huang and Fu (1982). Others have also worked on the relation between the stress and other types of gravity data. McKenzie (1967) mentioned that the long wavelength harmonics of the external gravity field cannot be supported by the strength of the lithosphere. Review of stress estimates based on gravity observations and isostatic compensation based on surface deformation was provided by McNutt (1980). Souriau and Souriau (1983) studied correlations between surface tectonics and geoid undulations in terms of harmonic expansion. In a similar way, Ricard *et al.* (1984) investigated the connection between the lithospheric stress and geoidal undulation in terms of spherical harmonics. Mathematical model in terms of a harmonic expansion for convection field of the mantle was provided by Fu (1986, 1990).

Pick and Charvátová-Jakubcová (1988) and Pick (1994) provided integral transformations between geoid or gravity anomaly and the stress components to reduce the contribution of the far-zone gravitational effects. Eshagh (2014) related gradiometric data, such as those provided by the Gravity field and steady-state Ocean Circulation Explorer (GOCE; ESA 1999) satellite mission, and the stress components by integral transformations. In fact, he proposed a scalar function, called stress function (SF), differentiating of which stress components may be obtained. Eshagh (2015) developed this theory further to use the existing model of Moho discontinuity for sub-crustal stress determination. Eshagh and Tenzer (2015) used this method for determination of the sub-crustal stress in different regions with proper geological interpretations. Tenzer and Eshagh (2015) investigated the subduction generated by sub-crustal stress under Taiwan. Eshagh (2016) presented some integral formulae for recovering the sub-crustal stress using terrestrial gravimetric data and applied them for stress modelling under Iran. Tenzer *et al.* (2015) applied this method for evaluation of stress fields on Mars. Eshagh and Romeshkani (2015) used the real GOCE data for recovering the sub-lithospheric stress field in Iran.

In this article, we discuss a model for recovery of the sub-crustal stress from satellite-to-satellite tracking (SST) data, *e.g.*, such as those available from the GRAVITY field and Climate Experiment (GRACE; Tapley *et al.* 2004). From the mathematical point of view, we provide two integral trans-

formations of the SF onto the SST data, which are in the form of gravitational potential differences and the line-of-sight (LOS) gravitational accelerations. Local recovery of the sub-crustal stress and/or any other type of gravity data is beneficial as: (a) global coverage of the data is not required; (b) the data with good coverage, uniform distribution and accuracies is used; and (c) local features are more visible. The third reason is highly dependent on the satellite altitude and type of the satellite data. The satellite data may represent the deep interior structure of the Earth much better than the local terrestrial data and especially the GRACE data are more sensitive to the long wavelength structure of the stress field occurring beneath the crust.

We extend the geophysical applications of the GRACE mission, such as those improving models of crustal thickness (Shin *et al.* 2007, Block *et al.* 2009, Tedla *et al.* 2011, Arabelos and Tsoulis 2013, Zhao 2013), lithospheric structures (Ricard *et al.* 2006, Kiamehr *et al.* 2008, Braitenberg and Ebbing 2009a, b; von Frese 2009, Kaban *et al.* 2010, Kaban and Trubitsyn 2012, Köther *et al.* 2012, Tondi *et al.* 2012), and our understanding of tectonic (Mikhailov *et al.* 2004, Kiamehr and Sjöberg 2006, Choi *et al.* 2006, McAdoo *et al.* 2008) and seismic processes (see, *e.g.*, Sun and Okubo 2004, Chen *et al.* 2007, Han *et al.* 2008, Panet *et al.* 2010, Matsuo and Heki 2011, Wang *et al.* 2012, Mikhailov *et al.* 2013, Dai *et al.* 2014, Li *et al.* 2014, Sun 2014).

In Section 2, we describe the SST data and present its relations to the disturbing gravitational field quantities such as disturbing potential differences (DPD) and LOS gravity disturbances. The new integral formulae are developed in Section 3 relating these quantities to the SF (Eshagh and Tenzer 2015). Here, we also study the behaviour of the corresponding sub-integral kernels in the spatial domain. The derived integral formulae are numerically tested in closed-loop simulations in Section 4. We investigate the accuracy of our algorithms by calculating the SST data assuming the SF is known, *i.e.*, the direct problem. We also consider the estimation of the SF and the stress components from known SST data, *i.e.*, the inverse problem. Contributions of this article are summarised in Section 5. Application of the mathematical apparatus to real SST data and geophysical interpretation of the recovered stress function is out of the scope of this paper.

2. SATELLITE-TO-SATELLITE TRACKING

SST is an inter-satellite measurement technique for precise determination of the satellite coordinates. It can be performed in two modes: (i) the high-low mode, when position of a low orbiting satellite is determined by satellites at higher altitudes; and (ii) the low-low mode, when two low-orbiting satellites follow each other in approximately the same orbit (see Reigber 1989, Seeber 2003). SST in high-low mode has been successfully realised by the satellite

mission CHAMP (Challenging Minisatellite Payload; Reigber *et al.* 2002). SST in low-low and high-low modes have been exploited on-board of GRACE mission mapping the Earth's gravitational field and GRAIL (Gravity Recovery and Interior Laboratory; Zuber *et al.* 2013) mission, which focused on the Moon's gravitational field.

The high-low SST provides position, velocity and acceleration vectors in the geocentric Earth-fixed frame, namely:

$$\mathbf{x} = [x, y, z]^T, \quad \dot{\mathbf{x}} = [\dot{x}, \dot{y}, \dot{z}]^T, \quad \ddot{\mathbf{x}} = [\ddot{x}, \ddot{y}, \ddot{z}]^T. \quad (1)$$

These are derived from Global Navigation Satellite Systems (GNSS) carrier phase observations. The inter-satellite ranging system between a pair of low-orbiting satellites provides the time series of the following quantities, *i.e.*, the low-low SST data:

$$\rho = |\delta \mathbf{x}|, \quad \dot{\rho} = \delta \dot{\mathbf{x}} \cdot \mathbf{e}, \quad \ddot{\rho} = \delta \ddot{\mathbf{x}} \cdot \mathbf{e} + \frac{|\delta \dot{\mathbf{x}}|^2 - \dot{\rho}^2}{\rho}. \quad (2)$$

Here, ρ is the inter-satellite range, $\dot{\rho}$ and $\ddot{\rho}$ represent its first and second time derivatives, *i.e.*, the inter-satellite velocity difference and inter-satellite acceleration difference (being derived from the inter-satellite range), $\mathbf{e}$ stands for the LOS inter-satellite unit vector and

$$\delta \mathbf{x} = \mathbf{x}_2 - \mathbf{x}_1, \quad \delta \dot{\mathbf{x}} = \dot{\mathbf{x}}_2 - \dot{\mathbf{x}}_1, \quad \delta \ddot{\mathbf{x}} = \ddot{\mathbf{x}}_2 - \ddot{\mathbf{x}}_1, \quad (3)$$

where the subscript 1 designates the leading satellite and the subscript 2 is assigned to the trailing satellite.

For gravity field modelling, the SST data should be mathematically related to some quantities of the field. The range-rates and velocity measurements have the simplest mathematical relations to the differences of the gravitational potential V between the pair of satellites (see, *e.g.*, Wolff 1969, Fischell and Pisacane 1978, Rummel 1980):

$$\delta V(\mathbf{x}) = |\dot{\mathbf{x}}_1| \dot{\rho}. \quad (4)$$

Note that Eq. 4 holds only approximately. A rigorous model for δV was provided and extensively analysed by Jekeli (1999). This model should be preferred for gravity field modelling purposes. Alternatively, the low-low and high-low SST data may rigorously be combined to provide differences of gravitational gradients projected onto the LOS inter-satellite unit vector (see, *e.g.*, Hajela 1974, Rummel 1980, Keller and Sharifi 2005):

$$\frac{|\delta \dot{\mathbf{x}}|^2 - \dot{\rho}^2 - \rho \ddot{\rho}}{\rho} = \frac{\delta \mathbf{x} \cdot \delta \nabla V(\mathbf{x})}{\rho} = \mathbf{e} \cdot \delta \nabla V(\mathbf{x}). \quad (5)$$

The right hand-side of Eq. 5 is also referred to as LOS gravitational acceleration.

In the following, we will derive the integral formulae between the SF, whose derivatives correspond to the components of the stress due to mantle convection, and the SST data in the form of Eqs. 4 and 5. We introduce the DPD by δT and LOS gravity disturbances by $\mathbf{e} \cdot \delta \nabla T$, where T stands for the disturbing gravitational potential. Both, the DPD and LOS gravity disturbances, can be obtained by subtracting the normal gravitational potential, *e.g.*, generated by the GRS80 reference ellipsoid (Moritz 2000), from both sides of Eqs. 4 and 5.

3. MATHEMATICAL DEVELOPMENTS

Here, we develop the mathematical relations between each one of δT and $\mathbf{e} \cdot \delta \nabla T$ and the SF, which are in fact of integral types. Also, we present the behaviour of the integral kernels to have an idea about the far-zone contribution of the integral formulae.

3.1 Integral transformation of the stress function onto disturbing gravitational potential

In this section, we focus on deriving the mathematical link between the SF and the disturbing gravitational potential. The spherical harmonic expansion of the disturbing gravitational potential is:

$$T(\mathbf{x}) = \sum_{n=2}^{\infty} \sum_{m=-n}^n \left(\frac{R}{r} \right)^{n+1} \bar{T}_{nm} \bar{Y}_{nm}(\Omega), \quad (6)$$

where R is the radius of the reference sphere, the triplet $r, \Omega = (\varphi, \lambda)$ represents the spherical coordinates, *i.e.*, the geocentric radius, spherical latitude φ and longitude λ , and they are presented by the vector $\mathbf{x} = \mathbf{x}(r, \Omega)$ on the left-hand side of Eq. 6. $\bar{T}_{nm}$ stand for the fully-normalised spherical harmonic coefficients of the disturbing gravitational potential of degree n and order m . These are related to the conventional dimensionless fully-normalised spherical harmonic coefficients $\bar{C}_{nm}$ by the following transformation:

$$\bar{T}_{nm} = \frac{GM}{R} \bar{C}_{nm}, \text{ where } G \text{ stands for the Newtonian gravitational constant and}$$

M is the mass of the Earth including oceans and atmosphere. The symbol $\bar{Y}_{nm}$ stands for the fully-normalised surface spherical harmonics. The summation in Eq. 6 starts from degree 2, meaning that the mass of the Earth equals to the mass of the reference ellipsoid and the frame in which the coordinates are defined is geocentric.

Similarly, the SF may be expanded into the following spherical harmonic series (Eshagh 2014):

$$S(\mathbf{x}, D) = \frac{R\kappa}{GM} \sum_{n=2}^{\infty} \sum_{m=-n}^n \nu_n \bar{T}_{nm} \bar{Y}_{nm}(\Omega) = \sum_{n=2}^{\infty} \sum_{m=-n}^n \bar{S}_{nm} \bar{Y}_{nm}(\Omega) , \quad (7)$$

where

$$\kappa = \frac{Mg}{4\pi[R-D]^2}, \quad \nu_n = \frac{2n+1}{n+1} \left[\frac{R}{R-D} \right]^{n+1}, \quad (8)$$

with g being the mean value of gravity, D is the normal or mean Moho depth and $\bar{S}_{nm}$ are the fully-normalised harmonic coefficients of the SF. By comparing Eqs. 6 and 7 it will not be difficult to find that:

$$\bar{T}_{nm} = \frac{GM}{R\kappa\nu_n} \bar{S}_{nm} . \quad (9)$$

Equation 9 shows the direct relation between the spherical harmonic coefficients of the SF and those of the disturbing gravitational potential. However, we can find an integral formula by transforming the SF onto the disturbing gravitational potential in the spatial domain. First, Eq. 9 is inserted back into Eq. 6, and by considering that:

$$\bar{S}_{nm} = \frac{1}{4\pi} \int_{\Omega'} S(\mathbf{x}', D) \bar{Y}_{nm}(\Omega') d\Omega(\mathbf{x}') , \quad (10)$$

where $\mathbf{x}' = \mathbf{x}(R, \Omega')$ assigns the position of an integration element on the surface of the reference sphere Ω' , we have:

$$T(\mathbf{x}) = \frac{GM}{4\pi R\kappa} \sum_{n=2}^{\infty} \frac{1}{\nu_n} \left(\frac{R}{r} \right)^{n+1} \int_{\Omega'} S(\mathbf{x}', D) \sum_{m=-n}^n \bar{Y}_{nm}(\Omega') \bar{Y}_{nm}(\Omega) d\Omega(\mathbf{x}') . \quad (11)$$

Now, according to the addition theorem of the surface spherical harmonics (Hofmann-Wellenhof and Moritz 2005, p. 26):

$$P_{n,0}(\cos \psi) = \frac{1}{2n+1} \sum_{m=-n}^n \bar{Y}_{nm}(\Omega) \bar{Y}_{nm}(\Omega') , \quad (12)$$

where $P_{n,0}$ are the associated Legendre functions of the first kind of degree n and order 0 (also known as Legendre polynomials) and ψ is the spherical distance between the geocentric directions Ω and Ω' , we get:

$$T(\mathbf{x}) = \frac{GM}{4\pi R\kappa} \int_{\Omega'} S(\mathbf{x}', D) K(t, u) d\Omega(\mathbf{x}') . \quad (13)$$

The series representation of the integral kernel $K(t, u)$ is of the form:

$$K(t, u) = \sum_{n=2}^{\infty} t^{n+1} (n+1) P_{n,0}(u) , \quad (14)$$

with variables t and u defined as follows:

$$t = t(\mathbf{x}, \mathbf{x}') = \frac{|\mathbf{x}'|}{|\mathbf{x}|} = \frac{R}{r}, \quad u = u(\mathbf{x}, \mathbf{x}') = \frac{\mathbf{x} \cdot \mathbf{x}'}{|\mathbf{x}| |\mathbf{x}'|} = \cos \psi . \quad (15)$$

We can also find a closed-form formula for the kernel of Eq. 14. To do so, we use the following identities, see, *e.g.*, (Pick *et al.* 1973):

$$\sum_{n=2}^{\infty} t^{n+1} P_{n,0}(u) = t \left(\frac{1}{g} - 1 - tu \right) , \quad (16)$$

$$\sum_{n=2}^{\infty} n t^{n+1} P_{n,0}(u) = t^2 \left(\frac{u-t}{g^3} - u \right) , \quad (17)$$

where

$$g = g(t, u) = \sqrt{1 - 2tu + t^2} . \quad (18)$$

Exploiting Eqs. 16 and 17, the infinite series of Eq. 14 may be written in the closed form:

$$K(t, u) = -t \left(1 + 2tu - \frac{1-tu}{g^3} \right) . \quad (19)$$

3.2 Integral transformation of the stress function onto SST data

By using the integral formula of Eq. 13 we can find corresponding integral transformations between the SF and the SST data in the form of either the DPD (δT) or LOS gravity disturbance ($\mathbf{e} \cdot \delta \nabla T$). In the following we will develop these transformations.

3.2.1 Integral transformation of the stress function onto disturbing potential differences

Considering two low-orbiting satellites, δT will be nothing else than a difference between disturbing gravitational potentials at the satellites' locations based on Eq. 13. In such a situation δT is:

$$\delta T(\mathbf{x}) = \frac{GM}{4\pi R \kappa} \int_{\Omega'} S(\mathbf{x}', D) \delta K(t, u) d\Omega(\mathbf{x}') . \quad (20)$$

It is seen that δT leads to an integral kernel difference $\delta K(t, u)$, which is a function of three points, *i.e.*, the integration element $\mathbf{x}'$ and the position vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ of both satellites; see also Eq. 15 for the definition of variables t and u .

To better understand the properties of the integral formula of Eq. 20 we have investigated the behaviour of the integral kernel $\delta K(t, u)$. For this purpose, we have selected one particular formation of GRACE twin satellites from the product GNV1B (GPS Navigation Data Format Record; Case *et al.* 2010) at GPS time $t_{\text{GPS}} = 316363415$ s. The spherical coordinates of the leading satellite are $r_1 = 6854183.3$ m, $\varphi_1 \approx 0.30^\circ$, $\lambda_1 \approx 5.05^\circ$, while the spherical coordinates of the trailing satellite are $r_2 = 6855126.8$ m, $\varphi_2 \approx -1.79^\circ$, $\lambda_2 \approx 5.10^\circ$. It follows that the barycentre of the satellite pair, *i.e.*, the arithmetic

Fig. 1. Graphical illustration of the integral kernel δK based on Eq. 20: (a) the total value of the kernel, (b) profile along the meridian with constant longitude $\lambda = 5.125^\circ$. The spherical coordinates of the leading satellite are $r_1 = 6854183.3$ m, $\varphi_1 \approx 0.30^\circ$, $\lambda_1 \approx 5.05^\circ$. The spherical coordinates of the trailing satellite are $r_2 = 6855126.8$ m, $\varphi_2 \approx -1.79^\circ$, $\lambda_2 \approx 5.10^\circ$. Value of the mean Moho depth is $D = 30\,000$ m and the radius of the reference sphere is $R = 6378136.3$ m. The black solid line in Fig. 1a indicates the zero crossings of the integral kernel.

tic average of angular spherical coordinates of both satellites, is defined by the position $\varphi_B \approx -0.75^\circ$, $\lambda_B \approx 5.08^\circ$. Next, we have assumed that the value of the mean Moho depth is $D = 30\,000$ m and the radius of the reference sphere is $R = 6\,378\,136.3$ m. Integral kernel $\delta K(t, u)$ in the closed-form based on Eq. 19 has been computed globally with step 0.25° in both angular directions and depicted in the area bounded by spherical coordinates $\varphi \in [-50.125^\circ, 50.125^\circ]$, $\lambda \in [-60.125^\circ, 60.125^\circ]$; see Fig. 1. Despite of the fact that the kernel $K(t, u)$ is isotropic, the difference $\delta K(t, u)$ is not isotropic anymore and leads to the azimuthal dependence illustrated in Fig. 1a. The extreme behaviour of the integral kernel is seen in the along-track direction that almost coincides with latitude direction. Obviously, value of the integral kernel is zero at the barycentre of the two satellites' positions. Moving from the barycentre along-track by approximately 2.5° , the total integral kernel reaches its extreme values. With increasing distance from the barycentre, one may observe the decay to zero. These facts are clearly demonstrated in Fig. 1b, which represents the behaviour of the integral kernel along the meridian with constant longitude $\lambda = 5.125^\circ$. The integral kernel is much smoother with significantly reduced magnitudes and values close to zero in the cross-track (longitude) direction. Implications of the integral kernel behaviour for the recovery of the SF are further discussed in Section 4.2.

3.2.2 Integral transformation of the stress function onto LOS gravity disturbances

To obtain the integral formula connecting the SF to the LOS gravity disturbance we use the following operator (Šprlak and Novák 2014a, b):

$$\mathbf{e} \cdot \delta \nabla = \frac{1}{R} \delta \left(e_x t \sqrt{1-u^2} \cos \alpha \frac{\partial}{\partial u} + e_y t \sqrt{1-u^2} \sin \alpha \frac{\partial}{\partial u} - e_z t^2 \frac{\partial}{\partial t} \right). \quad (21)$$

In the last equation, e_x , e_y , and e_z are the components of the LOS inter-satellite unit vector in the spherical local north-oriented reference frame, *i.e.*, the frame with origin at satellite's center of mass, with x axis directed north, y axis pointing west, and z axis pointing in the radial direction, $\alpha = \alpha(\Omega, \Omega')$ is the direct azimuth, and t, u are the substitutions defined by Eq. 15. Applying the differential operator of Eq. 21 onto Eq. 13 we get:

$$\mathbf{e} \cdot \delta \nabla T(\mathbf{x}) = \frac{GM}{4\pi R^2 \kappa} \int_{\Omega'} S(\mathbf{x}', D) \delta \left\{ K^u(t, u) \left[e_x \cos \alpha + e_y \sin \alpha \right] - e_z K'(t, u) \right\} d\Omega(\mathbf{x}'). \quad (22)$$

The last equation relates the SF to the LOS gravity disturbance. Obviously, the integral kernel is azimuth-dependent due to the presence of the trigonometric functions of the direct azimuth α . Similar to Eq. 20, the integral

kernel is a function of three points, *i.e.*, the integration element $\mathbf{x}'$ and the position vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ of both satellites. In addition, the integral kernel is composed of two isotropic kernels, which are defined as follows:

$$K^u(t, u) = t\sqrt{1-u^2} \frac{\partial K(t, u)}{\partial u} = \sum_{n=2}^{\infty} t^{n+2} (n+1) P_{n,1}(u) = -t^3 \sqrt{1-u^2} \left(2 - \frac{2-tu-t^2}{g^5} \right), \quad (23)$$

$$K^t(t, u) = t^2 \frac{\partial K(t, u)}{\partial t} = \sum_{n=2}^{\infty} t^{n+2} (n+1)^2 P_{n,0}(u) = -t^2 \left\{ 1 + 4tu - \frac{1}{g} - \frac{t}{g^3} \left[3u - 4t + \frac{3t(t-u)^2}{g^2} \right] \right\}. \quad (24)$$

Equations 23 and 24 define the spectral and closed forms of the isotropic kernels $K^u(t, u)$ and $K^t(t, u)$. The spectral forms immediately follow by applying the first equalities in Eqs. 23 and 24 to Eq. 14. On the other hand, the closed expressions may be derived making use of the identities of Eqs. 16 and 17 together with (see, *e.g.*, Pick *et al.* 1973, Šprlák and Novák 2014b):

$$\sum_{n=2}^{\infty} n^2 t^{n+1} P_{n,0}(u) = -t^2 \left[u - \frac{u-2t}{g^3} - \frac{3t(t-u)^2}{g^5} \right], \quad (25)$$

$$\sum_{n=2}^{\infty} t^{n+1} P_{n,1}(u) = -t^2 \sqrt{1-u^2} \left(1 - \frac{1}{g^3} \right), \quad (26)$$

$$\sum_{n=2}^{\infty} n t^{n+1} P_{n,1}(u) = -t^2 \sqrt{1-u^2} \left(1 - \frac{1}{g^3} - \frac{3t(u-t)}{g^5} \right). \quad (27)$$

We now demonstrate properties of the integral formula of Eq. 22 by investigating the behaviour of its integral kernel. The integral kernel has been evaluated globally with equiangular step of 0.25° and illustrated in the spatial domain bounded by spherical coordinates $\varphi \in [-30.125^\circ, 30.125^\circ]$, $\lambda \in [-30.125^\circ, -40.125^\circ]$. Note that the other kernel parameters, *i.e.*, positions of the satellite pair, the mean Moho depth D and the radius of the reference sphere R , have the same values as those already described for the integral kernel $\delta K(t, u)$; see Section 3.2.1.

The total integral kernel of Eq. 22 is illustrated in Fig. 2a, which shows its azimuth-dependent behaviour. The most significant values are again visible in the along-track direction. This direction is approximated by a meridian of constant longitude $\lambda = 5.125^\circ$ in Fig. 2b. We may observe that the integral kernel of Eq. 22 reaches its negative extreme value at the barycentre and then reaches local positive extremes approximately 4.5° from the barycentre. The kernel then decreases to zero with increasing distance from the barycen-

Fig. 2. Graphical illustration of the integral kernel of Eq. 22: (a) the total value of the kernel, (b) profile along the meridian with constant spherical longitude $\lambda = 5.125^\circ$, (c) profile along the parallel with constant spherical latitude $\varphi = -0.625^\circ$. The spherical coordinates of the leading satellite are $r_1 = 6\,854\,183.3$ m, $\varphi_1 \approx 0.30^\circ$, $\lambda_1 \approx 5.05^\circ$. The spherical coordinates of the trailing satellite are $r_2 = 6\,855\,126.8$ m, $\varphi_2 \approx -1.79^\circ$, $\lambda_2 \approx 5.10^\circ$. Value of the mean Moho depth is $D = 30\,000$ m and the radius of the reference sphere is $R = 6\,378\,136.3$ m. The black solid line in Fig. 2a indicates the zero crossings of the integral kernel.

tre. Figure 2c demonstrates the behaviour of the integral kernel along the parallel with constant latitude $\varphi = 0.625^\circ$. This direction represents approximately the cross-track direction and at the same time it is very close to the barycentre. It is seen that in this direction the integral kernel decays to zero faster since the local positive extremes are completely reduced. Behaviour of the integral kernel and its effect on the recovery of the SF are discussed in Section 4.2.

4. NUMERICAL EXPERIMENTS

Here, we divide our numerical studies into two parts. First, we will test the direct problem in which the generated SF by EGM2008 (Pavlis *et al.* 2012) is used to reproduce the SST data. In the second numerical experiment, we try to recover the SF by inverting δT and $\mathbf{e} \cdot \delta \nabla T$ based on the mathematical apparatus presented in the previous section.

4.1 Testing the direct problem

The presented theories have been practically tested in a numerical experiment. First, the correctness of our mathematical derivations has been checked as the direct problem, *i.e.*, direct integration of the SF for calculation of the SST data based on Eqs. 20 and 22. The SF has been generated from EGM2008 global gravity field model (GGFM) up to degree and order (d/o) 180 in a global equiangular grid with spacing of 1° . For evaluation of the SF, we assumed that the value of the mean Moho depth is $D = 30\,000$ m and the radius of the reference sphere is $R = 6\,378\,136.3$ m. Next, the true δT and the true $\mathbf{e} \cdot \delta \nabla T$ have been computed from EGM2008 up to d/o 180 at the locations given by real GRACE orbit available from the product GNV1B. Every fifth data record in the period from 1 January 2010 until 30 June 2010 have been considered and restricted to the area bounded by spherical coordinates $\varphi \in [5^\circ, 57^\circ]$ and $\lambda \in [55^\circ, 115^\circ]$. The restricted domain covers the area of India, Himalayas and Southeast Asia and contains 30 036 points in total. The statistics of the SST data is presented in Table 1 and their spatial behaviour is depicted in Fig. 3. We note that the values of the SST data were interpolated from the original locations along the orbit into an equiangular grid with sampling of 0.25° for better visualisation purposes. We also note that values of δT have opposite sign for descending and ascending tracks. In Fig. 3a we have opted for ascending tracks when interpolating δT and Fig. 3b is a similar map but for $\mathbf{e} \cdot \delta \nabla T$. We observe that the map of δT , presented in Fig. 3a is smoother than that of $\mathbf{e} \cdot \delta \nabla T$ in Fig. 3b as expected.

In a further step, we have performed the global integration of the SF based on integral formulae of Eqs. 20 and 22. For this purpose, numerical integration method has been implemented in a computer program. Subsequent-

Table 1

Statistics of the differences between the SST data
obtained by numerical integration and their true values

SST data	Equation	Units	Minimum	Maximum	Mean	Std. dev.
δT	Signal	$\text{m}^2 \text{s}^{-2}$	-44.938	46.892	0.602	18.123
	20		-0.005	0.005	0.000	0.001
$\mathbf{e} \cdot \delta \nabla T$	Signal	mGal	-10.196	8.135	0.894	2.449
	22		-0.096	0.102	0.004	0.012

Notes: Closed isotropic kernels were exploited in the numerical integration. The true values were synthesised from EGM2008 up to d/o 180. Statistics of the true values are given in the rows called “Signal”.

Fig. 3: (a) Disturbing potential differences (δT) [$\text{m}^2 \text{s}^{-2}$]; and (b) LOS gravity disturbance ($\mathbf{e} \cdot \delta \nabla T$) [mGal].

ly, δT and $\mathbf{e} \cdot \delta \nabla T$ from the global integration have been compared with the true values. Statistics of the differences is presented in Table 1. The standard deviation of differences reaches $0.001 \text{ m}^2 \text{s}^{-2}$ for δT and 0.012 mGal for $\mathbf{e} \cdot \delta \nabla T$. The relative accuracy, defined as the ratio of the standard deviation of differences to the standard deviation of the signal, is two orders higher for $\mathbf{e} \cdot \delta \nabla T$. This indicates that the performance of the numerical integration for δT is better. We suppose that the different performance is due to different spectral sensitivity of the quantities in Eqs. 20 and 22. Evaluation of δT is more precise as the SF is of similar spectral content. On the other hand, $\mathbf{e} \cdot \delta \nabla T$ is more sensitive to higher frequencies of the gravity field as compared to the SF. Evaluation of $\mathbf{e} \cdot \delta \nabla T$ may therefore be less accurate from a numerical point of view.

4.2 Solution of the inverse problem

The integrals 20 and 22 should be inverted to recover the SF. To do so, they are discretised with an equiangular resolution of 1° that corresponds to spherical harmonic expansion up to degree 180. Recovering SF with higher resolution is not meaningful as the GRACE satellite data are not sensitive to higher degrees. The synthesised GRACE data on their real orbit in the area $\varphi \in [5^\circ, 57^\circ]$ and $\lambda \in [55^\circ, 115^\circ]$ are considered as the observations. The discretised integrals represent observation equations connecting the observations to the SF in the same area. According to the size of the study area and the resolution of SF, 3120 values should be determined for SF and the number of synthesised data is 30 036, which means that we have 26 916 data more than the unknown values of SF. Therefore, the systems of equations, organised by the discretisation of the integral formulae, should be solved by the least-squares method. The determinant of the coefficients matrix of the system is very small which makes the inversion process numerically unstable with a condition number of 2×10^{14} . Regularisation is a method for solving such unstable systems. In this study, we use Tikhonov regularisation (Tikhonov 1963) for stabilising the system and L-curve (Hansen 1998) for estimating the regularisation parameter. For inverting our systems of equations we have used Regularisation Tools (Hansen 2007) of MATLAB. For more information about regularisation, see, *e.g.*, Hansen (1998), Bouman (1998) or Eshagh (2011a).

It should be noted that we have simulated δT and $\mathbf{e} \cdot \delta \nabla T$ using the EGM2008 model, while real GRACE orbits have been exploited as the purpose of this study is to investigate how successful our methods for recovering SF are for the noise-free scenario. In this way, we can judge better the theoretical apparatus derived in Section 3 rather than working with the real GRACE data which are of different nature and could make our primary investigations complicated. Recovery of SF from noisy SST data and application of the derived theoretical apparatus for real GRACE data is left for our future research.

4.2.1 Spatial truncation error

The spatial truncation error is a serious problem in inversion of the spherical integrals like those in Eqs. 20 and 22, in addition to the instability of system of equations organised from them. This is due to lacking data in the integration domain of integral formulae and occurs mainly in the marginal parts of the inversion area. The simplest way for reducing this error is to perform the inversion process in whole area and select the results located in the central part as the final result. Eshagh (2011b) showed that the truncation error may be reduced well when data and unknowns are located within the same area.

Fig. 4. Spatial truncation error (STE) of recovered SF by inverting integral of: (a) Eq. 20, and (b) Eq. 22.

Also, spatial behaviour of the corresponding integral kernel has a tremendous role to reduce the truncation error. Based on Eshagh (2011b), the truncation error may be considerably reduced for the well-behaving integral kernels, *i.e.*, kernels reaching their extremes at the computational point and decreasing uniformly to zero. Figures 1 and 2 show that the kernels of the integrals 20 and 22 decrease to zero after a geocentric angle of about 15° . Therefore, by considering the inversion area larger by 15° from each side, performing the inversion and selecting the result in the central area (smaller by 15°), the effect of spatial truncation error is reduced. Figures 4a and 4b present the maps of the recovered SF by both integral formulae 20 and 22, respectively, without removing the truncation error. As the figures show, the truncation error is considerably larger than the magnitude of SF in the marginal area. One issue which is visible in these figures is that the SF is

smoother when it is recovered from δT via the integral 20 as compared to the recovery by the integral 22.

4.2.2 Recovery of the stress function

Now, the SST data are computed on the real orbit of the twin-satellites. The integrals 20 and 22 are used to invert them to the SF. In order to reduce the effect of the spatial truncation error, we selected the recovered SF in the central part which is smaller by 10° . Figures 5a and 5b show the recovered SF by inverting the integrals 20 and 22. The coastal lines and the tectonic boundaries have been shown in the map by black and green lines. The values of SF are larger when they are recovered from δT , whilst smaller from $\mathbf{e} \cdot \delta \nabla T$. The maps clearly show that the SF is smoother in the former case than the latter. This could be expected as recovering high frequencies of the Earth gravity field from the potential observables is not possible, while first and second order derivatives of the potential are more capable for high resolution gravity field modelling from space.

Fig. 5. SF recovered from: (a) disturbing potential differences (δT), and (b) LOS gravity disturbances ($\mathbf{e} \cdot \delta \nabla T$). Unit: 1 MPa.

4.2.3 Determination of sub-crustal stress from the recovered stress function

Once SF is estimated, computation of its derivatives numerically towards the north and the east is straightforward. One can use the following formulae for generating the meridian and prime vertical sub-crustal stress components:

$$S_x = -\frac{\Delta S}{\Delta \varphi}, \quad S_y = \frac{1}{\cos \varphi} \frac{\Delta S}{\Delta \lambda}, \quad (28)$$

where S_x is the northward and S_y is the eastward component, ΔS is the difference of the stress function between two adjacent cells towards the north or the east, $\Delta \varphi$ is the resolution in the northward direction and $\Delta \lambda$ in the eastward directions. Note that both of them should be in the radian units.

Here, we compute the magnitudes of the sub-crustal stress by:

$$S_r = \sqrt{S_x^2 + S_y^2}, \quad (29)$$

Fig. 6. Magnitudes of sub-crustal stress derived from: (a) δT , and (b) $e \cdot \delta VT$. Unit: 1 MPa.

and present them in the maps; see Fig. 6. Figure 6a shows the map of S_T derived from the SF recovered from δT , and Fig. 6b that from $\mathbf{e} \cdot \delta \nabla T$. In spite of the fact that the values of SF presented in Fig. 5a are larger than those of Fig. 5b, the values of S_T are smaller in Fig. 6a when they are derived numerically from the SF of Fig. 5a. The reason is that when sub-crustal stress components are computed from SF, ΔS will be smaller in the case of using the SF generated from δT than $\mathbf{e} \cdot \delta \nabla T$, which is due to the smooth nature of SF in the first case. Both maps of S_T , presented in Figs. 6a and 6b, show large sub-crustal stress under the tectonic boundary in the southern part of Himalayas.

Table 2 presents the statistics of the recovered SF and the sub-crustal stress components recovered numerically from SF. According to the table, S_x ranges from -28.27 to 21.58 MPa when it is derived from δT , while for $\mathbf{e} \cdot \delta \nabla T$ the range is from -46.42 to 32.02 MPa. The corresponding extreme values for S_y are -9.12 to 14.11 MPa for the former and from -19.28 to 23.75 MPa for the latter functional.

Table 2

Statistics of recovered SF and sub-crustal stress components S_x and S_y
derived from δT and $\mathbf{e} \cdot \delta \nabla T$

SST data	Quantity	Minimum	Maximum	Mean	Std. dev.
δT	S	-1.12	1.42	-0.10	0.55
	S_x	-28.27	21.58	-1.09	6.98
	S_y	-9.12	14.11	-1.18	3.55
$\mathbf{e} \cdot \delta \nabla T$	S	-1.52	0.79	-0.68	0.45
	S_x	-46.42	32.02	-0.23	9.43
	S_y	-19.28	23.75	-0.98	4.93

Note: Unit: 1 MPa.

5. DISCUSSION

In this study, we have used the SST data for recovering the sub-crustal stress components. Generally, a GGFM can be used for modelling these components by the Runcorn's formula. However, Eshagh (2016) showed that Runcorn's formula is not convergent for high degrees. This means that the idea of working with high degree GGFM is not successful.

The mantle convection occurs inside the Earth and only by investigating the long wavelength structure of the Earth gravity field, we can model and investigate it; see, *e.g.*, Eshagh (2015), Eshagh and Tenzer (2015), and Tenzer and Eshagh (2015). The proper selection of the maximum degree of expansion and resolution of recovery play important roles in stress model-

ling. Eshagh (2016) presented a formula for estimating the maximum degree of the expansion and the mean Moho depth. For example, for a mean Moho depth of 35 km, the maximum expansion degree is 372. The larger the Moho depth is the lower the maximum expansion degree will be.

Exploiting terrestrial gravity data means considering all frequencies of the gravity field implicitly. Therefore, the divergence occurs in the computations. In such a case, either kernel functions should be limited to a specific maximum degree or the integral inversion should be performed locally, while the local diverting frequencies of the system of equations are removed, *e.g.*, using the truncated singular value decomposition technique; see Eshagh (2016).

The satellite data are more useful for stress modelling as the satellite altitudes filter out the high frequencies of the gravity field and represent the low frequencies better. For near sub-surface geophysical phenomena the lower altitude satellites are more suitable. On the other hand, for studying deeper structures of the Earth satellites flying at higher altitudes should be preferred. Consequently, SST data provided by GRACE should be more suitable for lithospheric stress modelling. Here, we have investigated two types of the SST data and concluded that $\mathbf{e} \cdot \delta \nabla T$ contains more frequencies of the gravity field and can present the stress better.

Eshagh (2015) reported that the sub-crustal stress reaches up to 100 MPa over Himalayas, when a GGFM is considered to degree 360 with mean Moho depth of 23 km. The magnitude of sub-crustal stress reaches to 5 MPa in Taiwan (Tenzer and Eshagh 2015) using a GGFM and Eshagh and Tenzer (2015) reported a maximum of 70 MPa when a mean Moho depth of 30 km is assumed. Eshagh (2016) reported maximum stress of about 60 MPa over Iran. Eshagh and Romeshkani (2015) used the real GOCE data to recover the stress at the depth of 100 km and found that the stress reaches to about 100 MPa. The values that we have obtained for the stress based on our approaches are about 30 and 45 MPa dependent on the use of δT and $\mathbf{e} \cdot \delta \nabla T$ which are consistent with what can be obtained from other data types.

We emphasise that both of our methods have been applied under the same conditions, *e.g.*, the same satellite orbits, spatial resolution, and the mean Moho depth. The difference between these two solutions is dependent on the type of observables and the mathematical models connecting the observables to the unknown SF. It is clear that the second method, which uses the LOS gravity disturbances, is capable of recovering higher frequencies of the sub-crustal stress as compared to disturbing potential differences. Consequently, we can conclude that exploiting the LOS gravity disturbances performs well and may be more suited for real applications. To prove this

finding, recovery of sub-crustal stress from SST data contaminated by noise have to be performed. This task is left for our future research.

6. CONCLUSIONS

In this article two integral transformations between the stress function, numerical differentiation of which provides components of the sub-crustal stress, and the SST data have been derived. The first integral transformation assumes that the SST data are in the form of disturbing potential differences. SST data in the form of LOS gravity disturbance are related to the stress function in the second integral transformation.

The corresponding integral kernels have been investigated in the spatial domain showing their azimuthal dependence. The integral kernels are well-behaving, thus suitable for solving the inverse problem, *i.e.*, estimation of the stress function from the SST data. Direct and inverse problems have been tested in the numerical experiments. The results suggest that the stress function (and thus the components of the sub-crustal stress) estimated from the disturbing potential differences is smoother as compared to the estimate from the LOS gravity disturbances. This is due to the type of data being inverted and by using LOS gravity disturbances higher frequencies of the stress field can be recovered. Inversion of such data is better as long as the divergence does not happen in the mathematical model of the stress function. The theoretical apparatus may be applied for the recovery of the sub-crustal stress from satellite mission GRACE and its planned successor.

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