Scaling Analysis of Time Distribution between Successive Earthquakes in Aftershock Sequences

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Abstract

The earthquake inter-event time distribution is studied, using catalogs for different recent aftershock sequences. For aftershock sequences following the Modified Omori’s Formula (MOF) it seems clear that the inter-event distribution is a power law. The parameters of this law are defined and they prove to be higher than the calculated value $(2 - 1/p)$. Based on the analysis of the catalogs, it is determined that the probability densities of the inter-event time distribution collapse into a single master curve when the data is rescaled with instantaneous intensity, $R(t; M_{th})$, defined by MOF. The curve is approximated by a gamma distribution. The collapse of the data provides a clear view of aftershock-occurrence self-similarity.

Key words: earthquakes, aftershocks, Omori’s law, inter-event time distribution, self-similarity.

1. INTRODUCTION

In seismology the distribution of the basic earthquake quantities is the most often studied; these include magnitude, time of occurrence, and localization. The simplest description of seismicity is as a point process in space, time and energy (or magnitude). At present, the distribution of derivative quantities such as times and distances between successive events is also studied

with increasing interest and ambition to understand the spatial and temporal complexity of the earthquake realization and interaction.

Nowadays, the attention paid to the statistics of the inter-event time (also called waiting time, interoccurrence time, *etc.*) between the consecutive earthquakes is continuously growing (Bak *et al.* 2002, Corral 2004a, b, 2005; Talbi and Yamazaki 2009, Lippiello *et al.* 2012, Davidsen and Kwiatek 2013).

Bak *et al.* (2002) studied the distribution of the inter-event times Δt between successive earthquakes for all earthquakes in different regions, by putting all events on the same footing (no distinction between mainshocks and aftershocks), therefore considering seismicity as a unique process.

After that, Corral (2004a, 2005) showed that under certain conditions the inter-event times in each region follow a specific probabilistic distribution, which is universal, *i.e.*, it depends neither on the geographic region nor on the mean seismic activity rate. This distribution can be written in the following form:

$$D(\Delta t, M_{th}) = R(M_{th}) f(R(M_{th}) \Delta t), \quad (1)$$

where $D(\Delta t; M_{th})$ is the inter-event time probability density, $R(M_{th})$ is the mean seismic rate, defined as the number of earthquakes per unit time, and M_{th} is the minimum magnitude threshold included in the analysis. It should be noted that $R(M_{th})$ controls the scale of $D(\Delta t; M_{th})$, but not its shape. The latter is defined by the scaling function f . In his studies Corral has shown that for stationary seismicity, *i.e.*, when $R(M_{th})$ is almost constant in time, the scaling function f well approximates with a gamma distribution:

$$f(x) \propto \frac{1}{x^{1-\gamma}} e^{-x/a}, \quad (2)$$

where $x \equiv R \cdot \Delta t$; γ and a are parameters. For a stationary case the best fit gives a value of approximately 0.7 for the parameter γ (Corral 2004b, 2005; Marekova 2012).

The validity of Eq. 2 has also been examined for non-stationary periods (Corral 2006a), Omori sequences (Shcherbakov *et al.* 2005, Bottiglieri *et al.* 2010), volcanic earthquakes (Bottiglieri *et al.* 2009a), and even picoseismicity (Davidsen and Kwiatek 2013). Talbi *et al.* (2013) used the analysis of the inter-event times to suggest a new measure of earthquake clustering. This can be used as a procedure for declustering catalogs.

All studies so far have demonstrated that the analysis of inter-event times gives important insight into the physical mechanisms of the earthquake process.

The existence of earthquake sequences (foreshocks, aftershocks, and swarms/clusters) considerably complicates the picture in terms of modelling the distribution of the events in time. A lot of attention is paid to the aftershock sequences, since they are the obvious manifestation of dependent events, following the main earthquake. The assumption for a Poisson distribution with intensity, different from constant, is basic in studying the aftershock activity (Utsu 2002). The decrease in this activity is most frequently approximated by the Modified Omori's Formula (MOF) (Utsu 2002):

$$n(t) = \frac{K}{(t+c)^p}, \quad (3)$$

where t is the time elapsed after the main shock; K , c , and p are constants.

The Japanese seismologist Tomoda (1954) stated that the time intervals τ between successive shocks in two aftershock sequences – 1927 Tango and 1948 Fukui – had the following type of distribution:

$$f(\tau) = k\tau^{-q}, \quad (4)$$

where k and q are constants. When the aftershock sequences are presented as a non-stationary Poisson process, whose intensity can be written in the form

$$n(t) = \frac{K}{t^p}, \quad (5)$$

then, for the above-mentioned sequences, the value of q is 1.23 and 1.48, respectively.

In this case another Japanese seismologist, Senshu (1959), showed that the distribution 4 is obtained directly from Eq. 3 and the relation between p and q has to be:

$$q = 2 - \frac{1}{p}. \quad (6)$$

The inter-event time distribution is analyzed for aftershock sequences of certain strong contemporary earthquakes in California: the $M7.3$ Landers, the $M6.7$ Hector Mine, and the $M7.1$ Northridge (Shcherbakov *et al.* 2005). When considering sequences with different magnitude thresholds within one and the same aftershock sequence, rescaling leads to collapse of data in accordance with Eq. 2, wherein f follows a power law with exponent $(2 - 1/p \approx 1)$, followed by an exponential cutoff.

Relation 6 was also obtained by Jonsdottir *et al.* (2006), Lindman (2009) and Shcherbakov *et al.* (2006) in an analytical way, based on the proposition that the aftershock sequences can be regarded as a Poisson process with in-

stantaneous intensity, defined on the basis of the Modified Omori's law at the considered moment t .

A number of studies have focused on the Modified Omori's Formula inter-event time probability density distributions (Utsu 1971, Utsu *et al.* 1995, Jonsdottir *et al.* 2006, Lindman 2009). They all have shown that individual aftershock sequences are characterized by a power law distribution of the intermediate inter-event times. Jonsdottir *et al.* (2006) analytically and numerically demonstrated that, by catalog incompleteness immediately after the main shock, the probability density distribution of the inter-event times is roughly constant for very short times (*i.e.*, for $\Delta t < c$), while a power law decay dominates for $\Delta t > c$. For the longest inter-event times, a fall-off related to the finiteness of the considered time window is observed.

The present work investigates the temporal organization of the aftershock occurrence. The analysis is also focused on the inter-event time distributions. Formula 4 is used, giving the distribution of the inter-event times as a power law. The parameters of this power law are defined for the studied aftershock sequences and it is established that they are higher than the calculated value $(2 - 1/p)$.

The inter-event time distributions evaluated for the different main shock magnitudes collapse onto a single master curve when the inter-event time is rescaled with instantaneous intensity $R(t; M_{ih})$, defined by the MOF. This implies that the aftershock temporal organization is independent of the main shock magnitude M_{ms} . The curve is approximated by a gamma distribution. The collapse of the data provides a clear illustration of aftershock-occurrence self-similarity. This result suggests again the existence of a universal mechanism for aftershock generation.

2. DATA

Aftershock sequences of relatively strong earthquakes are studied in the present paper. The data has been taken from California, Alaska, Canada, and Europe, from chosen seismic catalogs, on which detailed studies related to their completeness and homogeneity are focused. The identification of aftershock sequences is a complex problem which requires assumptions on the spatial and temporal clustering of aftershocks (see van Stiphout *et al.* 2012 for a complete review, Talbi *et al.* 2013). In this work it is assumed that all events which have occurred after a main shock within a given time interval T and a square (or polygon) area are aftershocks. The events in the spatial windows, described in Table 1, have been selected based on the published descriptions (References in Table 1). The maximum duration of the sequences is chosen to be $T_{\max} = 1$ year (where the catalog allows that).

Table 1

Spatial regions, data sources, and references about some of the studied sequences

Earthquake	Space window	Catalog source	References
Joshua Tree	33.75÷34.80°N; 116.0÷117.5°W	Advanced National Seismic System (ANSS) http://quake.geo.berkeley.edu/cnss	Jones and Helmberger 1998
Big Bear	33.5÷34.5°N; 116.40÷117.25°W		
Landers	34.75°N; 116.80°W 34.80°N; 116.50°W 33.85°N; 116.10°W 33.80°N; 116.40°W		
Andreanof Island 1986	50 ÷ 52 °N; 172÷177°W	Alaska Earthquake Center (AEC) http://giseis.alaska.edu/Seis/	Engdahl <i>et al.</i> 1989
Andreanof Island 1996	50.50÷51.75°N; 175.80÷179.0°W	ANSS http://quake.geo.berkeley.edu/cnss	Kisslinger and Kikuchi 1997
Hector Mine	35.0°N; -116.0°W 35.0°N; -116.5°W 34.5°N; -116.5°W 34.2°N; -116.0°W	Southern California Earthquake Data Center (SCEDC) http://www.scecdc.scec.org (or http://www.ncedc.org/anss/)	Wiemer <i>et al.</i> 2002, Felzer <i>et al.</i> 2002
Northridge	34.1÷34.5°N; 118.3÷ 119.9°W		Wiemer and Katsumata 1999
British Columbia	49.7 ÷ 50.9°N; 129.5÷130.9°W	Natural Resources Canada (now http://www.earthquakescanada.nrcan.gc.ca/index-eng.php)	Cassidy and Rogers 1995

The sequence of Joshua Tree earthquake is regarded as a foreshock sequence of the later Landers quake. It partially overlaps with the region of the quakes of the later Landers earthquake and therefore the end of this sequence coincides with the beginning of the Landers (Jones and Helmberger 1998).

The catalog published in an article by Stephens *et al.* (1980) is used as a data source about St. Elias sequence. One of the most striking features in the distribution of epicenters is that a relatively small number of aftershocks are localized near the epicenter of the main quake and the highest level of activity is observed about 50 km southeast from the epicenter of the main quake (Horner 1983).

It is characteristic of the Nahanni sequence that two earthquakes occurred in the region Mackenzie from the northwest territories of Canada – on 5 October 1985 ($M_s6.6$) and on 23 December 1985 ($M_s6.9$). The hypocenters

of both quakes are localized close to each other – near to the center of the aftershock region and at almost the same focal depth (Horner *et al.* 1990). The catalog is from a CD from WDC-A, Boulder, Colorado and it contains earthquakes with a magnitude $M \geq 3.0$. Only data from the first sequence to the beginning of the second sequence have been analyzed.

Part of the catalog data has been provided by colleagues or published and no further selection has been made in relation to them:

- the data on the earthquake in Strazhitza, Bulgaria, has been taken from the catalog prepared by the Geophysical Institute (now NIGGG), Bulgarian Academy of Science (Solakov and Simeonova 1993); data on the earthquake in Kresna, Bulgaria are from Ranguelov *et al.* (2001);
- short description of the sequences in Greece and Aegean has been found in Papazachos and Papazachou (1997); data about Friuli – in Zollo *et al.* (1997).

In seismology, the well-known Gutenberg–Richter (GR) law dominates the statistics of the occurrence of earthquakes. This law describes the rela-

Fig. 1. Frequency-magnitude distribution of events from 1992 Landers aftershock sequence (red squares). The green line represents the best fit to the observations in the area of complete recording (indicated by M_c). The slope of the green line is the b value of the Gutenberg and Richter relationship $\log N = a - bM$; the a parameter is determined by the intercept of the green line with the Y axis in the point where the magnitude equals zero.

Table 2

Description of the studied aftershock sequences

Earthquake	Date	Period	M_{ms}	M_{th}	Number of events	H [km]
St. Elias, Canada	28 Feb 1979	28 Feb 1979 – 30 Mar 1979	7.1	3.0	174	0-19
Joshua Tree, California	23 Apr 1992	23 Apr 1992 – 28 Jun 1992	6.1	3.0	234	0-18
Landers, California	28 Jun 1992	28 Jun 1992 – 28 Jul 1993	7.3	3.0	783	0-20
Big Bear, California	28 Jun 1992	28 Jun 1992 – 8 Jun 1993	6.4	3.0	404	0-20
Nahanni, Canada	5 Oct 1985	5 Oct 1985 – 20 Dec 1985	6.6	3.1	167	6-10
Andreanof Island '86, Alaska	7 May 1986	7 May 1986 – 6 Apr 1987	8.0	4.8	206	18-67
Andreanof Island '96, Alaska	10 Jun 1996	10 Jun 1996 – 23 May 1997	7.7	4.5	137	10-60
Aegean '81	19 Dec 1981	19 Dec 1981 – 14 Dec 1982	7.2	3.2	244	0-35
Aegean '82	18 Jan 1982	18 Jan 1982 – 30 Sep 1982	6.9	3.5	75	0-72
Aegean '83	6 Aug 1983	6 Aug 1983 – 12 Jun 1984	6.9	3.6	100	1-47
Albania	15 Apr 1979	15 Apr 1979 – 10 Oct 1979	7.2	3.0	461	0-22
Strazhitza, Bulgaria	7 Dec 1986	7 Dec 1986 – 11 May 1987	5.7	2.0	123	0-25
Kresna, Bulgaria	4 Apr 1904	6 Apr 2004 – 29 Mar 2005	7.8	2.7	1224	0-50
British Columbia, Canada	6 Apr 1992	6 Apr 1992 – 24 Mar 1993	6.8	2.7	59	0-10
Hector Mine, California	16 Oct 1999	16 Oct 1999 – 1 Nov 2000	7.1	3.0	529	0-18
Northridge, California	17 Jan 1994	17 Jan 1994 – 16 Jan 1995	6.7	3.0	439	0-23
Greece '81	24 Feb 1981	24 Dec 1981 – 25 Feb 1982	6.6	3.2	547	0-47
Greece '83	17 Jan 1983	17 Jan 1983 – 13 Jan 1984	7.0	3.6	324	0-58
Friuli, Italy	6 May 1976	6 May 1976 – 3 Apr 1977	6.5	3.0	276	0-58

Note: M_{ms} is the main earthquake magnitude, M_{th} is the chosen threshold magnitude, and H is the depth interval for the earthquakes in the sequences.

tionship between the frequency of occurrence and the magnitude of the earthquakes (Gutenberg and Richter 1944):

$$\log_{10} N = a - bM, \quad (7)$$

where N is the number of the earthquakes having magnitudes higher than M , and a and b are constants.

M_C is the magnitude at which the data departs from the linear trend (Fig. 1) and denoted the magnitude of completeness – a threshold above which all the events in the time-space window can be assumed as having been recorded.

A common practice when studying earthquake sequences is analyzing the GR law. For this purpose, in the present article, the Zmap software (Wiemer 2001) has been used. Zmap calculates the b value and the standard deviation using the maximum-likelihood method. The estimate of the b value based on the mean magnitude (Aki 1965) is as follows:

$$b = \frac{0.4343}{\bar{M} - M_{\min}}, \quad (8)$$

where $\bar{M}$ is the mean value of all magnitudes within the selected ranges ($M \geq M_{\min}$). The program Zmap also uses the maximum of the derivative of the frequency magnitude distribution to estimate M_C (Wiemer 2001).

Figure 1 illustrates the frequency-magnitude distribution for Landers aftershocks sequence. The magnitudes of completeness M_C for all studied aftershock sequences vary from 1.5 to 4.8. The chosen threshold magnitudes M_{th} , at which the aftershock sequences are studied are given in Table 2. It must be noted that in most cases the chosen thresholds M_{th} equal the magnitudes of completeness M_C . The values of M_C are lower only for the California earthquakes – Joshua Tree, Landers, Big Bear, Northridge, as well as British Columbia and Kresna earthquakes.

3. METHODOLOGY

The main objective of this work is the study and analysis of the distribution of time intervals between earthquakes from different aftershock sequences.

The pairs of earthquakes are formed by taking only the intervals of time Δt between the successive earthquakes. Thus, if the number of events is N , then $(N - 1)$ time differences are formed. Inter-event times are most often presented using histogram plots of the inter-event times probability distribution. In the present analysis, a histogram of the count using successive bins that are increasing in length is made. One could work with the logarithm of Δt but a more direct possibility is to define the bins over which the count is calculated as $A * c^{n-1}$, with $c > 1$ and n labeling consecutive bins. This en-

sures an appropriate bin size for each time scale ($c = 2$ and constant $A = 0.001$ days were assumed).

The relative frequencies p_i^* for each subinterval are calculated from $p_i^* = n_i / (N - 1)$, where n_i is the number of values in the i -th subinterval, and $(N - 1)$ is the total number of the time intervals used.

Such count histograms are equivalent to the statistical probability distributions (p.d.). In this work, normalized versions of the count histograms are presented, where they are divided by the length of the inter-event time intervals (bin size), as it was done by Jonsdottir *et al.* (2006). The author of this article will further use the term “Rescaled P.D.” for these curves.

The function approximating the Rescaled P.D. of the inter-event times Δt between the pairs of earthquakes has the form:

$$f(\Delta t) = a_t \Delta t^{-b_t}, \quad (9)$$

where a_t and b_t are parameters.

Another goal of the present work is to check whether the universal scaling law of the inter-event times distributions (Corral 2004a, 2005) is also applicable to aftershock sequences. The study of the inter-event time probability density $D(\Delta t; M_{th})$ from Eq. 1 is similar to the one described in Corral (2004b) and Marekova (2012). After a strong quake, sequences of weaker earthquakes follow, gradually decreasing with time. Thus, the analysis of the non-stationary seismicity can be generalized by replacing the average intensity $R(M_{th})$ by an instantaneous intensity $R(t; M_{th})$ as a scaling parameter in Eq. 1. The decrease in activity after the main shock can be described by an equation of the type:

$$R(t; M_{th}) = \frac{K}{(c + t)^p}. \quad (10)$$

The rescaled dimensionless time θ_i is obtained in the following way:

$$\theta_i = R(\Delta t_i; M_{th}) \Delta t_i. \quad (11)$$

As it was done by Corral (2006b), the results from fitting the real data with Eq. 10 for each sequence were also used here.

4. RESULTS AND DISCUSSION

The probability distributions of the time intervals (Fig. 2a) and the Rescaled probability distributions (Fig. 2b) for the studied aftershock sequences are obtained. The Rescaled P.D. are similar, as can be seen from Fig. 2b.

Fig. 2: (a) Probability distribution of the time intervals between successive events for the aftershock sequences, (b) Rescaled probability distributions (P.D.) for the same sequences.

Besides, for all sequences the following parameters are calculated: p , c , K from MOF Eq. 3, and a_i , b_i from the function 9, approximating the Rescaled P.D. They are presented in Table 3. The estimates for the model parameters of MOF are obtained by using the program Zmap.

Table 3

Parameters a_i and b_i of the approximating function 9 for the distributions of the inter-event times between the pairs of earthquakes for the studied aftershock sequences and parameters p , c , K of the Modified Omori's Formula (Eq. 3)

Earthquake	a_i	b_i	p	c	K
St. Elias, Canada	0.060 (± 0.104)	1.322 (± 0.078)	0.91 (± 0.06)	0.018 (± 0.013)	23.86 (± 2.43)
Joshua Tree, California	0.096 (± 0.087)	1.084 (± 0.052)	0.94 (± 0.05)	0.047 (± 0.026)	31.07 (± 3.58)
Landers, California	0.061 (± 0.071)	1.343 (± 0.055)	1.22 (± 0.03)	0.787 (± 0.147)	220.72 (± 29.01)
Big Bear, California	0.067 (± 0.062)	1.312 (± 0.046)	1.05 (± 0.03)	0.165 (± 0.054)	56.42 (± 6.57)
Nahanni, Canada	0.129 (± 0.083)	1.110 (± 0.071)	0.92 (± 0.06)	0.114 (± 0.075)	34.60 (± 5.43)
Andreanof Island '86, Alaska	0.089 (± 0.056)	1.183 (± 0.042)	0.90 (± 0.03)	0.005 (± 0.006)	18.53 (± 1.85)
Andreanof Island '96, Alaska	0.097 (± 0.061)	1.115 (± 0.046)	0.90 (± 0.03)	0.005 (± 0.010)	13.40 (± 1.65)
Aegean '81	0.085 (± 0.060)	1.205 (± 0.060)	0.91 (± 0.03)	0.020 (± 0.015)	22.65 (± 2.45)
Aegean '82	0.086 (± 0.056)	1.188 (± 0.040)	1.12 (± 0.07)	0.032 (± 0.022)	9.09 (± 1.60)
Aegean '83	0.110 (± 0.055)	1.133 (± 0.045)	0.98 (± 0.05)	0.008 (± 0.016)	10.15 (± 1.56)
Albania	0.070 (± 0.125)	1.282 (± 0.099)	1.05 (± 0.07)	1.822 (± 0.654)	114.96 (± 32.90)
Strazica, Bulgaria	0.090 (± 0.101)	1.180 (± 0.079)	1.19 (± 0.10)	0.399 (± 0.212)	28.74 (± 8.57)
Kresna, Bulgaria	0.585 (± 0.085)	1.278 (± 0.060)	0.93 (± 0.01)	0.070 (± 0.016)	124.70 (± 7.04)
British Columbia, Canada	0.117 (± 0.061)	0.901 (± 0.042)	0.63 (± 0.05)	0.000 (± 0.009)	2.67 (± 0.63)
Hector Mine, California	0.071 (± 0.042)	1.256 (± 0.033)	1.20 (± 0.04)	0.249 (± 0.057)	104.02 (± 11.68)
Northridge, California	0.066 (± 0.050)	1.269 (± 0.039)	1.20 (± 0.03)	0.137 (± 0.031)	80.28 (± 7.74)
Greece '81	0.059 (± 0.061)	1.360 (± 0.046)	1.30 (± 0.05)	1.314 (± 0.275)	222.61 (± 41.49)
Greece '83	0.105 (± 0.042)	1.322 (± 0.040)	1.14 (± 0.07)	3.008 (± 1.066)	106.70 (± 35.95)
Friuli, Italy	0.088 (± 0.084)	1.289 (± 0.069)	0.67 (± 0.03)	0.000 (± 0.022)	13.91 (± 1.77)

Note: The time taken into account in calculating the seismic activity is measured in days.

It should be noted that when rescaling P.D., deviation from the model line Eq. 9 is observed for almost all sequences at the short time intervals. Therefore, in the present work inter-event times of less than 0.002 days are discarded from the analysis. When calculating the parameters of Eq. 9 the distributions have been used without these values. Talbi and Yamazaki (2009) suggested a formula for inter-event time cutoff, but the values the formula determines exceed the chosen value leading to poor statistics significantly.

Examining whether there existed a relation of the type Eq. 6 between parameters p (from the Omori law) and b_t (from Eq. 9) it was proved that such a relation was not present. The results show that the obtained values for the parameter b_t are a little bit higher than the ones defined by relation 6 (Fig. 3).

The differences between the calculated q values and the values obtained from Eq. 6, were also discussed by Utsu (1971) and Utsu *et al.* (1995). The reasons for the difference between the theoretic relation 6 and the observed values of the b_t parameters can be various. When describing intensity decrease of the aftershocks, the Modified Omori's Formula (MOF) is not observed in all sequences. The decrease of the aftershock activity is better described for some sequences by the models ETAS (Ogata 1988) or RETAS

Fig. 3. Comparison between the values of b_t defined based on the real data, and the calculated values $(2 - 1/p)$ from Eq. 6. The green line corresponds to the case when the two parameters are equal.

Fig. 4. Real cumulative number of events in time (blue points), calculated expected cumulative number (thick red line) according to the model from the Modified Omori's Formula (MOF) and calculated standard deviation from the model used to present the error in defining the model number (dotted line). The lines of the model RETAS or ETAS are also shown for the sequences (thick blue line).

(Gospodinov and Rotondi 2001), as it is shown in Fig. 4. However, even if we take into account only those sequences for which the intensity decrease follows MOF, such a relation between parameters p and q cannot be obtained.

As it was described by Utsu (1971), the choice of duration of the after-shock sequence influences the parameter b_t – it presents the slope of the straight line, which depicts Eq. 9 in a double-logarithmic scale. When considering longer duration, the slope grows. To reduce this influence, the present study uses a maximum duration of the sequences of 1 year (except for some sequences, as it was discussed above).

When defining the parameters p and b_t , also standard errors are estimated. The errors calculated for these parameters are shown by error bars in Fig. 3.

Nowadays, the existence of the scaling behavior in Eq. 1 in models for seismic occurrence has been addressed by means of analytical and numerical investigations. A number of aftershock sequences are studied in the present work in order to show how the universal law is applied in these cases.

The probability densities of the rescaled times θ_i from Eq. 11, obtained for all sequences, are shown in Fig. 5. Although there is a certain scatter, a characteristic collapse of data in a curve is observed, similar to the cases of stationary seismicity. The obtained curve is approximated by a gamma distribution (Corral 2004a) in the form:

$$f(\theta) = a(b\theta)^{\gamma-1} \exp(-b\theta), \quad (12)$$

where a , b , and γ are the parameters whose values are:

$$a = 0.023 \pm 0.014; \quad b = 0.042 \pm 0.024; \quad \gamma = 0.449 \pm 0.086.$$

The analysis used in this paper differs from those made by Corral (2005), Bottiglieri *et al.* (2009b), and Shcherbakov *et al.* (2005).

The specific thing about the present study is the mixing of the data from aftershock sequences realized in different regions. Nevertheless, the distribution curves of the rescaled inter-event times collapse in a master curve and the curve is approximated by a gamma distribution, as described by Corral (2004a, 2005, 2006b).

It should be noted that in the distributions of the rescaled inter-event time intervals a deviation in the ends of the curves is observed. The explanation can be related to the way of identification of the aftershock sequences. Due to decrease in activity, the number of long time intervals between the earthquakes goes down. Besides, the duration of the aftershock sequences depends on the magnitude of the main shock. In such an analysis the time for realization of the sequence can be chosen depending on the magnitude of the

Fig. 5. Probability densities of recurrence times for the aftershock sequences listed in Table 2. The continuous red line is a gamma fit.

main event. The algorithm for defining the duration of the aftershock sequences in California was described in Bottiglieri *et al.* (2009b). According to it, for the Landers and Northridge sequences it was found that they lasted 140 and 110 days, respectively, which is much shorter than the maximum time of 1 year used in the present study.

It is a common practice to study individually main quakes and foreshocks or aftershocks. A reliable algorithm for separating the aftershocks is needed but not available yet (Talbi *et al.* 2013). Multiple seismic surveys use a number of declustering procedures based on subjective criteria to separate the observed clustering (van Stiphout *et al.* 2012).

The algorithms used for separation of the studied aftershock sequences are different and perhaps this leads to the stronger scattering of data around the model curve in Fig. 5.

5. CONCLUSION

The paper analyzes the scaling properties of aftershock sequences with different main shock magnitudes, realized in various regions. The study of the temporal properties of aftershocks shows that the aftershock sequences can be modeled as a point process, governed by non-homogeneous Poisson sta-

tistics. Seismic activity in this case decreases as a power dependence, what introduces a self-similar mode of occurrence.

For aftershock sequences following the Omori's law the inter-event times distribution is a power law. The results show that the power law exponents computed from the data are higher than the ones calculated theoretically from Modified Omori's Formula.

The distributions of the inter-event time intervals collapse in a master curve after rescaling by the so-called "instantaneous" intensity. This curve is approximated by a gamma distribution with exponent approximately equal to 0.45.

The probability densities of the rescaled inter-event times appear in good agreement with the results for stationary seismicity. This result suggests once again the existence of a universal mechanism for aftershock generation.

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