Varying Constants, Gravitation and Cosmology

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Abstract

Fundamental constants are a cornerstone of our physical laws. Any constant varying in space and/or time would reflect the existence of an almost massless field that couples to matter. This will induce a violation of the universality of free fall. Thus, it is of utmost importance for our understanding of gravity and of the domain of validity of general relativity to test for their constancy. We detail the relations between the constants, the tests of the local position invariance and of the universality of free fall. We then review the main experimental and observational constraints that have been obtained from atomic clocks, the Oklo phenomenon, solar system observations, meteorite dating, quasar absorption spectra, stellar physics, pulsar timing, the cosmic microwave background and big bang nucleosynthesis. At each step we describe the basics of each system, its dependence with respect to the constants, the known systematic effects and the most recent constraints that have been obtained. We then describe the main theoretical frameworks in which the low-energy constants may actually be varying and we focus on the unification mechanisms and the relations between the variation of different constants. To finish, we discuss the more speculative possibility of understanding their numerical values and the apparent fine-tuning that they confront us with.

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Contents

List of Tables

1   Introduction

Fundamental constants appear everywhere in the mathematical laws we use to describe the phenomena of Nature. They seem to contain some truth about the properties of the physical world while their real nature seem to evade us.

The question of the constancy of the constants of physics was probably first addressed by Dirac [155, 156] who expressed, in his "Large Numbers hypothesis", the opinion that very large (or small) dimensionless universal constants cannot be pure mathematical numbers and must not occur in the basic laws of physics. He suggested, on the basis of this numerological principle, that these large numbers should rather be considered as variable parameters characterizing the state of the universe. Dirac formed five dimensionless ratios among which[1] $\delta \equiv H_0\hbar/m_\mathrm{p}c^2 \sim 2h \times 10^{-42}$ and $\epsilon \equiv G\rho_0/H_0^2 \sim 5h^{-2} \times 10^{-4}$ and asked the question of which of these ratios is constant as the universe evolves. Usually, δ varies as the inverse of the cosmic time while ϵ varies also with time if the universe is not described by an Einstein–de Sitter solution (i.e., when a cosmological constant, curvature or radiation are included in the cosmological model). Dirac then noticed that $\alpha_\mathrm{G}/\mu\alpha_\mathrm{EM}$, representing the relative magnitude of electrostatic and gravitational forces between a proton and an electron, was of the same order as $H_0e^2/m_\mathrm{e}c^2 = \delta\alpha_\mathrm{EM}\mu$ representing the age of the universe in atomic units so that his five numbers can be "harmonized" if one assumes that α_G and δ vary with time and scale as the inverse of the cosmic time.

This argument by Dirac is indeed not a physical theory but it opened many doors in the investigation on physical constants, both on questioning whether they are actually constant and on trying to understand the numerical values we measure.

First, the implementation of Dirac's phenomenological idea into a field-theory framework was proposed by Jordan [268], who realized that the constants have to become dynamical fields and proposed a theory where both the gravitational and fine-structure constants can vary ([497] provides a summary of some earlier attempts to quantify the cosmological implications of Dirac's argument). Fierz [195] then realized that in such a case, atomic spectra will be spacetime-dependent so that these theories can be observationally tested. Restricting to the sub-case in which only G can vary led to definition of the class of scalar-tensor theories, which were further explored by Brans and Dicke [67]. This kind of theory was further generalized to obtain various functional dependencies for G in the formalization of scalar-tensor theories of gravitation (see, e.g., [124]).

Second, Dicke [151] pointed out that in fact the density of the universe is determined by its age, this age being related to the time needed to form galaxies, stars, heavy nuclei.... This led him to formulate that the presence of an observer in the universe places constraints on the physical laws that can be observed. In fact, what is meant by observer is the existence of (highly?) organized systems and this principle can be seen as a rephrasing of the question "why is the universe the way it is?" (see [252]). Carter [82, 83], who actually coined the term "anthropic principle" for it, showed that the numerological coincidences found by Dirac can be derived from physical models of stars and the competition between the weakness of gravity with respect to nuclear fusion. Carr and Rees [80] then showed how one can scale up from atomic to cosmological scales only by using combinations of α_EM, α_G and $m_\mathrm{e}/m_\mathrm{p}$.

To summarize, Dirac's insight was to question whether some numerical coincidences between very large numbers, that cannot be themselves explained by the theory in which they appear, was a mere coincidence or whether it can reveal the existence of some new physical laws. This gives three main roads of investigation

- how do we construct theories in which what were thought to be constants are in fact dynamical fields,

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[1] H_0 is the Hubble constant today, ρ_0 the mean matter energy density today. The other constants are defined in Tables 1 and 2 and the definitions of the cosmological quantities are summarized in Appendix A.3.

- how can we constrain, experimentally or observationally, the spacetime dependencies of the constants that appear in our physical laws

- how can we explain the values of the fundamental constants and the fine-tuning that seems to exist between their numerical values.

While "varying constants" may seem, at first glance, to be an oxymoron, it has to be considered merely as jargon to be understood as "revealing new degrees of freedom, and their coupling to the known fields of our theory". The tests on the constancy of the fundamental constants are indeed very important tests of fundamental physics and of the laws of Nature we are currently using. Detecting any such variation will indicate the need for new physical degrees of freedom in our theories, that is new physics.

The necessity of theoretical physics on deriving bounds on their variation is, at least, threefold:

1. it is necessary to understand and to model the physical systems used to set the constraints. In particular one needs to relate the effective parameters that can be observationally constrained to a set of fundamental constants;

2. it is necessary to relate and compare different constraints that are obtained at different spacetime positions. This often requires a spacetime dynamics and thus to specify a model as well as a cosmology;

3. it is necessary to relate the variations of different fundamental constants.

Therefore, we shall start in Section 2 by recalling the link between the constants of physics and the theories in which they appear, as well as with metrology. From a theoretical point of view, the constancy of the fundamental constants is deeply linked with the equivalence principle and general relativity. In Section 2 we will recall this relation and in particular the link with the universality of free fall. We will then summarize the various constraints that exist on such variations, mainly for the fine structure constant and for the gravitational constant in Sections 3 and 4, respectively. We will then turn to the theoretical implications in Section 5 in describing some of the arguments backing up the fact that constants are expected to vary, the main frameworks used in the literature and the various ways proposed to explain why they have the values we observe today. We shall finish by a discussion on their spatial variations in Section 6 and by discussing the possibility to understand their numerical values in Section 7.

Various reviews have been written on this topic. We will refer to the review [500] as FVC and we mention the following later reviews [31, 47, 72, 119, 226, 281, 278, 501, 395, 503, 505] and we refer to [356] for the numerical values of the constants adopted in this review.

2 Constants and Fundamental Physics

2.1 About constants

Our physical theories introduce various structures to describe the phenomena of Nature. They involve various fields, symmetries and constants. These structures are postulated in order to construct a mathematically-consistent description of the known physical phenomena in the most unified and simple way.

We define the fundamental constants of a physical theory as *any parameter that cannot be explained by this theory*. Indeed, we are often dealing with other constants that in principle can be expressed in terms of these fundamental constants. The existence of these two sets of constants is important and arises from two different considerations. From a theoretical point of view we would like to extract the minimal set of fundamental constants, but often these constants are not measurable. From a more practical point of view, we need to measure constants, or combinations of constants, which allow us to reach the highest accuracy.

Therefore, these fundamental constants are contingent quantities that can only be measured. Such parameters have to be assumed constant in this theoretical framework for two reasons:

- from a theoretical point of view: the considered framework does not provide any way to compute these parameters, i.e., it does not have any equation of evolution for them since otherwise it would be considered as a dynamical field,

- from an experimental point of view: these parameters can only be measured. If the theories in which they appear have been validated experimentally, it means that, at the precisions of these experiments, these parameters have indeed been checked to be constant, as required by the necessity of the reproducibility of experimental results.

This means that testing for the constancy of these parameters is a test of the theories in which they appear and allow to extend our knowledge of their domain of validity. This also explains the definition chosen by Weinberg [526] who stated that they cannot be calculated in terms of other constants "...not just because the calculation is too complicated (as for the viscosity of water) but because we do not know of anything more fundamental".

This has a series of implications. First, the list of fundamental constants to consider depends on our theories of physics and, thus, on time. Indeed, when introducing new, more unified or more fundamental, theories the number of constants may change so that this list reflects both our knowledge of physics and, more important, our ignorance. Second, it also implies that some of these fundamental constants can become dynamical quantities in a more general theoretical framework so that the tests of the constancy of the fundamental constants are tests of fundamental physics, which can reveal that what was thought to be a fundamental constant is actually a field whose dynamics cannot be neglected. If such fundamental constants are actually dynamical fields it also means that the equations we are using are only approximations of other and more fundamental equations, in an adiabatic limit, and that an equation for the evolution of this new field has to be obtained.

The reflections on the nature of the constants and their role in physics are numerous. We refer to the books [29, 215, 510, 509] as well as [59, 165, 216, 393, 521, 526, 538] for various discussions of this issue that we cannot develop at length here. This paragraph summarizes some of the properties of the fundamental constants that have attracted some attention.

2.1.1 Characterizing the fundamental constants

Physical constants seem to play a central role in our physical theories since, in particular, they determined the magnitudes of the physical processes. Let us sketch briefly some of their properties.

Table 1: List of the fundamental constants of our standard model. See Ref. [379] for further details on the measurements.

Constant	Symbol	Value
Speed of light	c	$299\,792\,458$ m s^{-1}
Planck constant (reduced)	$\hbar$	$1.054\,571\,628(53) \times 10^{-34}$ J s
Newton constant	G	$6.674\,28(67) \times 10^{-11}$ m^2 kg^{-1} s^{-2}
Weak coupling constant (at m_Z)	$g_2(m_Z)$	0.6520 ± 0.0001
Strong coupling constant (at m_Z)	$g_3(m_Z)$	1.221 ± 0.022
Weinberg angle	$\sin^2\theta_{\mathrm{W}}(91.2\text{ GeV})_{\overline{\mathrm{MS}}}$	0.23120 ± 0.00015
Electron Yukawa coupling	h_{e}	2.94×10^{-6}
Muon Yukawa coupling	h_μ	0.000607
Tauon Yukawa coupling	h_τ	0.0102156
Up Yukawa coupling	h_{u}	0.000016 ± 0.000007
Down Yukawa coupling	h_{d}	0.00003 ± 0.00002
Charm Yukawa coupling	h_{c}	0.0072 ± 0.0006
Strange Yukawa coupling	h_{s}	0.0006 ± 0.0002
Top Yukawa coupling	h_{t}	1.002 ± 0.029
Bottom Yukawa coupling	h_{b}	0.026 ± 0.003
Quark CKM matrix angle	$\sin\theta_{12}$	0.2243 ± 0.0016
	$\sin\theta_{23}$	0.0413 ± 0.0015
	$\sin\theta_{13}$	0.0037 ± 0.0005
Quark CKM matrix phase	δ_{CKM}	1.05 ± 0.24
Higgs potential quadratic coefficient	$\hat{\mu}^2$	?
Higgs potential quartic coefficient	λ	?
QCD vacuum phase	θ_{QCD}	$< 10^{-9}$

How many fundamental constants should be considered?   The set of constants, which are conventionally considered as fundamental [213] consists of the electron charge e, the electron mass m_{e}, the proton mass m_{p}, the reduced Planck constant $\hbar$, the velocity of light in vacuum c, the Avogadro constant N_{A}, the Boltzmann constant k_{B}, the Newton constant G, the permeability and permittivity of space, ε_0 and μ_0. The latter has a fixed value in the SI system of unit ($\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}), which is implicit in the definition of the Ampere; ε_0 is then fixed by the relation $\varepsilon_0\mu_0 = c^{-2}$.

However, it is clear that this cannot correspond to the list of the fundamental constants, as defined earlier as the free parameters of the theoretical framework at hand. To define such a list we must specify this framework. Today, gravitation is described by general relativity, and the three other interactions and the matter fields are described by the standard model of particle physics. It follows that one has to consider 22 unknown constants (i.e., 19 unknown dimensionless parameters): the Newton constant G, 6 Yukawa couplings for the quarks ($h_{\mathrm{u}}, h_{\mathrm{d}}, h_{\mathrm{c}}, h_{\mathrm{s}}, h_{\mathrm{t}}, h_{\mathrm{b}}$) and 3 for the leptons ($h_{\mathrm{e}}, h_\mu, h_\tau$), 2 parameters of the Higgs field potential ($\hat{\mu}, \lambda$), 4 parameters for the Cabibbo–Kobayashi–Maskawa matrix (3 angles θ_{ij} and a phase δ_{CKM}), 3 coupling constants for the gauge groups $SU(3)_c \times SU(2)_L \times U(1)_Y$ (g_1, g_2, g_3 or equivalently g_2, g_3 and the Weinberg angle θ_{W}), and a phase for the QCD vacuum (θ_{QCD}), to which one must add the speed of light c and the Planck constant h. See Table 1 for a summary and their numerical values.

Again, this list of fundamental constants relies on what we accept as a fundamental theory. Today we have many hints that the standard model of particle physics has to be extended, in particular to include the existence of massive neutrinos. Such an extension will introduce at least seven new constants (3 Yukawa couplings and 4 Maki–Nakagawa–Sakata (MNS) parameters,

similar to the CKM parameters). On the other hand, the number of constants can decrease if some unifications between various interaction exist (see Section 5.3.1 for more details) since the various coupling constants may be related to a unique coupling constant α_U and an energy scale of unification M_U through

$$\alpha_i^{-1}(E) = \alpha_U^{-1} + \frac{b_i}{2\pi} \ln \frac{M_U}{E},$$

where the b_i are numbers, which depend on the explicit model of unification. Note that this would also imply that the variations, if any, of various constants shall be correlated.

Relation to other usual constants. These parameters of the standard model are related to various constants that will appear in this review (see Table 2). First, the quartic and quadratic coefficients of the Higgs field potential are related to the Higgs mass and vev, $m_H = \sqrt{-\hat{\mu}^2/2}$ and $v = \sqrt{-\hat{\mu}^2/\lambda}$. The latter is related to the Fermi constant $G_F = (v^2\sqrt{2})^{-1}$, which imposes that $v = (246.7 \pm 0.2)$ GeV while the Higgs mass is badly constrained. The masses of the quarks and leptons are related to their Yukawa coupling and the Higgs vev by $m = hv/\sqrt{2}$. The values of the gauge couplings depend on energy via the renormalization group so that they are given at a chosen energy scale, here the mass of the Z-boson, m_Z. g_1 and g_2 are related by the Weinberg angle as $g_1 = g_2 \tan\theta_W$. The electromagnetic coupling constant is not g_1 since $SU(2)_L \times U(1)_Y$ is broken to $U(1)_{\mathrm{elec}}$ so that it is given by

$$g_{\mathrm{EM}}(m_Z) = e = g_2(m_Z)\sin\theta_W. \tag{1}$$

Defining the fine-structure constant as $\alpha_{\mathrm{EM}} = g_{\mathrm{EM}}^2/\hbar c$, the (usual) zero energy electromagnetic fine structure constant is $\alpha_{\mathrm{EM}} = 1/137.03599911(46)$ is related to $\alpha_{\mathrm{EM}}(m_Z) = 1/(127.918 \pm 0.018)$ by the renormalization group equations. In particular, it implies that $\alpha_{\mathrm{EM}} \sim \alpha(m_Z) + \frac{2}{9\pi} \ln \left(\frac{m_Z^{20}}{m_{\mathrm{u}}^4 m_{\mathrm{c}}^4 m_{\mathrm{d}} m_{\mathrm{s}} m_{\mathrm{b}} m_{\mathrm{e}}^3 m_{\mu}^3 m_{\tau}^3} \right)$. We define the QCD energy scale, Λ_{QCD}, as the energy at which the strong coupling constant diverges. Note that it implies that Λ_{QCD} also depends on the Higgs and fermion masses through threshold effects.

More familiar constants, such as the masses of the proton and the neutron are, as we shall discuss in more detail below (see Section 5.3.2), more difficult to relate to the fundamental parameters because they depend not only on the masses of the quarks but also on the electromagnetic and strong binding energies.

Are some constants more fundamental? As pointed-out by Lévy-Leblond [328], all constants of physics do not play the same role, and some have a much deeper role than others. Following [328], we can define three classes of fundamental constants, *class A* being the class of the constants characteristic of a particular system, *class B* being the class of constants characteristic of a class of physical phenomena, and *class C* being the class of universal constants. Indeed, the status of a constant can change with time. For instance, the velocity of light was initially a class A constant (describing a property of light), which then became a class B constant when it was realized that it was related to electromagnetic phenomena and, to finish, it ended as a type C constant (it enters special relativity and is related to the notion of causality, whatever the physical phenomena). It has even become a much more fundamental constant since it now enters in the definition of the meter [413] (see Ref. [510] for a more detailed discussion). This has to be contrasted with the proposition of Ref. [538] to distinguish the standard model free parameters as the gauge and gravitational couplings (which are associated to internal and spacetime curvatures) and the other parameters entering the accommodation of inertia in the Higgs sector.

Relation with physical laws. Lévy-Leblond [328] proposed to rank the constants in terms of their universality and he proposed that only three constants be considered to be of class C,

Table 2: List of some related constants that appear in our discussions. See Ref. [379].

Constant	Symbol	Value
Electromagnetic coupling constant	$g_{\rm EM} = e = g_2 \sin \theta_{\rm W}$	0.313429 ± 0.000022
Higgs mass	m_H	> 100 GeV
Higgs vev	v	(246.7 ± 0.2) GeV
Fermi constant	$G_{\rm F} = 1/\sqrt{2}v^2$	$1.166\,37(1) \times 10^{-5}$ GeV^{-2}
Mass of the $W^{\pm}$	m_W	80.398 ± 0.025 GeV
Mass of the Z	m_Z	91.1876 ± 0.0021 GeV
Fine structure constant	$\alpha_{\rm EM}$	$1/137.035\,999\,679(94)$
Fine structure constant at m_Z	$\alpha_{\rm EM}(m_Z)$	$1/(127.918 \pm 0.018)$
Weak structure constant at m_Z	$\alpha_{\rm W}(m_Z)$	0.03383 ± 0.00001
Strong structure constant at m_Z	$\alpha_{\rm S}(m_Z)$	0.1184 ± 0.0007
Gravitational structure constant	$\alpha_{\rm G} = Gm_{\rm p}^2/\hbar c$	$\sim 5.905 \times 10^{-39}$
Electron mass	$m_{\rm e} = h_{\rm e}v/\sqrt{2}$	510.998910 ± 0.000013 keV
Mu mass	$m_\mu = h_\mu v/\sqrt{2}$	105.658367 ± 0.000004 MeV
Tau mass	$m_\tau = h_\tau v/\sqrt{2}$	1776.84 ± 0.17 MeV
Up quark mass	$m_{\rm u} = h_{\rm u}v/\sqrt{2}$	$(1.5 - 3.3)$ MeV
Down quark mass	$m_{\rm d} = h_{\rm d}v/\sqrt{2}$	$(3.5 - 6.0)$ MeV
Strange quark mass	$m_{\rm s} = h_{\rm s}v/\sqrt{2}$	105^{+25}_{-35} MeV
Charm quark mass	$m_{\rm c} = h_{\rm c}v/\sqrt{2}$	$1.27^{+0.07}_{-0.11}$ GeV
Bottom quark mass	$m_{\rm b} = h_{\rm b}v/\sqrt{2}$	$4.20^{+0.17}_{-0.07}$ GeV
Top quark mass	$m_{\rm t} = h_{\rm t}v/\sqrt{2}$	171.3 ± 2.3 GeV
QCD energy scale	$\Lambda_{\rm QCD}$	$(190 - 240)$ MeV
Mass of the proton	$m_{\rm p}$	938.272013 ± 0.000023 MeV
Mass of the neutron	$m_{\rm n}$	939.565346 ± 0.000023 MeV
proton-neutron mass difference	$Q_{\rm np}$	1.2933321 ± 0.0000004 MeV
proton-to-electron mass ratio	$\mu = m_{\rm p}/m_{\rm e}$	1836.15
electron-to-proton mass ratio	$\bar{\mu} = m_{\rm e}/m_{\rm p}$	$1/1836.15$
$d - u$ quark mean mass	$m_{\rm q} = (m_{\rm u} + m_{\rm d})/2$	$(2.5 - 5.0)$ MeV
$d - u$ quark mass difference	$\delta m_{\rm q} = m_{\rm d} - m_{\rm u}$	$(0.2 - 4.5)$ MeV
proton gyromagnetic factor	$g_{\rm p}$	5.586
neutron gyromagnetic factor	$g_{\rm n}$	-3.826
Rydberg constant	R_∞	$10\,973\,731.568\,527(73)$ m^{-1}

namely G, $\hbar$ and c. He pointed out two important roles of these constants in the laws of physics. First, they act as *concept synthesizer* during the process of our understanding of the laws of nature: contradictions between existing theories have often been resolved by introducing new concepts that are more general or more synthetic than older ones. Constants build bridges between quantities that were thought to be incommensurable and thus allow new concepts to emerge. For example c underpins the synthesis of space and time while the Planck constant allowed to related the concept of energy and frequency and the gravitational constant creates a link between matter and space-time. Second, it follows that these constants are related to the domains of validity of these theories. For instance, as soon as velocity approaches c, relativistic effects become important, relativistic effects cannot be negligible. On the other hand, for speed much below c, Galilean kinematics is sufficient. Planck constant also acts as a referent, since if the action of a system greatly exceeds the value of that constant, classical mechanics will be appropriate to describe this system. While the place of c (related to the notion of causality) and $\hbar$ (related to the quantum) in this list are well argued, the place of G remains debated since it is thought that it will have to be replaced by some mass scale.

Evolution. There are many ways the list of constants can change with our understanding of physics. First, new constants may appear when new systems or new physical laws are discovered; this is, for instance, the case of the charge of the electron or more recently the gauge couplings of the nuclear interactions. A constant can also move from one class to a more universal class. An example is that of the electric charge, initially of class A (characteristic of the electron), which then became class B when it was understood that it characterizes the strength of the electromagnetic interaction. A constant can also disappear from the list, because it is either replaced by more fundamental constants (e.g., the Earth acceleration due to gravity and the proportionality constant entering Kepler law both disappeared because they were "explained" in terms of the Newton constant and the mass of the Earth or the Sun) or because it can happen that a better understanding of physics teaches us that two hitherto distinct quantities have to be considered as a single phenomenon (e.g., the understanding by Joule that heat and work were two forms of energy led to the fact that the Joule constant, expressing the proportionality between work and heat, lost any physical meaning and became a simple conversion factor between units used in the measurement of heat (calories) and work (Joule)). Nowadays the calorie has fallen in disuse. Indeed demonstrating that a constant is varying will have direct implications on our list of constants.

In conclusion, the evolution of the number, status of the constants can teach us a lot about the evolution of the ideas and theories in physics since it reflects the birth of new concepts, their evolution and unification with other ones.

2.1.2 Constants and metrology

Since we cannot compute them in the theoretical framework in which they appear, it is a crucial property of the fundamental constants (but in fact of all the constants) that their value can be measured. The relation between constants and metrology is a huge subject to which we just draw the attention on some selected aspects. For more discussions, see [56, 280, 278].

The introduction of constants in physical laws is also closely related to the existence of systems of units. For instance, Newton's law states that the gravitational force between two masses is proportional to each mass and inversely proportional to the square of their separation. To transform the proportionality to an equality one requires the use of a quantity with dimension of m^3 kg^{-1} s^{-2} independent of the separation between the two bodies, of their mass, of their composition (equivalence principle) and on the position (local position invariance). With an other system of units the numerical value of this constant could have simply been anything. Indeed, the numerical value of any constant crucially depends on the definition of the system of units.

Measuring constants. The determination of the laboratory value of constants relies mainly on the measurements of lengths, frequencies, times, ... (see [414] for a treatise on the measurement of constants and [213] for a recent review). Hence, any question on the variation of constants is linked to the definition of the system of units and to the theory of measurement. The behavior of atomic matter is determined by the value of many constants. As a consequence, if, e.g., the fine-structure constant is spacetime dependent, the comparison between several devices such as clocks and rulers will also be spacetime dependent. This dependence will also differ from one clock to another so that metrology becomes both device and spacetime dependent, a property that will actually be used to construct tests of the constancy of the constants.

Indeed a measurement is always a comparison between two physical systems of the same dimensions. This is thus a relative measurement, which will give as result a pure number. This trivial statement is oversimplifying since in order to compare two similar quantities measured separately, one needs to perform a number of comparisons. In order to reduce the number of comparisons (and in particular to avoid creating every time a chain of comparisons), a certain set of them has been included in the definitions of units. Each units can then be seen as an abstract physical sys-

tem, which has to be realized effectively in the laboratory, and to which another physical system is compared. A measurement in terms of these units is usually called an absolute measurement. Most fundamental constants are related to microscopic physics and their numerical values can be obtained either from a pure microscopic comparison (as is, e.g., the case for $m_{\mathrm{e}}/m_{\mathrm{p}}$) or from a comparison between microscopic and macroscopic values (for instance to deduce the value of the mass of the electron in kilogram). This shows that the choice of the units has an impact on the accuracy of the measurement since the pure microscopic comparisons are in general more accurate than those involving macroscopic physics. This implies that only the variation of dimensionless constants can be measured and in case such a variation is detected, it is impossible to determine, which dimensional constant is varying [183].

It is also important to stress that in order to deduce the value of constants from an experiment, one usually needs to use theories and models. An example [278] is provided by the Rydberg constant. It can easily be expressed in terms of some fundamental constants as $R_\infty = \alpha_{\mathrm{EM}}^2 m_{\mathrm{e}} c/2h$. It can be measured from, e.g., the triplet $1s - 2s$ transition in hydrogen, the frequency of which is related to the Rydberg constant and other constants by assuming QED so that the accuracy of R_∞ is much lower than that of the measurement of the transition. This could be solved by defining R_∞ as $4\nu_{\mathrm{H}}(1s - 2s)/3c$ but then the relation with more fundamental constants would be more complicated and actually not exactly known. This illustrates the relation between a practical and a fundamental approach and the limitation arising from the fact that we often cannot both exactly calculate and directly measure some quantity. Note also that some theoretical properties are plugged in the determination of the constants.

As a conclusion, let us recall that (i) in general, the values of the constants are not determined by a direct measurement but by a chain involving both theoretical and experimental steps, (ii) they depend on our theoretical understanding, (iii) the determination of a self-consistent set of values of the fundamental constants results from an adjustment to achieve the best match between theory and a defined set of experiments (which is important because we actually know that the theories are only good approximation and have a domain of validity) (iv) that the system of units plays a crucial role in the measurement chain, since for instance in atomic units, the mass of the electron could have been obtained directly from a mass ratio measurement (even more precise!) and (v) fortunately the test of the variability of the constants does not require *a priori* to have a high-precision value of the considered constants.

System of units. Thus, one needs to define a coherent system of units. This has a long, complex and interesting history that was driven by simplicity and universality but also by increasing stability and accuracy [29, 509].

Originally, the sizes of the human body were mostly used to measure the length of objects (e.g., the foot and the thumb gave feet and inches) and some of these units can seem surprising to us nowadays (e.g., the *span* was the measure of hand with fingers fully splayed, from the tip of the thumb to the tip of the little finger!). Similarly weights were related to what could be carried in the hand: the pound, the ounce, the dram.... Needless to say, this system had a few disadvantages since each country, region has its own system (for instance in France there was more than 800 different units in use in 1789). The need to define a system of units based on natural standard led to several propositions to define a standard of length (e.g., the *mille* by Gabriel Mouton in 1670 defined as the length of one angular minute of a great circle on the Earth or the length of the pendulum that oscillates once a second by Jean Picard and Christiaan Huygens). The real change happened during the French Revolution during which the idea of a universal and non anthropocentric system of units arose. In particular, the Assemblée adopted the principle of a uniform system of weights and measures on 8 May 1790 and, in March 1791 a decree (these texts are reprinted in [510]) was voted, stating that a quarter of the terrestrial meridian would be the basis of the definition of the *meter* (from the Greek metron, as proposed by Borda): a meter would

henceforth be one ten millionth part of a quarter of the terrestrial meridian. Similarly the *gram* was defined as the mass of one cubic centimeter of distilled water (at a precise temperature and pressure) and the second was defined from the property that a mean solar day must last 24 hours.

To make a long story short, this led to the creation of the metric system and then of the signature of *La convention du mètre* in 1875. Since then, the definition of the units have evolved significantly. First, the definition of the meter was related to more immutable systems than our planet, which, as pointed out by Maxwell in 1870, was an arbitrary and inconstant reference. He then suggested that atoms may be such a universal reference. In 1960, the International Bureau of Weights and Measures (BIPM) established a new definition of the meter as the length equal to 1650763 wavelengths, in a vacuum, of the transition line between the levels $2p_{10}$ and $5d_5$ of krypton-86. Similarly the rotation of the Earth was not so stable and it was proposed in 1927 by André Danjon to use the tropical year as a reference, as adopted in 1952. In 1967, the second was also related to an atomic transition, defined as the duration of 9 162 631 770 periods of the transition between the two hyperfine levels of the ground state of caesium-133. To finish, it was decided in 1983, that the meter shall be defined by fixing the value of the speed of light to $c = 299\,792\,458$ m s^{-1} and we refer to [55] for an up to date description of the SI system. Today, the possibility to redefine the kilogram in terms of a fixed value of the Planck constant is under investigation [279].

This summary illustrates that the system of units is a human product and all SI definitions are historically based on non-relativistic classical physics. The changes in the definition were driven by the will to use more stable and more fundamental quantities so that they closely follow the progress of physics. This system has been created for legal use and indeed the choice of units is not restricted to SI.

SI systems and the number of basic units.   The International System of Units defines seven basic units: the meter (m), second (s) and kilogram (kg), the Ampere (A), Kelvin (K), mole (mol) and candela (cd), from which one defines secondary units. While needed for pragmatic reasons, this system of units is unnecessarily complicated from the point of view of theoretical physics. In particular, the Kelvin, mole and candela are derived from the four other units since temperature is actually a measure of energy, the candela is expressed in terms of energy flux so that both can be expressed in mechanical units of length [L], mass [M] and time [T]. The mole is merely a unit denoting numbers of particles and has no dimension.

The status of the Ampere is interesting in itself. The discovery of the electric charge [Q] led to the introduction of a new units, the Coulomb. The Coulomb law describes the force between two charges as being proportional to the product of the two charges and to the inverse of the distance squared. The dimension of the force being known as [MLT^{-2}], this requires the introduction of a new constant ε_0 (which is only a conversion factor), with dimensions [Q^{2}M^{-1}L^{-3}T^2] in the Coulomb law, and that needs to be measured. Another route could have been followed since the Coulomb law tells us that no new constant is actually needed if one uses [M$^{1/2}$L$^{3/2}$T^{-1}] as the dimension of the charge. In this system of units, known as Gaussian units, the numerical value of ε_0 is 1. Hence the Coulomb can be expressed in terms of the mechanical units [L], [M] and [T], and so will the Ampere. This reduces the number of conversion factors, that need to be experimentally determined, but note that both choices of units assume the validity of the Coulomb law.

Natural units.   The previous discussion tends to show that all units can be expressed in terms of the three mechanical units. It follows, as realized by Johnstone Stoney in 1874[2], that these three basic units can be defined in terms of 3 independent constants. He proposed [27, 267] to

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[2] After studying electrolysis in 1874, Johnstone Stoney suggested the existence of a "single definite quantity of electricity". He was able to estimate the value of this elementary charge by means of Faraday's laws of electrolysis. He introduced the term "electron" in 1894 and it was identified as a particle in 1897 by Thomson.

use three constants: the Newton constant, the velocity of light and the basic units of electricity, i.e., the electron charge, in order to define, from dimensional analysis a "natural series of physical units" defined as

$$t_{\mathrm{S}} = \sqrt{\frac{Ge^2}{4\pi\varepsilon_0 c^6}} \sim 4.59 \times 10^{-45}\ \mathrm{s},$$

$$\ell_{\mathrm{S}} = \sqrt{\frac{Ge^2}{4\pi\varepsilon_0 c^4}} \sim 1.37 \times 10^{-36}\ \mathrm{m},$$

$$m_{\mathrm{S}} = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} \sim 1.85 \times 10^{-9}\ \mathrm{kg},$$

where the ε_0 factor has been included because we are using the SI definition of the electric charge. In such a system of units, by construction, the numerical value of G, e and c is 1, i.e., $c = 1 \times \ell_{\mathrm{S}} \cdot t_{\mathrm{S}}^{-1}$ etc.

A similar approach to the definition of the units was independently proposed by Planck [418] on the basis of the two constants a and b entering the Wien law and G, which he reformulated later [419] in terms of c, G and $\hbar$ as

$$t_{\mathrm{P}} = \sqrt{\frac{G\hbar}{c^5}} \sim 5.39056 \times 10^{-44}\ \mathrm{s},$$

$$\ell_{\mathrm{P}} = \sqrt{\frac{G\hbar}{c^3}} \sim 1.61605 \times 10^{-35}\ \mathrm{m},$$

$$m_{\mathrm{P}} = \sqrt{\frac{\hbar c}{G}} \sim 2.17671 \times 10^{-8}\ \mathrm{kg}.$$

The two systems are clearly related by the fine-structure constant since $e^2/4\pi\varepsilon_0 = \alpha_{\mathrm{EM}}hc$.

Indeed, we can construct many such systems since the choice of the 3 constants is arbitrary. For instance, we can construct a system based on (e, m_{e}, h), that we can call the *Bohr units*, which will be suited to the study of the atom. The choice may be dictated by the system, which is studied (it is indeed far fetched to introduce G in the construction of the units when studying atomic physics) so that the system is well adjusted in the sense that the numerical values of the computations are expected to be of order unity in these units.

Such constructions are very useful for theoretical computations but not adapted to measurement so that one needs to switch back to SI units. More important, this shows that, from a theoretical point of view, one can define the system of units from the laws of nature, which are supposed to be universal and immutable.

Do we actually need 3 natural units? is an issue debated at length. For instance, Duff, Okun and Veneziano [165] respectively argue for none, three and two (see also [535]). Arguing for no fundamental constant leads to consider them simply as conversion parameters. Some of them are, like the Boltzmann constant, but some others play a deeper role in the sense that when a physical quantity becomes of the same order as this constant, new phenomena appear; this is the case, e.g., of $\hbar$ and c, which are associated respectively to quantum and relativistic effects. Okun [392] considered that only three fundamental constants are necessary, as indicated by the International System of Units. In the framework of quantum field theory + general relativity, it seems that this set of three constants has to be considered and it allows to classify the physical theories (with the famous *cube of physical theories*). However, Veneziano [514] argued that in the framework of string theory one requires only two dimensionful fundamental constants, c and the

string length λ_s. The use of $\hbar$ seems unnecessary since it combines with the string tension to give λ_s. In the case of the Nambu–Goto action $S/\hbar = (T/\hbar) \int \mathrm{d}(Area) \equiv \lambda_s^{-2} \int \mathrm{d}(Area)$ and the Planck constant is just given by λ_s^{-2}. In this view, $\hbar$ has not disappeared but has been promoted to the role of a UV cut-off that removes both the infinities of quantum field theory and singularities of general relativity. This situation is analogous to pure quantum gravity [388] where $\hbar$ and G never appear separately but only in the combination $\ell_{\mathrm{Pl}} = \sqrt{G\hbar/c^3}$ so that only c and ℓ_{Pl} are needed. Volovik [520] made an analogy with quantum liquids to clarify this. There an observer knows both the effective and microscopic physics so that he can judge whether the fundamental constants of the effective theory remain fundamental constants of the microscopic theory. The status of a constant depends on the considered theory (effective or microscopic) and, more interestingly, on the observer measuring them, i.e., on whether this observer belongs to the world of low-energy quasi-particles or to the microscopic world.

Fundamental parameters.   Once a set of three independent constants has been chosen as natural units, then all other constants are dimensionless quantities. The values of these combinations of constants does not depend on the way they are measured, [110, 164, 437], on the definition of the units etc..... It follows that any variation of constants that will leave these numbers unaffected is actually just a redefinition of units.

These dimensionless numbers represent, e.g., the mass ratio, relative magnitude of strength etc..... Changing their values will indeed have an impact on the intensity of various physical phenomena, so that they encode some properties of our world. They have specific values (e.g., $\alpha_{\mathrm{EM}} \sim 1/137$, $m_{\mathrm{p}}/m_{\mathrm{e}} \sim 1836$, etc.) that we may hope to understand. Are all these numbers completely contingent, or are some (why not all?) of them related by relations arising from some yet unknown and more fundamental theories. In such theories, some of these parameters may actually be dynamical quantities and, thus, vary in space and time. These are our potential varying constants.

2.2   The constancy of constants as a test of general relativity

The previous paragraphs have yet emphasize why testing for the consistency of the constants is a test of fundamental physics since it can reveal the need for new physical degrees of freedom in our theory. We now want to stress the relation of this test with other tests of general relativity and with cosmology.

2.2.1   General relativity

The tests of the constancy of fundamental constants take all their importance in the realm of the tests of the equivalence principle [540]. Einstein general relativity is based on two independent hypotheses, which can conveniently be described by decomposing the action of the theory as $S = S_{\mathrm{grav}} + S_{\mathrm{matter}}$.

The equivalence principle has strong implication for the functional form of S_{grav}. This principle includes three hypotheses:

- the universality of free fall,

- the local position invariance,

- the local Lorentz invariance.

In its weak form (that is for all interactions but gravity), it is satisfied by any metric theory of gravity and general relativity is conjectured to satisfy it in its strong form (that is for all interactions including gravity). We refer to [540] for a detailed description of these principles. The

weak equivalence principle can be mathematically implemented by assuming that all matter fields are minimally coupled to a single metric tensor $g_{\mu\nu}$. This metric defines the length and times measured by laboratory clocks and rods so that it can be called the *physical metric*. This implies that the action for any matter field, ψ say, can be written as

$$S_{\text{matter}}(\psi, g_{\mu\nu}).$$

This *metric coupling* ensures in particular the validity of the universality of free-fall. Since locally, in the neighborhood of the worldline, there always exists a change of coordinates so that the metric takes a Minkowskian form at lowest order, the gravitational field can be locally "effaced" (up to tidal effects). If we identify this neighborhood to a small lab, this means that any physical properties that can be measured in this lab must be independent of where and when the experiments are carried out. This is indeed the assumption of *local position invariance*, which implies that the constants must take the same value independent of the spacetime point where they are measured. Thus, testing the constancy of fundamental constants is a direct test of this principle and therefore of the metric coupling. Interestingly, the tests we are discussing in this review allow one to extend them much further than the solar scales and even in the early universe, an important information to check the validity of relativity in cosmology.

As an example, the action of a point-particle reads

$$S_{\text{matter}} = - \int mc \sqrt{-g_{\mu\nu}(\mathbf{x}) v^{\mu} v^{\nu}} \, \mathrm{d}t, \tag{2}$$

with $v^{\mu} \equiv \mathrm{d}x^{\mu}/\mathrm{d}t$. The equation of motion that one derives from this action is the usual geodesic equation

$$a^{\mu} \equiv u^{\nu} \nabla_{\nu} u^{\mu} = 0, \tag{3}$$

where $u^{\mu} = \mathrm{d}x^{\mu}/c\mathrm{d}\tau$, τ being the proper time; ∇_{μ} is the covariant derivative associated with the metric $g_{\mu\nu}$ and a^{ν} is the 4-acceleration. Any metric theory of gravity will enjoy such a matter Lagrangian and the worldline of any test particle shall be a geodesic of the spacetime with metric $g_{\mu\nu}$, as long as there is no other long range force acting on it (see [190] for a detailed review of motion in alternative theories of gravity).

Note that in the Newtonian limit $g_{00} = -1 - 2\Phi_N/c^2$ where Φ_N is the Newtonian potential. It follows that, in the slow velocity limit, the geodesic equation reduces to

$$\dot{\mathbf{v}} = \mathbf{a} = -\nabla \Phi_N \equiv \mathbf{g}_N, \tag{4}$$

hence defining the Newtonian acceleration $\mathbf{g}_N$. Recall that the proper time of a clock is related to the coordinate time by $\mathrm{d}\tau = \sqrt{-g_{00}}\mathrm{d}t$. Thus, if one exchanges electromagnetic signals between two identical clocks in a stationary situation, the apparent difference between the two clocks rates will be

$$\frac{\nu_1}{\nu_2} = 1 + \frac{\Phi_N(2) - \Phi_N(1)}{c^2},$$

at lowest order. This is the *universality* of gravitational redshift.

The assumption of a metric coupling is actually well tested in the solar system:

- First, it implies that all non-gravitational constants are spacetime independent, which have been tested to a very high accuracy in many physical systems and for various fundamental constants; this is the subject of this review.

- Second, the isotropy has been tested from the constraint on the possible quadrupolar shift of nuclear energy levels [99, 304, 422] proving that different matter fields couple to a unique metric tensor at the 10^{-27} level.

Table 3: Summary of the constraints on the violation of the universality of free fall.

Constraint	Body 1	Body 2	Ref.
$(-1.9 \pm 2.5) \times 10^{-12}$	Be	Cu	[4]
$(0.1 \pm 2.7 \pm 1.7) \times 10^{-13}$	Earth-like rock	Moon-like rock	[23]
$(-1.0 \pm 1.4) \times 10^{-13}$	Earth	Moon	[543]
$(0.3 \pm 1.8) \times 10^{-13}$	Te	Bi	[450]
$(-0.2 \pm 2.8) \times 10^{-12}$	Be	Al	[481]
$(-1.9 \pm 2.5) \times 10^{-12}$	Be	Cu	[481]
$(5.1 \pm 6.7) \times 10^{-12}$	Si/Al	Cu	[481]

- Third, the universality of free fall can be tested by comparing the accelerations of two test bodies in an external gravitational field. The parameter η_{12} defined as

$$\eta_{12} \equiv 2\frac{|\mathbf{a}_1 - \mathbf{a}_2|}{|\mathbf{a}_1 + \mathbf{a}_2|}, \tag{5}$$

can be constrained experimentally, e.g., in the laboratory by comparing the acceleration of a beryllium and a copper mass in the Earth gravitational field [4] to get

$$\eta_{\mathrm{Be,Cu}} = (-1.9 \pm 2.5) \times 10^{-12}. \tag{6}$$

Similarly the comparison of Earth-core-like and Moon-mantle-like bodies gave [23]

$$\eta_{\mathrm{Earth,Moon}} = (0.1 \pm 2.7 \pm 1.7) \times 10^{-13}, \tag{7}$$

and experiments with torsion balance using test bodies composed of tellurium an bismuth allowed to set the constraint [450]

$$\eta_{\mathrm{Te,Bi}} = (0.3 \pm 1.8) \times 10^{-13}. \tag{8}$$

The Lunar Laser ranging experiment [543], which compares the relative acceleration of the Earth and Moon in the gravitational field of the Sun, also set the constraints

$$\eta_{\mathrm{Earth,Moon}} = (-1.0 \pm 1.4) \times 10^{-13}. \tag{9}$$

Note that since the core represents only 1/3 of the mass of the Earth, and since the Earth's mantle has the same composition as that of the Moon (and thus shall fall in the same way), one loses a factor of three, so that this constraint is actually similar to the one obtained in the lab. Further constraints are summarized in Table 3. The latter constraint also contains some contribution from the gravitational binding energy and thus includes the strong equivalence principle. When the laboratory result of [23] is combined with the LLR results of [542] and [365], one gets a constraints on the strong equivalence principle parameter, respectively

$$\eta_{\mathrm{SEP}} = (3 \pm 6) \times 10^{-13} \quad \text{and} \quad \eta_{\mathrm{SEP}} = (-4 \pm 5) \times 10^{-13}.$$

Large improvements are expected thanks to existence of two dedicated space mission projects: Microscope [493] and STEP [355].

- Fourth, the Einstein effect (or gravitational redshift) has been measured at the 2×10^{-4} level [517].

We can conclude that the hypothesis of metric coupling is extremely well-tested in the solar system.

The second building block of general relativity is related to the dynamics of the gravitational sector, assumed to be dictated by the Einstein–Hilbert action

$$S_{\mathrm{grav}} = \frac{c^3}{16\pi G} \int \sqrt{-g_*} R_* \mathrm{d}^4 x. \tag{10}$$

This defines the dynamics of a massless spin-2 field $g_{\mu\nu}^*$, called the Einstein metric. General relativity then assumes that both metrics coincide, $g_{\mu\nu} = g_{\mu\nu}^*$ (which is related to the strong equivalence principle), but it is possible to design theories in which this is indeed not the case (see the example of scalar-tensor theories below; Section 5.1.1) so that general relativity is one out of a large family of metric theories.

The variation of the total action with respect to the metric yields the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{11}$$

where $T^{\mu\nu} \equiv (2/\sqrt{-g}) \delta S_{\mathrm{matter}} / \delta g_{\mu\nu}$ is the matter stress-energy tensor. The coefficient $8\pi G/c^4$ is determined by the weak-field limit of the theory that should reproduce the Newtonian predictions.

The dynamics of general relativity can be tested in the solar system by using the parameterized post-Newtonian formalism (PPN). Its is a general formalism that introduces 10 phenomenological parameters to describe any possible deviation from general relativity at the first post-Newtonian order [540, 541] (see also [60] for a review on higher orders). The formalism assumes that gravity is described by a metric and that it does not involve any characteristic scale. In its simplest form, it reduces to the two Eddington parameters entering the metric of the Schwartzschild metric in isotropic coordinates

$$g_{00} = -1 + \frac{2Gm}{rc^2} - 2\beta^{\mathrm{PPN}} \left(\frac{2Gm}{rc^2} \right)^2, \qquad g_{ij} = \left(1 + 2\gamma^{\mathrm{PPN}} \frac{2Gm}{rc^2} \right) \delta_{ij}.$$

Indeed, general relativity predicts $\beta^{\mathrm{PPN}} = \gamma^{\mathrm{PPN}} = 1$.

These two phenomenological parameters are constrained (1) by the shift of the Mercury perihelion [457], which implies that $|2\gamma^{\mathrm{PPN}} - \beta^{\mathrm{PPN}} - 1| < 3 \times 10^{-3}$, (2) the Lunar laser ranging experiments [543], which implies that $|4\beta^{\mathrm{PPN}} - \gamma^{\mathrm{PPN}} - 3| = (4.4 \pm 4.5) \times 10^{-4}$ and (3) by the deflection of electromagnetic signals, which are all controlled by γ^{PPN}. For instance the very long baseline interferometry [459] implies that $|\gamma^{\mathrm{PPN}} - 1| = 4 \times 10^{-4}$, while the measurement of the time delay variation to the Cassini spacecraft [53] sets $\gamma^{\mathrm{PPN}} - 1 = (2.1 \pm 2.3) \times 10^{-5}$.

The PPN formalism does not allow to test finite range effects that could be caused, e.g., by a massive degree of freedom. In that case one expects a Yukawa-type deviation from the Newton potential,

$$V = \frac{Gm}{r} \left(1 + \alpha \mathrm{e}^{-r/\lambda} \right),$$

that can be probed by "fifth force" experimental searches. λ characterizes the range of the Yukawa deviation of strength α. The constraints on (λ, α) are summarized in [256], which typically shows that $\alpha < 10^{-2}$ on scales ranging from the millimeter to the solar system size.

General relativity is also tested with pulsars [125, 189] and in the strong field regime [425]. For more details we refer to [129, 495, 540, 541]. Needless to say that any extension of general relativity has to pass these constraints. However, deviations from general relativity can be larger in the past, as we shall see, which makes cosmology an interesting physical system to extend these constraints.

2.2.2   Varying constants and the universality of free fall

As the previous description shows, the constancy of the fundamental constants and the universality are two pillars of the equivalence principle. Dicke [152] realized that they are actually not independent and that if the coupling constants are spatially dependent then this will induce a violation of the universality of free fall.

The connection lies in the fact that the mass of any composite body, starting, e.g., from nuclei, includes the mass of the elementary particles that constitute it (this means that it will depend on the Yukawa couplings and on the Higgs sector parameters) but also a contribution, E_{binding}/c^2, arising from the binding energies of the different interactions (i.e., strong, weak and electromagnetic) but also gravitational for massive bodies. Thus, the mass of any body is a complicated function of all the constants, $m[\alpha_i]$.

It follows that the action for a point particle is no more given by Equation (2) but by

$$S_{\mathrm{matter}} = -\int m_A[\alpha_j] c \sqrt{-g_{\mu\nu}(\mathbf{x}) v^\mu v^\nu}\, \mathrm{d}t, \tag{12}$$

where α_j is a list of constants including α_{EM} but also many others and where the index A in m_A recalls that the dependency in these constants is a priori different for bodies of different chemical composition. The variation of this action gives the equation of motion

$$u^\nu \nabla_\nu u^\mu = -\left(\sum_i \frac{\partial \ln m_A}{\partial \alpha_i} \frac{\partial \alpha_i}{\partial x^\beta}\right) \left(g^{\beta\mu} + u^\beta u^\mu\right). \tag{13}$$

It follows that a test body will not enjoy a geodesic motion. In the Newtonian limit $g_{00} = -1 + 2\Phi_N/c^2$, and at first order in v/c, the equation of motion of a test particle reduces to

$$\mathbf{a} = \mathbf{g}_N + \delta\mathbf{a}_A, \qquad \delta\mathbf{a}_A = -c^2 \sum_i f_{A,i} \left(\nabla \alpha_i + \dot\alpha_i \frac{\mathbf{v}_A}{c^2}\right) \tag{14}$$

so that in the slow velocity (and slow variation) limit it reduces to

$$\delta\mathbf{a}_A = -c^2 \sum_i f_{A,i} \nabla \alpha_i.$$

where we have introduce the sensitivity of the mass A with respect to the variation of the constant α_i by

$$f_{A,i} \equiv \frac{\partial \ln m_A}{\partial \alpha_i}. \tag{15}$$

This simple argument shows that if the constants depend on time then there must exist an anomalous acceleration that will depend on the chemical composition of the body A.

This anomalous acceleration is generated by the change in the (electromagnetic, gravitational, ...) binding energies [152, 246, 386] but also in the Yukawa couplings and in the Higgs sector parameters so that the α_i-dependencies are a priori composition-dependent. As a consequence, any variation of the fundamental constants will entail a violation of the universality of free fall: the total mass of the body being space dependent, an anomalous force appears if energy is to be conserved. The variation of the constants, deviation from general relativity and violation of the weak equivalence principle are in general expected together.

On the other hand, the composition dependence of $\delta\mathbf{a}_A$ and thus of η_{AB} can be used to optimize the choice of materials for the experiments testing the equivalence principle [118, 120, 122] but also to distinguish between several models if data from the universality of free fall and atomic clocks are combined [143].

From a theoretical point of view, the computation of η_{AB} will requires the determination of the coefficients f_{Ai}. This can be achieved in two steps by first relating the new degrees of freedom of the theory to the variation of the fundamental constants and then relating them to the variation of the masses. As we shall see in Section 5, the first issue is very model dependent while the second is especially difficult, particularly when one wants to understand the effect of the quark mass, since it is related to the intricate structure of QCD and its role in low energy nuclear reactions.

As an example, the mass of a nuclei of charge Z and atomic number A can be expressed as

$$m(A, Z) = Zm_{\rm p} + (A - Z)m_{\rm n} + Zm_{\rm e} + E_{\rm S} + E_{\rm EM},$$

where $E_{\rm S}$ and $E_{\rm EM}$ are respectively the strong and electromagnetic contributions to the binding energy. The Bethe–Weizäcker formula allows to estimate the latter as

$$E_{\rm EM} = 98.25 \frac{Z(Z - 1)}{A^{1/3}} \alpha_{\rm EM} \text{ MeV}. \tag{16}$$

If we decompose the proton and neutron masses as [230] $m_{\rm (p,n)} = u_3 + b_{\rm (u,d)}m_{\rm u} + b_{\rm (d,u)}m_{\rm d} + B_{\rm (p,n)}\alpha_{\rm EM}$ where u_3 is the pure QCD approximation of the nucleon mass ($b_{\rm u}$, $b_{\rm d}$ and $B_{\rm (n,p)}/u_3$ being pure numbers), it reduces to

$$m(A, Z) = (Au_3 + E_{\rm S}) + (Zb_{\rm u} + Nb_{\rm d})m_{\rm u} + (Zb_{\rm d} + Nb_{\rm u})m_{\rm d}$$
$$+ \left(ZB_{\rm p} + NB_{\rm n} + 98.25 \frac{Z(Z - 1)}{A^{1/3}} \text{ MeV} \right) \alpha_{\rm EM}, \tag{17}$$

with $N = A - Z$, the neutron number. For an atom, one would have to add the contribution of the electrons, $Zm_{\rm e}$. This form depends on strong, weak and electromagnetic quantities. The numerical coefficients $B_{\rm (n,p)}$ are given explicitly by [230]

$$B_{\rm p}\alpha_{\rm EM} = 0.63 \text{ MeV} \quad B_{\rm n}\alpha_{\rm EM} = -0.13 \text{ MeV}. \tag{18}$$

Such estimations were used in the first analysis of the relation between variation of the constant and the universality of free fall [135, 166] but the dependency on the quark mass is still not well understood and we refer to [120, 122, 157, 159, 208] for some attempts to refine this description.

For macroscopic bodies, the mass has also a negative contribution

$$\Delta m(G) = -\frac{G}{2c^2} \int \frac{\rho(\vec{r})\rho(\vec{r'})}{|\vec{r} - \vec{r'}|} {\rm d}^3\vec{r}{\rm d}^3\vec{r'} \tag{19}$$

from the gravitational binding energy. As a conclusion, from (17) and (19), we expect the mass to depend on all the coupling constant, $m(\alpha_{\rm EM}, \alpha_{\rm W}, \alpha_{\rm S}, \alpha_{\rm G}, ...)$. We shall discuss this issue in more detail in Section 5.3.2.

Note that varying coupling constants can also be associated with violations of local Lorentz invariance and CPT symmetry [298, 52, 242].

2.2.3 Relations with cosmology

Most constraints on the time variation of the fundamental constants will not be local and related to physical systems at various epochs of the evolution of the universe. It follows that the comparison of different constraints requires a full cosmological model.

Our current cosmological model is known as the ΛCDM (see [409] for a detailed description, and Table 4 for the typical value of the cosmological parameters). It is important to recall that its construction relies on 4 main hypotheses: (H1) a theory of gravity; (H2) a description of the matter components contained in the Universe and their non-gravitational interactions; (H3)

symmetry hypothesis; and (H4) a hypothesis on the global structure, i.e., the topology, of the Universe. These hypotheses are not on the same footing since H1 and H2 refer to the physical theories. However, these hypotheses are not sufficient to solve the field equations and we must make an assumption on the symmetries (H3) of the solutions describing our Universe on large scales while H4 is an assumption on some global properties of these cosmological solutions, with same local geometry. But the last two hypothesis are unavoidable because the knowledge of the fundamental theories is not sufficient to construct a cosmological model [504].

The ΛCDM model assumes that gravity is described by general relativity (H1), that the Universe contains the fields of the standard model of particle physics plus some dark matter and a cosmological constant, the latter two having no physical explanation at the moment. It also deeply involves the Copernican principle as a symmetry hypothesis (H3), without which the Einstein equations usually cannot been solved, and assumes most often that the spatial sections are simply connected (H4). H2 and H3 imply that the description of the standard matter reduces to a mixture of a pressureless and a radiation perfect fluids. This model is compatible with all astronomical data, which roughly indicates that $\Omega_{\Lambda 0} \simeq 0.73$, $\Omega_{\mathrm{mat}0} \simeq 0.27$, and $\Omega_{K0} \simeq 0$. Thus, cosmology roughly imposes that $|\Lambda_0| \leq H_0^2$, that is $\ell_\Lambda \leq H_0^{-1} \sim 10^{26}$ m $\sim 10^{41}$ GeV^{-1}.

Hence, the analysis of the cosmological dynamics of the universe and of its large scale structures requires the introduction of a new constant, the *cosmological constant*, associated with a recent acceleration of the cosmic expansion, that can be introduced by modifying the Einstein–Hilbert action to

$$S_{\mathrm{grav}} = \frac{c^3}{16\pi G} \int \sqrt{-g}(R - 2\Lambda)\mathrm{d}^4 x.$$

This constant can equivalently be introduced in the matter action. Note, however, that it is disproportionately small compared to the natural scale fixed by the Planck length

$$\rho_{\Lambda_0} \sim 10^{-120} M_{\mathrm{Pl}}^4 \sim 10^{-47} \text{ GeV}^4. \qquad (20)$$

Classically, this value is no problem but it was pointed out that at the quantum level, the vacuum energy should scale as M^4, where M is some energy scale of high-energy physics. In such a case, there is a discrepancy of $60 - 120$ order of magnitude between the cosmological conclusions and the theoretical expectation. This is the *cosmological constant problem* [528].

Two approaches to solve this problem have been considered. Either one accepts such a constant and such a fine-tuning and tries to explain it on anthropic ground. Or, in the same spirit as Dirac, one interprets it as an indication that our set of cosmological hypotheses have to be extended, by either abandoning the Copernican principle [508] or by modifying the local physical laws (either gravity or the matter sector). The way to introduce such new physical degrees of freedom were classified in [502]. In that latter approach, the tests of the constancy of the fundamental constants are central, since they can reveal the coupling of this new degree of freedom to the standard matter fields. Note, however, that the cosmological data still favor a pure cosmological constant.

Among all the proposals quintessence involves a scalar field rolling down a runaway potential hence acting as a fluid with an effective equation of state in the range $-1 \leq w \leq 1$ if the field is minimally coupled. It was proposed that the quintessence field is also the dilaton [229, 434, 499]. The same scalar field then drives the time variation of the cosmological constant and of the gravitational constant and it has the property to also have tracking solutions [499]. Such models do not solve the cosmological constant problem but only relieve the coincidence problem. One of the underlying motivation to replace the cosmological constant by a scalar field comes from superstring models in which any dimensionful parameter is expressed in terms of the string mass scale and the vacuum expectation value of a scalar field. However, the requirement of slow roll (mandatory to have a negative pressure) and the fact that the quintessence field dominates today imply, if the minimum of the potential is zero, that it is very light, roughly of order $m \sim 10^{-33}$ eV [81].

Table 4: Main cosmological parameters in the standard Λ-CDM model. There are 7 main parameters (because $\sum \Omega_i = 0$) to which one can add 6 more to include dark energy, neutrinos and gravity waves. Note that often the spatial curvature is set to $\Omega_K = 0$. (See, e.g. Refs. [296, 409]).

Parameter	Symbol	Value
Reduced Hubble constant	h	0.73(3)
Baryon-to-photon ratio	$\eta = n_{\rm b}/n_\gamma$	$6.12(19) \times 10^{-10}$
Photon density	$\Omega_\gamma h^2$	2.471×10^{-5}
Dark matter density	$\Omega_{\rm CDM} h^2$	0.105(8)
Cosmological constant	Ω_Λ	0.73(3)
Spatial curvature	Ω_K	0.011(12)
Scalar modes amplitude	Q	$(2.0 \pm 0.2) \times 10^{-5}$
Scalar spectral index	n_S	0.958(16)
Neutrino density	$\Omega_\nu h^2$	$(0.0005 - 0.023)$
Dark energy equation of state	w	–0.97(7)
Scalar running spectral index	α_S	-0.05 ± 0.03
Tensor-to-scalar ratio	T/S	< 0.36
Tensor spectral index	n_T	< 0.001
Tensor running spectral index	α_T	?
Baryon density	$\Omega_{\rm b} h^2$	0.0223(7)

Such a light field can lead to observable violations of the universality of free fall if it is non-universally coupled to the matter fields. Carroll [81] considered the effect of the coupling of this very light quintessence field to ordinary matter via a coupling to the electromagnetic field as $\phi F^{\mu\nu} \widetilde{F}_{\mu\nu}$. Chiba and Kohri [96] also argued that an ultra-light quintessence field induces a time variation of the coupling constant if it is coupled to ordinary matter and studied a coupling of the form $\phi F^{\mu\nu} F_{\mu\nu}$, as, e.g., expected from Kaluza–Klein theories (see below). This was generalized to quintessence models with a couplings of the form $Z(\phi) F^{\mu\nu} F_{\mu\nu}$ [11, 112, 162, 315, 314, 347, 404, 531] and then to models of runaway dilaton [133, 132] inspired by string theory (see Section 5.4.1). The evolution of the scalar field drives both the acceleration of the universe at late time and the variation of the constants. As pointed in [96, 166, 532] such models are extremely constrained from the bound on the universality of free-fall (see Section 6.3).

We have two means of investigation:

- The field driving the time variation of the fundamental constants does not explain the acceleration of the universe (either it does not dominate the matter content today or its equation of state is not negative enough). In such a case, the variation of the constants is disconnected from the dark energy problem. Cosmology allows to determine the dynamics of this field during the whole history of the universe and thus to compare local constraints and cosmological constraints. An example is given by scalar-tensor theories (see Section 5.1.1) for which one can compare, e.g., primordial nucleosynthesis to local constraints [134]. However, in such a situation one should take into account the effect of the variation of the constants on the astrophysical observations since it can affect local physical processes and bias, e.g., the luminosity of supernovae and indirectly modify the distance luminosity-redshift relation derived from these observations [33, 435].

- The field driving the time variation of the fundamental constants is also responsible for the acceleration of the universe. It follows that the dynamics of the universe, the level of variation of the constants and the other deviations from general relativity are connected [348] so that the study of the variation of the constants can improve the reconstruction of the equation

state of the dark energy [20, 162, 389, 404].

In conclusion, cosmology seems to require a new constant. It also provides a link between the microphysics and cosmology, as foreseen by Dirac. The tests of fundamental constants can discriminate between various explanations of the acceleration of the universe. When a model is specified, cosmology also allows to set stringer constraints since it relates observables that cannot be compared otherwise.

3 Experimental and Observational Constraints

This section focuses on the experimental and observational constraints on the non-gravitational constants, that is assuming α_{G} remains constant. We use the convention that $\Delta\alpha = \alpha - \alpha_0$ for any constant α, so that $\Delta\alpha < 0$ refers to a value smaller than today.

The various physical systems that have been considered can be classified in many ways. We can classify them according to their look-back time and more precisely their space-time position relative to our actual position. This is summarized in Figure 1. Indeed higher redshift systems offer the possibility to set constraints on a larger time scale, but this is at the expense of usually involving other parameters such as the cosmological parameters. This is, in particular, the case of the cosmic microwave background or of primordial nucleosynthesis. The systems can also be classified in terms of the physics they involve. For instance, atomics clocks, quasar absorption spectra and the cosmic microwave background require only to use quantum electrodynamics to draw the primary constraints while the Oklo phenomenon, meteorites dating and nucleosynthesis require nuclear physics.

For any system, setting constraints goes through several steps. First we have some observable quantities from which we can draw constraints on primary constants, which may not be fundamental constants (e.g., the BBN parameters, the lifetime of β-decayers, ...). These primary parameters must then be related to some fundamental constants such as masses and couplings. In a last step, the number of constants can be reduced by relating them in some unification schemes. Indeed each step requires a specific modelization and hypothesis and has its own limitations. This is summarized on Table 5.

Table 5: Summary of the systems considered to set constraints on the variation of the fundamental constants. We summarize the observable quantities, the primary constants used to interpret the data and the other hypothesis required for this interpretation. All the quantities appearing in this table are defined in the text.

System	Observable	Primary constraints	Other hypothesis
Atomic clock	$\delta \ln \nu$	$g_i, \alpha_{\mathrm{EM}}, \mu$	–
Oklo phenomenon	isotopic ratio	E_r	geophysical model
Meteorite dating	isotopic ratio	λ	–
Quasar spectra	atomic spectra	$g_{\mathrm{p}}, \mu, \alpha_{\mathrm{EM}}$	cloud physical properties
Stellar physics	element abundances	B_D	stellar model
21 cm	T_b/T_{CMB}	$g_{\mathrm{p}}, \mu, \alpha_{\mathrm{EM}}$	cosmological model
CMB	$\Delta T/T$	$\mu, \alpha_{\mathrm{EM}}$	cosmological model
BBN	light element abundances	$Q_{\mathrm{np}}, \tau_{\mathrm{n}}, m_{\mathrm{e}}, m_{\mathrm{N}}, \alpha_{\mathrm{EM}}, B_D$	cosmological model

3.1 Atomic clocks

3.1.1 Atomic spectra and constants

The laboratory constraints on the time variation of fundamental constants are obtained by comparing the long-term behavior of several oscillators and rely on frequency measurements. The atomic transitions have various dependencies in the fundamental constants. For instance, for the hydrogen atom, the gross, fine and hyperfine-structures are roughly given by

$$2p - 1s : \ \nu \propto cR_\infty, \qquad 2p_{3/2} - 2p_{1/2} : \ \nu \propto cR_\infty\alpha_{\mathrm{EM}}^2, \qquad 1s : \ \propto cR_\infty\alpha_{\mathrm{EM}}^2 g_{\mathrm{p}}\bar{\mu},$$

respectively, where the Rydberg constant set the dimension. g_{p} is the proton gyromagnetic factor and $\bar{\mu} = m_{\mathrm{e}}/m_{\mathrm{p}}$. In the non-relativistic approximation, the transitions of all atoms have similar
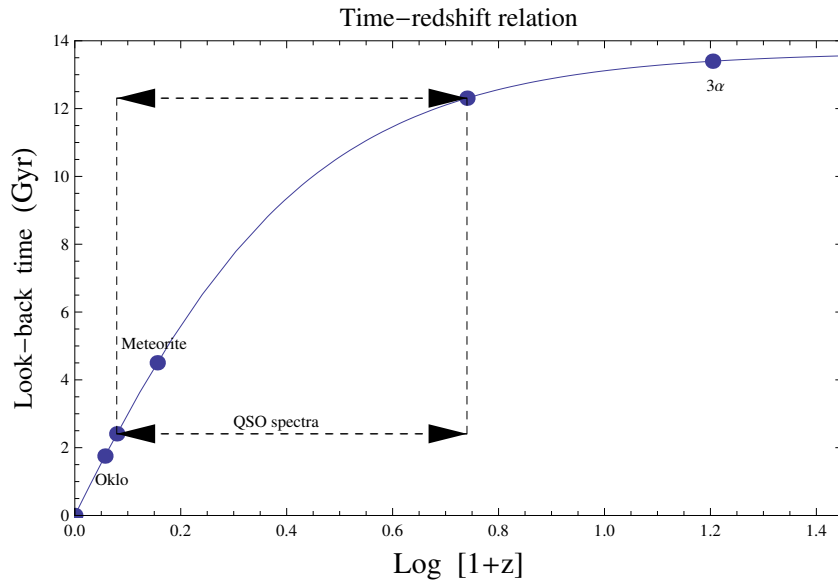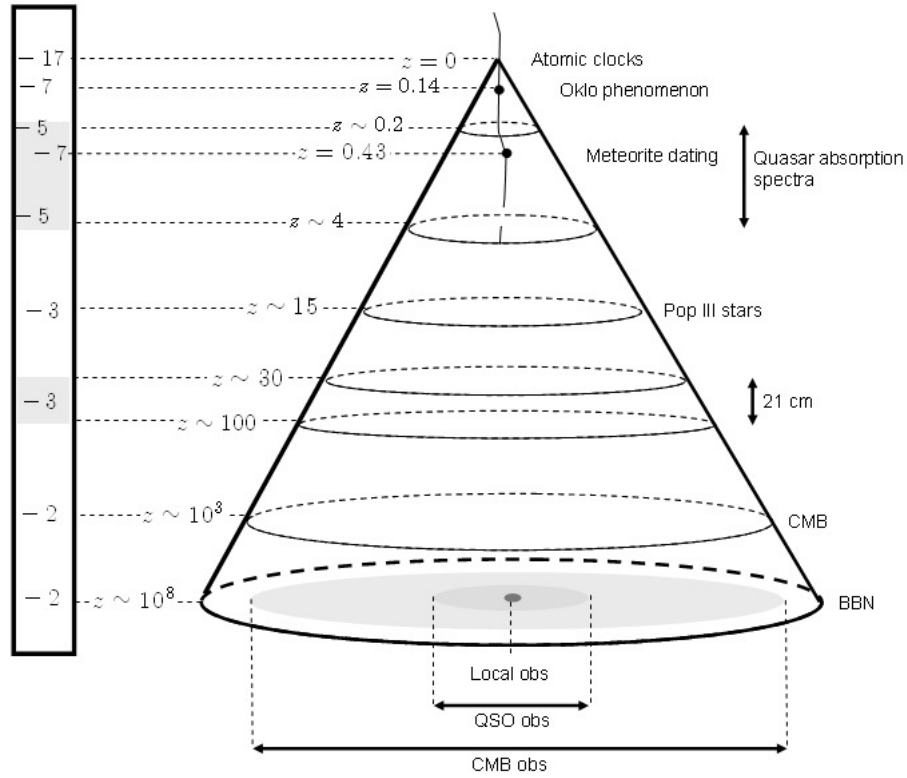
Figure 1: *Top:* Summary of the systems that have been used to probe the constancy of the fundamental constants and their position in a space-time diagram in which the cone represents our past light cone. The shaded areas represents the comoving space probed by different tests. *Bottom:* The look-back time-redshift relation for the standard ΛCDM model.

dependencies but two effects have to be taken into account. First, the hyperfine-structures involve a gyromagnetic factor g_i (related to the nuclear magnetic moment by $\mu_i = g_i \mu_N$, with $\mu_N = e\hbar/2m_p c$), which are different for each nuclei. Second, relativistic corrections (including the Casimir contribution), which also depend on each atom (but also on the type of the transition) can be included through a multiplicative function $F_{\rm rel}(\alpha_{\rm EM})$. It has a strong dependence on the atomic number Z, which can be illustrated on the case of alkali atoms, for which

$$F_{\rm rel}(\alpha_{\rm EM}) = \left[1 - (Z\alpha_{\rm EM})^2\right]^{-1/2} \left[1 - \frac{4}{3}(Z\alpha_{\rm EM})^2\right]^{-1} \simeq 1 + \frac{11}{6}(Z\alpha_{\rm EM})^2.$$

The developments of highly accurate atomic clocks using different transitions in different atoms offer the possibility to test a variation of various combinations of the fundamental constants.

It follows that at the lowest level of description, we can interpret all atomic clocks results in terms of the g-factors of each atoms, g_i, the electron to proton mass ration μ and the fine-structure constant $\alpha_{\rm EM}$. We shall parameterize the hyperfine and fine-structures frequencies as follows.

The hyperfine frequency in a given electronic state of an alkali-like atom, such as ^{133}Cs, ^{87}Rb, ^{199}Hg$^+$, is

$$\nu_{\rm hfs} \simeq R_\infty c \times A_{\rm hfs} \times g_i \times \alpha_{\rm EM}^2 \times \bar{\mu} \times F_{\rm hfs}(\alpha) \tag{21}$$

where $g_i = \mu_i/\mu_N$ is the nuclear g factor. $A_{\rm hfs}$ is a numerical factor depending on each particular atom and we have set $F_{\rm rel} = F_{\rm hfs}(\alpha)$. Similarly, the frequency of an electronic transition is well-approximated by

$$\nu_{\rm elec} \simeq R_\infty c \times A_{\rm elec} \times F_{\rm elec}(Z, \alpha), \tag{22}$$

where, as above, $A_{\rm elec}$ is a numerical factor depending on each particular atom and $F_{\rm elec}$ is the function accounting for relativistic effects, spin-orbit couplings and many-body effects. Even though an electronic transition should also include a contribution from the hyperfine interaction, it is generally only a small fraction of the transition energy and thus should not carry any significant sensitivity to a variation of the fundamental constants.

The importance of the relativistic corrections was probably first emphasized in [423] and their computation through relativistic N-body calculations was carried out for many transitions in [170, 174, 175, 198]. They can be characterized by introducing the sensitivity of the relativistic factors to a variation of $\alpha_{\rm EM}$,

$$\kappa_\alpha \equiv \frac{\partial \ln F}{\partial \ln \alpha_{\rm EM}}. \tag{23}$$

Table 6 summarizes the values of some of them, as computed in [175, 210]. Indeed a reliable knowledge of these coefficients at the 1% to 10% level is required to deduce limits to a possible variation of the constants. The interpretation of the spectra in this context relies, from a theoretical point of view, only on quantum electrodynamics (QED), a theory, which is well tested experimentally [280] so that we can safely obtain constraints on $(\alpha_{\rm EM}, \mu, g_i)$, still keeping in mind that the computation of the sensitivity factors required numerical N-body simulations.

From an experimental point of view, various combinations of clocks have been performed. It is important to analyze as many species as possible in order to rule-out species-dependent systematic effects. Most experiments are based on a frequency comparison to caesium clocks. The hyperfine splitting frequency between the $F = 3$ and $F = 4$ levels of its $^2S_{1/2}$ ground state at 9.192 GHz has been used for the definition of the second since 1967. One limiting effect, that contributes mostly to the systematic uncertainty, is the frequency shift due to cold collisions between the atoms. On this particular point, clocks based on the hyperfine frequency of the ground state of the rubidium at 6.835 GHz, are more favorable.

Table 6: Sensitivity of various transitions on a variation of the fine-structure constant.

Atom	Transition	sensitivity κ_α
^{1}H	$1s - 2s$	0.00
^{87}Rb	hf	0.34
^{133}Cs	$^2S_{1/2}(F=2) - (F=3)$	0.83
^{171}Yb$^+$	$^2S_{1/2} - {}^2D_{3/2}$	0.9
^{199}Hg$^+$	$^2S_{1/2} - {}^2D_{5/2}$	-3.2
^{87}Sr	$^1S_0 - {}^3P_0$	0.06
^{27}Al$^+$	$^1S_0 - {}^3P_0$	0.008

3.1.2 Experimental constraints

We present the latest results that have been obtained and refer to Section III.B.2 of FCV [500] for earlier studies. They all rely on the developments of new atomic clocks, with the primarily goal to define better frequency standards.

Table 7: Summary of the constraints obtained from the comparisons of atomic clocks. For each constraint on the relative drift of the frequency of the two clocks, we provide the dependence in the various constants, using the numbers of Table 6. From Ref. [379], which can be consulted for other constants.

Clock 1	Clock 2 $\frac{\mathrm{d}}{\mathrm{d}t} \ln\left(\frac{\nu_{\text{clock 1}}}{\nu_{\text{clock 2}}}\right)$	Constraint (yr^{-1})	Constants dependence	Reference
^{87}Rb	^{133}Cs	$(0.2 \pm 7.0) \times 10^{-16}$	$\frac{g_{\text{Cs}}}{g_{\text{Rb}}} \alpha_{\text{EM}}^{0.49}$	[346]
^{87}Rb	^{133}Cs	$(-0.5 \pm 5.3) \times 10^{-16}$		[58]
^{1}H	^{133}Cs	$(-32 \pm 63) \times 10^{-16}$	$g_{\text{Cs}}\bar{\mu}\alpha_{\text{EM}}^{2.83}$	[196]
^{199}Hg$^+$	^{133}Cs	$(0.2 \pm 7) \times 10^{-15}$	$g_{\text{Cs}}\bar{\mu}\alpha_{\text{EM}}^{6.05}$	[57]
^{199}Hg$^+$	^{133}Cs	$(3.7 \pm 3.9) \times 10^{-16}$		[214]
^{171}Yb$^+$	^{133}Cs	$(-1.2 \pm 4.4) \times 10^{-15}$	$g_{\text{Cs}}\bar{\mu}\alpha_{\text{EM}}^{1.93}$	[408]
^{171}Yb$^+$	^{133}Cs	$(-0.78 \pm 1.40) \times 10^{-15}$		[407]
^{87}Sr	^{133}Cs	$(-1.0 \pm 1.8) \times 10^{-15}$	$g_{\text{Cs}}\bar{\mu}\alpha_{\text{EM}}^{2.77}$	[61]
^{87}Dy	^{87}Dy	$(-2.7 \pm 2.6) \times 10^{-15}$	α_{EM}	[100]
^{27}Al$^+$	^{199}Hg$^+$	$(-5.3 \pm 7.9) \times 10^{-17}$	$\alpha_{\text{EM}}^{-3.208}$	[440]

- *Rubidium*: The comparison of the hyperfine frequencies of the rubidium and caesium in their electronic ground state between 1998 and 2003, with an accuracy of order 10^{-15}, leads to the constraint [346]

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln\left(\frac{\nu_{\text{Rb}}}{\nu_{\text{Cs}}}\right) = (0.2 \pm 7.0) \times 10^{-16} \text{ yr}^{-1}. \tag{24}$$

With one more year of experiment, the constraint dropped to [58]

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln\left(\frac{\nu_{\text{Rb}}}{\nu_{\text{Cs}}}\right) = (-0.5 \pm 5.3) \times 10^{-16} \text{ yr}^{-1}. \tag{25}$$

From Equation (21), and using the values of the sensitivities κ_α, we deduce that comparison constrains

$$\frac{\nu_{\text{Cs}}}{\nu_{\text{Rb}}} \propto \frac{g_{\text{Cs}}}{g_{\text{Rb}}} \alpha_{\text{EM}}^{0.49}.$$
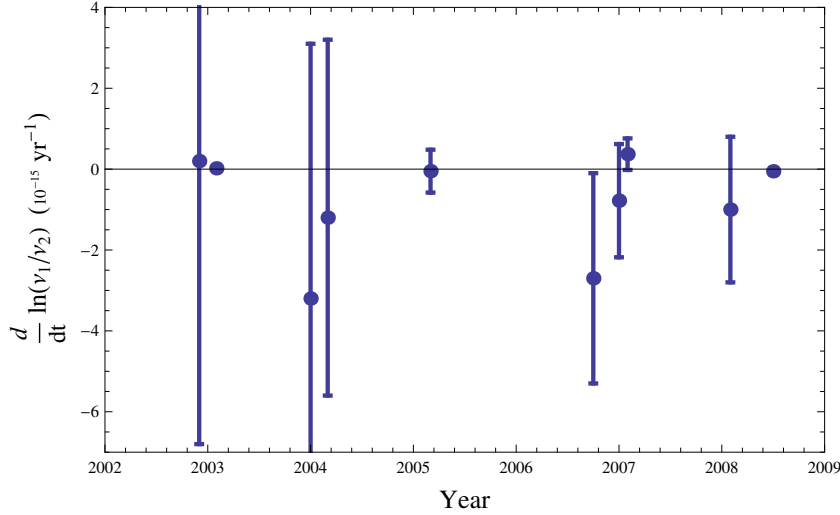
Figure 2: Evolution of the comparison of different atomic clocks summarized in Table 7.

- *Atomic hydrogen*: The $1s - 2s$ transition in atomic hydrogen was compared tp the ground state hyperfine splitting of caesium [196] in 1999 and 2003, setting an upper limit on the variation of ν_H of (-29 ± 57) Hz within 44 months. This can be translated in a relative drift

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln\left(\frac{\nu_H}{\nu_{Cs}}\right) = (-32 \pm 63) \times 10^{-16} \text{ yr}^{-1}. \qquad (26)$$

Since the relativistic correction for the atomic hydrogen transition nearly vanishes, we have $\nu_H \sim R_\infty$ so that

$$\frac{\nu_{Cs}}{\nu_H} \propto g_{Cs} \bar{\mu} \alpha_{EM}^{2.83}.$$

- *Mercury*: The $^{199}\text{Hg}^+$ $^2S_{1/2} - {}^2D_{5/2}$ optical transition has a high sensitivity to α_{EM} (see Table 6) so that it is well suited to test its variation. The frequency of the $^{199}\text{Hg}^+$ electric quadrupole transition at 282 nm was compared to the ground state hyperfine transition of caesium during a two year period, which lead to [57]

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln\left(\frac{\nu_{Hg}}{\nu_{Cs}}\right) = (0.2 \pm 7) \times 10^{-15} \text{ yr}^{-1}. \qquad (27)$$

This was improved by a comparison over a 6 year period [214] to get

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln\left(\frac{\nu_{Hg}}{\nu_{Cs}}\right) = (3.7 \pm 3.9) \times 10^{-16} \text{ yr}^{-1}. \qquad (28)$$

While ν_{Cs} is still given by Equation (21), ν_{Hg} is given by Equation (22). Using the sensitivities of Table 6, we conclude that this comparison test the stability of

$$\frac{\nu_{Cs}}{\nu_{Hg}} \propto g_{Cs} \bar{\mu} \alpha_{EM}^{6.05}.$$

- *Ytterbium*: The $^2S_{1/2} - {}^2D_{3/2}$ electric quadrupole transition at 688 THz of $^{171}\text{Yb}^+$ was compared to the ground state hyperfine transition of cesium. The constraint of [408] was

updated, after comparison over a six year period, which lead to [407]

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln \left(\frac{\nu_{\mathrm{Yb}}}{\nu_{\mathrm{Cs}}} \right) = (-0.78 \pm 1.40) \times 10^{-15} \ \mathrm{yr}^{-1}. \tag{29}$$

Proceeding as previously, this tests the stability of

$$\frac{\nu_{\mathrm{Cs}}}{\nu_{\mathrm{Yb}}} \propto g_{\mathrm{Cs}} \bar{\mu} \alpha_{\mathrm{EM}}^{1.93}.$$

- *Strontium*: The comparison of the $^{1}S_{0} - {}^{3}P_{0}$ transition in neutral ^{87}Sr with a cesium clock was performed in three independent laboratories. The combination of these three experiments [61] leads to the constraint

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln \left(\frac{\nu_{\mathrm{Sr}}}{\nu_{\mathrm{Cs}}} \right) = (-1.0 \pm 1.8) \times 10^{-15} \ \mathrm{yr}^{-1}. \tag{30}$$

Proceeding as previously, this tests the stability of

$$\frac{\nu_{\mathrm{Cs}}}{\nu_{\mathrm{Sr}}} \propto g_{\mathrm{Cs}} \bar{\mu} \alpha_{\mathrm{EM}}^{2.77}.$$

- *Atomic dyprosium*: It was suggested in [175, 174] (see also [173] for a computation of the transition amplitudes of the low states of dyprosium) that the electric dipole (E1) transition between two nearly degenerate opposite-parity states in atomic dyprosium should be highly sensitive to the variation of α_{EM}. It was then demonstrated [384] that a constraint of the order of 10^{-18}/yr can be reached. The frequencies of nearly of two isotopes of dyprosium were monitored over a 8 months period [100] showing that the frequency variation of the 3.1-MHz transition in ^{163}Dy and the 235-MHz transition in ^{162}Dy are 9.0 ± 6.7 Hz/yr and -0.6 ± 6.5 Hz/yr, respectively. These provide the constraint

$$\frac{\dot{\alpha_{\mathrm{EM}}}}{\alpha_{\mathrm{EM}}} = (-2.7 \pm 2.6) \times 10^{-15} \ \mathrm{yr}^{-1}, \tag{31}$$

at 1σ level, without any assumptions on the constancy of other fundamental constants.

- *Aluminium and mercury single-ion optical clocks*: The comparison of the $^{1}S_{0} - {}^{3}P_{0}$ transition in ^{27}Al^{+} and $^{2}S_{1/2} - {}^{2}D_{5/2}$ in ^{199}Hg^{+} over a year allowed to set the constraint [440]

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln \left(\frac{\nu_{\mathrm{Al}}}{\nu_{\mathrm{Hg}}} \right) = (-5.3 \pm 7.9) \times 10^{-17} \ \mathrm{yr}^{-1}. \tag{32}$$

Proceeding as previously, this tests the stability of

$$\frac{\nu_{\mathrm{Hg}}}{\nu_{\mathrm{Al}}} \propto \alpha_{\mathrm{EM}}^{-3.208},$$

which directly set the constraint

$$\frac{\dot{\alpha_{\mathrm{EM}}}}{\alpha_{\mathrm{EM}}} = (-1.6 \pm 2.3) \times 10^{-17} \ \mathrm{yr}^{-1}, \tag{33}$$

since it depends only on α_{EM}.

While the constraint (33) was obtained directly from the clock comparison, the other studies need to be combined to disentangle the contributions of the various constants. As an example, we first use the bound (33) on $\alpha_{\rm EM}$, we can then extract the two following bounds

$$\frac{\rm d}{{\rm d}t} \ln \left(\frac{g_{\rm Cs}}{g_{\rm Rb}}\right) = (0.48 \pm 6.68) \times 10^{-16} \ {\rm yr}^{-1}, \qquad \frac{\rm d}{{\rm d}t} \ln \left(g_{\rm Cs}\bar{\mu}\right) = (4.67 \pm 5.29) \times 10^{-16} \ {\rm yr}^{-1}, \quad (34)$$

on a time scale of a year. We cannot lift the degeneracies further with this clock comparison, since that would require a constraint on the time variation of μ. All these constraints are summarized in Table 7 and Figure 2.

A solution is to consider diatomic molecules since, as first pointed out by Thomson [488], molecular lines can provide a test of the variation of μ. The energy difference between two adjacent rotational levels in a diatomic molecule is inversely proportional to Mr^{-2}, r being the bond length and M the reduced mass, and the vibrational transition of the same molecule has, in first approximation, a $\sqrt{M}$ dependence. For molecular hydrogen $M = m_{\rm p}/2$ so that the comparison of an observed vibro-rotational spectrum with a laboratory spectrum gives an information on the variation of $m_{\rm p}$ and $m_{\rm n}$. Comparing pure rotational transitions with electronic transitions gives a measurement of μ. It follows that the frequency of vibro-rotation transitions is, in the Born–Oppenheimer approximation, of the form

$$\nu \simeq E_I \left(c_{\rm elec} + c_{\rm vib}\sqrt{\bar{\mu}} + c_{\rm rot}\bar{\mu}\right) \qquad (35)$$

where $c_{\rm elec}$, $c_{\rm vib}$ and $c_{\rm rot}$ are some numerical coefficients.

The comparison of the vibro-rotational transition in the molecule SF6 was compared to a caesium clock over a two-year period, leading to the constraint [464]

$$\frac{\rm d}{{\rm d}t} \ln \left(\frac{\nu_{\rm SF6}}{\nu_{\rm Cs}}\right) = (1.9 \pm 0.12 \pm 2.7) \times 10^{-14} \ {\rm yr}^{-1}, \qquad (36)$$

where the second error takes into account uncontrolled systematics. Now, using again Table 6, we deduce that

$$\frac{\nu_{\rm SF6}}{\nu_{\rm Cs}} \propto \bar{\mu}^{1/2}\alpha_{\rm EM}^{-2.83}(g_{\rm Cs}\bar{\mu})^{-1}.$$

It can be combined with the constraint (26), which enjoys the same dependence to cesium to infer that

$$\frac{\dot{\mu}}{\mu} = (-3.8 \pm 5.6) \times 10^{-14} \ {\rm yr}^{-1}. \qquad (37)$$

Combined with Equation (34), we can obtain independent constraints on the time variation of $g_{\rm Cs}$, $g_{\rm Rb}$ and μ.

3.1.3   Physical interpretation

The theoretical description must be pushed further if ones wants to extract constraints on constant more fundamental than the nuclear magnetic moments. This requires one to use quantum chromodynamics. In particular, it was argued than within this theoretical framework, one can relate the nucleon g-factors in terms of the quark mass and the QCD scale [198]. Under the assumption of a unification of the three non-gravitational interaction (see Section 6.3), the dependence of the magnetic moments on the quark masses was investigated in [210]. The magnetic moments, or equivalently the g-factors, are first related to the ones of the proton and a neutron to derive a relation of the form

$$g \propto g_{\rm p}^{a_{\rm p}} g_{\rm n}^{a_{\rm n}}.$$

[198, 210] argued that these g-factors mainly depend on the light quark mass $m_q = \frac{1}{2}(m_u + m_d)$ and m_s, respectively for the up, down and strange quarks, that is in terms of $X_q = m_q/\Lambda_{QCD}$ and $X_s = m_s/\Lambda_{QCD}$. Using a chiral perturbation theory, it was deduced, assuming Λ_{QCD} constant, that

$$g_p \propto X_q^{-0.087} X_s^{-0.013}, \qquad g_n \propto X_q^{-0.118} X_s^{0.0013},$$

so that for a hyperfine transition

$$\nu_{hfs} \propto \alpha_{EM}^{2+\kappa_\alpha} X_q^{\kappa_q} X_s^{\kappa_s} \bar{\mu}.$$

Both coefficients can be computed, leading to the possibility to draw constraints on the independent time variation of X_q, X_s and X_e.

To simplify, we may assume that $X_q \propto X_s$, which is motivated by the Higgs mechanism of mass generation, so that the dependence in the quark masses reduces to $\kappa = \frac{1}{2}(\kappa_q + \kappa_s)$. For instance, we have

$$\kappa_{Cs} = 0.009, \qquad \kappa_{Rb} = -0.016, \qquad \kappa_H = -0.10.$$

For hyperfine transition, one further needs to take into account the dependence in μ that can be described [204] by

$$m_p \sim 3\Lambda_{QCD} X_q^{0.037} X_s^{0.011},$$

so that the hyperfine frequencies behaves as

$$\nu_{hfs} \propto \alpha_{EM}^{2+\kappa_\alpha} X_q^{\kappa-0.048} X_e,$$

in the approximation $X_q \propto X_s$ and where $X_e \equiv m_e/\Lambda_{QCD}$. This allows one to get independent constraints on the independent time variation of X_e, X_q and α_{EM}. Indeed, these constraints are model-dependent and, as an example, Table III of [210] compares the values of the sensitivity κ when different nuclear effects are considered. For instance, it can vary from 0.127, 0.044 to 0.009 for the cesium according to whether one includes only valence nucleon, non-valence non-nucleon or effect of the quark mass on the spin-spin interaction. Thus, it is a very promising framework, which still needs to be developed and the accuracy of which must be quantified in detail.

3.1.4 Future evolutions

Further progresses in a near future are expected mainly through three types of developments:

- *New systems*: Many new systems with enhanced sensitivity [171, 200, 202, 205, 421] to some fundamental constants have recently been proposed. Other atomic systems are considered, such as, e.g., the hyperfine transitions in the electronic ground state of cold, trapped, hydrogen-like highly charged ions [44, 199, 448], or ultra-cold atom and molecule systems near the Feshbach resonances [98], where the scattering length is extremely sensitive to μ.

  Concerning diatomic molecules, it was shown that this sensitivity can be enhanced in transitions between narrow close levels of different nature [13, 15]. In such transitions, the fine structure mainly depends on the fine-structure constant, $\nu_{fs} \sim (Z\alpha_{EM})^2 R_\infty c$, while the vibrational levels depend mainly on the electron-to-proton mass ratio and the reduced mass of the molecule, $\nu_v \sim M_r^{-1/2} \bar{\mu}^{1/2} R_\infty c$. There could be a cancellation between the two frequencies when $\nu = \nu_{hf} - n\nu_v \sim 0$ with n a positive integer. It follows that $\delta\nu/\nu$ will be proportional to $K = \nu_{hf}/\nu$ so that the sensitivity to α_{EM} and μ can be enhanced for these particular transitions. A similar effect between transistions with hyperfine-structures, for which the sensitivity to α_{EM} can reach 600 for instance for $^{139}La^{32}S$ or silicon monobrid [42] that allows one to constrain $\alpha_{EM}\bar{\mu}^{-1/4}$.

Nuclear transitions, such as an optical clock based on a very narrow ultraviolet nuclear transition between the ground and first excited states in the ^{229}Th, are also under consideration. Using a Walecka model for the nuclear potential, it was concluded [199] that the sensitivity of the transition to the fine-structure constant and quark mass was typically

$$\frac{\delta\omega}{\omega} \sim 10^5 \left(4\frac{\delta\alpha_{\mathrm{EM}}}{\alpha_{\mathrm{EM}}} + \frac{\delta X_{\mathrm{q}}}{X_{\mathrm{q}}} - 10\frac{\delta X_{\mathrm{s}}}{X_{\mathrm{s}}} \right),$$

which roughly provides a five order of magnitude amplification, which can lead to a constraint at the level of 10^{-24}/yr on the time variation of X_{q}. Such a method is promising and would offer different sensitivities to systematic effects compared to atomic clocks. However, this sensitivity is not clearly established since different nuclear calculations do not agree [46, 247].

- *Atomic clocks in space (ACES)*: An improvement of at least an order of magnitude on current constraints can be achieved in space with the PHARAO/ACES project [433, 444] of the European Spatial Agency. PHARAO (Projet d'Horloge Atomique par Refroidissement d'Atomes en Orbite) combines laser cooling techniques and a microgravity environment in a satellite orbit. It aims at achieving time and frequency transfer with stability better than 10^{-16}.

  The SAGAS (Search for anomalous gravitation using atomic sensor) project aims at flying highly sensitive optical atomic clocks and cold atom accelerometers on a solar system trajectory on a time scale of 10 years. It could test the constancy of the fine-structure constant along the satellite worldline, which, in particular, can set a constraint on its spatial variation of the order of 10^{-9} [433, 547].

- *Theoretical developments*: We remind one more time that the interpretation of the experiments requires a good theoretical understanding of the systems but also that the constraints we draw on the fundamental constants such as the quark masses are conditional to our theoretical modeling, hence on hypothesis on a unification scheme as well as nuclear physics. The accuracy and the robustness of these steps need to be determined, e.g., by taking the dependence in the nuclear radius [154].

3.2 The Oklo phenomenon

3.2.1 A natural nuclear reactor

Oklo is the name of a town in the Gabon republic (West Africa) where an open-pit uranium mine is situated. About 1.8×10^9 yr ago (corresponding to a redshift of ~ 0.14 with the cosmological concordance model), in one of the rich vein of uranium ore, a natural nuclear reactor went critical, consumed a portion of its fuel and then shut a few million years later (see, e.g., [509] for more details). This phenomenon was discovered by the French Commissariat à l'Énergie Atomique in 1972 while monitoring for uranium ores [382]. Sixteen natural uranium reactors have been identified. Well studied reactors include the zone RZ2 (about 60 bore-holes, 1800 kg of ^{235}U fissioned during 8.5×10^5 yr) and zone RZ10 (about 13 bore-holes, 650 kg of ^{235}U fissioned during 1.6×10^5 yr).

The existence of such a natural reactor was predicted by P. Kuroda [303] who showed that under favorable conditions, a spontaneous chain reaction could take place in rich uranium deposits. Indeed, two billion years ago, uranium was naturally enriched (due to the difference of decay rate between ^{235}U and ^{238}U) and ^{235}U represented about 3.68% of the total uranium (compared with 0.72% today and to the $3-5\%$ enrichment used in most commercial reactors). Besides, in Oklo the conditions were favorable: (1) the concentration of neutron absorbers, which prevent the neutrons from being available for the chain fission, was low; (2) water played the role of moderator (the

zones RZ2 and RZ10 operated at a depth of several thousand meters, so that the water pressure and temperature was close to the pressurized water reactors of 20 Mpa and 300℃) and slowed down fast neutrons so that they can interact with other ^{235}U and (3) the reactor was large enough so that the neutrons did not escape faster than they were produced. It is estimated that the Oklo reactor powered 10 to 50 kW. This explanation is backed up by the substantial depletion of ^{235}U as well as a correlated peculiar distribution of some rare-earth isotopes. These rare-earth isotopes are abundantly produced during the fission of uranium and, in particular, the strong neutron absorbers like $^{149}_{62}$Sm, $^{151}_{63}$Eu, $^{155}_{64}$Gd and $^{155}_{64}$Gd are found in very small quantities in the reactor.

From the isotopic abundances of the yields, one can extract information about the nuclear reactions at the time the reactor was operational and reconstruct the reaction rates at that time. One of the key quantity measured is the ratio $^{149}_{62}$Sm/$^{147}_{62}$Sm of two light isotopes of samarium, which are not fission products. This ratio of order of 0.9 in normal samarium, is about 0.02 in Oklo ores. This low value is interpreted [465] by the depletion of $^{149}_{62}$Sm by thermal neutrons produced by the fission process and to which it was exposed while the reactor was active. The capture cross section of thermal neutron by $^{149}_{62}$Sm

$$n + {}^{149}_{62}\text{Sm} \longrightarrow {}^{150}_{62}\text{Sm} + \gamma \tag{38}$$

is dominated by a capture resonance of a neutron of energy of about 0.1 eV ($E_r = 97.3$ meV today). The existence of this resonance is a consequence of an almost cancellation between the electromagnetic repulsive force and the strong interaction.

Shlyakhter [465] pointed out that this phenomenon can be used to set a constraint on the time variation of fundamental constants. His argument can be summarized as follows.

- First, the cross section $\sigma_{(n,\gamma)}$ strongly depends on the energy of a resonance at $E_r = 97.3$ meV.

- Geochemical data allow to determine the isotopic composition of various element, such as uranium, neodynium, gadolinium and samarium. Gadolinium and neodium allow to determine the fluence (integrated flux over time) of the neutron while both gadolinium and samarium are strong neutron absorbers.

- From these data, one deduces the value of the averaged value of the cross section on the neutron flux, $\hat{\sigma}_{149}$. This value depends on hypothesis on the geometry of the reactor zone.

- The range of allowed value of $\hat{\sigma}_{149}$ was translated into a constraint on E_r. This step involves an assumption on the form and temperature of the neutron spectrum.

- E_r was related to some fundamental constant, which involve a model of the nucleus.

In conclusion, we have different steps, which all involve assumptions:

- Isotopic compositions and geophysical parameters are measured in a given set of bore-hold in each zone. A choice has to be made on the sample to use, in order, e.g., to ensure that they are not contaminated.

- With hypothesis on the geometry of the reactor, on the spectrum and temperature of the neutron flux, one can deduce the effective value of the cross sections of neutron absorbers (such as samarium and gadolinium). This requires one to solve a network of nuclear reactions describing the fission.

- One can then infer the value of the resonance energy E_r, which again depends on the assumptions on the neutron spectrum.

- E_r needs to be related to fundamental constant, which involves a model of the nucleus and high energy physics hypothesis.

We shall now detail the assumptions used in the various analyses that have been performed since the pioneering work of [465].

3.2.2   Constraining the shift of the resonance energy

Cross sections.   The cross section of the neutron capture (38) strongly depends on the energy of a resonance at $E_r = 97.3$ meV and is well described by the Breit–Wigner formula

$$\sigma_{(n,\gamma)}(E) = \frac{g_0 \pi}{2} \frac{\hbar^2}{m_\text{n} E} \frac{\Gamma_\text{n} \Gamma_\gamma}{(E - E_r)^2 + \Gamma^2/4} \tag{39}$$

where $g_0 \equiv (2J+1)(2s+1)^{-1}(2I+1)^{-1}$ is a statistical factor, which depends on the spin of the incident neutron $s = 1/2$, of the target nucleus I, and of the compound nucleus J. For the reaction (38), we have $g_0 = 9/16$. The total width $\Gamma \equiv \Gamma_\text{n} + \Gamma_\gamma$ is the sum of the neutron partial width $\Gamma_\text{n} = 0.533$ meV (at $E_r = 97.3$ meV and it scales as $\sqrt{E}$ in the center of mass) and of the radiative partial width $\Gamma_\gamma = 60.5$ meV. $^{155}_{64}$Gd has a resonance at $E_r = 26.8$ meV with $\Gamma_\text{n} = 0.104$ meV, $\Gamma_\gamma = 108$ meV and $g = 5/8$ while $^{157}_{64}$Gd has a resonance at $E_r = 31.4$ meV with $\Gamma_\text{n} = 0.470$ meV, $\Gamma_\gamma = 106$ meV and $g = 5/8$.

As explained in the previous Section 3.2.1, this cross section cannot be measured from the Oklo data, which allow only to measure its value averaged on the neutron flux $n(v, T)$, T being the temperature of the moderator. It is conventionally defined as

$$\hat{\sigma} = \frac{1}{n v_0} \int \sigma_{(n,\gamma)} n(v, T) v \, \mathrm{d}v, \tag{40}$$

where the velocity $v_0 = 2200 \text{ m} \cdot \text{s}^{-1}$ corresponds to an energy $E_0 = 25.3$ meV and $v = \sqrt{2E/m_\text{n}}$, instead of

$$\bar{\sigma} = \frac{\int \sigma_{(n,\gamma)} n(v, T) v \, \mathrm{d}v}{\int n(v, T) v \, \mathrm{d}v}.$$

When the cross section behaves as $\sigma = \sigma_0 v_0 / v$, which is the case for nuclei known as "$1/v$-absorbers", $\hat{\sigma} = \sigma_0$ and does not depend on the temperature, whatever the distribution $n(v)$. In a similar way, the effective neutron flux defined

$$\hat{\phi} = v_0 \int n(v, T) \mathrm{d}v, \tag{41}$$

which differs from the true flux

$$\phi = \int n(v, T) v \, \mathrm{d}v.$$

However, since $\bar{\sigma}\phi = \hat{\sigma}\hat{\phi}$, the reaction rates are not affected by these definitions.

Extracting the effective cross section from the data.   To "measure" the value of $\hat{\sigma}$ from the Oklo data, we need to solve the nuclear reaction network that controls the isotopic composition during the fission.

The samples of the Oklo reactors were exposed [382] to an integrated effective fluence $\int \hat{\phi} \mathrm{d}t$ of about 10^{21}neutron $\cdot$ cm$^{-2} = 1$ kb^{-1}. It implies that any process with a cross section smaller than 1 kb can safely be neglected in the computation of the abundances. This includes neutron capture by $^{144}_{62}$Sm and $^{148}_{62}$Sm, as well as by $^{155}_{64}$Gd and $^{157}_{64}$Gd. On the other hand, the fission of $^{235}_{92}$U, the

Table 8: Summary of the analysis of the Oklo data. The principal assumptions to infer the value of the resonance energy E_r are the form of the neutron spectrum and its temperature.

Ore	neutron spectrum	Temperature (°C)	$\hat{\sigma}_{149}$ (kb)	ΔE_r (meV)	Ref.
?	Maxwell	20	55 ± 8	0 ± 20	[465]
RZ2 (15)	Maxwell	$180 - 700$	75 ± 18	-1.5 ± 10.5	[123]
RZ10	Maxwell	$200 - 400$	91 ± 6	4 ± 16	[220]
RZ10				-97 ± 8	[220]
–	Maxwell + epithermal	327	91 ± 6	-45^{+7}_{-15}	[305]
RZ2	Maxwell + epithermal		73.2 ± 9.4	-5.5 ± 67.5	[416]
RZ2	Maxwell + epithermal	$200 - 300$	71.5 ± 10.0	–	[234]
RZ10	Maxwell + epithermal	$200 - 300$	85.0 ± 6.8	–	[234]
RZ2+RZ10				7.2 ± 18.8	[234]
RZ2+RZ10				90.75 ± 11.15	[234]

capture of neutron by $^{143}_{60}$Nd and by $^{149}_{62}$Sm with respective cross sections $\sigma_5 \simeq 0.6$ kb, $\sigma_{143} \sim 0.3$ kb and $\sigma_{149} \geq 70$ kb are the dominant processes. It follows that the equations of evolution for the number densities N_{147}, N_{148}, N_{149} and N_{235} of $^{147}_{62}$Sm, $^{148}_{62}$Sm, $^{149}_{62}$Sm and $^{235}_{92}$U takes the form

$$\frac{\mathrm{d}N_{147}}{\hat{\phi}\mathrm{d}t} = -\hat{\sigma}_{147}N_{147} + \hat{\sigma}_{f235}y_{147}N_{235} \tag{42}$$

$$\frac{\mathrm{d}N_{148}}{\hat{\phi}\mathrm{d}t} = \hat{\sigma}_{147}N_{147} \tag{43}$$

$$\frac{\mathrm{d}N_{149}}{\hat{\phi}\mathrm{d}t} = -\hat{\sigma}_{149}N_{149} + \hat{\sigma}_{f235}y_{149}N_{235} \tag{44}$$

$$\frac{\mathrm{d}N_{235}}{\hat{\phi}\mathrm{d}t} = -\sigma_5 N_{235}, \tag{45}$$

where y_i denotes the yield of the corresponding element in the fission of $^{235}_{92}$U and $\hat{\sigma}_5$ is the fission cross section. This system can be integrated under the assumption that the cross sections and the neutron flux are constant and the result compared with the natural abundances of the samarium to extract the value of $\hat{\sigma}_{149}$ at the time of the reaction. Here, the system has been closed by introducing a modified absorption cross section [123] σ_5^* to take into account both the fission, capture but also the formation from the α-decay of $^{239}_{94}$Pu. One can instead extend the system by considering $^{239}_{94}$Pu, and $^{235}_{92}$U (see [234]). While most studies focus on the samarium, [220] also includes the gadolinium even though it is not clear whether it can reliably be measured [123]. They give similar results.

By comparing the solution of this system with the measured isotopic composition, one can deduce the effective cross section. At this step, the different analyses [465, 415, 123, 220, 305, 416, 234] differ from the choice of the data. The measured values of $\hat{\sigma}_{149}$ can be found in these articles. They are given for a given zone (RZ2, RZ10 mainly) with a number that correspond to the number of the bore-hole and the depth (e.g., in Table 2 of [123], SC39-1383 means that we are dealing with the bore-hole number 39 at a depth of 13.83 m). Recently, another approach [416, 234] was proposed in order to take into account of the geometry and details of the reactor. It relies on a full-scale Monte-Carlo simulation and a computer model of the reactor zone RZ2 [416] and both RZ2 and RZ10 [234] and allows to take into account the spatial distribution of the neutron flux.

Determination of E_r. To convert the constraint on the effective cross section, one needs to specify the neutron spectrum. In the earlier studies [465, 415], a Maxwell distribution,

$$n_{\text{th}}(v, T) = \left(\frac{m_{\text{n}}}{2\pi T}\right)^{3/2} e^{-\frac{mv^2}{2k_{\text{B}}T}},$$

was assumed for the neutron with a temperature of 20°C, which is probably too small. Then v_0 is the mean velocity at a temperature $T_0 = m_{\text{n}}v_0^2/2k_{\text{B}} = 20.4$°C. [123, 220] also assume a Maxwell distribution but let the moderator temperature vary so that they deduce an effective cross section $\hat{\sigma}(R_r, T)$. They respectively restricted the temperature range to 180°C $< T < 700$°C and 200°C $< T < 400$°C, based on geochemical analysis. The advantage of the Maxwell distribution assumption is that it avoids to rely on a particular model of the Oklo reactor since the spectrum is determined solely by the temperature.

It was then noted [305, 416] that above an energy of several eV, the neutron spectrum shifted to a $1/E$ tail because of the absorption of neutrons in uranium resonances. Thus, the distribution was adjusted to include an epithermal distribution

$$n(v) = (1 - f)n_{\text{th}}(v, T) + f n_{\text{epi}}(v),$$

with $n_{\text{epi}} = v_c^2/v^2$ for $v > v_c$ and vanishing otherwise. v_c is a cut-off velocity that also needs to be specified. The effective cross section can then be parameterized [234] as

$$\hat{\sigma} = g(T)\sigma_0 + r_0 I, \tag{46}$$

where $g(T)$ is a measure of the departure of σ from the $1/v$ behavior, I is related to the resonance integral of the cross section and r_0 is the Oklo reactor spectral index. It characterizes the contribution of the epithermal neutrons to the cross section. Among the unknown parameters, the most uncertain is probably the amount of water present at the time of the reaction. [234] chooses to adjust it so that r_0 matches the experimental values.

These hypothesis on the neutron spectrum and on the temperature, as well as the constraint on the shift of the resonance energy, are summarized in Table 8. Many analyses [220, 416, 234] find two branches for $\Delta E_r = E_r - E_{r0}$, with one (the left branch) indicating a variation of E_r. Note that these two branches disappear when the temperature is higher since $\hat{\sigma}(E_r, T)$ is more peaked when T decreases but remain in any analysis at low temperature. This shows the importance of a good determination of the temperature. Note that the analysis of [416] indicates that the curves $\hat{\sigma}(T, E_r)$ lie appreciably lower than for a Maxwell distribution and that [220] argues that the left branch is hardly compatible with the gadolinium data.

3.2.3 From the resonance energy to fundamental constants

The energy of the resonance depends a priori on many constants since the existence of such resonance is mainly the consequence of an almost cancellation between the electromagnetic repulsive force and the strong interaction. But, since no full analytical understanding of the energy levels of heavy nuclei is available, the role of each constant is difficult to disentangle.

In his first analysis, Shlyakhter [465] stated that for the neutron, the nucleus appears as a potential well with a depth $V_0 \simeq 50$ MeV. He attributed the change of the resonance energy to a modification of the strong interaction coupling constant and concluded that $\Delta g_{\text{S}}/g_{\text{S}} \sim \Delta E_r/V_0$. Then, arguing that the Coulomb force increases the average inter-nuclear distance by about 2.5% for $A \sim 150$, he concluded that $\Delta \alpha_{\text{EM}}/\alpha_{\text{EM}} \sim 20\Delta g_{\text{S}}/g_{\text{S}}$, leading to $|\dot{\alpha}_{\text{EM}}/\alpha_{\text{EM}}| < 10^{-17}$ yr^{-1}, which can be translated to

$$|\Delta \alpha_{\text{EM}}/\alpha_{\text{EM}}| < 1.8 \times 10^{-8}. \tag{47}$$

The following analysis focused on the fine-structure constant and ignored the strong interaction. Damour and Dyson [123] related the variation of E_r to the fine-structure constant by taking into account that the radiative capture of the neutron by $^{149}_{62}\text{Sm}$ corresponds to the existence of an excited quantum state of $^{150}_{62}\text{Sm}$ (so that $E_r = E^*_{150} - E_{149} - m_n$) and by assuming that the nuclear energy is independent of α_{EM}. It follows that the variation of α_{EM} can be related to the difference of the Coulomb binding energy of these two states. The computation of this latter quantity is difficult and must be related to the mean-square radii of the protons in the isotopes of samarium. In particular this analysis [123] showed that the Bethe–Weizäcker formula overestimates by about a factor the 2 the α_{EM}-sensitivity to the resonance energy. It follows from this analysis that

$$\alpha_{\text{EM}} \frac{\Delta E_r}{\Delta \alpha_{\text{EM}}} \simeq -1.1 \text{ MeV}, \tag{48}$$

which, once combined with the constraint on ΔE_r, implies

$$-0.9 \times 10^{-7} < \Delta \alpha_{\text{EM}}/\alpha_{\text{EM}} < 1.2 \times 10^{-7} \tag{49}$$

at 2σ level, corresponding to the range $-6.7 \times 10^{-17} \text{ yr}^{-1} < \dot{\alpha}_{\text{EM}}/\alpha_{\text{EM}} < 5.0 \times 10^{-17} \text{ yr}^{-1}$ if $\dot{\alpha}_{\text{EM}}$ is assumed constant. This tight constraint arises from the large amplification between the resonance energy (~ 0.1 eV) and the sensitivity (~ 1 MeV). The re-analysis of these data and also including the data of [220] with gadolinium, found the favored result $\dot{\alpha}_{\text{EM}}/\alpha_{\text{EM}} = (-0.2 \pm 0.8) \times 10^{-17} \text{ yr}^{-1}$, which corresponds to

$$\Delta \alpha_{\text{EM}}/\alpha_{\text{EM}} = (-0.36 \pm 1.44) \times 10^{-8} \tag{50}$$

and the other branch (indicating a variation; see Table 8) leads to $\dot{\alpha}_{\text{EM}}/\alpha_{\text{EM}} = (4.9 \pm 0.4) \times 10^{-17} \text{ yr}^{-1}$. This non-zero result cannot be eliminated.

The more recent analysis, based on a modification of the neutron spectrum lead respectively to [416]

$$\Delta \alpha_{\text{EM}}/\alpha_{\text{EM}} = (3.85 \pm 5.65) \times 10^{-8} \tag{51}$$

and [234]

$$\Delta \alpha_{\text{EM}}/\alpha_{\text{EM}} = (-0.65 \pm 1.75) \times 10^{-8}, \tag{52}$$

at a 95% confidence level, both using the formalism of [123].

Olive et al. [399], inspired by grand unification model, reconsider the analysis of [123] by letting all gauge and Yukawa couplings vary. Working within the Fermi gas model, the over-riding scale dependence of the terms, which determine the binding energy of the heavy nuclei was derived. Parameterizing the mass of the hadrons as $m_i \propto \Lambda_{\text{QCD}}(1 + \kappa_i m_q/\Lambda_{\text{QCD}} + \ldots)$, they estimate that the nuclear Hamiltonian was proportional to m_q/Λ_{QCD} at lowest order, which allows to estimate that the energy of the resonance is related to the quark mass by

$$\frac{\Delta E_r}{E_r} \sim (2.5 - 10) \times 10^{17} \Delta \ln \left(\frac{m_q}{\Lambda_{\text{QCD}}} \right). \tag{53}$$

Using the constraint (48), they first deduced that

$$\left| \Delta \ln \left(\frac{m_q}{\Lambda_{\text{QCD}}} \right) \right| < (1 - 4) \times 10^{-8}.$$

Then, assuming that $\alpha_{\text{EM}} \propto m_q^{50}$ on the basis of grand unification (see Section 6.3 for details), they concluded that

$$|\Delta \alpha_{\text{EM}}/\alpha_{\text{EM}}| < (2 - 8) \times 10^{-10}. \tag{54}$$

Similarly, [207, 467, 212] related the variation of the resonance energy to the quark mass. Their first estimate [207] assumes that it is related to the pion mass, m_π, and that the main variation

arises from the variation of the radius $R \sim 5\mathrm{fm} + 1/m_\pi$ of the nuclear potential well of depth V_0, so that

$$\delta E_r \sim -2V_0 \frac{\delta R}{R} \sim 3 \times 10^8 \frac{\delta m_\pi}{m_\pi},$$

assuming that $R \simeq 1.2 A^{1/3} r_0$, r_0 being the inter-nucleon distance.

Then, in [467], the nuclear potential was described by a Walecka model, which keeps only the σ (scalar) and ω (vector) exchanges in the effective nuclear force. Their masses was related to the mass m_s of the strange quark to get $m_\sigma \propto m_s^{0.54}$ and $m_\omega \propto m_s^{0.15}$. It follows that the variation of the potential well can be related to the variation of m_σ and m_ω and thus on m_q by $V \propto m_q^{-3.5}$. The constraint (48) then implies that

$$\left| \Delta \ln \left(\frac{m_s}{\Lambda_{\mathrm{QCD}}} \right) \right| < 1.2 \times 10^{-10}.$$

By extrapolating from light nuclei where the N-body calculations can be performed more accurately, it was concluded [208] that the resonance energy scales as $\Delta E_r \simeq 10(\Delta \ln X_q - 0.1 \Delta \ln \alpha_{\mathrm{EM}})$, so that the the constraints from [416] would imply that $\Delta \ln(X_q/\alpha_{\mathrm{EM}}^{0.1}) < 7 \times 10^{-9}$.

In conclusion, these last results illustrate that a detailed theoretical analysis and quantitative estimates of the nuclear physics (and QCD) aspects of the resonance shift still remain to be carried out. In particular, the interface between the perturbative QCD description and the description in term of hadron is not fully understand: we do not know the exact dependence of hadronic masses and coupling constant on Λ_{QCD} and quark masses. The second problem concerns modeling nuclear forces in terms of the hadronic parameters.

At present, the Oklo data, while being stringent and consistent with no variation, have to be considered carefully. While a better understanding of nuclear physics is necessary to understand the full constant-dependence, the data themselves require more insight, particularly to understand the existence of the left-branch.

3.3 Meteorite dating

Long-lived α- or β-decay isotopes may be sensitive probes of the variation of fundamental constants on geological times ranging typically to the age of the solar system, $t \sim (4-5)$ Gyr, corresponding to a mean redshift of $z \sim 0.43$. Interestingly, it can be compared with the shallow universe quasar constraints. This method was initially pointed out by Wilkinson [539] and then revived by Dyson [168]. The main idea is to extract the α_{EM}-dependence of the decay rate and to use geological samples to bound its time variation.

The sensitivity of the decay rate of a nucleus to a change of the fine-structure constant is defined, in a similar way as for atomic clocks [Equation (23)], as

$$s_\alpha \equiv \frac{\partial \ln \lambda}{\partial \ln \alpha_{\mathrm{EM}}}. \tag{55}$$

λ is a function of the decay energy Q. When Q is small, mainly due to an accidental cancellation between different contributions to the nuclear binding energy, the sensitivity s_α maybe strongly enhanced. A small variation of the fundamental constants can either stabilize or destabilize certain isotopes so that one can extract bounds on the time variation of their lifetime by comparing laboratory data to geophysical and solar system probes.

Assume some meteorites containing an isotope X that decays into Y are formed at a time t_*. It follows that

$$N_X(t) = N_{X*} \mathrm{e}^{-\lambda(t-t_*)}, \qquad N_Y(t) = N_{X*} \left[1 - \mathrm{e}^{-\lambda(t-t_*)} \right] + N_{Y*} \tag{56}$$

if one assumes the decay rate constant. If it is varying then these relations have to be replaced by

$$N_X(t) = N_{X*}\mathrm{e}^{\int_{t_*}^{t} \lambda(t')\mathrm{d}t'}$$

so that the value of N_X today can be interpreted with Equation (56) but with an effective decay rate of

$$\bar{\lambda} = \frac{1}{t_0 - t_*} \int_{t_*}^{t_0} \lambda(t')\mathrm{d}t'. \tag{57}$$

From a sample of meteorites, we can measure $\{N_X(t_0), N_Y(t_0)\}$ for each meteorite. These two quantities are related by

$$N_Y(t_0) = \left[\mathrm{e}^{\bar{\lambda}(t_0 - t_*)} - 1\right] N_X(t_0) + N_{Y*},$$

so that the data should lie on a line (since N_{X*} is a priori different for each meteorite), called an "isochron", the slope of which determines $\bar{\lambda}(t_0 - t_*)$. It follows that meteorites data only provides an *average* measure of the decay rate, which complicates the interpretation of the constraints (see [219, 218] for explicit examples). To derive a bound on the variation of the constant we also need a good estimation of $t_0 - t_*$, which can be obtained from the same analysis for an isotope with a small sensitivity s_α, as well as an accurate laboratory measurement of the decay rate.

3.3.1   Long lived α-decays

The α-decay rate, λ, of a nucleus $_Z^A\mathrm{X}$ of charge Z and atomic number A,

$$_{Z+2}^{A+4}\mathrm{X} \longrightarrow {_Z^A}\mathrm{X} + {_2^4}\mathrm{He}, \tag{58}$$

is governed by the penetration of the Coulomb barrier that can be described by the Gamow theory. It is well approximated by

$$\lambda \simeq \Lambda(\alpha_{\mathrm{EM}}, v) \exp\left(-4\pi Z \alpha_{\mathrm{EM}} \frac{c}{v}\right), \tag{59}$$

where $v/c = \sqrt{Q/2m_{\mathrm{p}}c^2}$ is the escape velocity of the α particle. Λ is a function that depends slowly on α_{EM} and Q. It follows that the sensitivity to the fine-structure constant is

$$s_\alpha \simeq -4\pi Z \frac{\alpha_{\mathrm{EM}}}{\sqrt{Q/2m_{\mathrm{p}}}} \left(1 - \frac{1}{2}\frac{\mathrm{d}\ln Q}{\mathrm{d}\ln \alpha_{\mathrm{EM}}}\right). \tag{60}$$

The decay energy is related to the nuclear binding energies $B(A, Z)$ of the different nuclei by

$$Q = B(A, Z) + B_\alpha - B(A + 4, Z + 2)$$

with $B_\alpha = B(4, 2)$. Physically, an increase of α_{EM} induces an increase in the height of the Coulomb barrier at the nuclear surface while the depth of the nuclear potential well below the top remains the same. It follows that α-particle escapes with a greater energy but at the same energy below the top of the barrier. Since the barrier becomes thiner at a given energy below its top, the penetrability increases. This computation indeed neglects the effect of a variation of α_{EM} on the nucleus that can be estimated to be dilated by about 1% if α_{EM} increases by 1%.

As a first insight, when focusing on the fine-structure constant, one can estimate s_α by varying only the Coulomb term of the binding energy. Its order of magnitude can be estimated from the Bethe–Weizäcker formula

$$E_{\mathrm{EM}} = 98.25 \frac{Z(Z - 1)}{A^{1/3}} \alpha_{\mathrm{EM}} \text{ MeV}. \tag{61}$$

Table 9: Summary of the main nuclei and their physical properties that have been used in α-decay studies.

Element	Z	A	Lifetime (yr)	Q (MeV)	s_α
Sm	62	147	1.06×10^{11}	2.310	774
Gd	64	152	1.08×10^{14}	2.204	890
Dy	66	154	3×10^{6}	2.947	575
Pt	78	190	6.5×10^{11}	3.249	659
Th	90	232	1.41×10^{10}	4.082	571
U	92	235	7.04×10^{8}	4.678	466
U	92	238	4.47×10^{9}	4.270	548

Table 9 summarizes the most sensitive isotopes, with the sensitivities derived from a semi-empirical analysis for a spherical nucleus [399]. They are in good agreement with the ones derived from Equation (61) (e.g., for ^{238}U, one would obtain $s_\alpha = 540$ instead of $s_\alpha = 548$).

The sensitivities of all the nuclei of Table 9 are similar, so that the best constraint on the time variation of the fine-structure constant will be given by the nuclei with the smaller $\Delta\lambda/\lambda$.

Wilkinson [539] considered the most favorable case, that is the decay of $^{238}_{92}$U for which $s_\alpha = 548$ (see Table 9). By comparing the geological dating of the Earth by different methods, he concluded that the decay constant λ of ^{238}U, ^{235}U and ^{232}Th have not changed by more than a factor 3 or 4 during the last $3-4 \times 10^9$ years from which it follows

$$|\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}| < 8 \times 10^{-3}. \tag{62}$$

This constraint was revised by Dyson [168] who claimed that the decay rate has not changed by more than 20%, during the past 2×10^9 years, which implies

$$|\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}| < 4 \times 10^{-4}. \tag{63}$$

Uranium has a short lifetime so that it cannot be used to set constraints on longer time scales. It is also used to calibrate the age of the meteorites. Therefore, it was suggested [399] to consider ^{147}Sm. Assuming that $\Delta\lambda_{147}/\lambda_{147}$ is smaller than the fractional uncertainty of 7.5×10^{-3} of its half-life

$$|\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}| \lesssim \times 10^{-5}. \tag{64}$$

As for the Oklo phenomena, the effect of other constants has not been investigated in depth. It is clear that at lowest order both Q and m_{p} scales as Λ_{QCD} so that one needs to go beyond such a simple description to determine the dependence in the quark masses. Taking into account the contribution of the quark masses, in the same way as for Equation (53), it was argued that $\lambda \propto X_{\mathrm{q}}^{300-2000}$, which leads to $|\Delta \ln X_{\mathrm{q}}| \lesssim 10^{-5}$. In a grand unify framework, that could lead to a constraint of the order of $|\Delta \ln \alpha_{\mathrm{EM}}| \lesssim 2 \times 10^{-7}$.

3.3.2 Long lived β-decays

Dicke [150] stressed that the comparison of the rubidium-strontium and potassium-argon dating methods to uranium and thorium rates constrains the variation of α_{EM}.

As long as long-lived β-decay isotopes are concerned for which the decay energy Q is small, we can use a non-relativistic approximation for the decay rate

$$\lambda = \Lambda_\pm Q^{p_\pm} \tag{65}$$

respectively for β^--decay and electron capture. $\Lambda_\pm$ are functions that depend smoothly on α_{EM} and which can thus be considered constant, $p_+ = \ell + 3$ and $p_- = 2\ell + 2$ are the degrees of

forbiddenness of the transition. For high-Z nuclei with small decay energy Q, the exponent p becomes $p = 2 + \sqrt{1 - \alpha_{\rm EM}^2 Z^2}$ and is independent of ℓ. It follows that the sensitivity to a variation of the fine-structure constant is

$$s_\alpha = p \frac{{\rm d} \ln Q}{{\rm d} \ln \alpha_{\rm EM}}. \tag{66}$$

The second factor can be estimated exactly as for α-decay. We note that $\Lambda_\pm$ depends on the Fermi constant and on the mass of the electron as $\Lambda_\pm \propto G_{\rm F}^2 m_{\rm e}^5 Q^p$. This dependence is the same for any β-decay so that it will disappear in the comparison of two dating methods relying on two different β-decay isotopes, in which case only the dependence on the other constants appear again through the nuclear binding energy. Note, however, that comparing a α- to a β-decay may lead to interesting constraints.

We refer to Section III.A.4 of FVC [500] for earlier constraints derived from rubidium-strontium, potassium-argon and we focus on the rhenium-osmium case,

$$^{187}_{75}{\rm Re} \longrightarrow {}^{187}_{76}{\rm Os} + \bar{\nu}_e + e^- \tag{67}$$

first considered by Peebles and Dicke [406]. They noted that the very small value of its decay energy $Q = 2.6$ keV makes it a very sensitive probe of the variation of $\alpha_{\rm EM}$. In that case $p \simeq 2.8$ so that $s_\alpha \simeq -18000$; a change of $10^{-2}\%$ of $\alpha_{\rm EM}$ will induce a change in the decay energy of order of the keV, that is of the order of the decay energy itself. Peebles and Dicke [406] did not have reliable laboratory determination of the decay rate to put any constraint. Dyson [167] compared the isotopic analysis of molybdenite ores ($\lambda_{187} = (1.6 \pm 0.2) \times 10^{-11}$ yr^{-1}), the isotopic analysis of 14 iron meteorites ($\lambda_{187} = (1.4 \pm 0.3) \times 10^{-11}$ yr^{-1}) and laboratory measurements of the decay rate ($\lambda_{187} = (1.1 \pm 0.1) \times 10^{-11}$ yr^{-1}). Assuming that the variation of the decay energy comes entirely from the variation of $\alpha_{\rm EM}$, he concluded that $|\Delta\alpha_{\rm EM}/\alpha_{\rm EM}| < 9 \times 10^{-4}$ during the past 3×10^9 years. Note that the discrepancy between meteorite and lab data could have been interpreted as a time-variation of $\alpha_{\rm EM}$, but the laboratory measurement were complicated by many technical issues so that Dyson only considered a conservative upper limit.

The modelization and the computation of s_α were improved in [399], following the same lines as for α-decay.

$$\frac{\Delta\lambda_{187}}{\lambda_{187}} = p \frac{\Delta Q}{Q} \simeq p \left(\frac{20 \text{ MeV}}{Q} \right) \frac{\Delta\alpha_{\rm EM}}{\alpha_{\rm EM}} \sim -2.2 \times 10^4 \frac{\Delta\alpha_{\rm EM}}{\alpha_{\rm EM}}$$

if one considers only the variation of the Coulomb energy in Q. A similar analysis [147] leads to $\Delta \ln \lambda_{187} \simeq 10^4 \Delta \ln[\alpha_{\rm EM}^{-2.2} X_{\rm q}^{-1.9} (X_{\rm d} - X_{\rm u})^{0.23} X_{\rm e}^{-0.058}]$.

The dramatic improvement in the meteoric analysis of the Re/Os ratio [468] led to a recent re-analysis of the constraints on the fundamental constants. The slope of the isochron was determined with a precision of 0.5%. However, the Re/Os ratio is inferred from iron meteorites the age of which is not determined directly. Models of formation of the solar system tend to show that iron meteorites and angrite meteorites form within the same 5 million years. The age of the latter can be estimated from the ^{207}Pb-^{208}Pb method, which gives 4.558 Gyr [337] so that $\lambda_{187} = (1.666 \pm 0.009) \times 10^{-11}$ yr^{-1}. Thus, we could adopt [399]

$$\left| \frac{\Delta\lambda_{187}}{\lambda_{187}} \right| < 5 \times 10^{-3}.$$

However, the meteoritic ages are determined mainly by ^{238}U dating so that effectively we have a constraint on the variation of $\lambda_{187}/\lambda_{238}$. Fortunately, since the sensitivity of ^{238}U is much smaller than the one of the rhenium, it is safe to neglect its effect. Using the recent laboratory measurement [333] ($\lambda_{187} = (-1.639 \pm 0.025) \times 10^{-11}$ yr^{-1}), the variation of the decay rate is not

given by the dispersion of the meteoritic measurement, but by comparing to its value today, so that

$$\left| \frac{\Delta\lambda_{187}}{\lambda_{187}} \right| = -0.016 \pm 0.016. \tag{68}$$

The analysis of Ref. [400], following the assumption of [399], deduced that

$$\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}} = (-8 \pm 16) \times 10^{-7}, \tag{69}$$

at a 95% confidence level, on a typical time scale of 5 Gyr (or equivalently a redshift of order $z \sim 0.2$).

   As pointed out in [219, 218], these constraints really represents a bound on the average decay rate $\bar{\lambda}$ since the formation of the meteorites. This implies in particular that the redshift at which one should consider this constraint depends on the specific functional dependence $\lambda(t)$. It was shown that well-designed time dependence for λ can obviate this limit, due to the time average.

3.3.3 Conclusions

Meteorites data allow to set constraints on the variation of the fundamental constants, which are comparable to the ones set by the Oklo phenomenon. Similar constraints can also bet set from spontaneous fission (see Section III.A.3 of FVC [500]) but this process is less well understood and less sensitive than the α- and β- decay processes and.

   From an experimental point of view, the main difficulty concerns the dating of the meteorites and the interpretation of the effective decay rate.

   As long as we only consider α_{EM}, the sensitivities can be computed mainly by considering the contribution of the Coulomb energy to the decay energy, that reduces to its contribution to the nuclear energy. However, as for the Oklo phenomenon, the dependencies in the other constants, X_{q}, G_{F}, $\mu\ldots$, require a nuclear model and remain very model-dependent.

3.4 Quasar absorption spectra

3.4.1 Generalities

Quasar (QSO) absorption lines provide a powerful probe of the variation of fundamental constants. Absorption lines in intervening clouds along the line of sight of the QSO give access to the spectra of the atoms present in the cloud, that it is to paleo-spectra. The method was first used by Savedoff [447] who constrained the time variation of the fine-structure constraint from the doublet separations seen in galaxy emission spectra. For general introduction to these observations, we refer to [412, 474, 271].

   Indeed, one cannot use a single transition compared to its laboratory value since the expansion of the universe induces a global redshifting of all spectra. In order to tackle down a variation of the fundamental constants, one should resort on various transitions and look for chromatic effects that can indeed not be reproduce by the expansion of the universe, which acts chromatically on all wavelengths.

   To achieve such a test, one needs to understand the dependencies of different types of transitions, in a similar way as for atomic clock experiments. [175, 169] suggested to use the convenient formulation

$$\omega = \omega_0 + q\left[\left(\frac{\alpha_{\mathrm{EM}}}{\alpha_{\mathrm{EM}}^{(0)}}\right)^2 - 1\right] + q_2\left[\left(\frac{\alpha_{\mathrm{EM}}}{\alpha_{\mathrm{EM}}^{(0)}}\right)^4 - 1\right], \tag{70}$$

in order to take into account the dependence of the spectra on the fine-structure constant. ω is the energy in the rest-frame of the cloud, that is at a redshift z, ω_0 is the energy measured today in the

laboratory. q and q_2 are two coefficients that determine the frequency dependence on a variation of α_{EM} and that arise from the relativistic corrections for the transition under consideration. The coefficient q is typically an order of magnitude larger than q_2 so that the possibility to constrain a variation of the fine-structure constant is mainly determined by q. These coefficients were computed for a large set of transitions, first using a relativistic Hartree–Fock method and then using many-body perturbation theory. We refer to [175, 45, 14] for an extensive discussion of the computational methods and a list of the q-coefficients for various transitions relevant for both quasar spectra and atomic clock experiments. Figure 3 summarizes some of these results. The uncertainty in q are typically smaller than 30 cm^{-1} for Mg, Si, Al and Zn, but much larger for Cr, Fe and Ni due to their more complicated electronic configurations. The accuracy for ω_0 from dedicated laboratory measurements now reach 0.004 cm^{-1}. It is important to stress that the form (70) ensures that errors in the q-coefficients cannot lead to a non zero detection of $\Delta\alpha_{EM}$.
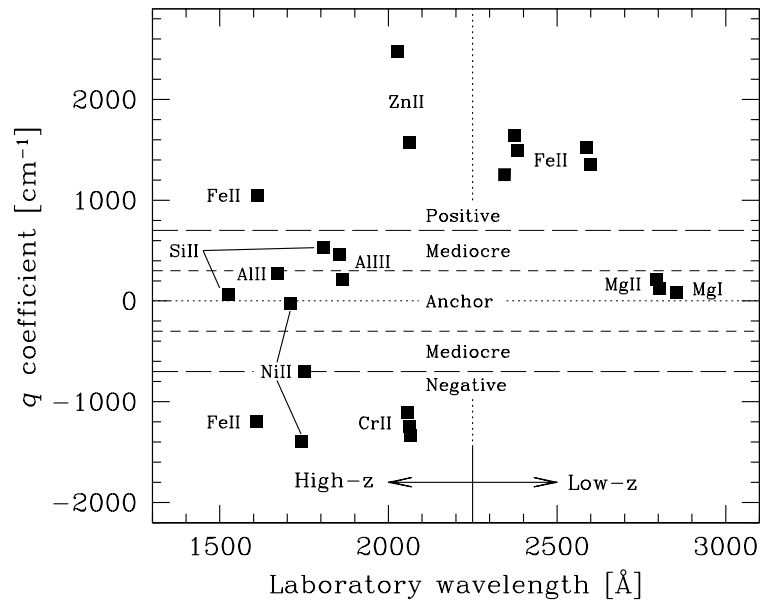
Figure 3: Summary of the values of some coefficients entering the parameterization (70) and necessary to interpret the QSO absorption spectra data. From [367]

The shift between two lines is easier to measure when the difference between the q-coefficients of the two lines is large, which occurs, e.g., for two levels with large q of opposite sign. Many methods were developed to take this into account. The *alkali doublet method* (AD) focuses on the fine-structure doublet of alkali atoms. It was then generalized to the *many-multiplet method* (MM), which uses correlations between various transitions in different atoms. As can be seen on Figure 3, some transitions are almost insensitive to a variation of α_{EM}. This is the case of Mg II, which can be used as an *anchor*, i.e., a reference point. To obtain strong constraints one can either compare transitions of light atoms with those of heavy atoms (because the α_{EM} dependence of the ground state scales as Z^2) or compare $s-p$ and $d-p$ transitions in heavy elements (in that case, the relativistic correction will be of opposite signs). This latter effect increases the sensitivity and strengthens the method against systematic errors. However, the results of this method rely on two assumptions: (i) ionization and chemical homogeneity and (ii) isotopic abundance of Mg II close to the terrestrial value. Even though these are reasonable assumptions, one cannot completely rule out systematic biases that they could induce. The AD method completely avoids the assumption of homogeneity because, by construction, the two lines of the doublet must have the same profile.

Indeed the AD method avoids the implicit assumption of the MM method that chemical and ionization inhomogeneities are negligible. Another way to avoid the influence of small spectral shift due to ionization inhomogeneities within the absorber and due to possible non-zero offset between different exposures was to rely on different transitions of a single ion in individual exposure. This method has been called the *Single ion differential alpha measurement method* (SIDAM).

Most studies are based on *optical techniques* due to the profusion of strong UV transitions that are redshifted into the optical band (this includes AD, MM, SIDAM and it implies that they can be applied only above a given redshift, e.g., Si IV at $z > 1.3$, Fe IIλ1608 at $z > 1$) or on *radio techniques* since radio transitions arise from many different physical effects (hyperfine splitting and in particular H I 21 cm hyperfine transition, molecular rotation, Lambda-doubling, etc). In the latter case, the line frequencies and their comparisons yield constraints on different sets of fundamental constants including α_{EM}, g_{p} and μ. Thus, these techniques are complementary since systematic effects are different in optical and radio regimes. Also the radio techniques offer some advantages: (1) to reach high spectral resolution (< 1 km/s), alleviating in particular problems with line blending and the use of, e.g., masers allow to reach a frequency calibration better than roughly 10 m/s; (2) in general, the sensitivity of the line position to a variation of a constant is higher; (3) the isotopic lines are observed separately, while in optical there is a blend with possible differential saturations (see, e.g., [109] for a discussion).

Let us first emphasize that the shifts in the absorption lines to be detected are extremely small. For instance a change of α_{EM} of order 10^{-5} corresponds a shift of at most 20 mÅ for a redshift of $z \sim 2$, which would corresponds to a shift of order ~ 0.5 km/s, or to about a third of a pixel at a spectral resolution of $R \sim 40000$, as achieved with Keck/HIRES or VLT/UVES. As we shall discuss later, there are several sources of uncertainty that hamper the measurement. In particular, the absorption lines have complex profiles (because they result from the propagation of photons through a highly inhomogeneous medium) that are fitted using a combination of Voigt profiles. Each of these components depends on several parameters including the redshift, the column density and the width of the line (Doppler parameter) to which one now needs to add the constants that are assumed to be varying. These parameters are constrained assuming that the profiles are the same for all transitions, which is indeed a non-trivial assumption for transitions from different species (this was one of the driving motivations to use the transition from a single species and of the SIDAM method). More important, the fit is usually not unique. This is not a problem when the lines are not saturated but it can increase the error on α_{EM} by a factor 2 in the case of strongly saturated lines [91].

3.4.2   Alkali doublet method (AD)

The first method used to set constraint on the time variation of the fine-structure constant relies on fine-structure doublets splitting for which

$$\Delta\nu \propto \frac{\alpha_{\mathrm{EM}}^2 Z^4 R_\infty}{2n^3}.$$

It follows that the relative separation is proportional α_{EM}, $\Delta\nu/\bar{\nu} \propto \alpha_{\mathrm{EM}}^2$ so that the variation of the fine structure constant at a redshift z can be obtained as

$$\left(\frac{\Delta\alpha_{\mathrm{EM}}}{\alpha_{\mathrm{EM}}}\right)(z) = \frac{c_r}{2}\left[\left(\frac{\Delta\lambda}{\bar{\lambda}}\right)_z \bigg/ \left(\frac{\Delta\lambda}{\bar{\lambda}}\right)_0 - 1\right],$$

where $c_r \sim 1$ is a number taking into account the relativistic corrections. This expression is indeed a simple approach of the alkali doublet since one should, as for atomic clocks, take into account the relativistic corrections more precisely. Using the formulation (70), one can deduce that

$$c_r = \frac{\delta q + \delta q_2}{\delta q + 2\delta q_2},$$

where the δq are the differences between the q-coefficients for the doublet transitions.

Several authors have applied the AD method to doublets of several species such as, e.g., C IV, N V, O VI, Mg II, Al III, Si II, Si IV. We refer to Section III.3 of FVC [500] for a summary of their results (see also [318]) and focus on the three most recent analysis, based on the Si IV doublet. In this particular case, $q = 766$ (resp. 362) cm^{-1} and $q_2 = 48$ (resp. –8) cm^{-1} for Si IV $\lambda 1393$ (resp. $\lambda 1402$) so that $c_r = 0.8914$. The method is based on a χ^2 minimization of multiple component Voigt profile fits to the absorption features in the QSO spectra. In general such a profile depends on three parameters, the column density N, the Doppler width (b) and the redshift. It is now extended to include $\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$. The fit is carried out by simultaneously varying these parameters for each component.

- Murphy et al. [377] analyzed 21 Keck/HIRES Si IV absorption systems toward 8 quasars to obtain the weighted mean of the sample,

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-0.5 \pm 1.3) \times 10^{-5}, \qquad 2.33 < z < 3.08, \tag{71}$$

  with a mean redshift of $z = 2.6$. The S/N ratio of these data is in the range $15 - 40$ per pixel and the spectral resolution is $R \sim 34000$.

- Chand et al. [91] analyzed 15 Si IV absorption systems selected from a ESO-UVES sample containing 31 systems (eliminating contaminated, saturated or very broad systems; in particular a lower limit on the column density was fixed so that both lines of the doublets are detected at more than 5σ) to get the weighted mean,

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-0.15 \pm 0.43) \times 10^{-5}, \qquad 1.59 < z < 2.92. \tag{72}$$

  The improvement of the constraint arises mainly from a better S/N ratio, of order $60 - 80$ per pixel, and resolution $R \sim 45000$. Note that combining this result with the previous one (71 in a weighted mean would lead to $\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-0.04 \pm 0.56) \times 10^{-5}$ in the range $1.59 < z < 3.02$

- The analysis [349] of seven C IV systems and two Si IV systems in the direction of a single quasar, obtained by the VLT-VES (during the science verification) has led to

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-3.09 \pm 8.46) \times 10^{-5}, \qquad 1.19 < z < 1.84. \tag{73}$$

  This is less constraining than the two previous analyses, mainly because the q-coefficients are smaller for C IV (see [410] for the calibration of the laboratory spectra)

One limitation may arise from the isotopic composition of silicium. Silicium has three naturally occurring isotopes with terrestrial abundances ^{28}Si:^{29}Si:^{30}Si = 92.23:4.68:3.09 so that each absorption line is a composite of absorption lines from the three isotopes. However, it was shown that this effect of isotopic shifts [377] is negligible in the case of Si IV.

3.4.3 Many multiplet method (MM)

A generalization of the AD method, known as the many-mulptiplet was proposed in [176]. It relies on the combination of transitions from different species. In particular, as can be seen on Figure 3, some transitions are fairly unsensitive to a change of the fine-structure constant (e.g., Mg II or Mg I, hence providing good anchors) while others such as Fe II are more sensitive. The first implementation [522] of the method was based on a measurement of the shift of the Fe II (the rest wavelengths of which are very sensitive to $\alpha_{\rm EM}$) spectrum with respect to the one of Mg II. This comparison increases the sensitivity compared with methods using only alkali doublets. Two series of analyses were performed during the past ten years and lead to contradictory conclusions. The accuracy of the measurements depends on how well the absorption line profiles are modeled.

Keck/HIRES data. The MM-method was first applied in [522] who analyzed one transition of the Mg II doublet and five Fe II transitions from three multiplets. Using 30 absorption systems toward 17 quasars, they obtained

$$\Delta\alpha_{EM}/\alpha_{EM} = (-0.17 \pm 0.39) \times 10^{-5}, \qquad 0.6 < z < 1$$
$$\Delta\alpha_{EM}/\alpha_{EM} = (-1.88 \pm 0.53) \times 10^{-5}, \qquad 1 < z < 1.6.$$

This was the first claim that a constant may have varied during the evolution of the universe. It was later confirmed in a re-analysis [376, 524] of the initial sample and by including new optical QSO data to reach 28 absorption systems with redshift $z = 0.5-1.8$ plus 18 damped Lyman-α absorption systems towards 13 QSO plus 21 Si IV absorption systems toward 13 QSO. The analysis used mainly the multiplets of Ni II, Cr II and Zn II and Mg I, Mg I, Al II, Al III and Fe II was also included. The most recent analysis [369] relies on 128 absorption spectra, later updated [367] to include 143 absorption systems. The more robust estimates is the weighted mean

$$\Delta\alpha_{EM}/\alpha_{EM} = (-0.57 \pm 0.11) \times 10^{-5}, \qquad 0.2 < z < 4.2. \tag{74}$$

The resolution for most spectra was $R \sim 45000$ and the S/N per pixel ranges from 4 to 240, with most spectral regions with S/N$\sim$ 30. The wavelength scale was calibrated by mean of a thorium-argon emission lamp. This calibration is crucial and its quality is discussed in [368, 374] for the Keck/HIRES (see also [236]) as well as [534] for the VLT/UVES measurements.

The low-z ($z < 1.8$) and high-z rely on different ions and transitions with very different α_{EM}-dependencies. At low-z, the Mg transitions are used as anchors against which the large positive shifts in the Fe II can be measured. At high-z, different transitions are fitted (Fe II, S II, Cr II, Ni II, Zn II, Al II, Al III). The two sub-samples respond differently to simple systematic errors due to their different arrangement of q-coefficients in wavelength space. The analysis for each sample give the weighted mean

$$\Delta\alpha_{EM}/\alpha_{EM} = (-0.54 \pm 0.12) \times 10^{-5}, \qquad 0.2 < z < 1.8$$
$$\Delta\alpha_{EM}/\alpha_{EM} = (-0.74 \pm 0.17) \times 10^{-5}, \qquad 1.8 < z < 4.2, \tag{75}$$

with respectively 77 and 66 systems.

Hunting systematics. While performing this kind of observations a number of problems and systematic effects have to be taken into account and controlled. (1) Errors in the determination of laboratory wavelengths to which the observations are compared. (2) While comparing wavelengths from different atoms one has to take into account that they may be located in different regions of the cloud with different velocities and hence with different Doppler shifts. (3) One has to ensure that there is no transition not blended by transitions of another system. (4) The differential isotopic saturation has to be controlled. Usually quasar absorption systems are expected to have lower heavy element abundances. The spatial inhomogeneity of these abundances may also play a role. (5) Hyperfine splitting can induce a saturation similar to isotopic abundances. (6) The variation of the velocity of the Earth during the integration of a quasar spectrum can also induce differential Doppler shift. (7) Atmospheric dispersion across the spectral direction of the spectrograph slit can stretch the spectrum. It was shown that, on average, this can, for low redshift observations, mimic a negative $\Delta\alpha_{EM}/\alpha_{EM}$, while this is no more the case for high redshift observations (hence emphasizing the complementarity of these observations). (8) The presence of a magnetic field will shift the energy levels by Zeeman effect. (9) Temperature variations during the observation will change the air refractive index in the spectrograph. In particular, flexures in the instrument are dealt with by recording a calibration lamp spectrum before and after the science exposure and the signal-to-noise and stability of the lamp is crucial (10) Instrumental effects such as variations of the intrinsic instrument profile have to be controlled.

All these effects have been discussed in detail in [374, 376] to argue that none of them can explain the current detection. This was recently complemented by a study on the calibration since a distortion of the wavelength scale could lead to a non-zero value of $\Delta\alpha_{EM}$. The quality of the calibration is discussed in [368] and shown to have a negligible effect on the measurements (a similar result has been obtained for the VLT/UVES data [534]).

As we pointed out earlier, one assumption of the method concerns the isotopic abundances of Mg II that can affect the low-z sample since any changes in the isotopic composition will alter the value of effective rest-wavelengths. This isotopic composition is assumed to be close to terrestrial ^{24}Mg:^{25}Mg:^{26}Mg = 79:10:11. No direct measurement of $r_{Mg} = (^{26}\text{Mg} + {}^{25}\text{Mg})/^{24}\text{Mg}$ in QSO absorber is currently feasible due to the small separation of the isotopic absorption lines. However, it was shown [231], on the basis of molecular absorption lines of MgH that r_{Mg} generally decreases with decreasing metallicity. In standard models it should be near 0 at zero metallicity since type II supernovae are primarily producers of ^{24}Mg. It was also argued that ^{13}C is a tracer of ^{25}Mg and was shown to be low in the case of HE 0515-4414 [321]. However, contrary to this trend, it was found [552] that r_{Mg} can reach high values for some giant stars in the globular cluster NGC 6752 with metallicity [Fe/H]~ -1.6. This led Ashenfelter et al. [18] to propose a chemical evolution model with strongly enhanced population of intermediate $(2-8\,M_\odot)$ stars, which in their asymptotic giant branch phase are the dominant factories for heavy Mg at low metallicities typical of QSO absorption systems, as a possible explanation of the low-z Keck/HIRES observations without any variation of α_{EM}. It would require that r_{Mg} reaches 0.62, compared to 0.27 (but then the UVES/VLT constraints would be converted to a detection). Care needs to be taken since the star formation history can be different ine each region, even in each absorber, so that one cannot a priori use the best-fit obtained from the Keck data to the UVES/VLT data. However, such modified nucleosynthetic history will lead to an overproduction of elements such as P, Si, Al, P above current constraints [192], but this later model is not the same as the one of Ref. [18] that was tuned to avoid these problems.

In conclusion, no compelling evidence for a systematic effect has been raised at the moment.

VLT/UVES data. The previous results, and their importance for fundamental physics, led another team to check this detection using observations from UVES spectrograph operating on the VLT. In order to avoid as much systematics as possible, and based on numerical simulations, they apply a series of selection criteria [90] on the systems used to constrain the time variation of the fine-structure constant: (1) consider only lines with similar ionization potentials (Mg II, Fe II, Si II and Al II) as they are most likely to originate from similar regions in the cloud; (2) avoid absorption lines contaminated by atmospheric lines; (3) consider only systems with hight enough column density to ensure that all the mutiplets are detected at more than 5σ; (4) demand than at least one of the anchor lines is not saturated to have a robust measurement of the redshift; (5) reject strongly saturated systems with large velocity spread; (6) keep only systems for which the majority of the components are separated from the neighboring by more than the Doppler shift parameter.

The advantage of this choice is to reject most complex or degenerate systems, which could result in uncontrolled systematics effects. The drawback is indeed that the analysis will be based on less systems.

Refs. [90, 470] analyzed the observations of 23 absorption systems, fulfilling the above criteria, in direction of 18 QSO with a S/N ranging between 50 and 80 per pixel and a resolution $R > 44000$. They concluded that

$$\Delta\alpha_{EM}/\alpha_{EM} = (-0.06 \pm 0.06) \times 10^{-5}, \qquad 0.4 < z < 2.3,$$

hence giving a 3σ constraint on a variation of α_{EM}.

This analysis was challenged by Murphy, Webb and Flambaum [372, 371, 370]. Using (quoting them) the same reduced data, using the same fits to the absorption profiles, they claim to find different individual measurements of $\Delta\alpha_{EM}/\alpha_{EM}$ and a weighted mean,

$$\Delta\alpha_{EM}/\alpha_{EM} = (-0.44 \pm 0.16) \times 10^{-5}, \qquad 0.4 < z < 2.3,$$

which differs from the above cited value. The main points that were raised are (1) the fact that some of the uncertainties on $\Delta\alpha_{EM}/\alpha_{EM}$ are smaller than a minimum uncertainty that they estimated and (2) the quality of the statistical analysis (in particular on the basis of the χ^2 curves). These arguments were responded in [471] The revision [471] of the VLT/UVES constraint rejects two more than 4σ deviant systems that were claimed to dominate the re-analysis [371, 370] and concludes that

$$\Delta\alpha_{EM}/\alpha_{EM} = (0.01 \pm 0.15) \times 10^{-5}, \qquad 0.4 < z < 2.3, \tag{76}$$

emphasizing that the errors are probably larger.

On the basis of the articles [372, 371, 370] and the answer [471], it is indeed difficult (without having played with the data) to engage one of the parties. This exchange has enlightened some differences in the statistical analysis.

To finish, let us mention that [361] reanalyzed some systems of [90, 470] by means of the SIDAM method (see below) and disagree with some of them, claiming for a problem of calibration. They also claim that the errors quoted in [367] are underestimated by a factor 1.5.

Regressional MM (RMM). The MM method was adapted to use a linear regression method [427]. The idea is to measure the redshift z_i deduced from the transition i and plot z_i as a function of the sensitivity coefficient. If $\Delta\alpha_{EM} \neq 0$ then there should exist a linear relation with a slope proportional to $\Delta\alpha_{EM}/\alpha_{EM}$. On a single absorption system (VLT/UVES), on the basis of Fe II transition, they concluded that

$$\Delta\alpha_{EM}/\alpha_{EM} = (-0.4 \pm 1.9 \pm 2.7_{syst}) \times 10^{-6}, \qquad z = 1.15, \tag{77}$$

compared to $\Delta\alpha_{EM}/\alpha_{EM} = (0.1 \pm 1.7) \times 10^{-6}$ that is obtained with the standard MM technique on the same data. This is also consistent with the constraint (79) obtained on the same system with the HARPS spectrograph.

Open controversy. At the moment, we have to face a situation in which two teams have performed two independent analyses based on data sets obtained by two instruments on two telescopes. Their conclusions do not agree, since only one of them is claiming for a detection of a variation of the fine-structure constant. This discrepancy between VLT/UVES and Keck/Hires results is yet to be resolved. In particular, they use data from a different telescopes observing a different (Southern/Northern) hemisphere.

Ref. [236] provides an analysis of the wavelength accuracy of the Keck/HIRES spectrograph. An absolute uncertainty of $\Delta z \sim 10^{-5}$, corresponding to $\Delta\lambda \sim 0.02$ Å with daily drift of $\Delta z \sim 5\times10^{-6}$ and multiday drift of $\Delta z \sim 2 \times 10^{-5}$. While the cause of this drift remains unknown, it is argued [236] that this level of systematic uncertainty makes it difficult to use the Keck/HIRES to constrain the time variation of α_{EM} (at least for a single system or a small sample since the distortion pattern pertains to the echelle orders as they are recorded on the CDD, that is it is similar from exposure to exposure, the effect on $\Delta\alpha_{EM}/\alpha_{EM}$ for an ensemble of absorbers at different redshifts would be random since the transitions fall in different places with respect to the pattern of the disortion). This needs to be confirmed and investigated in more detail. We refer to [373] for a discussion on the Keck wavelength calibration error and [534] for the VLT/UVES as well as [86] for a discussion on the ThAr calibration.

On the one hand, it is appropriate that one team has reanalyzed the data of the other and challenged its analysis. This would indeed lead to an improvement in the robustness of these results. Indeed a similar reverse analysis would also be appropriate. On the other hand both teams have achieved an amazing work in order to understand and quantify all sources of systematics. Both developments, as well as the new techniques, which are appearing, should hopefully set this observational issue. Today, it is unfortunately premature to choose one data set compared to the other.

A recent data [523] set of 60 quasar spectra (yielding 153 absorption systems) for the VLT was used and split at $z = 1.8$ to get

$$(\Delta\alpha_{\rm EM}/\alpha_{\rm EM})_{\rm VLT;\, z<1.8} = (-0.06 \pm 0.16) \times 10^{-5},$$

in agreement with the former study [471], while at higher redshift

$$(\Delta\alpha_{\rm EM}/\alpha_{\rm EM})_{\rm VLT\, z>1.8} = (+0.61 \pm 0.20) \times 10^{-5}.$$

This higher component exhibits a positive variation of $\alpha_{\rm EM}$, that is of opposite sign with respect to the previous Keck/HIRES detection [367]

$$(\Delta\alpha_{\rm EM}/\alpha_{\rm EM})_{\rm Keck;\, z<1.8} = (-0.54 \pm 0.12) \times 10^{-5}, \quad (\Delta\alpha_{\rm EM}/\alpha_{\rm EM})_{\rm Keck;\, z>1.8} = (-0.74 \pm 0.17) \times 10^{-5}.$$

It was pointed out that the Keck/HIRES and VLT/UVES observations can be made consistent in the case the fine structure constant is spatially varying [523]. Indeed, one can note that they do not correspond to the same hemisphere and invoke a spatial variation. [523] concludes that the distribution of $\alpha_{\rm EM}$ is well represented by a spatial dipole, significant at 4.1σ, in the direction right ascension 17.3 ± 0.6 hours and declination -61 ± 9 deg (see also [50, 48]). This emphasizes the difficulty in comparing different data sets and shows that the constraints can easily be combined as long as they are compatible with no variation but one must care about a possible spatial variation otherwise.

3.4.4   Single ion differential measurement (SIDAM)

This method [320] is an adaptation of the MM method in order to avoid the influence of small spectral shifts due to ionization inhomogeneities within the absorbers as well as to non-zero offsets between different exposures. It was mainly used with Fe II, which provides transitions with positive and negative q-coefficients (see Figure 3). Since it relies on a single ion, it is less sensitive to isotopic abundances, and in particular not sensitive to the one of Mg.

The first analysis relies on the QSO HE 0515-4414 that was used in [427] to get the constraint (77). An independent analysis [361] of the same system gave a weighted mean

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-0.12 \pm 1.79) \times 10^{-6}, \qquad z = 1.15, \tag{78}$$

at 1σ. The same system was studied independently, using the HARPS spectrograph mounted on the 3.6 m telescope at La Silla observatory [92]. The HARPS spectrograph has a higher resolution that UVES; $R \sim 112000$. Observations based on Fe II with a S/N of about $30-40$ per pixel set the constraint

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (0.5 \pm 2.4) \times 10^{-6}, \qquad z = 1.15. \tag{79}$$

The second constraint [325, 361] is obtained from an absorption system toward Q 1101-264,

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (5.66 \pm 2.67) \times 10^{-6}, \qquad z = 1.84, \tag{80}$$

These constraints do not seem to be compatible with the results of the Keck/HIRES based on the MM method. A potential systematic uncertainty, which can affect these constraints is the relative shift of the wavelength calibration in the blue and the red arms of UVES where the distant Fe lines are recorded simultaneously (see, e.g., [359] for a discussion of the systematics of this analysis).

3.4.5   H I-21 cm vs. UV: $x = \alpha_{\mathrm{EM}}^2 g_{\mathrm{p}}/\mu$

The comparison of UV heavy element transitions with the hyperfine H I transition allows to extract [496]

$$x \equiv \alpha_{\mathrm{EM}}^2 g_{\mathrm{p}}/\mu,$$

since the hyperfine transition is proportional to $\alpha_{\mathrm{EM}}^2 g_{\mathrm{p}} \mu^{-1} R_\infty$ while optical transitions are simply proportional to R_∞. It follows that constraints on the time variation of x can be obtained from high resolution 21 cm spectra compared to UV lines, e.g., of Si II, Fe II and/or Mg II, as first performed in [548] in $z \sim 0.524$ absorber.

Using 9 absorption systems, there was no evidence for any variation of x [494],

$$\Delta x/x = (-0.63 \pm 0.99) \times 10^{-5}, \qquad 0.23 < z < 2.35, \tag{81}$$

This constraint was criticised in [275] on the basis that the systems have multiple components and that it is not necessary that the strongest absorption arises in the same component in both type of lines. However, the error analysis of [494] tries to estimate the effect of the assumption that the strongest absorption arises in the same component.

Following [147], we note that the systems lie in two widely-separated ranges and that the two samples have completely different scatter. Therefore it can be split into two samples of respectively 5 and 4 systems to get

$$\Delta x/x = (1.02 \pm 1.68) \times 10^{-5}, \qquad 0.23 < z < 0.53, \tag{82}$$
$$\Delta x/x = (0.58 \pm 1.94) \times 10^{-5}, \qquad 1.7 < z < 2.35. \tag{83}$$

In such an approach two main difficulties arise: (1) the radio and optical source must coincide (in the optical QSO can be considered pointlike and it must be checked that this is also the case for the radio source), (2) the clouds responsible for the 21 cm and UV absorptions must be localized in the same place. Therefore, the systems must be selected with care and today the number of such systems is small and are actively looked for [411].

The recent detection of 21 cm and molecular hydrogen absorption lines in the same damped Lyman-α system at $z_{\mathrm{abs}} = 3.174$ towards SDSS J1337+3152 constrains [472] the variation x to

$$\Delta x/x = -(1.7 \pm 1.7) \times 10^{-6}, \qquad z = 3.174. \tag{84}$$

This system is unique since it allows for 21 cm, H_2 and UV observation so that in principle one can measure α_{EM}, x and μ independently. However, as the H_2 column density was low, only Werner band absorption lines are seen so that the range of sensitivity coefficients is too narrow to provide a stringent constraint, $\Delta \mu/\mu < 4 \times 10^{-4}$. It was also shown that the H_2 and 21 cm are shifted because of the inhomogeneity of the gas, hence emphasizing this limitation. [411] also mentioned that 4 systems at $z = 1.3$ sets $\Delta x/x = (0.0 \pm 1.5) \times 10^{-6}$ and that another system at $z = 3.1$ gives $\Delta x/x = (0.2 \pm 0.5) \times 10^{-6}$. Note also that the comparison [274] with C I at $z \sim 1.4\text{--}1.6$ towards Q0458-020 and Q2337-011, yields $\Delta x/x = (6.8 \pm 1.0) \times 10^{-6}$ over the band o redshift $0 < \langle z \rangle \leq 1.46$, but this analysis ignores an important wavelength calibration estimated to be of the order of 6.7×10^{-6}. It was argued that, using the existing constraints on $\Delta \mu/\mu$, this measurement is inconsistent with claims of a smaller value of α_{EM} from the many-multiplet method, unless fractional changes in g_p are larger than those in α_{EM} and μ.

3.4.6   H I vs. molecular transitions: $y \equiv g_{\mathrm{p}} \alpha_{\mathrm{EM}}^2$

The H I 21 cm hyperfine transition frequency is proportional to $g_{\mathrm{p}} \mu^{-1} \alpha_{\mathrm{EM}}^2 R_\infty$ (see Section 3.1.1). On the other hand, the rotational transition frequencies of diatomic are inversely proportional to

their reduced mass M. As on the example of Equation (35) where we compared an electronic transition to a vibro-rotational transition, the comparison of the hyperfine and rotational frequencies is proportional to

$$\frac{\nu_{\rm hf}}{\nu_{\rm rot}} \propto g_{\rm p}\alpha_{\rm EM}^2 \frac{M}{m_{\rm p}} \simeq g_{\rm p}\alpha_{\rm EM}^2 \equiv y,$$

where the variation of $M/m_{\rm p}$ is usually suppressed by a large factor of the order of the ratio between the proton mass and nucleon binding energy in nuclei, so that we can safely neglect it.

The constraint on the variation of y is directly determined by comparing the redshift as determined from H I and molecular absorption lines,

$$\frac{\Delta y}{y} = \frac{z_{\rm mol} - z_{\rm H}}{1 + z_{\rm mol}}.$$

This method was first applied [513] to the CO molecular absorption lines [536] towards PKS 1413+135 to get

$$\Delta y/y = (-4 \pm 6) \times 10^{-5} \qquad z = 0.247.$$

The most recent constraint [375] relies on the comparison of the published redshifts of two absorption systems determined both from H I and molecular absorption. The first is a system at $z = 0.6847$ in the direction of TXS 0218+357 for which the spectra of CO(1-2), ^{13}CO(1-2), C^{18}O(1-2), CO(2-3), HCO$^+$(1-2) and HCN(1-2) are available. They concluded that

$$\Delta y/y = (-0.16 \pm 0.54) \times 10^{-5} \qquad z = 0.6847. \tag{85}$$

The second system is an absorption system in direction of PKS 1413+135 for which the molecular lines of CO(1-2), HCO$^+$(1-2) and HCO$^+$(2-3) have been detected. The analysis led to

$$\Delta y/y = (-0.2 \pm 0.44) \times 10^{-5}, \qquad z = 0.247. \tag{86}$$

[78] obtains the constraints $|\Delta y/y| < 3.4 \times 10^{-5}$ at $z \sim 0.25$ and $z \sim 0.685$.

The radio domain has the advantage of heterodyne techniques, with a spectral resolution of 10^6 or more, and dealing with cold gas and narrow lines. The main systematics is the kinematical bias, i.e., that the different lines do not come exactly from the same material along the line of sight, with the same velocity. To improve this method one needs to find more sources, which may be possible with the radio telescope ALMA [3].

3.4.7 OH - 18 cm: $F = g_{\rm p}(\alpha_{\rm EM}^2 \mu)^{1.57}$

Using transitions originating from a single species, like with SIDAM, allows to reduce the systematic effects. The 18 cm lines of the OH radical offers such a possibility [95, 272].

The ground state, $^2\Pi_{3/2} J = 3/2$, of OH is split into two levels by Λ-doubling and each of these doubled level is further split into two hyperfine-structure states. Thus, it has two "main" lines ($\Delta F = 0$) and two "satellite" lines ($\Delta F = 1$). Since these four lines arise from two different physical processes (Λ-doubling and hyperfine splitting), they enjoy the same Rydberg dependence but different $g_{\rm p}$ and $\alpha_{\rm EM}$ dependencies. By comparing the four transitions to the H I hyperfine line, one can have access to

$$F \equiv g_{\rm p}(\alpha_{\rm EM}^2 \mu)^{1.57} \tag{87}$$

and it was also proposed to combine them with HCO$^+$ transitions to lift the degeneracy.

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[3] http://www.eso.org/sci/facilities/alma/

Using the four 18 cm OH lines from the gravitational lens at $z \sim 0.765$ toward PMN J0134-0931 and comparing the H I 21 cm and OH absorption redshifts of the different components allowed to set the constraint [276]

$$\Delta F/F = (-0.44 \pm 0.36 \pm 1.0_{\mathrm{syst}}) \times 10^{-5}, \qquad z = 0.765, \tag{88}$$

where the second error is due to velocity offsets between OH and H I assuming a velocity dispersion of 3 km/s. A similar analysis [138] in a system in the direction of PKS 1413+135 gave

$$\Delta F/F = (0.51 \pm 1.26) \times 10^{-5}, \qquad z = 0.2467. \tag{89}$$

3.4.8   Far infrared fine-structure lines: $F' = \alpha_{\mathrm{EM}}^2 \mu$

Another combination [300] of constants can be obtained from the comparison of far infrared fine-structure spectra with rotational transitions, which respectively behaves as $R_\infty \alpha_{\mathrm{EM}}^2$ and $R_\infty \bar{\mu} = R_\infty/\mu$ so that they give access to

$$F' = \alpha_{\mathrm{EM}}^2 \mu.$$

A good candidate for the rotational lines is CO since it is the second most abundant molecule in the Universe after H_2.

Using the C II fine-structure and CO rotational emission lines from the quasars J1148+5251 and BR 1202-0725, it was concluded that

$$\Delta F'/F' = (0.1 \pm 1.0) \times 10^{-4}, \qquad z = 6.42, \tag{90}$$
$$\Delta F'/F' = (1.4 \pm 1.5) \times 10^{-5}, \qquad z = 4.69, \tag{91}$$

which represents the best constraints at high redshift. As usual, when comparing the frequencies of two different species, one must account for random Doppler shifts caused by non-identical spatial distributions of the two species. Several other candidates for microwave and FIR lines with good sensitivities are discussed in [299].

3.4.9   "Conjugate" satellite OH lines: $G = g_{\mathrm{p}}(\alpha_{\mathrm{EM}}\mu)^{1.85}$

The satellite OH 18 cm lines are conjugate so that the two lines have the same shape, but with one line in emission and the other in absorption. This arises due to an inversion of the level of populations within the ground state of the OH molecule. This behavior has recently been discovered at cosmological distances and it was shown [95] that a comparison between the sum and difference of satellite line redshifts probes $G = g_{\mathrm{p}}(\alpha_{\mathrm{EM}}\mu)^{1.85}$.

From the analysis of the two conjugate satellite OH systems at $z \sim 0.247$ towards PKS 1413+135 and at $z \sim 0.765$ towards PMN J0134-0931, it was concluded [95] that

$$|\Delta G/G| < 7.6 \times 10^{-5}. \tag{92}$$

It was also applied to a nearby system, Centaurus A, to give $|\Delta G/G| < 1.6 \times 10^{-5}$ at $z \sim 0.0018$. A more recent analysis [273] claims for a tentative evidence (with 2.6σ significance, or at 99.1% confidence) for a smaller value of G

$$\Delta G/G = (-1.18 \pm 0.46) \times 10^{-5} \tag{93}$$

for the system at $z \sim 0.247$ towards PKS 1413+135.

One strength of this method is that it guarantees that the satellite lines arise from the same gas, preventing from velocity offset between the lines. Also, the shape of the two lines must agree if they arise from the same gas.

3.4.10   Molecular spectra and the electron-to-proton mass ratio

As was pointed out in Section 3.1, molecular lines can provide a test of the variation[4] [488] of μ since rotational and vibrational transitions are respectively inversely proportional to their reduce mass and its square-root [see Equation (35)].

Constraints with H_2

H_2 is the most abundant molecule in the universe and there were many attempts to use its absorption spectra to put constraints on the time variation of μ despite the fact that H_2 is very difficult to detect [387].

As proposed in [512], the sensitivity of a vibro-rotational wavelength to a variation of μ can be parameterized as

$$\lambda_i = \lambda_i^0 (1 + z_{\text{abs}}) \left(1 + K_i \frac{\Delta\mu}{\mu}\right),$$

where λ_i^0 is the laboratory wavelength (in the vacuum) and λ_i is the wavelength of the transition i in the rest-frame of the cloud, that is at a redshift z_{abs} so that the observed wavelength is $\lambda_i/(1+z_{\text{abs}})$. K_i is a sensitivity coefficient analogous to the q-coefficient introduced in Equation (70), but with different normalization since in the parameterization we would have $q_i = \omega_i^0 K_i/2$,

$$K_i \equiv \frac{\text{d}\ln\lambda_i}{\text{d}\ln\mu}$$

corresponding to the Lyman and Werner bands of molecular hydrogen. From this expression, one can deduce that the observed redshift measured from the transition i is simply

$$z_i = z_{\text{abs}} + bK_i, \qquad b \equiv -(1 + z_{\text{abs}})\frac{\Delta\mu}{\mu},$$

which implies in particular that z_{abs} is not the mean of the z_i if $\Delta\mu \neq 0$. Indeed z_i is measured with some uncertainty of the astronomical measurements λ_i and by errors of the laboratory measurements λ_i^0. But if $\Delta\mu \neq 0$ there must exist a correlation between z_i and K_i so that a linear regression of z_i (measurement) as a function of K_i (computed) allows to extract (z_{abs}, b) and their statistical significance.

We refer to Section V.C of FVC [500] for earlier studies and we focus on the latest results. The recent constraints are mainly based on the molecular hydrogen of two damped Lyman-α absorption systems at $z = 2.3377$ and 3.0249 in the direction of two quasars (Q 1232+082 and Q 0347-382) for which a first analysis of VLT/UVES data showed [262] a slight indication of a variation,

$$\Delta\mu/\mu = (5.7 \pm 3.8) \times 10^{-5}$$

at 1.5σ for the combined analysis. The lines were selected so that they are isolated, unsaturated and unblended. It follows that the analysis relies on 12 lines (over 50 detected) for the first quasar and 18 (over 80) for second but the two selected spectra had no transition in common. The authors performed their analysis with two laboratory catalogs and got different results. They point out that the errors on the laboratory wavelengths are comparable to those of the astronomical measurements.

It was further improved with an analysis of two absorption systems at $z = 2.5947$ and $z = 3.0249$ in the directions of Q 0405-443 and Q 0347-383 observed with the VLT/UVES spectrograph. The data have a resolution $R = 53000$ and a S/N ratio ranging between 30 and 70. The same selection

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[4] Again, μ is used either from $m_{\text{e}}/m_{\text{p}}$ or $m_{\text{p}}/m_{\text{e}}$. I have chosen to use $\mu = m_{\text{p}}/m_{\text{e}}$ and $\bar{\mu} = m_{\text{e}}/m_{\text{p}}$.

criteria where applied, letting respectively 39 (out of 40) and 37 (out of 42) lines for each spectrum and only 7 transitions in common. The combined analysis of the two systems led [261]

$$\Delta\mu/\mu = (1.65 \pm 0.74) \times 10^{-5} \qquad \text{or} \qquad \Delta\mu/\mu = (3.05 \pm 0.75) \times 10^{-5},$$

according to the laboratory measurements that were used. The same data were reanalyzed with new and highly accurate measurements of the Lyman bands of H_2, which implied a reevaluation of the sensitivity coefficient K_i. It leads to the two constraints [431]

$$\Delta\mu/\mu = (2.78 \pm 0.88) \times 10^{-5}, \qquad z = 2.59, \tag{94}$$
$$\Delta\mu/\mu = (2.06 \pm 0.79) \times 10^{-5}, \qquad z = 3.02, \tag{95}$$

leading to a 3.5σ detection for the weighted mean $\Delta\mu/\mu = (2.4 \pm 0.66) \times 10^{-5}$. The authors of [431] do not claim for a detection and are cautious enough to state that systematics dominate the measurements. The data of the $z = 3.02$ absorption system were re-analyzed in [529], which claim that they lead to the bound $|\Delta\mu/\mu| < 4.9 \times 10^{-5}$ at a 2σ level, instead of Equation (95). Adding a new set of 6 spectra, it was concluded that $\Delta\mu/\mu = (15 \pm 14) \times 10^{-6}$ for the weighted fit [530].

These two systems were reanalyzed [289], adding a new system in direction of Q 0528-250,

$$\Delta\mu/\mu = (1.01 \pm 0.62) \times 10^{-5}, \qquad z = 2.59, \tag{96}$$
$$\Delta\mu/\mu = (0.82 \pm 0.74) \times 10^{-5}, \qquad z = 2.8, \tag{97}$$
$$\Delta\mu/\mu = (0.26 \pm 0.30) \times 10^{-5}, \qquad z = 3.02, \tag{98}$$

respectively with 52, 68 and 64 lines. This gives a weighted mean of $(2.6 \pm 3.0) \times 10^{-6}$ at $z \sim 2.81$. To compare with the previous data, the analysis of the two quasars in common was performed by using the same lines (this implies adding 3 and removing 16 for Q 0405-443 and adding 4 and removing 35 for Q 0347-383) to get respectively $(-1.02 \pm 0.89) \times 10^{-5}$ ($z = 2.59$) and $(-1.2 \pm 1.4) \times 10^{-5}$ ($z = 3.02$). Both analyses disagree and this latter analysis indicates a systematic shift of $\Delta\mu/\mu$ toward 0. A second re-analysis of the same data was performed in [490, 489] using a different analysis method to get

$$\Delta\mu/\mu = (-7 \pm 8) \times 10^{-6}. \tag{99}$$

Recently discovered molecular transitions at $z = 2.059$ toward the quasar J2123-0050 observed by the Keck telescope allow to obtain 86 H_2 transitions and 7 HD transitions to conclude [342]

$$\Delta\mu/\mu = (5.6 \pm 5.5_{\text{stat}} \pm 2.7_{\text{syst}}) \times 10^{-6}, \qquad z = 2.059. \tag{100}$$

This method is subject to important systematic errors among which (1) the sensitivity to the laboratory wavelengths (since the use of two different catalogs yield different results [431]), (2) the molecular lines are located in the Lyman-α forest where they can be strongly blended with intervening H I Lyman-α absorption lines, which requires a careful fitting of the lines [289] since it is hard to find lines that are not contaminated. From an observational point of view, very few damped Lyman-α systems have a measurable amount of H_2 so that only a dozen systems is actually known even though more systems will be obtained soon [411]. To finish, the sensitivity coefficients are usually low, typically of the order of 10^{-2}. Some advantages of using H_2 arise from the fact there are several hundred available H_2 lines so that many lines from the same ground state can be used to eliminate different kinematics between regions of different excitation temperatures. The overlap between Lyman and Werner bands also allow to reduce the errors of calibration.

To conclude, the combination of all the existing observations indicate that μ is constant at the 10^{-5} level during the past 11 Gigayrs while an improvement of a factor 10 can be expected in the five coming years.

Other constraints

It was recently proposed [201, 202] that the inversion spectrum of ammonia allows for a better sensitivity to μ. The inversion vibro-rotational mode is described by a double well with the first two levels below the barrier. The tunneling implies that these two levels are split in inversion doublets. It was concluded that the inversion transitions scale as $\nu_{\rm inv} \sim \bar{\mu}^{4.46}$, compared with a rotational transition, which scales as $\nu_{\rm rot} \sim \bar{\mu}$. This implies that the redshifts determined by the two types of transitions are modified according to $\delta z_{\rm inv} = 4.46(1 + z_{\rm abs})\Delta\mu/\mu$ and $\delta z_{\rm rot} \sim (1 + z_{\rm abs})\Delta\mu/\mu$ so that

$$\Delta\mu/\mu = 0.289 \frac{z_{\rm inv} - z_{\rm rot}}{1 + z_{\rm abs}}.$$

Only one quasar absorption system, at $z = 0.68466$ in the direction of B 0218+357, displaying NH_3 is currently known and allows for this test. A first analysis [201] estimated from the published redshift uncertainties that a precision of $\sim 2 \times 10^{-6}$ on $\Delta\mu/\mu$ can be achieved. A detailed measurement [366] of the ammonia inversion transitions by comparison to HCN and HCO^+ rotational transitions concluded that

$$|\Delta\mu/\mu| < 1.8 \times 10^{-6}, \qquad z = 0.685, \tag{101}$$

at a 2σ level. Recently the analysis of the comparison of NH_3 to HC_3N spectra was performed toward the gravitational lens system PKS 1830-211 ($z \simeq 0.89$), which is a much more suitable system, with 10 detected NH_3 inversion lines and a forest of rotational transitions. It reached the conclusion that

$$|\Delta\mu/\mu| < 1.4 \times 10^{-6}, \qquad z = 0.89, \tag{102}$$

at a 3σ level [250]. From a comparison of the ammonia inversion lines with the NH_3 rotational transitions, it was concluded [353]

$$|\Delta\mu/\mu| < 3.8 \times 10^{-6}, \qquad z = 0.89, \tag{103}$$

at 95% C.L. One strength of this analysis is to focus on lines arising from only one molecular species but it was mentioned that the frequencies of the inversion lines are about 25 times lower than the rotational ones, which might cause differences in the absorbed background radio continuum.

This method was also applied [323] in the Milky Way, in order to constrain the spatial variation of μ in the galaxy (see Section 6.1.3). Using ammonia emission lines from interstellar molecular clouds (Perseus molecular core, the Pipe nebula and the infrared dark clouds) it was concluded that $\Delta\mu = (4 - 14) \times 10^{-8}$. This indicates a positive velocity offset between the ammonia inversion transition and rotational transitions of other molecules. Two systems being located toward the galactic center while one is in the direction of the anti-center, this may indicate a spatial variation of μ on galactic scales.

New possibilities

The detection of several deuterated molecular hydrogen HD transitions makes it possible to test the variation of μ in the same way as with H_2 but in a completely independent way, even though today it has been detected only in 2 places in the universe. The sensitivity coefficients have been published in [263] and HD was first detected by [387].

HD was recently detected [473] together with CO and H_2 in a DLA cloud at a redshift of 2.418 toward SDSS1439+11 with 5 lines of HD in 3 components together with several H_2 lines in 7 components. It allowed to set the 3σ limit of $|\Delta\mu/\mu| < 9 \times 10^{-5}$ [412].

Even though the small number of lines does not allow to reach the level of accuracy of H_2 it is a very promising system in particular to obtain independent measurements.

3.4.11   Emission spectra

Similar analysis to constrain the time variation of the fundamental constants were also performed with emission spectra. Very few such estimates have been performed, since it is less sensitive and harder to extend to sources with high redshift. In particular, emission lines are usually broad as compared to absorption lines and the larger individual errors need to be beaten by large statistics.

The O III doublet analysis [24] from a sample of 165 quasars from SDSS gave the constraint

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (12 \pm 7) \times 10^{-5}, \qquad 0.16 < z < 0.8. \tag{104}$$

The method was then extended straightforwardly along the lines of the MM method and applied [238] to the fine-structure transitions in Ne III, Ne V, O III, O I and S II multiplets from a sample of 14 Seyfert 1.5 galaxies to derive the constraint

$$\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (150 \pm 70) \times 10^{-5}, \qquad 0.035 < z < 0.281. \tag{105}$$

3.4.12   Conclusion and prospects

This subsection illustrates the diversity of methods and the progresses that have been achieved to set robust constraints on the variation of fundamental constants. Many systems are now used, giving access to different combinations of the constants. It exploits a large part of the electromagnetic spectrum from far infrared to ultra violet and radio bands and optical and radio techniques have played complementary roles. The most recent and accurate constraints are summarized in Table 10 and Figure 4.
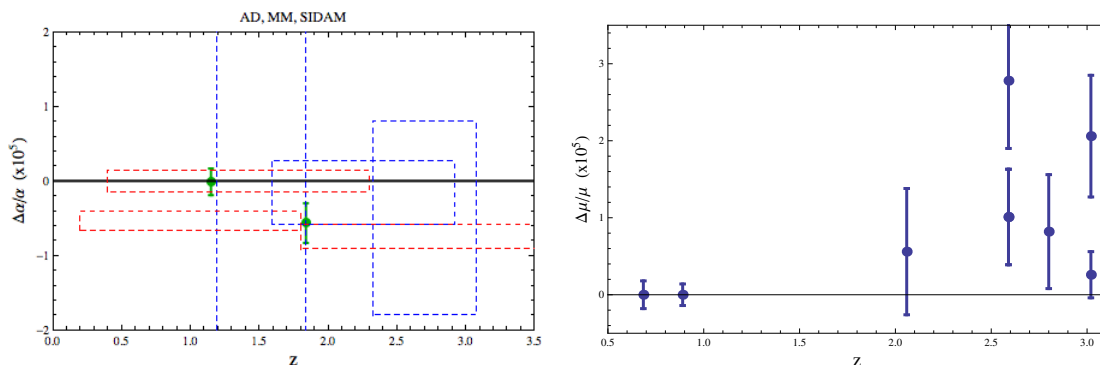
Figure 4: Summary of the direct constraints on $\alpha_{\rm EM}$ obtained from the AD (blue), MM (red) and AD (green) methods (left) and on μ (right) that are summarized in Table 10.

At the moment, only one analysis claims to have detected a variation of the fine structure constant (Keck/HIRES) while the VLT/UVES points toward no variation of the fine structure constant. It has led to the proposition that $\alpha_{\rm EM}$ may be space dependent and exhibit a dipole, the origin of which is not explained. Needless to say that such a controversy and hypotheses are sane since it will help improve the analysis of this data, but it is premature to conclude on the issue of this debate and the jury is still out. Most of the systematics have been investigated in detail and now seem under control.

Let us what we can learn on the physics from these measurement. As an example, consider the constraints obtained on μ, y and F in the redshift band $0.6 - 0.8$ (see Table 10). They can be used to extract independent constraints on $g_{\rm p}$, $\alpha_{\rm EM}$ and μ

$$\Delta\mu/\mu = (0 \pm 0.18) \times 10^{-5}, \quad \Delta\alpha_{\rm EM}/\alpha_{\rm EM} = (-0.27 \pm 2.09) \times 10^{-5}, \quad \Delta g_{\rm p}/g_{\rm p} = (0.38 \pm 4.73) \times 10^{-5}.$$

Table 10: Summary of the latest constraints on the variation of fundamental constants obtained from the analysis of quasar absorption spectra. We recall that $y \equiv g_p\alpha_{EM}^2$, $F \equiv g_p(\alpha_{EM}^2\mu)^{1.57}$, $x \equiv \alpha_{EM}^2 g_p/\mu$, $F' \equiv \alpha_{EM}^2\mu$ and $\mu \equiv m_p/m_e$, $G = g_p(\alpha\mu)^{1.85}$.

Constant	Method	System	Constraint ($\times 10^{-5}$)	Redshift	Ref.
α_{EM}	AD	21	(-0.5 ± 1.3)	$2.33 - 3.08$	[377]
	AD	15	(-0.15 ± 0.43)	$1.59 - 2.92$	[91]
	AD	9	(-3.09 ± 8.46)	$1.19 - 1.84$	[349]
	MM	143	(-0.57 ± 0.11)	$0.2 - 4.2$	[367]
	MM	21	(0.01 ± 0.15)	$0.4 - 2.3$	[90]
	SIDAM	1	(-0.012 ± 0.179)	1.15	[361]
	SIDAM	1	(0.566 ± 0.267)	1.84	[361]
y	H I - mol	1	(-0.16 ± 0.54)	0.6847	[375]
	H I - mol	1	(-0.2 ± 0.44)	0.247	[375]
	CO, CHO$^+$		(-4 ± 6)	0.247	[536]
F	OH - H I	1	$(-0.44 \pm 0.36 \pm 1.0_{syst})$	0.765	[276]
	OH - H I	1	(0.51 ± 1.26)	0.2467	[138]
x	H I - UV	9	(-0.63 ± 0.99)	$0.23 - 2.35$	[494]
	H I - UV	2	(-0.17 ± 0.17)	3.174	[472]
F'	C II - CO	1	(1 ± 10)	4.69	[327]
	C II - CO	1	(14 ± 15)	6.42	[327]
G	OH	1	< 1.1	0.247, 0.765	[95]
	OH	1	< 1.16	0.0018	[95]
	OH	1	(-1.18 ± 0.46)	0.247	[273]
μ	H$_2$	1	(2.78 ± 0.88)	2.59	[431]
	H$_2$	1	(2.06 ± 0.79)	3.02	[431]
	H$_2$	1	(1.01 ± 0.62)	2.59	[289]
	H$_2$	1	(0.82 ± 0.74)	2.8	[289]
	H$_2$	1	(0.26 ± 0.30)	3.02	[289]
	H$_2$	1	(0.7 ± 0.8)	3.02, 2.59	[490]
	NH$_3$	1	< 0.18	0.685	[366]
	NH$_3$	1	< 0.38	0.685	[353]
	HC$_3$N	1	< 0.14	0.89	[250]
	HD	1	< 9	2.418	[412]
	HD	1	$(0.56 \pm 0.55_{stat} \pm 0.27_{syst})$	2.059	[342]

This shows that one can test the compatibility of the constraints obtained from different kind of systems. Independently of these constraints, we have seen in Section 6.3 that in grand unification theory the variation of the constants are correlated. The former constraints show that if $\Delta \ln \mu = R \Delta \ln \alpha_{\mathrm{EM}}$ then the constraint (101) imposes that $|R \Delta \ln \alpha_{\mathrm{EM}}| < 1.8 \times 10^{-6}$. In general R is expected to be of the order of $30 - 50$. Even if its value its time-dependent, that would mean that $\Delta \ln \alpha_{\mathrm{EM}} \sim (1 - 5) \times 10^{-7}$, which is highly incompatible with the constraint (74) obtained by the same team on α_{EM}, but also on the constraints (71) and (72) obtained from the AD method and on which both teams agree. This illustrates how important the whole set of data is since one will probably be able to constrain the order of magnitude of R in a near future, which would be a very important piece of information for the theoretical investigations.

We mention in the course of this paragraph many possibilities to improve these constraints.

Since the AD method is free of the two main assumptions of the MM method, it seems important to increase the precision of this method as well as any method relying only on one species. This can be achieved by increasing the S/N ratio and spectral resolution of the data used or by increasing the sample size and including new transitions (e.g., cobalt [172, 187]).

The search for a better resolution is being investigated in many direction. With the current resolution of $R \sim 40000$, the observed line positions can be determined with an accuracy of $\sigma_\lambda \sim 1$ mÅ. This implies that the accuracy on $\Delta \alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}$ is of the order of 10^{-5} for lines with typical q-coefficients. As we have seen this limit can be improved to 10^{-6} when more transitions or systems are used together. Any improvement is related to the possibility to measure line positions more accurately. This can be done by increasing R up to the point at which the narrowest lines in the absorption systems are resolved. The Bohlin formula [62] gives the estimates

$$
\sigma_\lambda \sim \Delta \lambda_{\mathrm{pix}} \left(\frac{\Delta \lambda_{\mathrm{pix}}}{W_{\mathrm{obs}}} \right) \frac{1}{\sqrt{N_e}} \left(\frac{M^{3/2}}{\sqrt{12}} \right),
$$

where $\Delta \lambda_{\mathrm{pix}}$ is the pixel size, W_{obs} is the observed equivalent width, N_e is the mean number of photoelectron at the continuum level and M is the number of pixel covering the line profile. The metal lines have intrinsic width of a few km/s. Thus, one can expect improvements from higher spectral resolution. Progresses concerning the calibration are also expected, using, e.g., laser comb [478]. Let us just mention, the EXPRESSO (Echelle Spectrograph for PREcision Super Stable Observation) project [115] on 4 VLT units or the CODEX (COsmic Dynamics EXplorer) on E-ELT projects [360, 357, 507]. They shall provide a resolving power of $R = 150000$ to be compared to the HARPS[5] (High Accuracy Radial velocity planet Searcher) spectrograph ($R \sim 112000$) has been used but it is operating on a 3.6 m telescope.

The limitation may then lie in the statistics and the calibration and it would be useful to use more than two QSO with overlapping spectra to cross-calibrate the line positions. This means that one needs to discover more absorption systems suited for these analyses. Much progress is expected. For instance, the FIR lines are expected to be observed by a new generation of telescopes such as HERSCHEL[6]. While the size of the radio sample is still small, surveys are being carried out so that the number of known redshift OH, HI and HCO$^+$ absorption systems will increase. For instance the future Square Kilometer Array (SKA) will be able to detect relative changes of the order of 10^{-7} in α_{EM}.

In conclusion, it is clear that these constraints and the understanding of the absorption systems will increase in the coming years.

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[5] http://obswww.unige.ch/Instruments/HARPS/
[6] http://sci.esa.int/science-e/www/area/index.cfm?fareaid=16

3.5 Stellar constraints

Stars start to accumulate helium produced by the pp-reaction and the CNO cycle in their core. Furthermore, the products of further nuclear reactions of helium with either helium or hydrogen lead to isotopes with $A = 5$ or $A = 8$, which are highly unstable. In order to produce elements heavier than $A > 7$ by fusion of lighter isotopes, the stars need to reach high temperatures and densities. In these conditions, newly produced ^{12}C would almost immediately be fused further to form heavier elements so that one expects only a tiny amount of ^{12}C to be produced, in contradiction with the observed abundances. This led Hoyle [257] to conclude that a then unknown excited state of the ^{12}C with an energy close to the 3α-threshold should exist since such a resonance would increase the probability that ^{8}Be captures an α-particle. It follows that the production of ^{12}C in stars relies on the three conditions:

- the decay lifetime of ^{8}Be, of order 10^{-16} s, is four orders of magnitude longer than the time for two α particles to scatter, so that a macroscopic amount of beryllium can be produced, which is sufficient to lead to considerable production of carbon,

- an excited state of ^{12}C lies just above the energy of ^{8}Be$+\alpha$, which allows for

$$^4\text{He} + {}^4\text{He} \leftrightarrow {}^8\text{Be}, \qquad {}^8\text{Be} + {}^4\text{He} \leftrightarrow {}^{12}\text{C}^* \rightarrow {}^{12}\text{C} + 7.367 \text{ MeV},$$

- the energy level of ^{16}O at 7.1197 MeV is non resonant and below the energy of ^{12}C $+ \alpha$, of order 7.1616 MeV, which ensures that most of the carbon synthesized is not destroyed by the capture of an α-particle. The existence of this resonance, the $J_l^\pi = 0_2^+$-state of ^{12}C was actually discovered [111] experimentally later, with an energy of 372 ± 4 keV [today, $E_{0_2^+} = 379.47 \pm 0.15$ keV], above the ground state of three α-particles (see Figure 5).
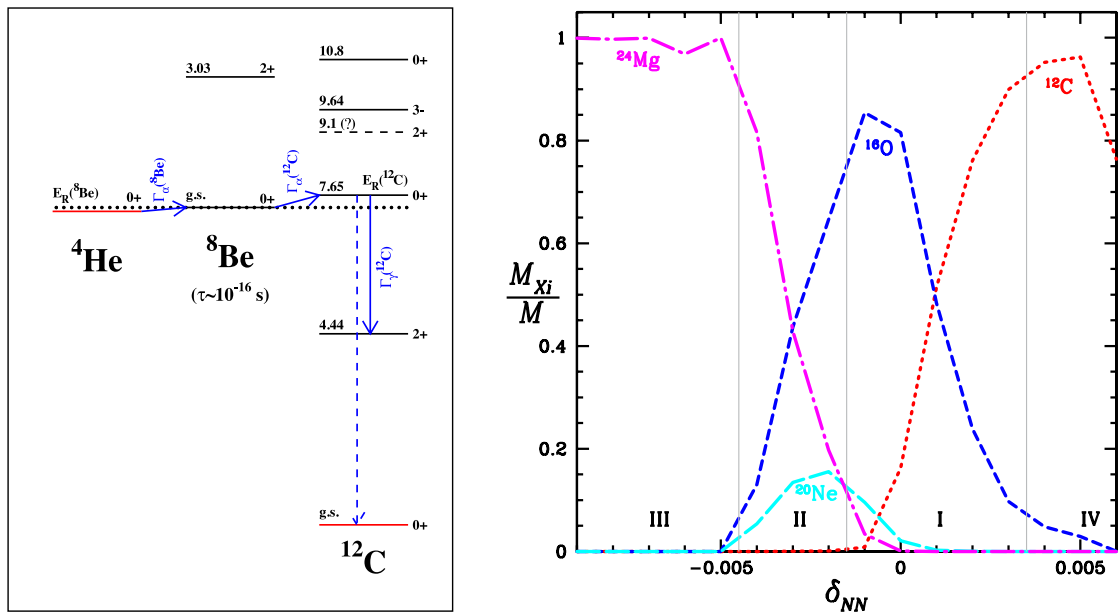
Figure 5: *Left:* Level scheme of nuclei participating to the ^{4}He$(\alpha\alpha, \gamma)^{12}$C reaction. *Right:* Central abundances at the end of the CHe burning as a function of δ_{NN} for a $60\,M_\odot$ star with $Z = 0$. From [103].

The variation of any constant that would modify the energy of this resonance would also endanger the stellar nucleosynthesis of carbon, so that the possibility for carbon production has

often been used in anthropic arguments. Qualitatively, if $E_{0_2^+}$ is increased then the carbon would be rapidly processed to oxygen since the star would need to be hotter for the triple-α process to start. On the other hand, if $E_{0_2^+}$ is decreased, then all α-particles would produce carbon so that no oxygen would be synthesized. It was estimated [334] that the carbon production in intermediate and massive stars is suppressed if the various of the energy of the resonance is outside the range -250 keV $\lesssim \Delta E_{0_2^+} \lesssim 60$ keV, which was further improved [451] to, -5 keV $\lesssim \Delta E_{0_2^+} \lesssim 50$ keV in order for the C/O ratio to be larger than the error in the standard yields by more than 50%. Indeed, in such an analysis, the energy of the resonance was changed by hand. However, we expect that if $E_{0_2^+}$ is modified due to the variation of a constant other quantities, such as the resonance of the oxygen, the binding energies and the cross sections will also be modified in a complex way.

In practice, to draw a constraint on the variation of the fundamental constants from the stellar production of carbon, one needs to go through different steps, any of them involving assumptions,

1. to determine the effective parameters, e.g., cross sections, which affects the stellar evolution. The simplest choice is to modify only the energy of the resonance but it may not be realistic since all cross sections and binding energies should also be affected. This requires one to use a stellar evolutionary model;

2. relate these parameters to nuclear parameters. This involves the whole nuclear physics machinery;

3. to relate the nuclear parameters to fundamental constants. As for the Oklo phenomenon, it requires to link QCD to nuclear physics.

A first analysis [390, 391, 451] used a model that treats the carbon nucleus by solving the 12-nucleon Schrödinger equation using a three-cluster wavefunction representing the three-body dynamics of the ^{12}C state. The NN interaction was described by the Minnesota model [297, 491] and its strength was modified by multiplying the effective NN-potential by an arbitrary number p. This allows to relate the energy of the Hoyle level relative to the triple alpha threshold, $\varepsilon \equiv Q_{\alpha\alpha\alpha}$, and the gamma width, Γ_γ, as a function of the parameter p, the latter being almost not affected. The modified 3α-reaction rate was then given by

$$r_\alpha = 3^{3/2} N_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha k_B T} \right)^3 \frac{\Gamma}{\hbar} \exp\left[-\frac{\varepsilon(p)}{k_B T} \right], \tag{106}$$

where M_α and N_α are the mass and number density of the α-particle, The resonance width $\Gamma = \Gamma_\alpha \Gamma_\gamma / (\Gamma_\alpha + \Gamma_\gamma) \sim \Gamma_\gamma$. This was included in a stellar code and ran for red giant stars with 1.3, 5 and 20 $M_\odot$ with solar metallicity up to thermally pulsating asymptotic giant branch [390] and in low, intermediate and high mass $(1.3, 5, 15, 25 \, M_\odot)$ with solar metallicity also up to TP-AGB [451] to conclude that outside a window of respectively 0.5% and 4% of the values of the strong and electromagnetic forces, the stellar production of carbon or oxygen will be reduced by a factor 30 to 1000.

In order to compute the resonance energy of the ^{8}Be and ^{12}C a microscopic cluster model was developed [297]. The Hamiltonian of the system is then of the form $H = \sum_i^A T(\mathbf{r}_i + \sum_{j<i}^A V(\mathbf{r}_{ij})$, where A is the nucleon number, T the kinetic energy and V the NN interaction potential. In order to implement the variation of the strength of the nuclear interaction with respect to the electromagnetic interaction, it was taken as

$$V(\mathbf{r}_{ij}) = V_C(\mathbf{r}_{ij}) + (1 + \delta_{NN}) V_N(\mathbf{r}_{ij}),$$

where δ_{NN} is a dimensionless parameter that describes the change of the nuclear interaction, V_N being described in [491]. When $A > 4$ no exact solution can be found and approximate solutions

in which the wave function of the ^{8}Be and ^{12}C are described by clusters of respectively 2 and 3 α-particle is well adapted.

First, δ_{NN} can be related to the deuterium binding energy as

$$\Delta B_D/B_D = 5.7701 \times \delta_{NN}, \tag{107}$$

which, given the discussion in Section 3.8.3, allows to relate δ_{NN} to fundamental constants, as, e.g., in [104]. Then, the resonance energy of the ^{8}Be and ^{12}C scale as

$$E_R(^8\text{Be}) = (0.09208 - 12.208 \times \delta_{NN}) \text{ Mev}, \quad E_R(^{12}\text{C}) = (0.2877 - 20.412 \times \delta_{NN}) \text{ Mev}, \tag{108}$$

so that the energy of the Hoyle level relative to the triple alpha threshold is $Q_{\alpha\alpha\alpha} = E_R(^8\text{Be}) + E_R(^{12}\text{C})$.

This was implemented in [103, 180] to population III stars with typical masses, 15 and 60 $M_\odot$ with zero metallicity, in order to compute the central abundances at the end of the core He burning. From Figure 5, one can distinguish 4 regimes (I) the star ends the CHe burning phase with a core composed of a mixture of ^{12}C and ^{16}O, as in the standard case; (II) if the 3α rate is weaker, ^{12}C is produced slower, the reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ becomes efficient earlier so that the star ends the CHe burning phase with a core composed mostly of ^{16}O; (III) for weaker rates, the ^{16}O is further processed to ^{20}Ne and then ^{24}Mg so that the star ends the CHe burning phase with a core composed of ^{24}Mg and (IV) if the 3α rate is stronger, the ^{12}C is produced more rapidly and the star ends the CHe burning phase with a core composed mostly of ^{12}C. Typically this imposes that

$$-5 \times 10^{-4} < \delta_{NN} < 1.5 \times 10^{-3}, \qquad -3 \times 10^{-4} < \Delta B_D/B_D < 9 \times 10^{-3}, \tag{109}$$

at a redshift of order $z \sim 15$, to ensure the ratio C/O to be of order unity.

To finish, a recent study [3] focus on the existence of stars themselves, by revisiting the stellar equilibrium when the values of some constants are modified. In some sense, it can be seen as a generalization of the work by Gamow [224] to constrain the Dirac model of a varying gravitational constant by estimating its effect on the lifetime of the Sun. In this semi-analytical stellar structure model, the effect of the fundamental constants was reduced phenomenologically to 3 parameters, G, which enters mainly on the hydrostatic equilibrium, α_{EM}, which enters in the Coulomb barrier penetration through the Gamow energy, and a composite parameter $\mathcal{C}$, which describes globally the modification of the nuclear reaction rates. The underlying idea is to assume that the power generated per unit volume, $\varepsilon(r)$, and which determines the luminosity of the star, is proportional to the fudge factor $\mathcal{C}$, which would arise from a modification of the nuclear fusion factor, or equivalently of the cross section. Thus, it assumes that all cross sections are affected is a similar way. The parameter space for which stars can form and for which stable nuclear configurations exist was determined, showing that no fine-tuning seems to be required.

This new system is very promising and will provide new information on the fundamental constants at redshifts smaller than $z \sim 15$ where no constraints exist at the moment, even though drawing a robust constraint seems to be difficult at the moment. In particular, an underlying limitation arises from the fact that the composition of the interstellar media is a mixture of ejecta from stars with different masses and it is not clear which type of stars contribute the most the carbon and oxygen production. Besides, one would need to include rotation and mass loss [181]. As for the Oklo phenomenon, another limitation arises from the complexity of nuclear physics.

3.6   Cosmic Microwave Background

The CMB radiation is composed of photons emitted at the time of the recombination of hydrogen and helium when the universe was about 300,000 years old [see, e.g., [409] for details on the physics of the CMB]. This radiation is observed to be a black-body with a temperature $T_0 = 2.725$ K with

small anisotropies of order of the μK. The temperature fluctuation in a direction (ϑ, φ) is usually decomposed on a basis of spherical harmonics as

$$\frac{\delta T}{T}(\vartheta, \varphi) = \sum_{\ell} \sum_{m=-\ell}^{m=+\ell} a_{\ell m} Y_{\ell m}(\vartheta, \varphi). \tag{110}$$

The angular power spectrum multipole $C_\ell = \langle |a_{lm}|^2 \rangle$ is the coefficient of the decomposition of the angular correlation function on Legendre polynomials. Given a model of structure formation and a set of cosmological parameters, this angular power spectrum can be computed and compared to observational data in order to constrain this set of parameters.

The CMB temperature anisotropies mainly depend on three constants: G, $\alpha_{\rm EM}$ and $m_{\rm e}$.

The gravitational constant enters in the Friedmann equation and in the evolution of the cosmological perturbations. It has mainly three effects [435] that are detailed in Section 4.4.1. $\alpha_{\rm EM}$, $m_{\rm e}$ affect the dynamics of the recombination. Their influence is complex and must be computed numerically. However, we can trace their main effects since they mainly modify the CMB spectrum through the change in the differential optical depth of photons due to the Thomson scattering

$$\dot{\tau} = x_{\rm e} n_{\rm e} c \sigma_{\rm T}, \tag{111}$$

which enters in the collision term of the Boltzmann equation describing the evolution of the photon distribution function and where $x_{\rm e}$ is the ionization fraction (i.e., the number density of free electrons with respect to their total number density $n_{\rm e}$).

The first dependence arises from the Thomson scattering cross section given by

$$\sigma_{\rm T} = \frac{8\pi}{3} \frac{\hbar^2}{m_{\rm e}^2 c^2} \alpha_{\rm EM}^2 \tag{112}$$

and the scattering by free protons can be neglected since $m_{\rm e}/m_{\rm p} \sim 5 \times 10^{-4}$.

The second, and more subtle dependence, comes from the ionization fraction. Recombination proceeds via 2-photon emission from the $2s$ level or via the Ly-α photons, which are redshifted out of the resonance line [405] because recombination to the ground state can be neglected since it leads to immediate re-ionization of another hydrogen atom by the emission of a Ly-α photons. Following [405, 338] and taking into account, for the sake of simplicity, only the recombination of hydrogen, the equation of evolution of the ionization fraction takes the form

$$\frac{{\rm d}x_{\rm e}}{{\rm d}t} = \mathcal{C} \left[\beta \left(1 - x_{\rm e} \right) \exp\left(-\frac{B_1 - B_2}{k_{\rm B} T_M} \right) - \mathcal{R} n_{\rm p} x_{\rm e}^2 \right], \tag{113}$$

where T_M is the temperature. At high redshift, T_M is identical to the one of the photons $T_\gamma = T_0(1+z)$ but evolves according to

$$\frac{{\rm d}T_M}{{\rm d}t} = -\frac{8\sigma_{\rm T} a_R}{3 m_{\rm e}} T_R^4 \frac{x_{\rm e}}{1 + x_{\rm e}} (T_M - T_\gamma) - 2HT_M \tag{114}$$

where the radiation constant $a_R = 4\sigma_{\rm SB}/c$ with $\sigma_{\rm SB} = k_{\rm B}^4 \pi^2/(60\pi c^2 \hbar^3)$ the Stefan–Boltzmann constant. In Equation (113), $B_n = -E_I/n^2$ is the energy of the nth hydrogen atomic level, β is the ionization coefficient, $\mathcal{R}$ the recombination coefficient, $\mathcal{C}$ the correction constant due to the redshift of Ly-α photons and to 2-photon decay and $n_p = n_e$ is the number density of protons. β is related to $\mathcal{R}$ by the principle of detailed balance so that

$$\beta = \mathcal{R} \left(\frac{2\pi m_{\rm e} k_{\rm B} T_M}{h^2} \right)^{3/2} \exp\left(-\frac{B_2}{k_{\rm B} T_M} \right). \tag{115}$$

The recombination rate to all other excited levels is

$$\mathcal{R} = \frac{8\pi}{c^2} \left(\frac{k_{\mathrm{B}}T}{2\pi m_{\mathrm{e}}}\right)^{3/2} \sum_{n,l}^{*} (2l+1) \mathrm{e}^{B_n/k_{\mathrm{B}}T} \int_{B_n/k_{\mathrm{B}}T}^{\infty} \sigma_{nl} \frac{y^2 \mathrm{d}y}{\mathrm{e}^y - 1}$$

where σ_{nl} is the ionization cross section for the (n, l) excited level of hydrogen. The star indicates that the sum needs to be regularized and the α_{EM}-, m_{e}-dependence of the ionization cross section is complicated to extract. However, it can be shown to behave as $\sigma_{nl} \propto \alpha_{\mathrm{EM}}^{-1} m_{\mathrm{e}}^{-2} f(h\nu/B_1)$. Finally, the factor $\mathcal{C}$ is given by

$$\mathcal{C} = \frac{1 + K\Lambda_{2s}(1 - x_e)}{1 + K(\beta + \Lambda_{2s})(1 - x_e)} \tag{116}$$

where Λ_{2s} is the rate of decay of the $2s$ excited level to the ground state via 2 photons; it scales as $m_{\mathrm{e}}\alpha_{\mathrm{EM}}^8$. The constant K is given in terms of the Ly-α photon $\lambda_\alpha = 16\pi\hbar/(3m_e\alpha_{\mathrm{EM}}^2 c)$ by $K = n_p\lambda_\alpha^3/(8\pi H)$ and scales as $m_{\mathrm{e}}^{-3}\alpha_{\mathrm{EM}}^{-6}$.

In summary, both the temperature of the decoupling and the residual ionization after recombination are modified by a variation of α_{EM} or m_{e}. This was first discussed in [36, 277]. The last scattering surface can roughly be determined by the maximum of the visibility function $g = \dot{\tau} \exp(-\tau)$, which measures the differential probability for a photon to be scattered at a given redshift. Increasing α_{EM} shifts g to a higher redshift at which the expansion rate is faster so that the temperature and x_e decrease more rapidly, resulting in a narrower g. This induces a shift of the C_ℓ spectrum to higher multipoles and an increase of the values of the C_ℓ. The first effect can be understood by the fact that pushing the last scattering surface to a higher redshift leads to a smaller sound horizon at decoupling. The second effect results from a smaller Silk damping.

Most studies have introduced those modifications in the RECFAST code [454] including similar equations for the recombination of helium. Our previous analysis shows that the dependences in the fundamental constants have various origins, since the binding energies B_i scale has $m_{\mathrm{e}}\alpha_{\mathrm{EM}}^2$, σ_T as $\alpha_{\mathrm{EM}}^2 m_{\mathrm{e}}^{-2}$, K as $m_{\mathrm{e}}^{-3}\alpha_{\mathrm{EM}}^{-6}$, the ionisation coefficients β as α_{EM}^3, the transition frequencies as $m_{\mathrm{e}}\alpha_{\mathrm{EM}}^2$, the Einstein's coefficients as $m_{\mathrm{e}}\alpha_{\mathrm{EM}}^5$, the decay rates Λ as $m_{\mathrm{e}}\alpha_{\mathrm{EM}}^8$ and $\mathcal{R}$ has complicated dependence, which roughly reduces to $\alpha_{\mathrm{EM}}^{-1} m_{\mathrm{e}}^{-2}$. Note that a change in the fine-structure constant and in the mass of the electron are degenerate according to $\Delta\alpha_{\mathrm{EM}} \approx 0.39\Delta m_{\mathrm{e}}$ but this degeneracy is broken for multipoles higher than 1500 [36]. In earlier works [244, 277] it was approximated by the scaling $\mathcal{R} \propto \alpha_{\mathrm{EM}}^{2(1+\xi)}$ with $\xi \sim 0.7$.

The first studies [244, 277] focused on the sensitivity that can be reached by WMAP[7] and Planck[8]. They concluded that they should provide a constraint on α_{EM} at recombination, i.e., at a redshift of about $z \sim 1,000$, with a typical precision $|\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}| \sim 10^{-2} - 10^{-3}$.

The first attempt [21] to actually set a constraint was performed on the first release of the data by BOOMERanG and MAXIMA. It concluded that a value of α_{EM} smaller by a few percents in the past was favored but no definite bound was obtained, mainly due to the degeneracies with other cosmological parameters. It was later improved [22] by a joint analysis of BBN and CMB data that assumes that only α_{EM} varies and that included 4 cosmological parameters $(\Omega_{\mathrm{mat}}, \Omega_{\mathrm{b}}, h, n_s)$ assuming a universe with Euclidean spatial section, leading to $-0.09 < \Delta\alpha_{\mathrm{EM}} < 0.02$ at 68% confidence level. A similar analysis [307], describing the dependence of a variation of the fine-structure constant as an effect on recombination the redshift of which was modeled to scale as $z_* = 1080[1 + 2\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}]$, set the constraint $-0.14 < \Delta\alpha_{\mathrm{EM}} < 0.02$, at a 2σ level, assuming a spatially flat cosmological models with adiabatic primordial fluctuations that. The effect of re-ionisation was discussed in [350]. These works assume that only α_{EM} is varying but, as can been seen from Eqs. (110–116), assuming the electron mass constant.

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[7] http://map.gsfc.nasa.gov/
[8] http://astro.estec.esa.nl/SA-general/Projects/Planck/

With the WMAP first year data, the bound on the variation of $\alpha_{\rm EM}$ was sharpened [438] to $-0.05 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 0.02$, after marginalizing over the remaining cosmological parameters $(\Omega_{\rm mat}h^2, \Omega_{\rm b}h^2, \Omega h^2, n_s, \alpha_s, \tau)$ assuming a universe with Euclidean spatial sections. Restricting to a model with a vanishing running of the spectral index ($\alpha_s \equiv {\rm d}n_s/{\rm d}\ln k = 0$), it gives $-0.06 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 0.01$, at a 95% confidence level. In particular it shows that a lower value of $\alpha_{\rm EM}$ makes $\alpha_s = 0$ more compatible with the data. These bounds were obtained without using other cosmological data sets. This constraint was confirmed by the analysis of [259], which got $-0.097 < \Delta\alpha_{\rm EM}\alpha_{\rm EM} < 0.034$, with the WMAP-1yr data alone and $-0.042 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 0.026$, at a 95% confidence level, when combined with constraints on the Hubble parameter from the HST Hubble Key project.

The analysis of the WMAP-3yr data allows to improve [476] this bound to $-0.039 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 0.010$, at a 95% confidence level, assuming $(\Omega_{\rm mat}, \Omega_{\rm b}, h, n_s, z_{\rm re}, A_s)$ for the cosmological parameters (Ω_Λ being derived from the assumption $\Omega_K = 0$, as well as τ from the re-ionisation redshift, $z_{\rm re}$) and using both temperature and polarization data (TT, TE, EE).

The WMAP 5-year data were analyzed, in combination with the 2dF galaxy redshift survey, assuming that both $\alpha_{\rm EM}$ and $m_{\rm e}$ can vary and that the universe was spatially Euclidean. Letting 6 cosmological parameters $[(\Omega_{\rm mat}h^2, \Omega_{\rm b}h^2, \Theta, \tau, n_s, A_s)$, Θ being the ratio between the sound horizon and the angular distance at decoupling] and 2 constants vary they, it was concluded [452, 453] $-0.012 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 0.018$ and $-0.068 < \Delta m_{\rm e}/m_{\rm e} < 0.044$, the bounds fluctuating slightly depending on the choice of the recombination scenario. A similar analyis [381] not including $m_{\rm e}$ gave $-0.050 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 0.042$, which can be reduced by taking into account some further prior from the HST data. Including polarisation data data from ACBAR, QUAD and BICEP, it was also obtained [352] $-0.043 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 0.038$ at 95% C.L. and $-0.013 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 0.015$ including HST data, also at 95% C.L. Let us also emphasize the work by [351] trying to include the variation of the Newton constant by assuming that $\Delta\alpha_{\rm EM}/\alpha_{\rm EM} = Q\Delta G/G$, Q being a constant and the investigation of [380] taking into account $\alpha_{\rm EM}$, $m_{\rm e}$ and μ, G being kept fixed. Considering $(\Omega_{\rm mat}, \Omega_{\rm b}, h, n_s, \tau)$ for the cosmological parameters they concluded from WMAP-5 data (TT, TE, EE) that $-8.28 \times 10^{-3} < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < 1.81 \times 10^{-3}$ and $-0.52 < \Delta\mu/\mu < 0.17$

The analysis of [452, 453] was updated [310] to the WMAP-7yr data, including polarisation and SDSS data. It leads to $-0.025 < \Delta\alpha_{\rm EM}/\alpha_{\rm EM} < -0.003$ and $0.009 < \Delta m_{\rm e}/m_{\rm e} < 0.079$ at a 1σ level.

The main limitation of these analyses lies in the fact that the CMB angular power spectrum depends on the evolution of both the background spacetime and the cosmological perturbations. It follows that it depends on the whole set of cosmological parameters as well as on initial conditions, that is on the shape of the initial power spectrum, so that the results will always be conditional to the model of structure formation. The constraints on $\alpha_{\rm EM}$ or $m_{\rm e}$ can then be seen mostly as constraints on a delayed recombination. A strong constraint on the variation of $\alpha_{\rm EM}$ can be obtained from the CMB only if the cosmological parameters are independently known. [438] forecasts that CMB alone can determine $\alpha_{\rm EM}$ to a maximum accuracy of 0.1%.

3.7   21 cm

After recombination, the CMB photons are redshifted and their temperature drops as $(1 + z)$. However, the baryons are prevented from cooling adiabatically since the residual amount of free electrons, that can couple the gas to the radiation through Compton scattering, is too small. It follows that the matter decouples thermally from the radiation at a redshift of order $z \sim 200$.

The intergalactic hydrogen atoms after recombination are in their ground state, which hyperfine-structure splits into a singlet and a triple states ($1s_{1/2}$ with $F = 0$ and $F = 1$ respectively, see Section III.B.1 of FCV [500]). It was recently proposed [284] that the observation of the 21 cm emission can provide a test on the fundamental constants. We refer to [221] for a detailed review

Table 11: Summary of the latest constraints on the variation of fundamental constants obtained from the analysis of cosmological data and more particularly of CMB data. All assume $\Omega_K = 0$.

Constraint ($\alpha_{\rm EM} \times 10^2$)	Data	Comment	Ref.
[−9, 2]	BOOMERanG-DASI-COBE + BBN	BBN with $\alpha_{\rm EM}$ only $(\Omega_{\rm mat}, \Omega_{\rm b}, h, n_s)$	[22]
[−1.4, 2]	COBE-BOOMERanG-MAXIMA	$(\Omega_{\rm mat}, \Omega_{\rm b}, h, n_s)$	[307]
[−5, 2]	WMAP-1	$(\Omega_{\rm mat}h^2, \Omega_{\rm b}h^2, \Omega_\Lambda h^2, \tau, n_s, \alpha_s)$	[438]
[−6, 1]	WMAP-1	same + $\alpha_s = 0$	[438]
[−9.7, 3.4]	WMAP-1	$(\Omega_{\rm mat}, \Omega_{\rm b}, h, n_s, \tau, m_{\rm e})$	[259]
[−4.2, 2.6]	WMAP-1 + HST	same	[259]
[−3.9, 1.0]	WMAP-3 (TT,TE,EE) + HST	$(\Omega_{\rm mat}, \Omega_{\rm b}, h, n_s, z_{\rm re}, A_s)$	[476]
[−1.2, 1.8]	WMAP-5 + ACBAR + CBI + 2df	$(\Omega_{\rm mat}h^2, \Omega_{\rm b}h^2, \Theta, \tau, n_s, A_s, m_{\rm e})$	[452]
[−1.9, 1.7]	WMAP-5 + ACBAR + CBI + 2df	$(\Omega_{\rm mat}h^2, \Omega_{\rm b}h^2, \Theta, \tau, n_s, A_s, m_{\rm e})$	[453]
[−5.0, 4.2]	WMAP-5 + HST	$(\Omega_{\rm mat}h^2, \Omega_{\rm b}h^2, h, \tau, n_s, A_s)$	[381]
[−4.3, 3.8]	WMAP-5 + ACBAR + QUAD + BICEP	$(\Omega_{\rm mat}h^2, \Omega_{\rm b}h^2, h, \tau, n_s)$	[352]
[−1.3, 1.5]	WMAP-5 + ACBAR + QUAD + BICEP+HST	$(\Omega_{\rm mat}h^2, \Omega_{\rm b}h^2, h, \tau, n_s)$	[352]
[−0.83, 0.18]	WMAP-5 (TT,TE,EE)	$(\Omega_{\rm mat}h^2, \Omega_{\rm b}h^2, h, \tau, n_s, A_s, m_{\rm e}, \mu)$	[380]
[−2.5, −0.3]	WMAP-7 + H_0 + SDSS	$(\Omega_{\rm mat}h^2, \Omega_{\rm b}h^2, \Theta, \tau, n_s, A_s, m_{\rm e}$	[310]

on 21 cm.

The fraction of atoms in the excited (triplet) state versus the ground (singlet) state is conventionally related by the spin temperature $T_{\rm s}$ defined by the relation

$$\frac{n_t}{n_s} = 3 \exp\left(-\frac{T_*}{T_{\rm s}}\right) \tag{117}$$

where $T_* \equiv hc/(\lambda_{21} k_{\rm B}) = 68.2$ mK is the temperature corresponding to the 21 cm transition and the factor 3 accounts for the degeneracy of the triplet state (note that this is a very simplified description since the assumption of a unique spin temperature is probably not correct [221]. The population of the two states is determined by two processes, the radiative interaction with CMB photons with a wavelength of $\lambda_{21} = 21.1$ cm (i.e., $\nu_{21} = 1420$ MHz) and spin-changing atomic collision. Thus, the evolution of the spin temperature is dictated by [221].

$$\frac{{\rm d}T_{\rm s}}{{\rm d}t} = 4C_{10}\left(\frac{1}{T_{\rm s}} - \frac{1}{T_{\rm g}}\right)T_{\rm s}^2 + (1+z)HA_{10}\left(\frac{1}{T_{\rm s}} - \frac{1}{T_\gamma}\right)\frac{T_\gamma}{T_*} \tag{118}$$

The first term corresponds to the collision de-excitation rate from triplet to singlet and the coefficient C_{10} is decomposed as

$$C_{10} = \kappa_{10}^{HH} n_p + \kappa_{10}^{eH} x_{\rm e} n_p$$

with the respective contribution of H-H and e-H collisions. The second term corresponds to spontaneous transition and A_{10} is the Einstein coefficient. The equation of evolution for the gas temperature $T_{\rm g}$ is given by Equation (114) with $T_M = T_{\rm g}$ (we recall that we have neglected the contribution of helium) and the electronic density satisfies Equation (113).

It follows [284, 285] that the change in the brightness temperature of the CMB at the corresponding wavelength scales as $T_{\rm b} \propto A_{12}/\nu_{21}^2$, where the Einstein coefficient A_{12} is defined below. Observationally, we can deduce the brightness temperature from the brightness I_ν, that is the energy received in a given direction per unit area, solid angle and time, defined as the temperature of the black-body radiation with spectrum I_ν. Thus, $k_{\rm B}T_{\rm b} \simeq I_\nu c^2/2\nu^2$. It has a mean value, $\bar{T}_{\rm b}(z_{\rm obs})$ at various redshift where $1 + z_{\rm obs} = \nu_{21}^{\rm today}/\nu_{\rm obs}$. Besides, as for the CMB, there will also be fluctuation in $T_{\rm b}$ due to imprints of the cosmological perturbations on n_p and $T_{\rm g}$. It follows that we also have access to an angular power spectrum $C_\ell(z_{\rm obs})$ at various redshift (see [329] for details on this computation).

Both quantities depend on the value of the fundamental constants. Beside the same dependencies of the CMB that arise from the Thomson scattering cross section, we have to consider those arising from the collision terms. In natural units, the Einstein coefficient scaling is given by $A_{12} = \frac{2}{3}\pi\alpha_{EM}\nu_{21}^3 m_e^{-2} \sim 2.869 \times 10^{-15}$ s^{-1}. It follows that it scales as $A_{10} \propto g_p^3 \mu^3 \alpha_{EM}^{13} m_e$. The brightness temperature depends on the fundamental constant as $T_b \propto g_p \mu \alpha_{EM}^5 / m_e$. Note that the signal can also be affected by a time variation of the gravitational constant through the expansion history of the universe. [284] (see also [221] for further discussions), focusing only on α_{EM}, showed that this was the dominant effect on a variation of the fundamental constant (the effect on C_{10} is much complicated to determine but was argued to be much smaller). It was estimated that a single station telescope like LWA[9] or LOFAR[10] can lead to a constraint of the order of $\Delta\alpha_{EM}/\alpha_{EM} \sim 0.85\%$, improving to 0.3% for the full LWA. The fundamental challenge for such a measurement is the subtraction of the foreground.

The 21 cm absorption signal in a available on a band of redshift typically ranging from $z \lesssim 1000$ to $z \sim 20$, which is between the CMB observation and the formation of the first stars, that is during the "dark age". Thus, it offers an interesting possibility to trace the constraints on the evolution of the fundamental constants between the CMB epoch and the quasar absorption spectra.

As for CMB, the knowledge of the cosmological parameters is a limitation since a change of 1% in the baryon density or the Hubble parameter implies a 2% (3% respectively) on the mean bolometric temperature. The effect on the angular power spectrum have been estimated but still require an in depth analysis along the lines of, e.g., [329]. It is motivating since $C_\ell(z_{obs})$ is expected to depend on the correlators of the fundamental constants, e.g., $\langle \alpha_{EM}(\mathbf{x}, z_{obs})\alpha_{EM}(\mathbf{x}', z_{obs})\rangle$ and thus in principle allows to study their fluctuation, even though it will also depend on the initial condition, e.g., power spectrum, of the cosmological perturbations.

In conclusion, the 21 cm observation opens a observational window on the fundamental at redshifts ranging typically from 30 to 100, but full in-depth analysis is still required (see [206, 286] for a critical discussion of this probe).

3.8 Big bang nucleosynthesis

3.8.1 Overview

The amount of ^{4}He produced during the big bang nucleosynthesis is mainly determined by the neutron to proton ratio at the freeze-out of the weak interactions that interconvert neutrons and protons. The result of Big Bang nucleosynthesis (BBN) thus depends on G, α_W, α_{EM} and α_S respectively through the expansion rate, the neutron to proton ratio, the neutron-proton mass difference and the nuclear reaction rates, besides the standard parameters such as, e.g., the number of neutrino families.

The standard BBN scenario [117, 409] proceeds in three main steps:

1. for $T > 1$ MeV, $(t < 1$ s) a first stage during which the neutrons, protons, electrons, positrons an neutrinos are kept in statistical equilibrium by the (rapid) weak interaction

$$n \longleftrightarrow p + e^- + \bar{\nu}_e, \qquad n + \nu_e \longleftrightarrow p + e^-, \qquad n + e^+ \longleftrightarrow p + \bar{\nu}_e. \tag{119}$$

As long as statistical equilibrium holds, the neutron to proton ratio is

$$(n/p) = e^{-Q_{np}/k_B T} \tag{120}$$

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[9] http://lwa.unm.edu
[10] http://www.lofar.org

where $Q_{\mathrm{np}} \equiv (m_{\mathrm{n}} - m_{\mathrm{p}})c^2 = 1.29$ MeV. The abundance of the other light elements is given by [409]

$$Y_A = g_A \left(\frac{\zeta(3)}{\sqrt{\pi}} \right)^{A-1} 2^{(3A-5)/2} A^{5/2} \left[\frac{k_{\mathrm{B}}T}{m_{\mathrm{N}}c^2} \right]^{3(A-1)/2} \eta^{A-1} Y_{\mathrm{p}}^Z Y_{\mathrm{n}}^{A-Z} \mathrm{e}^{B_A/k_{\mathrm{B}}T}, \quad (121)$$

where g_A is the number of degrees of freedom of the nucleus $^A_Z\mathrm{X}$, m_{N} is the nucleon mass, η the baryon-photon ratio and $B_A \equiv (Zm_{\mathrm{p}} + (A-Z)m_{\mathrm{n}} - m_A)c^2$ the binding energy.

2. Around $T \sim 0.8$ MeV ($t \sim 2$ s), the weak interactions freeze out at a temperature T_{f} determined by the competition between the weak interaction rates and the expansion rate of the universe and thus roughly determined by $\Gamma_{\mathrm{w}}(T_{\mathrm{f}}) \sim H(T_{\mathrm{f}})$ that is

$$G_{\mathrm{F}}^2 (k_{\mathrm{B}}T_{\mathrm{f}})^5 \sim \sqrt{GN_*}(k_{\mathrm{B}}T_{\mathrm{f}})^2 \qquad (122)$$

where G_{F} is the Fermi constant and N_* the number of relativistic degrees of freedom at T_{f}. Below T_{f}, the number of neutrons and protons change only from the neutron β-decay between T_{f} to $T_{\mathrm{N}} \sim 0.1$ MeV when $p + n$ reactions proceed faster than their inverse dissociation.

3. For 0.05 MeV $< T < 0.6$ MeV (3 s $< t <$ 6 min), the synthesis of light elements occurs only by two-body reactions. This requires the deuteron to be synthesized ($p + n \to D$) and the photon density must be low enough for the photo-dissociation to be negligible. This happens roughly when

$$\frac{n_{\mathrm{d}}}{n_\gamma} \sim \eta^2 \exp(-B_D/T_{\mathrm{N}}) \sim 1 \qquad (123)$$

with $\eta \sim 3 \times 10^{-10}$. The abundance of ^{4}He by mass, Y_{p}, is then well estimated by

$$Y_{\mathrm{p}} \simeq 2 \frac{(n/p)_{\mathrm{N}}}{1 + (n/p)_{\mathrm{N}}} \qquad (124)$$

with

$$(n/p)_{\mathrm{N}} = (n/p)_{\mathrm{f}} \exp(-t_{\mathrm{N}}/\tau_{\mathrm{n}}) \qquad (125)$$

with $t_{\mathrm{N}} \propto G^{-1/2} T_{\mathrm{N}}^{-2}$ and $\tau_{\mathrm{n}}^{-1} = 1.636\, G_{\mathrm{F}}^2 (1 + 3g_A^2) m_{\mathrm{e}}^5/(2\pi^3)$, with $g_A \simeq 1.26$ being the axial/vector coupling of the nucleon. Assuming that $B_D \propto \alpha_{\mathrm{S}}^2$, this gives a dependence $t_{\mathrm{N}}/\tau_{\mathrm{p}} \propto G^{-1/2} \alpha_{\mathrm{S}}^2 G_{\mathrm{F}}^2$.

4. The abundances of the light element abundances, Y_i, are then obtained by solving a series of nuclear reactions

$$\dot{Y}_i = J - \Gamma Y_i,$$

where J and Γ are time-dependent source and sink terms.

From an observational point of view, the light elements abundances can be computed as a function of η and compared to their observed abundances. Figure 6 summarizes the observational constraints obtained on helium-4, helium-3, deuterium and lithium-7. On the other hand, η can be determined independently from the analysis of the cosmic microwave background anisotropies and the WMAP data [296] have led to to the conclusion that

$$\eta = \eta_{\mathrm{WMAP}} = (6.19 \pm 0.15) \times 10^{-10}.$$

This number being fixed, all abundances can be computed. At present, there exists a discrepancy between the predicted abundance of lithium-7 based on the WMAP results [108, 107] for η, ^{7}Li/H $= (5.14 \pm 0.50) \times 10^{-10}$ and its values measured in metal-poor halo stars in our galaxy [63], ^{7}Li/H $=$

$(1.26\pm0.26)\times10^{-10}$, which is a factor of three lower, at least [116] (see also [469]), than the predicted value. No solution to this *Lithium-7* problem is known. A back of the envelope estimates shows that we can mimic a lower η parameter, just by modifying the deuterium binding energy, letting T_N unchanged, since from Equation (123), one just need $\Delta B_D/T_N \sim -\ln 9$ so that the effective η parameter, assuming no variation of constant, is three times smaller than η_{WMAP}. This rough rule of thumb explains that the solution of the lithium-7 problem may lie in a possible variation of the fundamental constants (see below for details).

3.8.2 Constants everywhere...

In complete generality, the effect of varying constants on the BBN predictions is difficult to model because of the intricate structure of QCD and its role in low energy nuclear reactions. Thus, a solution is to proceed in *two steps*, first by determining the dependencies of the light element abundances on the BBN parameters and then by relating those parameters to the fundamental constants.

The analysis of the previous Section 3.8.1, that was restricted to the helium-4 case, clearly shows that the abundances will depend on: (1) α_{G}, which will affect the Hubble expansion rate at the time of nucleosynthesis in the same way as extra-relativistic degrees of freedom do, so that it modifies the freeze-out time T_{f}. This is the only gravitational sector parameter. (2) τ_{n}, the neutron lifetime dictates the free neutron decay and appears in the normalization of the proton-neutron reaction rates. It is the only weak interaction parameter and it is related to the Fermi constant G_{F}, or equivalently the Higgs vev. (3) α_{EM}, the fine-structure constant. It enters in the Coulomb barriers of the reaction rates through the Gamow factor, in all the binding energies. (4) Q_{np}, the neutron-proton mass difference enters in the neutron-proton ratio and we also have a dependence in (5) m_{N} and m_{e} and (6) the binding energies.

Clearly all these parameters are not independent but their relation is often model-dependent. If we focus on helium-4, its abundance mainly depends on Q_{np}, T_{f} and T_{N} (and hence mainly on the neutron lifetime, τ_{n}). Early studies (see Section III.C.2 of FVC [500]) generally focused on one of these parameters. For instance, Kolb et al. [295] calculated the dependence of primordial ^{4}He on G, G_{F} and Q_{np} to deduce that the helium-4 abundance was mostly sensitive in the change in Q_{np} and that other abundances were less sensitive to the value of Q_{np}, mainly because ^{4}He has a larger binding energy; its abundances is less sensitive to the weak reaction rate and more to the parameters fixing the value of (n/p). To extract the constraint on the fine-structure constant, they decomposed Q_{np} as $Q_{\mathrm{np}} = \alpha_{\mathrm{EM}}Q_\alpha + \beta Q_\beta$ where the first term represents the electromagnetic contribution and the second part corresponds to all non-electromagnetic contributions. Assuming that Q_α and Q_β are constant and that the electromagnetic contribution is the dominant part of Q, they deduced that $|\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}| < 10^{-2}$. Campbell and Olive [77] kept track of the changes in T_{f} and Q_{np} separately and deduced that $\frac{\Delta Y_{\mathrm{p}}}{Y_{\mathrm{p}}} \simeq \frac{\Delta T_{\mathrm{f}}}{T_{\mathrm{f}}} - \frac{\Delta Q_{\mathrm{np}}}{Q_{\mathrm{np}}}$ while more recently the analysis [308] focused on α_{EM} and v.

Let us now see how the effect of all these parameters are now accounted for in BBN codes.

Bergström et al. [51] started to focus on the α_{EM}-dependence of the thermonuclear rates (see also Ref. [260]). In the non-relativistic limit, it is obtained as the thermal average of the product of the cross, the relative velocity and the the number densities. Charged particles must tunnel through a Coulomb barrier to react. Changing α_{EM} modifies these barriers and thus the reaction rates. Separating the Coulomb part, the low-energy cross section can be written as

$$\sigma(E) = \frac{S(E)}{E}\mathrm{e}^{-2\pi\eta(E)} \tag{126}$$

where $\eta(E)$ arises from the Coulomb barrier and is given in terms of the charges and the reduced

mass M_r of the two interacting particles as

$$\eta(E) = \alpha_{\mathrm{EM}} Z_1 Z_2 \sqrt{\frac{M_r c^2}{2E}}. \tag{127}$$

The form factor $S(E)$ has to be extrapolated from experimental nuclear data but its α_{EM}-dependence as well as the one of the reduced mass were neglected. Keeping all other constants fixed, assuming no exotic effects and taking a lifetime of 886.7 s for the neutron, it was deduced that $|\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}| < 2 \times 10^{-2}$. This analysis was then extended [385] to take into account the α_{EM}-dependence of the form factor to conclude that

$$\sigma(E) = \frac{2\pi\eta(E)}{\exp^{2\pi\eta(E)} - 1} \simeq 2\pi\alpha_{\mathrm{EM}} Z_1 Z_2 \sqrt{\frac{M_r c^2}{c^2}} \exp^{-2\pi\eta(E)}.$$

Ref. [385] also took into account (1) the effect that when two charged particles are produced they must escape the Coulomb barrier. This effect is generally weak because the Q_i-values (energy release) of the different reactions are generally larger than the Coulomb barrier at the exception of two cases, ${}^3\mathrm{He}(n,p){}^3\mathrm{H}$ and ${}^7\mathrm{Be}(n,p){}^7\mathrm{Li}$. The rate of these reactions must be multiplied by a factor $(1 + a_i \Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}})$. (2) The radiative capture (photon emitting processes) are proportional to α_{EM} since it is the strength of the coupling of the photon and nuclear currents. All these rates need to be multiplied by $(1 + \Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}})$. (3) The electromagnetic contribution to all masses was taken into account, which modify the Q_i-values as $Q_i \to Q_i + q_i \Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}$). For helium-4 abundance these effects are negligible since the main α_{EM}-dependence arises from Q_{np}. Equipped with these modifications, it was concluded that $\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}} = -0.007^{+0.010}_{-0.017}$ using only deuterium and helium-4 since the lithium-7 problem was still present.

Then the focus fell on the deuterium binding energy, B_D. Flambaum and Shuryak [207, 208, 158, 157] illustrated the sensitivity of the light element abundances on B_D. Its value mainly sets the beginning of the nucleosynthesis, that is of T_N since the temperature must low-enough in order for the photo-dissociation of the deuterium to be negligible (this is at the origin of the deuterium bottleneck). The importance of B_D is easily understood by the fact that the equilibrium abundance of deuterium and the reaction rate $p(n,\gamma)\mathrm{D}$ depends exponentially on B_D and on the fact that the deuterium is in a shallow bound state. Focusing on the T_N-dependence, it was concluded [207] that $\Delta B_D/B_D < 0.075$.

This shows that the situation is more complex and that one cannot reduce the analysis to a single varying parameter. Many studies then tried to determinate the sensitivity to the variation of many independent parameters.

The sensitivity of the helium-4 abundance to the variation of 7 parameters was first investigated by Müller et al. [364] considering the dependence on the parameters $\{X_i\} \equiv \{G, \alpha_{\mathrm{EM}}, v, m_{\mathrm{e}}, \tau_{\mathrm{n}}, Q_{\mathrm{np}}, B_D\}$ independently,

$$\Delta \ln Y_{\mathrm{He}} = \sum_i c_i^{(X)} \Delta \ln X_i$$

and assuming Λ_{QCD} fixed (so that the seven parameters are in fact dimensionless quantities). The $c_i^{(X)}$ are the sensitivities to the BBN parameters, assuming the six others are fixed. It was concluded that $Y_{\mathrm{He}} \propto \alpha_{\mathrm{EM}}^{-0.043} v^{2.4} m_{\mathrm{e}}^{0.024} \tau_{\mathrm{n}}^{0.24} Q_{\mathrm{np}}^{-1.8} B_D^{0.53} G^{0.405}$ for independent variations. They further related $(\tau_{\mathrm{n}}, Q_{\mathrm{np}}, B_D)$ to $(\alpha_{\mathrm{EM}}, v, m_{\mathrm{e}}, m_{\mathrm{N}}, m_{\mathrm{d}} - m_{\mathrm{u}})$, as we shall discuss in the next Section 3.8.3.

This was generalized by Landau et al. [309] up to lithium-7 considering the parameters $\{\alpha_{\mathrm{EM}}, G_{\mathrm{F}}, \Lambda_{\mathrm{QCD}}, \Omega_b h^2\}$, assuming G constant where the variation of τ_{n} and the variation of the masses where tied to these parameters but the effect on the binding energies were not considered.

Coc et al. [104] considered the effect of a variation of $(Q_{\mathrm{np}}, B_D, \tau_{\mathrm{n}}, m_{\mathrm{e}})$ on the abundances of the light elements up to lithium-7, neglecting the effect of α_{EM} on the . Their dependence on the
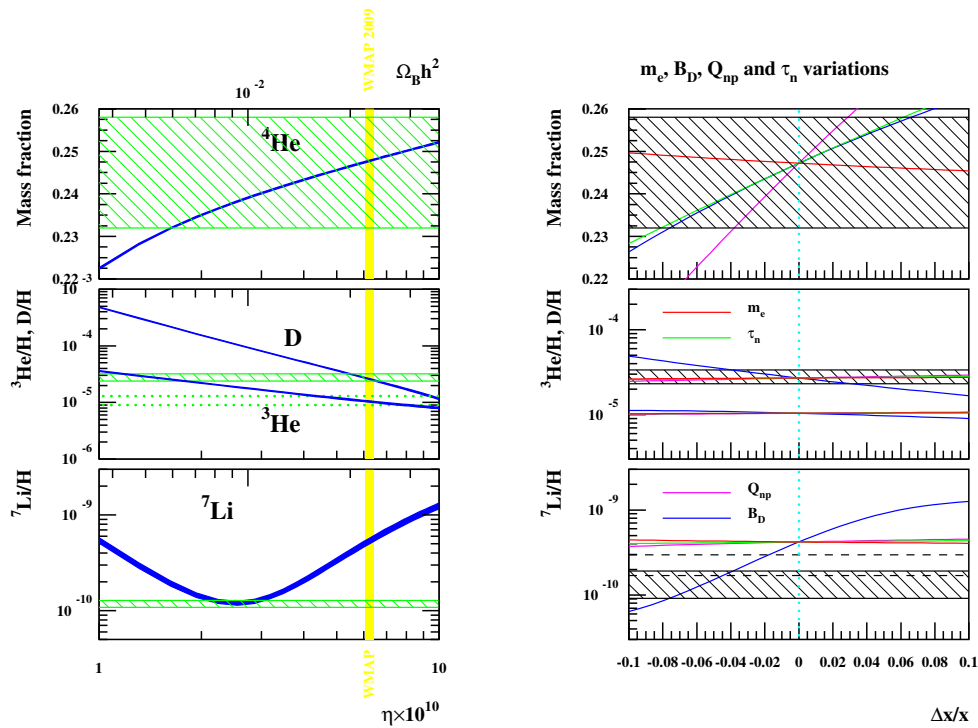
Figure 6: (Left): variation of the light element abundances in function of η compared to the spectroscopic abundances. The vertical line depicts the constraint obtained on η from the study of the cosmic microwave background data. The lithium-7 problem lies in the fact that $\eta_{\mathrm{spectro}} < \eta_{\mathrm{WMAP}}$. From [107]. (right): Dependence of the light element abundance on the independent variation of the BBN parameters, assuming $\eta = \eta_{\mathrm{WMAP}}$. From [105]

independent variation of each of these parameters is depicted on Figure 6. It confirmed the result of [207, 394] that the deuterium binding energy is the most sensitive parameter. From the helium-4 data alone, the bounds

$$-8.2 \times 10^{-2} \lesssim \frac{\Delta \tau_{\rm n}}{\tau_{\rm n}} \lesssim 6 \times 10^{-2}, \quad -4 \times 10^{-2} \lesssim \frac{\Delta Q_{\rm np}}{Q_{\rm np}} \lesssim 2.7 \times 10^{-2}, \tag{128}$$

and

$$-7.5 \times 10^{-2} \lesssim \frac{\Delta B_D}{B_D} \lesssim 6.5 \times 10^{-2}, \tag{129}$$

at a 2σ level, were set (assuming $\eta_{\rm WMAP}$). The deuterium data set the tighter constraint $-4 \times 10^{-2} \lesssim \Delta \ln B_D \lesssim 3 \times 10^{-2}$. Note also on Figure 6 that the lithium-7 abundance can be brought in concordance with the spectroscopic observations provided that B_D was smaller during BBN

$$-7.5 \times 10^{-2} \lesssim \frac{\Delta B_D}{B_D} \lesssim -4 \times 10^{-2},$$

so that B_D may be the most important parameter to resolve the lithium-7 problem. The effect of the quark mass on the binding energies was described in [49]. They then concluded that a variation of $\Delta m_{\rm q}/m_{\rm q} = 0.013 \pm 0.002$ allows to reconcile the abundance of lithium-7 and the value of η deduced from WMAP.

This analysis was extended [146] to incorporate the effect of 13 independent BBN parameters including the parameters considered before plus the binding energies of deuterium, tritium, helium-3, helium-4, lithium-6, lithium-7 and beryllium-7. The sensitivity of the light element abundances to the independent variation of these parameters is summarized in Table I of [146]. These BBN parameters were then related to the same 6 "fundamental" parameters used in [364].

All these analyses demonstrate that the effects of the BBN parameters on the light element abundances are now under control. They have been implemented in BBN codes and most results agree, as well as with semi-analytical estimates. As long as these parameters are assume to vary independently, no constraints sharper than 10^{-2} can be set. One should also not forget to take into account standard parameters of the BBN computation such as η and the effective number of relativistic particle.

3.8.3 From BBN parameters to fundamental constants

To reduce the number parameters, we need to relate the BBN parameters to more fundamental ones, keeping in mind that this can usually be done only in a model-dependent way. We shall describe some of the relations that have been used in many studies. They mainly concern $Q_{\rm np}$, $\tau_{\rm n}$ and B_D.

At lowest order, all dimensional parameters of QCD, e.g., masses, nuclear energies etc., are to a good approximation simply proportional to some powers of $\Lambda_{\rm QCD}$. One needs to go beyond such a description and takes the effects of the masses of the quarks into account.

$Q_{\rm np}$ can be expressed in terms of the mass on the quarks u and d and the fine-structure constant as

$$Q_{\rm np} = a\alpha_{\rm EM}\Lambda_{\rm QCD} + (m_{\rm d} - m_{\rm u}),$$

where the electromagnetic contribution today is $(a\alpha_{\rm EM}\Lambda_{\rm QCD})_0 = -0.76$ MeV and therefore the quark mass contribution today is $(m_{\rm d} - m_{\rm u}) = 2.05$ [230] so that

$$\frac{\Delta Q_{\rm np}}{Q_{\rm np}} = -0.59 \frac{\Delta \alpha_{\rm EM}}{\alpha_{\rm EM}} + 1.59 \frac{\Delta (m_{\rm d} - m_{\rm u})}{(m_{\rm d} - m_{\rm u})}. \tag{130}$$

All the analyses cited above agree on this dependence.

The neutron lifetime can be well approximated by

$$\tau_{\rm n}^{-1} = \frac{1 + 3g_A^2}{120\pi^3} G_{\rm F}^2 m_{\rm e}^5 \left[\sqrt{q^2 - 1}(2q^4 - 9q^2 - 8) + 15\ln\left(q + \sqrt{q^2 - 1}\right) \right],$$

with $q \equiv Q_{\rm np}/m_{\rm e}$ and $G_{\rm F} = 1/\sqrt{2}v^2$. Using the former expression for $Q_{\rm np}$ we can express $\tau_{\rm n}$ in terms of $\alpha_{\rm EM}$, v and the u, d and electron masses. It follows

$$\frac{\Delta\tau_{\rm n}}{\tau_{\rm n}} = 3.86\frac{\Delta\alpha_{\rm EM}}{\alpha_{\rm EM}} + 4\frac{\Delta v}{v} + 1.52\frac{\Delta m_{\rm e}}{m_{\rm e}} - 10.4\frac{\Delta(m_{\rm d} - m_{\rm u})}{(m_{\rm d} - m_{\rm u})}. \tag{131}$$

Again, all the analyses cited above agree on this dependence.

Let us now turn to the binding energies, and more particularly to B_D that, as we have seen, is a crucial parameter. This is one the better known quantities in the nuclear domain and it is experimentally measured to a precision better than 10^{-6} [19]. Two approaches have been followed.

- *Pion mass.* A first route is to use the dependence of the binding energy on the pion mass [188, 38], which is related to the u and d quark masses by

$$m_\pi^2 = m_{\rm q}\langle\bar{u}u + \bar{d}d\rangle f_\pi^{-2} \simeq \hat{m}\Lambda_{\rm QCD},$$

  where $m_{\rm q} \equiv \frac{1}{2}(m_{\rm u} + m_{\rm d})$ and assuming that the leading order of $\langle\bar{u}u + \bar{d}d\rangle f_\pi^{-2}$ depends only on $\Lambda_{\rm QCD}$, f_π being the pion decay constant. This dependence was parameterized [553] as

$$\frac{\Delta B_D}{B_D} = -r\frac{\Delta m_\pi}{m_\pi},$$

  where r is a fitting parameter found to be between 6 [188] and 10 [38]. Prior to this result, the analysis of [207] provides two computations of this dependence, which respectively lead to $r = -3$ and $r = 18$ while, following the same lines, [88] got $r = 0.082$.

  [364], following the computations of [426], adds an electromagnetic contribution $-0.0081\Delta\alpha_{\rm EM}/\alpha_{\rm EM}$ so that

$$\frac{\Delta B_D}{B_D} = -\frac{r}{2}\frac{\Delta m_{\rm q}}{m_{\rm q}} - 0.0081\frac{\Delta\alpha_{\rm EM}}{\alpha_{\rm EM}}, \tag{132}$$

  but this latter contribution has not been included in other work.

- *Sigma model.* In the framework of the Walecka model, where the potential for the nuclear forces keeps only the σ and ω meson exchanges,

$$V = -\frac{g_s^2}{4\pi r}\exp(-m_\sigma r) + \frac{g_v^2}{4\pi r}\exp(-m_\omega r),$$

  where g_s and g_v are two coupling constants. Describing σ as a SU(3) singlet state, its mass was related to the mass of the strange quark. In this way one can hope to take into account the effect of the strange quark, both on the nucleon mass and the binding energy. In a second step B_D is related to the meson and nucleon mass by

$$\frac{\Delta B_D}{B_D} = -48\frac{\Delta m_\sigma}{m_\sigma} + 50\frac{\Delta m_\omega}{m_\omega} + 6\frac{\Delta m_{\rm N}}{m_{\rm N}}$$

  so that $\Delta B_D/B_D \simeq -17\Delta m_{\rm s}m_{\rm s}$ [208]. Unfortunately, a complete treatment of all the nuclear quantities on $m_{\rm s}$ has not been performed yet.

The case of the binding energies of the other elements has been less studied. [146] follows a route similar than for B_D and relates them to pion mass and assumes that

$$\frac{\partial B_i}{\partial m_\pi} = f_i(A_i - 1)\frac{B_D}{m_\pi}r \simeq -0.13f_i(A_i - 1),$$

where f_i are unknown coefficients assumed to be of order unity and A_i is the number of nucleons. No other estimates has been performed. Other nuclear potentials (such as Reid 93 potential, Nijmegen potential, Argonne $v18$ potential and Bonn potential) have been used in [101] to determine the dependence of B_D on v and agree with previous studies.

These analyses allow one to reduce all the BBN parameter to the physical constants (α_{EM}, v, m_e, $m_d - m_u$, m_q) and G that is not affected by this discussion. This set can be further reduced, since all the masses can be expressed in terms of v as $m_i = h_i v$, where h_i are Yukawa couplings.

To go further, one needs to make more assumption, such as grand unification, or by relating the Yukawa coupling of the top to v by assuming that weak scale is determined by dimensional transmutation [104], or that the variation of the constant is induced by a string dilaton [77]. At each step, one gets more stringent constraints, which can reach the 10^{-4} [146] to 10^{-5} [104] level but indeed more model-dependent!

3.8.4   Conclusion

Primordial nucleosynthesis offers a possibility to test almost all fundamental constants of physics at a redshift of $z \sim 10^8$. This makes it very rich but indeed the effect of each constant is more difficult to disentangle. The effect of the BBN parameters has been quantified with precision and they can be constrained typically at a 10^{-2} level, and in particular it seems that the most sensitive parameter is the deuterium binding energy.

The link with more fundamental parameters is better understood but the dependence of the deuterium binding energy still left some uncertainties and a good description of the effect of the strange quark mass is missing.

We have not considered the variation of G in this section. Its effect is disconnected from the other parameters. Let us just stress that assuming the BBN sensitivity on G by just modifying its value may be misleading. In particular G can vary a lot during the electron-positron annihilation so that the BBN constraints can in general not be described by an effective speed-up factor [105, 134].

4   The Gravitational Constant

The gravitational constant was the first constant whose constancy was questioned [155]. From a theoretical point of view, theories with a varying gravitational constant can be designed to satisfy the equivalence principle in its weak form but not in its strong form [540] (see also Section 5). Most theories of gravity that violate the strong equivalence principle predict that the locally measured gravitational constant may vary with time.

The value of the gravitational constant is $G = 6.674\,28(67) \times 10^{-11}$ m^3 kg^{-1} s^{-2} so that its relative standard uncertainty fixed by the CODATA[11] in 2006 is 0.01%. Interestingly, the disparity between different experiments led, in 1998, to a temporary increase of this uncertainty to 0.15% [241], which demonstrates the difficulty in measuring the value of this constant. This explains partly why the constraints on the time variation are less stringent than for the other constants.

A variation of the gravitational constant, being a pure gravitational phenomenon, does not affect the local physics, such as, e.g., the atomic transitions or the nuclear physics. In particular, it is equivalent at stating that the masses of all particles are varying in the same way to that their ratios remain constant. Similarly all absorption lines will be shifted in the same way. It follows that most constraints are obtained from systems in which gravity is non-negligible, such as the motion of the bodies of the Solar system, astrophysical and cosmological systems. They are mostly related in the comparison of a gravitational time scale, e.g., period of orbits, to a non-gravitational time scale. It follows that in general the constraints assume that the values of the other constants are fixed. Taking their variation into account would add degeneracies and make the constraints cited below less stringent.

We refer to Section IV of FVC [500] for earlier constraints based, e.g., on the determination of the Earth surface temperature, which roughly scales as $G^{2.25} M_\odot^{1.75}$ and gives a constraint of the order of $|\Delta G/G| < 0.1$ [224], or on the estimation of the Earth radius at different geological epochs. We also emphasize that constraints on the variation of G are meant to be constraints on the dimensionless parameter $\alpha_{\rm G}$.

4.1   Solar systems constraints

Monitoring the orbits of the various bodies of the solar system offers a possibility to constrain deviations from general relativity, and in particular the time variation of G. This accounts for comparing a gravitational time scale (related to the orbital motion) and an atomic time scale and it is thus assumed that the variation of atomic constants is negligible over the time of the experiment.

The first constraint arises from the Earth-Moon system. A time variation of G is then related to a variation of the mean motion ($n = 2\pi/P$) of the orbit of the Moon around the Earth. A decrease in G would induce both the Lunar mean distance and period to increase. As long as the gravitational binding energy is negligible, one has

$$\frac{\dot{P}}{P} = -2\frac{\dot{G}}{G}. \tag{133}$$

Earlier constraints rely on paleontological data and ancient eclipses observations (see Section IV.B.1 of FVC [500]) and none of them are very reliable. A main difficulty arises from tidal dissipation that also causes the mean distance and orbital period to increase (for tidal changes $2\dot{n}/n + 3\dot{a}/a = 0$), but not as in the same ratio as for $\dot{G}$.

The Lunar Laser Ranging (LLR) experiment has measured the relative position of the Moon with respect to the Earth with an accuracy of the order of 1 cm over 3 decades. An early analysis

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[11] The CODATA is the COmmittee on Data for Science and Technology, see http://www.codata.org/.

of this data [544] assuming a Brans–Dicke theory of gravitation gave that $|\dot{G}/G| \leq 3 \times 10^{-11}$ yr^{-1}. It was improved [365] by using 20 years of observation to get $|\dot{G}/G| \leq 1.04 \times 10^{-11}$ yr^{-1}, the main uncertainty arising from Lunar tidal acceleration. With, 24 years of data, one reached [542] $|\dot{G}/G| \leq 6 \times 10^{-12}$ yr^{-1} and finally, the latest analysis of the Lunar laser ranging experiment [543] increased the constraint to

$$\left.\frac{\dot{G}}{G}\right|_0 = (4 \pm 9) \times 10^{-13} \text{ yr}^{-1}. \tag{134}$$

Similarly, Shapiro et al. [458] compared radar-echo time delays between Earth, Venus and Mercury with a caesium atomic clock between 1964 and 1969. The data were fitted to the theoretical equation of motion for the bodies in a Schwarzschild spacetime, taking into account the perturbations from the Moon and other planets. They concluded that $|\dot{G}/G| < 4 \times 10^{-10}$ yr^{-1}. The data concerning Venus cannot be used due to imprecision in the determination of the portion of the planet reflecting the radar. This was improved to $|\dot{G}/G| < 1.5 \times 10^{-10}$ yr^{-1} by including Mariner 9 and Mars orbiter data [429]. The analysis was further extended [457] to give $\dot{G}/G = (-2 \pm 10) \times 10^{-12}$ yr^{-1}. The combination of Mariner 10 an Mercury and Venus ranging data gives [12]

$$\left.\frac{\dot{G}}{G}\right|_0 = (0.0 \pm 2.0) \times 10^{-12} \text{ yr}^{-1}. \tag{135}$$

Reasenberg et al. [430] considered the 14 months data obtained from the ranging of the Viking spacecraft and deduced, assuming a Brans–Dicke theory, $|\dot{G}/G| < 10^{-12}$ yr^{-1}. Hellings et al. [249] using all available astrometric data and in particular the ranging data from Viking landers on Mars deduced that

$$\left.\frac{\dot{G}}{G}\right|_0 = (2 \pm 4) \times 10^{-12} \text{ yr}^{-1}. \tag{136}$$

The major contribution to the uncertainty is due to the modeling of the dynamics of the asteroids on the Earth-Mars range. Hellings et al. [249] also tried to attribute their result to a time variation of the atomic constants. Using the same data but a different modeling of the asteroids, Reasenberg [428] got $|\dot{G}/G| < 3 \times 10^{-11}$ yr^{-1}, which was then improved by Chandler et al. [93] to $|\dot{G}/G| < 10^{-11}$ yr^{-1}.

4.2 Pulsar timing

Contrary to the solar system case, the dependence of the gravitational binding energy cannot be neglected while computing the time variation of the period. Here two approaches can be followed; either one sticks to a model (e.g., scalar-tensor gravity) and compute all the effects in this model or one has a more phenomenological approach and tries to put some model-independent bounds.

Eardley [177] followed the first route and discussed the effects of a time variation of the gravitational constant on binary pulsar in the framework of the Brans–Dicke theory. In that case, both a dipole gravitational radiation and the variation of G induce a periodic variation in the pulse period. Nordtvedt [386] showed that the orbital period changes as

$$\frac{\dot{P}}{P} = -\left[2 + \frac{2(m_1 c_1 + m_2 c_2) + 3(m_1 c_2 + m_2 c_1)}{m_1 + m_2}\right] \frac{\dot{G}}{G} \tag{137}$$

where $c_i \equiv \delta \ln m_i / \delta \ln G$. He concluded that for the pulsar PSR 1913+16 ($m_1 \simeq m_2$ and $c_1 \simeq c_2$) one gets

$$\frac{\dot{P}}{P} = -[2 + 5c]\frac{\dot{G}}{G}, \tag{138}$$

the coefficient c being model dependent. As another application, he estimated that $c_{\mathrm{Earth}} \sim -5 \times 10^{-10}$, $c_{\mathrm{Moon}} \sim -10^{-8}$ and $c_{\mathrm{Sun}} \sim -4 \times 10^{-6}$ justifying the formula used in the solar system.

Damour et al. [127] used the timing data of the binary pulsar PSR 1913+16. They implemented the effect of the time variation of G by considering the effect on $\dot{P}/P$. They defined, in a phenomenological way, that $\dot{G}/G = -0.5\delta\dot{P}/P$, where $\delta\dot{P}$ is the part of the orbital period derivative that is not explained otherwise (by gravitational waves radiation damping). This theory-independent definition has to be contrasted with the theory-dependent result (138) by Nordtvedt [386]. They got

$$\dot{G}/G = (1.0 \pm 2.3) \times 10^{-11} \text{ yr}^{-1}. \tag{139}$$

Damour and Taylor [137] then reexamined the data of PSR 1913+16 and established the upper bound

$$\dot{G}/G < (1.10 \pm 1.07) \times 10^{-11} \text{ yr}^{-1}. \tag{140}$$

Kaspi et al. [282] used data from PSR B1913+16 and PSR B1855+09 respectively to get

$$\dot{G}/G = (4 \pm 5) \times 10^{-12} \text{ yr}^{-1} \tag{141}$$

and

$$\dot{G}/G = (-9 \pm 18) \times 10^{-12} \text{ yr}^{-1}, \tag{142}$$

the latter case being more "secure" since the orbiting companion is not a neutron star.

All the previous results concern binary pulsars but isolated ones can also be used. Heintzmann and Hillebrandt [248] related the spin-down of the pulsar JP1953 to a time variation of G. The spin-down is a combined effect of electromagnetic losses, emission of gravitational waves, possible spin-up due to matter accretion. Assuming that the angular momentum is conserved so that $I/P = \text{constant}$, one deduces that

$$\left.\frac{\dot{P}}{P}\right|_G = \left(\frac{\mathrm{d}\ln I}{\mathrm{d}\ln G}\right)\frac{\dot{G}}{G}. \tag{143}$$

The observational spin-down can be decomposed as

$$\left.\frac{\dot{P}}{P}\right|_{\mathrm{obs}} = \left.\frac{\dot{P}}{P}\right|_{\mathrm{mag}} + \left.\frac{\dot{P}}{P}\right|_{\mathrm{GW}} + \left.\frac{\dot{P}}{P}\right|_G. \tag{144}$$

Since $\dot{P}/P_{\mathrm{mag}}$ and $\dot{P}/P_{\mathrm{GW}}$ are positive definite, it follows that $\dot{P}/P_{\mathrm{obs}} \geq \dot{P}/P_G$ so that a bound on $\dot{G}$ can be inferred if the main pulse period is the period of rotation. Heintzmann and Hillebrandt [248] then modelled the pulsar by a polytropic ($P \propto \rho^n$) white dwarf and deduced that $\mathrm{d}\ln I/\mathrm{d}\ln G = 2 - 3n/2$ so that $|\dot{G}/G| < 10^{-10} \text{ yr}^{-1}$. Mansfield [344] assumed a relativistic degenerate, zero temperature polytropic star and got that, when $\dot{G} < 0$, $0 \leq -\dot{G}/G < 6.8 \times 10^{-11} \text{ yr}^{-1}$ at a 2σ level. He also noted that a positive $\dot{G}$ induces a spin-up counteracting the electromagnetic spin-down, which can provide another bound if an independent estimate of the pulsar magnetic field can be obtained. Goldman [233], following Eardley [177], used the scaling relations $N \propto G^{-3/2}$ and $M \propto G^{-5/2}$ to deduce that $2\mathrm{d}\ln I/\mathrm{d}\ln G = -5 + 3\mathrm{d}\ln I/\mathrm{d}\ln N$. He used the data from the pulsar PSR 0655+64 to deduce that the rate of decrease of G was smaller than

$$0 \leq -\dot{G}/G < 5.5 \times 10^{-11} \text{ yr}^{-1}. \tag{145}$$

The analysis [516] of 10 years high precision timing data on the millisecond pulsar PSR J0437-4715 has allowed to improve the constraint to

$$|\dot{G}/G| < 2.3 \times 10^{-11} \text{ yr}^{-1}. \tag{146}$$

Recently, it was argued [266, 432] that a variation of G would induce a departure of the neutron star matter from β-equilibrium, due to the changing hydrostatic equilibrium. This would force non-equilibrium β-processes to occur, which release energy that is invested partly in neutrino emission and partly in heating the stellar interior. Eventually, the star arrives at a stationary state in which the temperature remains nearly constant, as the forcing through the change of G is balanced by the ongoing reactions. Comparing the surface temperature of the nearest millisecond pulsar, PSR J0437-4715, inferred from ultraviolet observations, two upper limits for variation were obtained, $|\dot{G}/G| < 2 \times 10^{-10}$ yr^{-1}, direct Urca reactions operating in the neutron star core are allowed, and $|\dot{G}/G| < 4 \times 10^{-12}$ yr^{-1}, considering only modified Urca reactions. This was extended in [302] in order to take into account the correlation between the surface temperatures and the radii of some old neutron stars to get $|\dot{G}/G| < 2.1 \times 10^{-11}$ yr^{-1}.

4.3 Stellar constraints

Early works, see Section IV.C of FVC [500], studied the Solar evolution in presence of a time varying gravitational constant, concluding that under the Dirac hypothesis, the original nuclear resources of the Sun would have been burned by now. This results from the fact that an increase of the gravitational constant is equivalent to an increase of the star density (because of the Poisson equation).

The idea of using stellar evolution to constrain the possible value of G was originally proposed by Teller [487], who stressed that the evolution of a star was strongly dependent on G. The luminosity of a main sequence star can be expressed as a function of Newton's gravitational constant and its mass by using homology relations [224, 487]. In the particular case that the opacity is dominated by free-free transitions, Gamow [224] found that the luminosity of the star is given approximately by $L \propto G^{7.8} M^{5.5}$. In the case of the Sun, this would mean that for higher values of G, the burning of hydrogen will be more efficient and the star evolves more rapidly, therefore we need to increase the initial content of hydrogen to obtain the present observed Sun. In a numerical test of the previous expression, Delg'Innocenti et al. [140] found that low-mass stars evolving from the Zero Age Main Sequence to the red giant branch satisfy $L \propto G^{5.6} M^{4.7}$, which agrees to within 10% of the numerical results, following the idea that Thomson scattering contributes significantly to the opacity inside such stars. Indeed, in the case of the opacity being dominated by pure Thomson scattering, the luminosity of the star is given by $L \propto G^4 M^3$. It follows from the previous analysis that the evolution of the star on the main sequence is highly sensitive to the value of G.

The driving idea behind the stellar constraints is that a secular variation of G leads to a variation of the gravitational interaction. This would affect the hydrostatic equilibrium of the star and in particular its pressure profile. In the case of non-degenerate stars, the temperature, being the only control parameter, will adjust to compensate the modification of the intensity of the gravity. It will then affect the nuclear reaction rates, which are very sensitive to the temperature, and thus the nuclear time scales associated to the various processes. It follows that the main stage of the stellar evolution, and in particular the lifetimes of the various stars, will be modified. As we shall see, basically two types of methods have been used, the first in which on relate the variation of G to some physical characteristic of a star (luminosity, effective temperature, radius), and a second in which only a statistical measurement of the change of G can be inferred. Indeed, the first class of methods are more reliable and robust but is usually restricted to nearby stars. Note also that they usually require to have a precise distance determination of the star, which may depend on G.

4.3.1 Ages of globular clusters

The first application of these idea has been performed with globular clusters. Their ages, determined for instance from the luminosity of the main-sequence turn-off, have to be compatible with

the estimation of the age of the galaxy. This gives the constraint [140]

$$\dot{G}/G = (-1.4 \pm 2.1) \times 10^{-11} \text{ yr}^{-1}. \tag{147}$$

The effect of a possible time dependence of G on luminosity has been studied in the case of globular cluster H-R diagrams but has not yielded any stronger constraints than those relying on celestial mechanics

4.3.2   Solar and stellar seismology

A side effect of the change of luminosity is a change in the depth of the convection zone so that the inner edge of the convecting zone changes its location. This induces a modification of the vibration modes of the star and particularly to the acoustic waves, i.e., p-modes [141].

Helioseismology. These waves are observed for our star, the Sun, and helioseismology allows one to determine the sound speed in the core of the Sun and, together with an equation of state, the central densities and abundances of helium and hydrogen. Demarque et al. [141] considered an ansatz in which $G \propto t^{-\beta}$ and showed that $|\beta| < 0.1$ over the last 4.5×10^9 years, which corresponds to $|\dot{G}/G| < 2 \times 10^{-11}$ yr^{-1}. Guenther et al. [240] also showed that g-modes could provide even much tighter constraints but these modes are up to now very difficult to observe. Nevertheless, they concluded, using the claim of detection by Hill and Gu [251], that $|\dot{G}/G| < 4.5 \times 10^{-12}$ yr^{-1}. Guenther et al. [239] then compared the p-mode spectra predicted by different theories with varying gravitational constant to the observed spectrum obtained by a network of six telescopes and deduced that

$$\left| \dot{G}/G \right| < 1.6 \times 10^{-12} \text{ yr}^{-1}. \tag{148}$$

The standard solar model depends on few parameters and G plays a important role since stellar evolution is dictated by the balance between gravitation and other interactions. Astronomical observations determines $GM_\odot$ with an accuracy better than 10^{-7} and a variation of G with $GM_\odot$ fixed induces a change of the pressure ($P = GM_\odot^2/R_\odot^2$) and density ($\rho = M_\odot/R_\odot^3$). The experimental uncertainties in G between different experiments have important implications for helioseismology. In particular the uncertainties for the standard solar model lead to a range in the value of the sound speed in the nuclear region that is as much as 0.15% higher than the inverted helioseismic sound speed [335]. While a lower value of G is preferred for the standard model, any definite prediction is masked by the uncertainties in the solar models available in the literature. Ricci and Villante [436] studied the effect of a variation of G on the density and pressure profile of the Sun and concluded that present data cannot constrain G better than 10^{-2}%. It was also shown [335] that the information provided by the neutrino experiments is quite significant because it constitutes an independent test of G complementary to the one provided by helioseismology.

White dwarfs. The observation of the period of non-radial pulsations of white dwarf allows to set similar constraints. White dwarfs represent the final stage of the stellar evolution for stars with a mass smaller to about $10 \, M_\odot$. Their structure is supported against gravitational collapse by the pressure of degenerate electrons. It was discovered that some white dwarfs are variable stars and in fact non-radial pulsator. This opens the way to use seismological techniques to investigate their internal properties. In particular, their non-radial oscillations is mostly determined by the Brunt–Väisälä frequency

$$N^2 = g \frac{\mathrm{d}\ln P^{1/\gamma_1}/\rho}{\mathrm{d}r}$$

where g is the gravitational acceleration, Γ_1 the first adiabatic exponent and P and ρ the pressure and density (see, e.g., [283] for a white dwarf model taking into account a varying G). A variation of G induces a modification of the degree of degeneracy of the white dwarf, hence on the frequency N

as well as the cooling rate of the star, even though this is thought to be negligible at the luminosities where white dwarfs are pulsationally unstable[54]. Using the observation of G117-B15A that has been monitored during 20 years, it was concluded [43] that

$$-2.5 \times 10^{-10} \text{ yr}^{-1} < \dot{G}/G < 4.0 \times 10^{-11} \text{ yr}^{-1}, \tag{149}$$

at a 2σ-level. The same observations were reanalyzed in [54] to obtain

$$|\dot{G}/G| < 4.1 \times 10^{-11} \text{ yr}^{-1}. \tag{150}$$

4.3.3   Late stages of stellar evolution and supernovae

A variation of G can influence the white dwarf cooling and the light curves ot Type Ia supernovae.

García-Berro et al. [225] considered the effect of a variation of the gravitational constant on the cooling of white dwarfs and on their luminosity function. As first pointed out by Vila [518], the energy of white dwarfs, when they are cool enough, is entirely of gravitational and thermal origin so that a variation of G will induce a modification of their energy balance and thus of their luminosity. Restricting to cold white dwarfs with luminosity smaller than ten solar luminosity, the luminosity can be related to the star binding energy B and gravitational energy, E_{grav}, as

$$L = -\frac{\mathrm{d}B}{\mathrm{d}t} + \frac{\dot{G}}{G} E_{\text{grav}}, \tag{151}$$

which simply results from the hydrostatic equilibrium. Again, the variation of the gravitational constant intervenes via the Poisson equation and the gravitational potential. The cooling process is accelerated if $\dot{G}/G < 0$, which then induces a shift in the position of the cut-off in the luminosity function. García-Berro et al. [225] concluded that

$$0 \leq -\dot{G}/G < (1 \pm 1) \times 10^{-11} \text{ yr}^{-1}. \tag{152}$$

The result depends on the details of the cooling theory, on whether the C/O white dwarf is stratified or not and on hypothesis on the age of the galactic disk. For instance, with no stratification of the C/O binary mixture, one would require $\dot{G}/G = -(2.5 \pm 0.5) \times 10^{-11} \text{ yr}^{-1}$ if the solar neighborhood has a value of 8 Gyr (i.e., one would require a variation of G to explain the data). In the case of the standard hypothesis of an age of 11 Gyr, one obtains that $0 \leq -\dot{G}/G < 3 \times 10^{-11} \text{ yr}^{-1}$.

The late stages of stellar evolution are governed by the Chandrasekhar mass $(\hbar c/G)^{3/2} m_{\text{n}}^{-2}$ mainly determined by the balance between the Fermi pressure of a degenerate electron gas and gravity.

Simple analytical models of the light curves of Type Ia supernovae predict that the peak of luminosity is proportional to the mass of nickel synthesized. In a good approximation, it is a fixed fraction of the Chandrasekhar mass. In models allowing for a varying G, this would induce a modification of the luminosity distance-redshift relation [227, 232, 435]. However, it was shown that this effect is small. Note that it will be degenerate with the cosmological parameters. In particular, the Hubble diagram is sensitive to the whole history of $G(t)$ between the highest redshift observed and today so that one needs to rely on a better defined model, such as, e.g., scalar-tensor theory [435] (the effect of the Fermi constant was also considered in [194]).

In the case of Type II supernovae, the Chandrasekhar mass also governs the late evolutionary stages of massive stars, including the formation of neutron stars. Assuming that the mean neutron star mass is given by the Chandrasekhar mass, one expects that $\dot{G}/G = -2\dot{M}_{\text{NS}}/3\,M_{\text{NS}}$. Thorsett [492] used the observations of five neutron star binaries for which five Keplerian parameters can be determined (the binary period P_b, the projection of the orbital semi-major axis $a_1 \sin i$, the eccentricity e, the time and longitude of the periastron T_0 and ω) as well as the relativistic

advance of the angle of the periastron $\dot{\omega}$. Assuming that the neutron star masses vary slowly as $M_{\mathrm{NS}} = M_{\mathrm{NS}}^{(0)} - \dot{M}_{\mathrm{NS}} t_{\mathrm{NS}}$, that their age was determined by the rate at which P_b is increasing (so that $t_{NS} \simeq 2P_b/\dot{P}_b$) and that the mass follows a normal distribution, Thorsett [492] deduced that, at 2σ,

$$\dot{G}/G = (-0.6 \pm 4.2) \times 10^{-12} \ \mathrm{yr}^{-1}. \tag{153}$$

4.3.4 New developments

It has recently been proposed that the variation of G inducing a modification of the binary's binding energy, it should affect the gravitational wave luminosity, hence leading to corrections in the chirping frequency [554]. For instance, it was estimated that a LISA observation of an equal-mass inspiral event with total redshifted mass of $10^5 \, M_\odot$ for three years should be able to measure $\dot{G}/G$ at the time of merger to better than $10^{-11}/\mathrm{yr}$. This method paves the way to constructing constraints in a large band of redshifts as well as in different directions in the sky, which would be an invaluable constraint for many models.

More speculative is the idea [25] that a variation of G can lead a neutron to enter into the region where strange or hybrid stars are the true ground state. This would be associated with gamma-ray bursts that are claimed to be able to reach the level of $10^{-17}/\mathrm{yr}$ on the time variation of G.

4.4 Cosmological constraints

Cosmological observations are more difficult to use in order to set constraints on the time variation of G. In particular, they require to have some ideas about the whole history of G as a function of time but also, as the variation of G reflects an extension of General relativity, it requires to modify all equations describing the evolution (of the universe and of the large scale structure) in a consistent way. We refer to [504, 502, 506] for a discussion of the use of cosmological data to constrain deviations from general relativity.

4.4.1 Cosmic microwave background

A time-dependent gravitational constant will have mainly three effects on the CMB angular power spectrum (see [435] for discussions in the framework of scalar-tensor gravity in which G is considered as a field):

1. The variation of G modifies the Friedmann equation and therefore the age of the Universe (and, hence, the sound horizon). For instance, if G is larger at earlier time, the age of the Universe is smaller at recombination, so that the peak structure is shifted towards higher angular scales.

2. The amplitude of the Silk damping is modified. At small scales, viscosity and heat conduction in the photon-baryon fluid produce a damping of the photon perturbations. The damping scale is determined by the photon diffusion length at recombination, and therefore depends on the size of the horizon at this epoch, and hence, depends on any variation of the Newton constant throughout the history of the Universe.

3. The thickness of the last scattering surface is modified. In the same vein, the duration of recombination is modified by a variation of the Newton constant as the expansion rate is different. It is well known that CMB anisotropies are affected on small scales because the last scattering "surface" has a finite thickness. The net effect is to introduce an extra, roughly exponential, damping term, with the cutoff length being determined by the thickness of the last scattering surface. When translating redshift into time (or length), one has to use the

Friedmann equations, which are affected by a variation of the Newton constant. The relevant quantity to consider is the visibility function g. In the limit of an infinitely thin last scattering surface, τ goes from ∞ to 0 at recombination epoch. For standard cosmology, it drops from a large value to a much smaller one, and hence, the visibility function still exhibits a peak, but it is much broader.

In full generality, the variation of G on the CMB temperature anisotropies depends on many factors: (1) modification of the background equations and the evolution of the universe, (2) modification of the perturbation equations, (3) whether the scalar field inducing the time variation of G is negligible or not compared to the other matter components, (4) on the time profile of G that has to be determine to be consistent with the other equations of evolution. This explains why it is very difficult to state a definitive constraint. For instance, in the case of scalar-tensor theories (see below), one has two arbitrary functions that dictate the variation of G. As can be seen, e.g., from [435, 378], the profiles and effects on the CMB can be very different and difficult to compare. Indeed, the effects described above are also degenerate with a variation of the cosmological parameters.

In the case of Brans–Dicke theory, one just has a single constant parameter ω_{BD} characterizing the deviation from general relativity and the time variation of G. Thus, it is easier to compare the different constraints. Chen and Kamionkowski [94] showed that CMB experiments such as WMAP will be able to constrain these theories for $\omega_{\mathrm{BD}} < 100$ if all parameters are to be determined by the same CMB experiment, $\omega_{\mathrm{BD}} < 500$ if all parameters are fixed but the CMB normalization and $\omega_{\mathrm{BD}} < 800$ if one uses the polarization. For the Planck mission these numbers are respectively, 800, 2500 and 3200. [2] concluded from the analysis of WMAP, ACBAR, VSA and CBI, and galaxy power spectrum data from 2dF, that $\omega_{\mathrm{BD}} > 120$, in agreement with the former analysis of [378]. An analysis [549] indictates that The 'WMAP-5yr data' and the 'all CMB data' both favor a slightly non-zero (positive) $\dot{G}/G$ but with the addition of the SDSS poser spectrum data, the best-fit value is back to zero, concluding that $-0.083 < \Delta G/G < 0.095$ between recombination and today, which corresponds to $-1.75 \times 10^{-12}\ \mathrm{yr}^{-1} < \dot{G}/G < 1.05 \times 10^{-12}\ \mathrm{yr}^{-1}$.

From a more phenomenological prospect, some works modeled the variation of G with time in a purely ad-hoc way, for instance [89] by assuming a linear evolution with time or a step function.

4.4.2 BBN

As explained in detail in Section 3.8.1, changing the value of the gravitational constant affects the freeze-out temperature T_{f}. A larger value of G corresponds to a higher expansion rate. This rate is determined by the combination $G\rho$ and in the standard case the Friedmann equations imply that $G\rho t^2$ is constant. The density ρ is determined by the number N_* of relativistic particles at the time of nucleosynthesis so that nucleosynthesis allows to put a bound on the number of neutrinos N_ν. Equivalently, assuming the number of neutrinos to be three, leads to the conclusion that G has not varied from more than 20% since nucleosynthesis. But, allowing for a change both in G and N_ν allows for a wider range of variation. Contrary to the fine structure constant the role of G is less involved.

The effect of a varying G can be described, in its most simple but still useful form, by introducing a speed-up factor, $\xi = H/H_{GR}$, that arises from the modification of the value of the gravitational constant during BBN. Other approaches considered the full dynamics of the problem but restricted themselves to the particular class of Jordan–Fierz–Brans–Dicke theory [1, 16, 26, 84, 102, 128, 441, 551] (Casas et al. [84] concluded from the study of helium and deuterium that $\omega_{\mathrm{BD}} > 380$ when $N_\nu = 3$ and $\omega_{\mathrm{BD}} > 50$ when $N_\nu = 2$.), of a massless dilaton with a quadratic coupling [105, 106, 134, 446] or to a general massless dilaton [455]. It should be noted that a combined analysis of BBN and CMB data was investigated in [113, 292]. The former considered G constant during BBN while the latter focused on a nonminimally quadratic coupling and a runaway potential. It

was concluded that from the BBN in conjunction with WMAP determination of η set that $\Delta G/G$ has to be smaller than 20%. However, we stress that the dynamics of the field can modify CMB results (see previous Section 4.4.1) so that one needs to be careful while inferring Ω_b from WMAP unless the scalar-tensor theory has converged close to general relativity at the time of decoupling.

In early studies, Barrow [26] assumed that $G \propto t^{-n}$ and obtained from the helium abundances that $-5.9 \times 10^{-3} < n < 7 \times 10^{-3}$, which implies that $|\dot{G}/G| < (2 \pm 9.3)\,h \times 10^{-12}$ yr^{-1}, assuming a flat universe. This corresponds in terms of the Brans–Dicke parameter to $\omega_{\mathrm{BD}} > 25$. Yang et al. [551] included the deuterium and lithium to improve the constraint to $n < 5 \times 10^{-3}$, which corresponds to $\omega_{\mathrm{BD}} > 50$. It was further improved by Rothman and Matzner [441] to $|n| < 3 \times 10^{-3}$ implying $|\dot{G}/G| < 1.7 \times 10^{-13}$ yr^{-1}. Accetta et al. [1] studied the dependence of the abundances of D, ^{3}He, ^{4}He and ^{7}Li upon the variation of G and concluded that $-0.3 < \Delta G/G < 0.4$, which roughly corresponds to $|\dot{G}/G| < 9 \times 10^{-13}$ yr^{-1}. All these investigations assumed that the other constants are kept fixed and that physics is unchanged. Kolb et al. [295] assumed a correlated variation of G, α_{EM} and G_{F} and got a bound on the variation of the radius of the extra dimensions.

Although the uncertainty in the helium-4 abundance has been argued to be significantly larger that what was assumed in the past [401], interesting bounds can still be derived [117]. In particular translating the bound on extra relativistic degress of freedom ($-0.6 < \delta N_\nu < 0.82$) to a constraint on the speed-up factor ($0.949 < \xi < 1.062$), it was concluded [117], since $\Delta G/G = \xi^2 - 1 = 7\delta N_\nu/43$, that

$$-0.10 < \frac{\Delta G}{G} < 0.13. \tag{154}$$

The relation between the speed-up factor, or an extra number of relativistic degrees of freedom, with a variation of G is only approximate since it assumes that the variation of G affects only the Friedmann equation by a renormalization of G. This is indeed accurate only when the scalar field is slow-rolling. For instance [105], the speed-up factor is given (with the notations of Section 5.1.1) by

$$\xi = \frac{A(\varphi_*)}{A_0} \frac{1 + \alpha(\varphi_*)\varphi_*'}{\sqrt{1 - \varphi_*^{2\prime}/3}} \frac{1}{\sqrt{1 + \alpha_0^2}}$$

so that

$$\xi^2 = \frac{G}{G_0} \frac{(1 + \alpha(\varphi_*)\varphi_*')^2}{(1 + \alpha^2)(1 - \varphi_*^{2\prime}/3)}, \tag{155}$$

so that $\Delta G/G_0 = \xi^2 - 1$ only if $\alpha \ll 1$ (small deviation from general relativity) and $\varphi_*' \ll 1$ (slow rolling dilaton). The BBN in scalar-tensor theories was investigated [105, 134] in the case of a two-parameter family involving a non-linear scalar field-matter coupling function. They concluded that even in the cases where before BBN the scalar-tensor theory was far from general relativity, BBN enables to set quite tight constraints on the observable deviations from general relativity today. In particular, neglecting the cosmological constant, BBN imposes $\alpha_0^2 < 10^{-6.5}\beta^{-1}(\Omega_{\mathrm{mat}}h^2/0.15)^{-3/2}$ when $\beta > 0.5$ (with the definitions introduced below Equation (164)).

5   Theories With Varying Constants

As explained in the introduction, Dirac postulated that G varies as the inverse of the cosmic time. Such an hypothesis is indeed not a theory since the evolution of G with time is postulated and not derived from an equation of evolution[12] consistent with the other field equations, that have to take into account that G is no more a constant (in particular in a Lagrangian formulation one needs to take into account that G is no more constant when varying.

The first implementation of Dirac's phenomenological idea into a field-theory framework (i.e., modifying Einstein's gravity and incorporating non-gravitational forces and matter) was proposed by Jordan [268]. He realized that the constants have to become dynamical fields and proposed the action

$$S = \int \sqrt{-g} \mathrm{d}^4 \mathbf{x} \phi^\eta \left[R - \xi \left(\frac{\nabla \phi}{\phi} \right)^2 - \frac{\phi}{2} F^2 \right], \tag{156}$$

η and ξ being two parameters. It follows that both G and the fine-structure constant have been promoted to the status of a dynamical field.

Fierz [195] realized that with such a Lagrangian, atomic spectra will be space-time-dependent, and he proposed to fix η to the value -1 to prevent such a space-time dependence. This led to the definition of a one-parameter (ξ) class of scalar-tensor theories in which only G is assumed to be a dynamical field. This was then further explored by Brans and Dicke [67] (with the change of notation $\xi \to \omega$). In this Jordan–Fierz–Brans–Dicke theory the gravitational constant is replaced by a scalar field, which can vary both in space and time. It follows that, for cosmological solutions, $G \propto t^{-n}$ with $n^{-1} = 2 + 3\omega_{\mathrm{BD}}/2$. Thus, Einstein's gravity is recovered when $\omega_{\mathrm{BD}} \to \infty$. This kind of theory was further generalized to obtain various functional dependencies for G in the formalisation of scalar-tensor theories of gravitation (see, e.g., Damour and Esposito-Farèse [124] or Will [540]).

5.1   Introducing new fields: generalities

5.1.1   The example of scalar-tensor theories

Let us start to remind how the standard general relativistic framework can be extended to make G dynamical on the example of scalar-tensor theories, in which gravity is mediated not only by a massless spin-2 graviton but also by a spin-0 scalar field that couples universally to matter fields (this ensures the universality of free fall). In the Jordan frame, the action of the theory takes the form

$$S = \int \frac{\mathrm{d}^4 x}{16\pi G_*} \sqrt{-g} \left[F(\varphi) R - g^{\mu\nu} Z(\varphi) \varphi_{,\mu} \varphi_{,\nu} - 2U(\varphi) \right] + S_{\mathrm{matter}}[\psi; g_{\mu\nu}] \tag{157}$$

where G_* is the bare gravitational constant. This action involves three arbitrary functions (F, Z and U) but only two are physical since there is still the possibility to redefine the scalar field. F needs to be positive to ensure that the graviton carries positive energy. S_{matter} is the action of the matter fields that are coupled minimally to the metric $g_{\mu\nu}$. In the Jordan frame, the matter is universally coupled to the metric so that the length and time as measured by laboratory apparatus are defined in this frame.

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[12] Note that the Dirac hypothesis can also be achieved by assuming that e varies as $t^{1/2}$. Indeed this reflects a choice of units, either atomic or Planck units. However, there is a difference: assuming that only G varies violates the strong equivalence principle while assuming a varying e results in a theory violating the weak equivalence principle. It does not mean that we are detecting the variation of a dimensionful constant but simply that either $e^2/\hbar c$ or $Gm_{\mathrm{e}}^2/\hbar c$ is varying. This shows that many implementations of this idea are a priori possible.

The variation of this action gives the following field equations

$$F(\varphi)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = 8\pi G_* T_{\mu\nu} + Z(\varphi)\left[\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}(\partial_\alpha\varphi)^2\right]$$

$$+ \nabla_\mu\partial_\nu F(\varphi) - g_{\mu\nu}\Box F(\varphi) - g_{\mu\nu}U(\varphi) , \tag{158}$$

$$2Z(\varphi)\,\Box\varphi = -\frac{dF}{d\varphi}\,R - \frac{dZ}{d\varphi}\,(\partial_\alpha\varphi)^2 + 2\frac{dU}{d\varphi} , \tag{159}$$

$$\nabla_\mu T^\mu_\nu = 0 , \tag{160}$$

where $T \equiv T^\mu_\mu$ is the trace of the matter energy-momentum tensor $T^{\mu\nu} \equiv (2/\sqrt{-g}) \times \delta S_m/\delta g_{\mu\nu}$. As expected [183], we have an equation, which reduces to the standard Einstein equation when φ is constant and a new equation to describe the dynamics of the new degree of freedom while the conservation equation of the matter fields is unchanged, as expected from the weak equivalence principle.

It is useful to define an Einstein frame action through a conformal transformation of the metric $g^*_{\mu\nu} = F(\varphi)g_{\mu\nu}$. In the following all quantities labeled by a star (*) will refer to Einstein frame. Defining the field φ_* and the two functions $A(\varphi_*)$ and $V(\varphi_*)$ (see, e.g., [191]) by

$$\left(\frac{\mathrm{d}\varphi_*}{\mathrm{d}\varphi}\right)^2 = \frac{3}{4}\left(\frac{\mathrm{d}\ln F(\varphi)}{\mathrm{d}\varphi}\right)^2 + \frac{1}{2F(\varphi)}, \quad A(\varphi_*) = F^{-1/2}(\varphi), \quad 2V(\varphi_*) = U(\varphi)F^{-2}(\varphi),$$

the action (157) reads as

$$S = \frac{1}{16\pi G_*}\int \mathrm{d}^4x\sqrt{-g_*}\left[R_* - 2g_*^{\mu\nu}\partial_\mu\varphi_*\partial_\nu\varphi_* - 4V\right] + S_{\mathrm{matter}}[A^2 g^*_{\mu\nu};\psi]. \tag{161}$$

The kinetic terms have been diagonalized so that the spin-2 and spin-0 degrees of freedom of the theory are perturbations of $g^*_{\mu\nu}$ and φ_* respectively. In this frame the field equations are given by

$$R^*_{\mu\nu} - \frac{1}{2}R^* g^*_{\mu\nu} = 8\pi G_* T^*_{\mu\nu} + 2\partial_\mu\varphi_*\partial_\nu\varphi_* - g^*_{\mu\nu}(g_*^{\alpha\beta}\partial_\alpha\varphi_*\partial_\beta\varphi_*) - 2V(\varphi)g^*_{\mu\nu} , \tag{162}$$

$$\Box_*\varphi_* = -4\pi G_*\alpha(\varphi_*)\,T_* + dV(\varphi)/d\varphi_* , \tag{163}$$

$$\nabla^*_\mu T^\mu_{*\nu} = \alpha(\varphi_*)\,T_*\partial_\nu\varphi_* , \tag{164}$$

with $\alpha \equiv \mathrm{d}\ln A/\mathrm{d}\varphi_*$ and $\beta \equiv \mathrm{d}\alpha/\mathrm{d}\varphi_*$. In this version, the Einstein equations are not modified, but since the theory can now be seen as the theory in which all the mass are varying in the same way, there is a source term to the conservation equation. This shows that the same theory can be interpreted as a varying G theory or a universally varying mass theory, but remember that whatever its form the important parameter is the dimensionless quantity $Gm^2/\hbar c$.

The action (157) defines an effective gravitational constant $G_{\mathrm{eff}} = G_*/F = G_*A^2$. This constant does not correspond to the gravitational constant effectively measured in a Cavendish experiment. The Newton constant measured in this experiment is

$$G_{\mathrm{cav}} = G_* A_0^2(1 + \alpha_0^2) = \frac{G_*}{F}\left(1 + \frac{F_\phi^2}{2F + 3F_\phi^2}\right) \tag{165}$$

where the first term, $G_*A_0^2$ corresponds to the exchange of a graviton while the second term $G_*A_0^2\alpha_0^2$ is related to the long range scalar force, a subscript 0 referring to the quantity evaluated today. The gravitational constant depends on the scalar field and is thus dynamical.

This illustrates the main features that will appear in any such models: (i) new dynamical fields appear (here a scalar field), (ii) some constant will depend on the value of this scalar field (here G

is a function of the scalar field). It follows that the Einstein equations will be modified and that there will exist a new equation dictating the propagation of the new degree of freedom.

In this particular example, the coupling of the scalar field is universal so that no violation of the universality of free fall is expected. The deviation from general relativity can be quantified in terms of the post-Newtonian parameters, which can be expressed in terms of the values of α and β today as

$$\gamma^{\text{PPN}} - 1 = -\frac{2\alpha_0^2}{1 + \alpha_0^2}, \qquad \beta^{\text{PPN}} - 1 = \frac{1}{2}\frac{\beta_0\alpha_0^2}{(1 + \alpha_0^2)^2}. \tag{166}$$

These expressions are valid only if the field is light on the solar system scales. If this is not the case then these conclusions may be changed [287]. The solar system constraints imply α_0 to be very small, typically $\alpha_0^2 < 10^{-5}$ while β_0 can still be large. Binary pulsar observations [125, 189] impose that $\beta_0 > -4.5$. The time variation of G is then related to α_0, β_0 and the time variation of the scalar field today

$$\frac{\dot{G}_{\text{cav}}}{G_{\text{cav}}} = 2\alpha_0 \left(1 + \frac{\beta_0}{1 + \alpha_0^2}\right)\dot{\varphi}_{*0}. \tag{167}$$

This example shows that the variation of the constant and the deviation from general relativity quantified in terms of the PPN parameters are of the same magnitude, because they are all driven by the same new scalar field.

The example of scalar-tensor theories is also very illustrative to show how deviation from general relativity can be fairly large in the early universe while still being compatible with solar system constraints. It relies on the attraction mechanism toward general relativity [130, 131].

Consider the simplest model of a massless dilaton ($V(\varphi_*) = 0$) with quadratic coupling ($\ln A = a = \frac{1}{2}\beta\varphi_*^2$). Note that the linear case correspond to a Brans–Dicke theory with a fixed deviation from general relativity. It follows that $\alpha_0 = \beta\varphi_{0*}$ and $\beta_0 = \beta$. As long as $V = 0$, the Klein–Gordon equation can be rewritten in terms of the variable $p = \ln a$ as

$$\frac{2}{3 - \varphi_*'^2}\varphi_*'' + (1 - w)\varphi_*' = -\alpha(\varphi_*)(1 - 3w). \tag{168}$$

As emphasized in [130], this is the equation of motion of a point particle with a velocity dependent inertial mass, $m(\varphi_*) = 2/(3 - \varphi_*'^2)$ evolving in a potential $\alpha(\varphi_*)(1 - 3w)$ and subject to a damping force, $-(1 - w)\varphi_*'$. During the cosmological evolution the field is driven toward the minimum of the coupling function. If $\beta > 0$, it drives φ_* toward 0, that is $\alpha \to 0$, so that the scalar-tensor theory becomes closer and closer to general relativity. When $\beta < 0$, the theory is driven way from general relativity and is likely to be incompatible with local tests unless φ_* was initially arbitrarily close from 0.

It follows that the deviation from general relativity remains constant during the radiation era (up to threshold effects in the early universe [108, 134] and quantum effects [85]) and the theory is then attracted toward general relativity during the matter era. Note that it implies that postulating a linear or inverse variation of G with cosmic time is actually not realistic in this class of models. Since the theory is fully defined, one can easily compute various cosmological observables (late time dynamics [348], CMB anisotropy [435], weak lensing [449], BBN [105, 106, 134]) in a consistent way and confront them with data.

5.1.2   Making other constants dynamical

Given this example, it seems a priori simple to cook up a theory that will describe a varying fine-structure constant by coupling a scalar field to the electromagnetic Faraday tensor as

$$S = \int \left[\frac{R}{16\pi G} - 2(\partial_\mu\phi)^2 - \frac{1}{4}B(\phi)F_{\mu\nu}^2\right]\sqrt{-g}\mathrm{d}^4x \tag{169}$$

so that the fine-structure will evolve according to $\alpha = B^{-1}$. However, such an simple implementation may have dramatic implications. In particular, the contribution of the electromagnetic binding energy to the mass of any nucleus can be estimated by the Bethe–Weizäcker formula so that

$$m_{(A,Z)}(\phi) \supset 98.25\,\alpha(\phi)\,\frac{Z(Z-1)}{A^{1/3}}\ \mathrm{MeV}.$$

This implies that the sensitivity of the mass to a variation of the scalar field is expected to be of the order of

$$f_{(A,Z)} = \partial_\phi m_{(A,Z)}(\phi) \sim 10^{-2}\frac{Z(Z-1)}{A^{4/3}}\alpha'(\phi). \tag{170}$$

It follows that the level of the violation of the universality of free fall is expected to be of the level of $\eta_{12} \sim 10^{-9} X(A_1, Z_1; A_2, Z_2)(\partial_\phi \ln B)_0^2$. Since the factor $X(A_1, Z_1; A_2, Z_2)$ typically ranges as $\mathcal{O}(0.1-10)$, we deduce that $(\partial_\phi \ln B)_0$ has to be very small for the solar system constraints to be satisfied. It follows that today the scalar field has to be very close to the minimum of the coupling function $\ln B$. This led to the idea of the *least coupling mechanism* [135, 136] discussed in Section 5.4.1. This is indeed very simplistic because this example only takes into account the effect of the electromagnetic binding energy (see Section 6.3).

Let us also note that such a simple coupling cannot be eliminated by a conformal rescaling $g_{\mu\nu} = A^2(\phi)g^*_{\mu\nu}$ since

$$\int B(\phi)g^{\mu\rho}g^{\mu\nu}F_{\nu\sigma}F_{\rho\sigma}\sqrt{-g}\mathrm{d}^4x \longrightarrow \int B(\phi)A^{D-4}(\phi)g^{\mu\rho}_*g^{\mu\nu}_*F_{\nu\sigma}F_{\rho\sigma}\sqrt{-g_*}\mathrm{d}^4x$$

so that the action is invariant in $D = 4$ dimensions.

This example shows that we cannot couple a field blindly to, e.g., the Faraday tensor to make the fine-structure constant dynamics and that some mechanism for reconciling this variation with local constraints, and in particular the university of free fall, will be needed.

5.2 High-energy theories and varying constants

5.2.1 Kaluza–Klein

Such coupling terms naturally appear when compactifying a higher-dimensional theory. As an example, let us recall the compactification of a 5-dimensional Einstein–Hilbert action ([409], chapter 13)

$$S = \frac{1}{12\pi^2 G_5}\int \bar{R}\sqrt{-\bar{g}}\mathrm{d}^5x.$$

Decomposing the 5-dimensional metric $\bar{g}_{AB}$ as

$$\bar{g}_{AB} = \begin{pmatrix} g_{\mu\nu} + \frac{A_\mu A_\nu}{M^2}\phi^2 & \frac{A_\mu}{M}\phi^2 \\ \frac{A_\nu}{M}\phi^2 & \phi^2 \end{pmatrix},$$

where M is a mass scale, we obtain

$$S = \frac{1}{16\pi G_*}\int \left(R - \frac{\phi^2}{4M^2}F^2\right)\phi\sqrt{-g}\mathrm{d}^4x, \tag{171}$$

where the 4-dimensional gravitational constant is $G_* = 3\pi G_5/4\int \mathrm{d}y$. The scalar field couples explicitly to the kinetic term of the vector field and cannot be eliminated by a redefinition of the metric: again, this is the well-known conformal invariance of electromagnetism in four dimensions. Such a term induces a variation of the fine-structure constant as well as a violation of the universality of free-fall. Such dependencies of the masses and couplings are generic for higher-dimensional

theories and in particular string theory. It is actually one of the definitive predictions for string theory that there exists a dilaton, that couples directly to matter [484] and whose vacuum expectation value determines the string coupling constants [546].

In the models by Kaluza [269] and Klein [291] the 5-dimensional spacetime was compactified assuming that one spatial extra-dimension S^1, of radius R_{KK}. It follows that any field $\chi(x^\mu, y)$ can be Fourier transformed along the compact dimension (with coordinate y), so that, from a 4-dimensional point of view, it gives rise to a tower of of fields $\chi^{(n)}(x^\mu)$ of mas $m_n = nR_{KK}$. At energies small compared to R_{KK}^{-1} only the y-independent part of the field remains and the physics looks 4-dimensional.

Assuming that the action (171) corresponds to the Jordan frame action, as the coupling ϕR may suggest, it follows that the gravitational constant and the Yang–Mills coupling associated with the vector field A^μ must scale as

$$G \propto \phi^{-1}, \qquad g_{YM}^{-2} \propto \phi^2/G \propto \phi^3. \tag{172}$$

Note that the scaling of G with ϕ (or time) is not the one of the gravitational constant that would be measured in a Cavendish experiment since Eq. (165) tells us that $G_{\rm cav} \propto G_* \phi^{-1} \left(1 + \frac{1}{2\phi+3}\right)$.

This can be generalized to the case of D extra-dimensions [114] to

$$G \propto \phi^{-D}, \quad \alpha_i(m_{\rm KK}) = K_i(D)G\phi^{-2} \tag{173}$$

where the constants K_i depends only on the dimension and topology of the compact space [525] so that the only fundamental constant of the theory is the mass scale M_{4+D} entering the $4 + D$-dimensional theory. A theory on $\mathcal{M}_4 \times \mathcal{M}_D$ where $\mathcal{M}_D$ is a D-dimensional compact space generates a low-energy quantum field theory of the Yang–Mills type related to the isometries of $\mathcal{M}_D$ [for instance [545] showed that for $D = 7$, it can accommodate the Yang–Mills group $SU(3) \times SU(2) \times U(1)$]. The two main problems of these theories are that one cannot construct chiral fermions in four dimensions by compactification on a smooth manifold with such a procedure and that gauge theories in five dimensions or more are not renormalizable.

In such a framework the variation of the gauge couplings and of the gravitational constant arises from the variation of the size of the extra dimensions so that one can derives stronger constraints that by assuming independent variation, but at the expense of being more model-dependent. Let us mention the works by Marciano [345] and Wu and Wang [550] in which the structure constants at lower energy are obtained by the renormalization group, and the work by Veneziano [515] for a toy model in $D \geq 4$ dimensions, endowed with an invariant UV cut-off Λ, and containing a large number N of non-self-interacting matter species.

Ref. [295] used the variation (173) to constrain the time variation of the radius of the extra dimensions during primordial nucleosynthesis to conclude that $|\Delta R_{\rm KK}/R_{\rm KK}| < 1\%$. [28] took the effects of the variation of $\alpha_{\rm S} \propto R_{\rm KK}^{-2}$ and deduced from the helium-4 abundance that $|\Delta R_{\rm KK}/R_{\rm KK}| < 0.7\%$ and $|\Delta R_{\rm KK}/R_{\rm KK}| < 1.1\%$ respectively for $D = 2$ and $D = 7$ Kaluza–Klein theory and that $|\Delta R_{\rm KK}/R_{\rm KK}| < 3.4 \times 10^{-10}$ from the Oklo data. An analysis of most cosmological data (BBN, CMB, quasar etc..) assuming that the extra dimension scales as $R_0(1 + \Delta t^{-3/4})$ and $R_0[1 + \Delta](1 - \cos\omega(t - t_0))$ concluded that Δ has to be smaller than 10^{-16} and 10^{-8} respectively [311], while [330] assumes that gauge fields and matter fields can propagate in the bulk, that is in the extra dimensions. Ref. [336] evaluated the effect of such a couple variation of G and the structures constants on distant supernova data, concluding that a variation similar to the one reported in [524] would make the distant supernovae brighter.

5.2.2 String theory

There exist five anomaly-free, supersymmetric perturbative string theories respectively known as type I, type IIA, type IIB, SO(32) heterotic and $E_8 \times E_8$ heterotic theories (see, e.g., [420]).

One of the definitive predictions of these theories is the existence of a scalar field, the dilaton, that couples directly to matter [484] and whose vacuum expectation value determines the string coupling constant [546]. There are two other excitations that are common to all perturbative string theories, a rank two symmetric tensor (the graviton) $g_{\mu\nu}$ and a rank two antisymmetric tensor $B_{\mu\nu}$. The field content then differs from one theory to another. It follows that the 4-dimensional couplings are determined in terms of a string scale and various dynamical fields (dilaton, volume of compact space, ...). When the dilaton is massless, we expect *three* effects: (i) a scalar admixture of a scalar component inducing deviations from general relativity in gravitational effects, (ii) a variation of the couplings and (iii) a violation of the weak equivalence principle. Our purpose is to show how the 4-dimensional couplings are related to the string mass scale, to the dilaton and the structure of the extra dimensions mainly on the example of heterotic theories.

To be more specific, let us consider an example. The two *heterotic theories* originate from the fact that left- and right-moving modes of a closed string are independent. This reduces the number of supersymmetry to $N = 1$ and the quantization of the left-moving modes imposes that the gauge group is either $SO(32)$ or $E_8 \times E_8$ depending on the fermionic boundary conditions. The effective tree-level action is (see, e.g., Ref. [237])

$$S_H = \int \mathrm{d}^{10}\mathbf{x}\sqrt{-g_{10}}\mathrm{e}^{-2\Phi}\left[M_H^8\left\{R_{10} + 4\Box\Phi - 4(\nabla\Phi)^2\right\} - \frac{M_H^6}{4}F_{AB}F^{AB} + \dots\right]. \quad (174)$$

When compactified on a 6-dimensional Calabi–Yau space, the effective 4-dimensional action takes the form

$$S_H = \int \mathrm{d}^4\mathbf{x}\sqrt{-g_4}\phi\left[M_H^8\left\{R_4 + \left(\frac{\nabla\phi}{\phi}\right)^2 - \frac{1}{6}\left(\frac{\nabla V_6}{V_6}\right)^2\right\} - \frac{M_H^6}{4}F^2\right] + \dots \quad (175)$$

where $\phi \equiv V_6 \mathrm{e}^{-2\Phi}$ couples identically to the Einstein and Yang–Mills terms. It follows that

$$M_4^2 = M_H^8\phi, \qquad g_{\mathrm{YM}}^{-2} = M_H^6\phi \quad (176)$$

at tree-level. Note that to reach this conclusion, one has to assume that the matter fields (in the 'dots' of Equation (175) are minimally coupled to g_4; see, e.g., [340]).

The strongly coupled $SO(32)$ heterotic string theory is equivalent to the weakly coupled type I string theory. *Type I superstring* admits open strings, the boundary conditions of which divide the number of supersymmetries by two. It follows that the tree-level effective bosonic action is $N = 1$, $D = 10$ supergravity, which takes the form, in the string frame,

$$S_I = \int \mathrm{d}^{10}\mathbf{x}\sqrt{-g_{10}}M_I^6\mathrm{e}^{-\Phi}\left[\mathrm{e}^{-\Phi}M_I^2 R_{10} - \frac{F^2}{4} + \dots\right] \quad (177)$$

where the dots contains terms describing the dynamics of the dilaton, fermions and other form fields. At variance with (174), the field Φ couples differently to the gravitational and Yang–Mills terms because the graviton and Yang–Mills fields are respectively excitation of close and open strings. It follows that M_I can be lowered even to the weak scale by simply having $\exp\Phi$ small enough. Type I theories require $D9$-branes for consistency. When V_6 is small, one can use T-duality (to render V_6 large, which allows to use a quantum field theory approach) and turn the $D9$-brane into a $D3$-brane so that

$$S_I = \int \mathrm{d}^{10}\mathbf{x}\sqrt{-g_{10}}\mathrm{e}^{-2\Phi}M_I^8 R_{10} - \int \mathrm{d}^4\mathbf{x}\sqrt{-g_4}\mathrm{e}^{-\Phi}\frac{1}{4}F^2 + \dots \quad (178)$$

where the second term describes the Yang–Mills fields localized on the $D3$-brane. It follows that

$$M_4^2 = \mathrm{e}^{-2\Phi}V_6 M_I^8, \qquad g_{\mathrm{YM}}^{-2} = \mathrm{e}^{-\Phi} \quad (179)$$

at tree-level. If one compactifies the $D9$-brane on a 6-dimensional orbifold instead of a 6-torus, and if the brane is localized at an orbifold fixed point, then gauge fields couple to fields M_i living only at these orbifold fixed points with a (calculable) tree-level coupling c_i so that

$$M_4^2 = \mathrm{e}^{-2\Phi} V_6 M_I^8, \qquad g_{\mathrm{YM}}^{-2} = \mathrm{e}^{-\Phi} + c_i M_i. \tag{180}$$

The coupling to the field c_i is a priori non universal. At strong coupling, the 10-dimensional $E_8 \times E_8$ heterotic theory becomes M-theory on $R^{10} \times S^1/Z_2$ [255]. The gravitational field propagates in the 11-dimensional space while the gauge fields are localized on two 10-dimensional branes.

At one-loop, one can derive the couplings by including Kaluza–Klein excitations to get [163]

$$g_{\mathrm{YM}}^{-2} = M_H^6 \phi - \frac{b_a}{2}(R M_H)^2 + \dots \tag{181}$$

when the volume is large compared to the mass scale and in that case the coupling is no more universal. Otherwise, one would get a more complicated function. Obviously, the 4-dimensional effective gravitational and Yang–Mills couplings depend on the considered superstring theory, on the compactification scheme but in any case they depend on the dilaton.

As an example, [340] considered the $(N = 1, D = 10)$-supergravity model derived from the heterotic superstring theory in the low energy limit and assumed that the 10-dimensional spacetime is compactified on a 6-torus of radius $R(x^\mu)$ so that the effective 4-dimensional theory described by (175) is of the Brans–Dicke type with $\omega = -1$. Assuming that ϕ has a mass μ, and couples to the matter fluid in the universe as $S_{\mathrm{matter}} = \int \mathrm{d}^{10}\mathbf{x}\sqrt{-g_{10}}\exp(-2\Phi)\mathcal{L}_{\mathrm{matter}}(g_{10})$, the reduced 4-dimensional matter action is

$$S_{\mathrm{matter}} = \int \mathrm{d}^4\mathbf{x}\sqrt{-g}\phi\mathcal{L}_{\mathrm{matter}}(g). \tag{182}$$

The cosmological evolution of ϕ and R can then be computed to deduce that $\dot{\alpha}_{\mathrm{EM}}/\alpha_{\mathrm{EM}} \simeq 10^{10}$ $(\mu/1\ \mathrm{eV})^{-2}\ \mathrm{yr}^{-1}$. considered the same model but assumed that supersymmetry is broken by non-perturbative effects such as gaugino condensation. In this model, and contrary to [340], ϕ is stabilized and the variation of the constants arises mainly from the variation of R in a runaway potential.

Ref. [290] considers a probe D3-brane probe in the context of AdS/CFT correspondence at finite temperature and provides the predictions for the running electric and magnetic effective couplings, beyond perturbation theory. It allows to construct a varying speed of light model.

To conclude, superstring theories offer a natural theoretical framework to discuss the value of the fundamental constants since they become expectation values of some fields. This is a first step towards their understanding but yet, no complete and satisfactory mechanism for the stabilization of the extra dimensions and dilaton is known.

It has paved the way for various models that we detail in Section 5.4.

5.3   Relations between constants

There are different possibilities to relate the variations of different constants. First, in quantum field theory, we have to take into account the running of coupling constants with energy and the possibilities of grand unification to relate them. It will also give a link between the QCD scale, the coupling constants and the mass of the fundamental particles (i.e., the Yukawa couplings and the Higgs vev). Second, one can compute the binding energies and the masses of the proton, neutron and different nuclei in terms of the gauge couplings and the quark masses. This step involves QCD and nuclear physics. Third, one can relate the gyromagnetic factor in terms of the quark masses. This is particularly important to interpret the constraints from the atomic clocks and the QSO spectra. This allows one to set stronger constraints on the varying parameters at the expense of a model-dependence.

5.3.1   Implication of gauge coupling unification

The first theoretical implication of high-energy physics arises from the unification of the non-gravitational interactions. In these unification schemes, the three standard model coupling constants derive from one unified coupling constant.

In quantum field, the calculation of scattering processes include higher order corrections of the coupling constants related to loop corrections that introduce some integrals over internal 4-momenta. Depending on the theory, these integrals may be either finite or diverging as the logarithm or power law of a UV cut-off. In a class of theories, called renormalizable, among which the standard model of particle physics, the physical quantities calculated at any order do not depend on the choice of the cut-off scale. But the result may depend on $\ln E/m$ where E is the typical energy scale of the process. It follows that the values of the coupling constants of the standard model depend on the energy at which they are measured (or of the process in which they are involved). This running arises from the screening due to the existence of virtual particles, which are polarized by the presence of a charge. The renormalization group allows to compute the dependence of a coupling constants as a function of the energy E as

$$\frac{\mathrm{d}g_i(E)}{\mathrm{d}\ln E} = \beta_i(E),$$

where the beta functions, β_i, depend on the gauge group and on the matter content of the theory and may be expended in powers of g_i. For the SU(2) and U(1) gauge couplings of the standard model, they are given by

$$\beta_2(g_2) = -\frac{g_2^3}{4\pi^2}\left(\frac{11}{6} - \frac{n_g}{3}\right), \qquad \beta_1(g_1) = +\frac{g_1^3}{4\pi^2}\frac{5n_g}{9}$$

where n_g is the number of generations for the fermions. We remind that the fine-structure constant is defined in the limit of zero momentum transfer so that cosmological variation of α_{EM} are independent of the issue of the renormalization group dependence. For the SU(3) sector, with fundamental Dirac fermion representations,

$$\beta_3(g_3) = -\frac{g_3^3}{4\pi^2}\left(\frac{11}{4} - \frac{n_f}{6}\right),$$

n_f being the number of quark flavors with mass smaller than E. The negative sign implies that (1) at large momentum transfer the coupling decreases and loop corrections become less and less significant: QCD is said to be asymptotically free; (2) integrating the renormalization group equation for α_3 gives

$$\alpha_3(E) = \frac{6\pi}{(33 - n_f)\ln(E/\Lambda_c)}$$

so that it diverges as the energy scale approaches Λ_c from above, that we decided to call Λ_{QCD}. This scale characterizes all QCD properties and in particular the masses of the hadrons are expected to be proportional to Λ_{QCD} up to corrections of order $m_{\mathrm{q}}/\Lambda_{\mathrm{QCD}}$.

It was noticed quite early that these relations imply that the weaker gauge coupling becomes stronger at high energy, while the strong coupling becomes weaker so that one can thought the three non-gravitational interactions may have a single common coupling strength above a given energy. This is the driving idea of Grand Unified Theories (GUT) in which one introduces a mechanism of symmetry-breaking from a higher symmetry group, such, e.g., as SO(10) or SU(5), at high energies. It has two important consequences for our present considerations. First there may exist algebraic relations between the Yukawa couplings of the standard model. Second, the structure constants of the standard model unify at an energy scale M_U

$$\alpha_1(M_U) = \alpha_2(M_U) = \alpha_3(M_U) \equiv \alpha_U(M_U). \tag{183}$$

We note that the electroweak mixing angle, i.e., the can also be time dependent parameter, but only for $E \neq M_U$ since at $E = M_U$, it is fixed by the symmetry to have the value $\sin^2 \theta = 3/8$, from which we deduce that

$$\alpha_{\mathrm{EM}}^{-1}(M_Z) = \frac{5}{3}\alpha_1^{-1}(M_Z) + \alpha_2^{-1}(M_Z).$$

It follows from the renormalization group relations that

$$\alpha_i^{-1}(E) = \alpha_i^{-1}(M_U) - \frac{b_i}{2\pi}\ln\frac{E}{M_U}, \tag{184}$$

where the beta-function coefficients are given by $b_i = (41/10, -19/6, 7)$ for the standard model (or below the SUSY scale Λ_{SUSY}) and by $b_i = (33/5, 1, -3)$ for $N = 1$ supersymmetric theory. Given a field decoupling at m_{th}, one has

$$\alpha_i^{-1}(E_-) = \alpha_i^{-1}(E_+) - \frac{b_i^{(-)}}{2\pi}\ln\frac{E_-}{E_+} - \frac{b_i^{(\mathrm{th})}}{2\pi}\ln\frac{m_{\mathrm{th}}}{E_+}$$

where $b_i^{(\mathrm{th})} = b^{(+)} - b^{(-)}$ with $b^{(+/-)}$ the beta-function coefficients respectively above and below the mass threshold, with tree-level matching at m_{th}. In the case of multiple thresholds, one must sum the different contributions. The existence of these thresholds implies that the running of α_3 is complicated since it depends on the masses of heavy quarks and colored superpartner in the case of supersymmetry. For non-supersymmetric theories, the low-energy expression of the QCD scale is

$$\Lambda_{\mathrm{QCD}} = E\left(\frac{m_{\mathrm{c}}m_{\mathrm{b}}m_{\mathrm{t}}}{E}\right)^{2/27}\exp\left(-\frac{2\pi}{9\alpha_3(E)}\right) \tag{185}$$

for $E > m_{\mathrm{t}}$. This implies that the variation of Yukawa couplings, gauge couplings, Higgs vev and $\Lambda_{\mathrm{QCD}}/M_{\mathrm{P}}$ are correlated. A second set of relations arises in models in which the weak scale is determined by dimensional transmutation [184, 185]. In such cases, the Higgs vev is related to the Yukawa constant of the top quark by [77]

$$v = M_p \exp\left(-\frac{8\pi^2 c}{h_{\mathrm{t}}^2}\right), \tag{186}$$

where c is a constant of order unity. This would imply that $\delta \ln v = S\delta \ln h$ with $S \sim 160$ [104].

The first consequences of this unification were investigated in Refs. [77, 74, 75, 135, 136, 185, 313] where the variation of the 3 coupling constants was reduced to the one of α_U and M_U/M_{P}. It was concluded that, setting

$$R \equiv \delta \ln \Lambda_{\mathrm{QCD}}/\delta \ln \alpha_{\mathrm{EM}}, \tag{187}$$

$R \sim 34$ with a stated accuracy of about 20% [312, 313] (assuming only α_U can vary), $R \sim 40.82$ in the string dilaton model assuming Grand Unification [135, 136] (see Section 5.4.1), $R = 38 \pm 6$ [74] and then $R = 46$ [75, 76], the difference arising from the quark masses and their associated thresholds. However, these results implicitly assume that the electroweak symmetry breaking and supersymmetry breaking mechanisms, as well as the fermion mass generation, are not affected by the variation of the unified coupling. It was also mentioned in [75] that R can reach -235 in unification based on SU(5) and SO(10). The large value of R arises from the exponential dependence of Λ_{QCD} on α_3. In the limit in which the quark masses are set to zero, the proton mass, as well as all other hadronic masses are proportional to Λ_{QCD}, i.e., $m_{\mathrm{p}} \propto \Lambda_{\mathrm{QCD}}(1 + \mathcal{O}(m_{\mathrm{q}}/\Lambda_{\mathrm{QCD}}))$. [313] further relates the Higgs vev to α_{EM} by $\mathrm{d}\ln v/\mathrm{d}\ln\alpha_{\mathrm{EM}} \equiv \kappa$ and estimated that $\kappa \sim 70$ so that, assuming that the variation of the Yukawa couplings is negligible, it could be concluded that

$$\delta \ln \frac{m}{\Lambda_{\mathrm{QCD}}} \sim 35\delta \ln \alpha_{\mathrm{EM}},$$

for the quark and electron masses. This would also imply that the variation of μ and α_{EM} are correlated, still in a very model-dependent way, typically one can conclude [104] that

$$\frac{\delta\mu}{\mu} = -0.8R\frac{\delta\alpha_{\mathrm{EM}}}{\alpha_{\mathrm{EM}}} + 0.6(S+1)\frac{\delta h}{h},$$

with $S \sim 160$. The running of α_U can be extrapolated to the Planck mass, M_{P}. Assuming $\alpha_U(M_{\mathrm{P}})$ fixed and letting M_U/M_{P} vary, it was concluded [153] that $R = 2\pi(b_U+3)/[9\alpha_{\mathrm{EM}}(8b_U/3-12)]$ where b_U is the beta-function coefficient describing the running of α_U. This shows that a variation of α_{EM} and μ can open a windows on GUT theories. A similar analysis [142] assuming that electroweak symmetry breaking was triggered by nonperturbative effects in such a way that v and α_U are related, concludes that $\delta\mu/\mu = (13\pm7)\delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}$ in a theory with soft SUSY breaking and $\delta\mu/\mu = (-4\pm5)\delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}}$ otherwise.

From a phenomenological point of view, [147] making an assumption of proportionality with fixed "unification coefficients" assumes that the variations of the constants at a given redshift z depend on a unique evolution factor $\ell(z)$ and that the variation of all the constants can be derived from those of the unification mass scale (in Planck units), M_U, the unified gauge coupling α_U, the Higgs vev, v and in the case of supersymmetric theories the soft supersymmetry breaking mass, $\tilde{m}$. Introducing the coefficients d_i by

$$\Delta\ln\frac{M_U}{M_{\mathrm{P}}} = d_M\ell, \quad \Delta\ln\alpha_U = d_U\ell, \quad \Delta\ln\frac{v}{M_U} = d_H\ell, \quad \Delta\ln\frac{\tilde{m}}{M_{\mathrm{P}}} = d_S\ell,$$

($d_S = 0$ for non-supersymmetric theories) and assuming that the masses of the standard model fermions all vary with v so that the Yukawa couplings are assumed constant, it was shown that the variations of all constants can be related to (d_M, d_U, d_H, d_S) and $\ell(z)$, using the renormalization group equations (neglecting the effects induced by the variation of α_U on the RG running of fermion masses). This decomposition is a good approximation provided that the time variation is slow, which is actually backed up by the existing constraints, and homogeneous in space (so that it may not be applied as such in the case a chameleon mechanism is at work [69]).

This allowed to be defined six classes of scenarios: (1) varying gravitational constant ($d_H = d_S = d_X = 0$) in which only M_U/M_{P} or equivalently $G\Lambda_{\mathrm{QCD}}^2$ is varying; (2) varying unified coupling ($d_U = 1, d_H = d_S = d_M = 0$); (3) varying Fermi scale defined by ($d_H = 1, d_U = d_S = d_M = 0$) in which one has $\mathrm{d}\ln\mu/\mathrm{d}\ln\alpha_{\mathrm{EM}} = -325$; (4) varying Fermi scale and SUSY-breaking scale ($d_S = d_H = 1, d_U = d_M = 0$) and for which $\mathrm{d}\ln\mu/\mathrm{d}\ln\alpha_{\mathrm{EM}} = -21.5$; (5) varying unified coupling and Fermi scale ($d_X = 1, d_H = \tilde{\gamma}d_X, d_S = d_M = 0$) and for which $\mathrm{d}\ln\mu/\mathrm{d}\ln\alpha_{\mathrm{EM}} = (23.2 - 0.65\tilde{\gamma})/(0.865 + 0.02\tilde{\gamma})$; (6) varying unified coupling and Fermi scale with SUSY ($d_X = 1, d_S \simeq d_H = \tilde{\gamma}d_X, d_M = 0$) and for which $\mathrm{d}\ln\mu/\mathrm{d}\ln\alpha_{\mathrm{EM}} = (14 - 0.28\tilde{\gamma})/(0.52 + 0.013\tilde{\gamma})$.

Each scenario can be compared to the existing constraints to get sharper bounds on them [146, 147, 149, 364] and emphasize that the correlated variation between different constants (here μ and α_{EM}) depends strongly on the theoretical hypothesis that are made.

5.3.2   Masses and binding energies

The previous Section 5.3.1 described the unification of the gauge couplings. When we consider "composite" systems such as proton, neutron, nuclei or even planets and stars, we need to compute their mass, which requires to determine their binding energy. As we have already seen, the electromagnetic binding energy induces a direct dependence on α_{EM} and can be evaluated using, e.g., the Bethe–Weizäcker formula (61). The dependence of the masses on the quark masses, via nuclear interactions, and the determination of the nuclear binding energy are especially difficult to estimate.

In the chiral limit of QCD in which all quark masses are negligible compared to Λ_{QCD} all dimensionful quantities scale as some power of Λ_{QCD}. For instance, concerning the nucleon mass, $m_{\mathrm{N}} = c\Lambda_{\mathrm{QCD}}$ with $c \sim 3.9$ being computed from lattice QCD. This predicts a mass of order 860 MeV, smaller than the observed value of 940 MeV. The nucleon mass can be computed in chiral perturbation theory and expressed in terms of the pion mass as [316] $m_{\mathrm{N}} = a_0 + a_2 m_\pi^2 + a_4 m_\pi^4 + a_6 m_\pi^6 + \sigma_{N\pi} + \sigma_{\Delta\pi} + \sigma_{\mathrm{tad}}$ (where all coefficients of this expansion are defined in [316]), which can be used to show [204] that the nucleon mass is scaling as

$$m_{\mathrm{N}} \propto \Lambda_{\mathrm{QCD}} X_{\mathrm{q}}^{0.037} X_{\mathrm{s}}^{0.011}. \tag{188}$$

(Note, however, that such a notation is dangerous since it would imply that m_{N} vanishes in the chiral limit but it is a compact way to give $\delta m_{\mathrm{N}}/\delta X_{\mathrm{q}}$ etc.). It was further extended [208] by using a sigma model to infer that $m_{\mathrm{N}} \propto \Lambda_{\mathrm{QCD}} X_{\mathrm{q}}^{0.045} X_{\mathrm{s}}^{0.19}$. These two examples explicitly show the strong dependence in nuclear modeling.

To go further and determine the sensitivity of the mass of a nucleus to the various constant,

$$m(A, Z) = Z m_{\mathrm{p}} + (A - Z) m_{\mathrm{n}} + Z m_{\mathrm{e}} + E_{\mathrm{S}} + E_{\mathrm{EM}}$$

one should determine the strong binding energy [see related discussion below Eq. (17)] in function of the atomic number Z and the mass number A.

The case of the deuterium binding energy B_D has been discussed in different ways (see Section 3.8.3). Many models have been created. A first route relies on the use of the dependence of B_D on the pion mass [188, 38, 426, 553], which can then be related to m_{u}, m_{d} and Λ_{QCD}. A second avenue is to use a sigma model in the framework of the Walecka model [456] in which the potential for the nuclear forces keeps only the σ, ρ and ω meson exchanges [208]. We also emphasize that the deuterium is only produced during BBN, as it is too weakly bound to survive in the regions of stars where nuclear processes take place. The fact that we do observe deuterium today sets a non-trivial constraint on the constants by imposing that the deuterium remains stable from BBN time to today. Since it is weakly bound, it is also more sensitive to a variation of the nuclear force compared to the electromagnetic force. This was used in [145] to constrain the variation of the nuclear strength in a sigma-model.

For larger nuclei, the situation is more complicated since there is no simple modeling. For large mass number A, the strong binding energy can be approximated by the liquid drop model

$$\frac{E_{\mathrm{S}}}{A} = a_V - \frac{a_S}{A^{1/3}} - a_A \frac{(A - 2Z)^2}{A^2} + a_P \frac{(-1)^A + (-1)^Z}{A^{3/2}} \tag{189}$$

with $(a_V, a_S, a_A, a_P) = (15.7, 17.8, 23.7, 11.2)$ MeV [439]. It has also been suggested [129] that the nuclear binding energy can be expressed as

$$E_{\mathrm{S}} \simeq A a_3 + A^{2/3} b_3 \qquad \text{with} \qquad a_3 = a_3^{\text{chiral limit}} + m_\pi^2 \frac{\partial a_3}{\partial m_\pi^2}. \tag{190}$$

In the chiral limit, a_3 has a non-vanishing limit to which we need to add a contribution scaling like $m_\pi^2 \propto \Lambda_{\mathrm{QCD}} m_{\mathrm{q}}$. [129] also pointed out that the delicate balance between attractive and repulsive nuclear interactions [456] implies that the binding energy of nuclei is expected to depend strongly on the quark masses [159]. Recently, a fitting formula derived from effective field theory and based of the semi-empirical formula derived in [222] was proposed [120] as

$$\frac{E_{\mathrm{S}}}{A} = -\left(120 - \frac{97}{A^{1/3}}\right)\eta_S + \left(67 - \frac{57}{A^{1/3}}\right)\eta_V + \dots \tag{191}$$

where η_S and η_V are the strength of respectively the scalar (attractive) and vector (repulsive) nuclear contact interactions normalized to their actual value. These two parameters need to be

related to the QCD parameters [159]. We also refer to [211] for the study of the dependence of the binding of light ($A \leq 8$) nuclei on possible variations of hadronic masses, including meson, nucleon, and nucleon-resonance masses.

These expressions allow to compute the sensitivity coefficients that enter in the decomposition of the mass [see Equation (201)]. They also emphasize one of the most difficult issue concerning the investigation about constant related to the intricate structure of QCD and its role in low energy nuclear physics, which is central to determine the masses of nuclei and the binding energies, quantities that are particularly important for BBN, the universality of free fall and stellar physics.

5.3.3 Gyromagnetic factors

The constraints arising from the comparison of atomic clocks (see Section 3.1) involve the fine-structure constant $\alpha_{\rm EM}$, the proton-to-electron mass ratio μ and various gyromagnetic factors. It is important to relate these factors to fundamental constants.

The proton and neutron gyromagnetic factors are respectively given by $g_{\rm p} = 5.586$ and $g_{\rm n} = -3.826$ and are expected to depend on $X_{\rm q} = m_{\rm q}/\Lambda_{\rm QCD}$ [197]. In the chiral limit in which $m_{\rm u} = m_{\rm d} = 0$, the nucleon magnetic moments remain finite so that one could have thought that the finite quark mass effects should be small. However, it is enhanced by π-meson loop corrections, which are proportional to $m_\pi \propto \sqrt{m_{\rm q}\Lambda_{\rm QCD}}$. Following [316], this dependence can be described by the approximate formula

$$g(m_\pi) = \frac{g(0)}{1 + am_\pi + bm_\pi^2}.$$

The coefficients are given by $a = (1.37, 1.85)/$ GeV and $b = (0.452, 0.271)/$ GeV^2 respectively for the proton an neutron. This lead [197] to $g_{\rm p} \propto m_\pi^{-0.174} \propto X_{\rm q}^{-0.087}$ and $g_{\rm n} \propto m_\pi^{-0.213} \propto X_{\rm q}^{-0.107}$. This was further extended in [204] to take into account the dependence with the strange quark mass $m_{\rm s}$ to obtain

$$g_{\rm p} \propto X_{\rm q}^{-0.087} X_{\rm s}^{-0.013}, \qquad g_{\rm n} \propto X_{\rm q}^{-0.118} X_{\rm s}^{0.0013}. \tag{192}$$

All these expressions assumes $\Lambda_{\rm QCD}$ constant in their derivations.

This allows one to express the results of atomic clocks (see Section 3.1.3) in terms of $\alpha_{\rm EM}$, $X_{\rm q}$, $X_{\rm s}$ and $X_{\rm e}$. Similarly, for the constants constrained by QSO observation, we have (see Table 10)

$$\begin{aligned}
x &\propto \alpha_{\rm EM}^2 X_{\rm q}^{-0.087} X_{\rm s}^{-0.013}, \\
y &\propto \alpha_{\rm EM}^2 X_{\rm q}^{-0.124} X_{\rm s}^{-0.024} X_{\rm e}, \\
\bar{\mu} &\propto X_{\rm q}^{-0.037} X_{\rm s}^{-0.011} X_{\rm e}, \\
F &\propto \alpha_{\rm EM}^{3.14} X_{\rm q}^{-0.0289} X_{\rm s}^{0.0043} X_{\rm e}^{-1.57}, \\
F' &\propto \alpha_{\rm EM}^2 X_{\rm q}^{0.037} X_{\rm s}^{0.011} X_{\rm e}^{-1}, \\
G &\propto \alpha_{\rm EM}^{1.85} X_{\rm q}^{-0.0186} X_{\rm s}^{0.0073} X_{\rm e}^{-1.85},
\end{aligned} \tag{193}$$

once the scaling of the nucleon mass as $m_{\rm N} \propto \Lambda_{\rm QCD} X_{\rm q}^{0.037} X_{\rm s}^{0.011}$ (see Section 5.3.2). This shows that the seven observable quantities that are constrained by current QSO observations can be reduced to only 4 parameters.

5.4 Models with varying constants

The models that can be constructed are numerous and cannot all be reviewed here. Thus, we focus on the string dilaton model in Section 5.4.1 and then discuss the chameleon mechanism in Section 5.4.2 and the Bekenstein framework in Section 5.4.3.

5.4.1 String dilaton and Runaway dilaton models

Damour and Polyakov [135, 136] argued that the effective action for the massless modes taking into account the full string loop expansion should be of the form

$$S = \int \mathrm{d}^4\mathbf{x}\sqrt{-\hat{g}}\left[M_s^2\left\{B_g(\Phi)\hat{R} + 4B_\Phi(\Phi)\left[\hat{\Box}\Phi - (\hat{\nabla}\Phi)^2\right]\right\} - B_F(\Phi)\frac{k}{4}\hat{F}^2\right.$$
$$\left. - B_\psi(\Phi)\bar{\hat{\psi}}\hat{\slashed{D}}\hat{\psi} + \dots\right] \tag{194}$$

in the string frame, M_s being the string mass scale. The functions B_i are not known but can be expanded (from the genus expansion of string theory) in the limit $\Phi \to -\infty$ as

$$B_i(\Phi) = \mathrm{e}^{-2\Phi} + c_0^{(i)} + c_1^{(i)}\mathrm{e}^{2\Phi} + c_2^{(i)}\mathrm{e}^{4\Phi} + \dots \tag{195}$$

where the first term is the tree level term. It follows that these functions can exhibit a local maximum. After a conformal transformation $(g_{\mu\nu} = CB_g\hat{g}_{\mu\nu}, \psi = (CB_g)^{-3/4}B_\psi^{1/2}\hat{\psi})$, the action in Einstein frame takes the form

$$S = \int \frac{\mathrm{d}^4\mathbf{x}}{16\pi G}\sqrt{-g}\left[R - 2(\nabla\phi)^2 - \frac{k}{4}B_F(\phi)F^2 - \bar{\psi}\slashed{D}\psi + \dots\right] \tag{196}$$

where the field ϕ is defined as

$$\phi \equiv \int\left[\frac{3}{4}\left(\frac{B_g'}{B_g}\right)^2 + 2\frac{B_\Phi'}{B_\Phi} + 2\frac{B_\Phi'}{B_g}\right]\mathrm{d}\Phi.$$

It follows that the Yang–Mills coupling behaves as $g_{\mathrm{YM}}^{-2} = kB_F(\phi)$. This also implies that the QCD mass scale is given by

$$\Lambda_{\mathrm{QCD}} \sim M_s(CB_g)^{-1/2}\mathrm{e}^{-8\pi^2 kB_F/b} \tag{197}$$

where b depends on the matter content. It follows that the mass of any hadron, proportional to Λ_{QCD} in first approximation, depends on the dilaton, $m_A(B_g, B_F, \dots)$.

If, as allowed by the ansatz (195), $m_A(\phi)$ has a minimum ϕ_m then the scalar field will be driven toward this minimum during the cosmological evolution. However, if the various coupling functions have different minima then the minima of $m_A(\phi)$ will depend on the particle A. To avoid violation of the equivalence principle at an unacceptable level, it is thus necessary to assume that all the minima coincide in $\phi = \phi_m$, which can be implemented by setting $B_i = B$. This can be realized by assuming that ϕ_m is a special point in field space, for instance it could be associated to the fixed point of a Z_2 symmetry of the T- or S-duality [129].

Expanding $\ln B$ around its maximum ϕ_m as $\ln B \propto -\kappa(\phi - \phi_m)^2/2$, Damour and Polyakov [135, 136] constrained the set of parameters $(\kappa, \phi_0 - \phi_m)$ using the different observational bounds. This toy model allows one to address the unsolved problem of the dilaton stabilization, to study all the experimental bounds together and to relate them in a quantitative manner (e.g., by deriving a link between equivalence-principle violations and time-variation of α_{EM}). This model was compared to astrophysical data in [306] to conclude that $|\Delta\phi| < 3.4\kappa 10^{-6}$.

An important feature of this model lies in the fact that at lowest order the masses of all nuclei are proportional to Λ_{QCD} so that at this level of approximation, the coupling is universal and the theory reduces to a scalar-tensor theory and there will be no violation of the universality of free fall. It follows that the deviation from general relativity are characterized by the PPN parameters

$$\gamma^{\mathrm{PPN}} - 1 \simeq -2f_A^2 = -2\beta_s^2\kappa^2\Delta\phi_0^2, \qquad \beta^{\mathrm{PPN}} - 1 \simeq \frac{1}{2}f_A^2\frac{\mathrm{d}f_A}{\mathrm{d}\phi} = \frac{1}{2}\beta_s^3\kappa^3\Delta\phi_0^2$$

with

$$f_A = \frac{\partial \ln \Lambda_{\mathrm{QCD}}(\phi)}{\partial \phi} = -\left[\ln \frac{M_s}{m_A} + \frac{1}{2}\right] \frac{\mathrm{d} \ln B}{\mathrm{d}\phi} \equiv -\beta_s \frac{\mathrm{d} \ln B}{\mathrm{d}\phi} = \beta_s \kappa \Delta\phi_0 \qquad (198)$$

with $\Delta\phi_0 = \phi_0 - \phi_m$ and $\beta_s \sim 40$ [135]. The variation of the gravitational constant is, from Equation (167), simply

$$\frac{\dot{G}}{G} = 2f_A\dot{\phi}_0 = -2\left[\ln \frac{M_s}{m_A} + \frac{1}{2}\right] \frac{\mathrm{d} \ln B}{\mathrm{d}\phi}\dot{\phi}_0.$$

The value of $\dot{\phi}_0 = H_0\phi'_0$ is obtained from the Klein–Gordon equation (168) and is typically given by $\phi'_0 = -Z\beta_s\kappa H_0\Delta\phi_0$ were Z is a number that depends on the equation of state of the fluid dominating the matter content of the universe in the last e-fold and the cosmological parameters so that

$$\left.\frac{\dot{G}}{G}\right|_0 = 2f_A\dot{\phi}_0 = -2ZH_0\beta_s^2\kappa^2\Delta\phi_0^2. \qquad (199)$$

The factor Z is model-dependent and another way to estimate $\dot{\phi}_0$ is to use the Friedmann equations, which imply that $\dot{\phi}_0 = H_0\sqrt{1 + q_0 - \frac{3}{3}\Omega_{\mathrm{m}0}}$ where q is the deceleration parameter.

When one considers the effect of the quark masses and binding energies, various composition-dependent effects appear. First, the fine-structure constant scales as $\alpha_{\mathrm{EM}} \simeq B^{-1}$ so that

$$\left.\frac{\dot{\alpha}}{\alpha}\right|_0 = \kappa\Delta\phi_0\dot{\phi}_0 = -ZH_0\beta_s\kappa^2\Delta\phi_0^2. \qquad (200)$$

The second effect is, as pointed out earlier, a violation of the universality of free fall. In full generality, we expect that

$$m_A(\phi) = N\Lambda_{\mathrm{QCD}}(\phi)\left[1 + \sum_{\mathrm{q}} \epsilon_A^q \frac{m_{\mathrm{q}}}{\Lambda_{\mathrm{QCD}}} + \epsilon_A^{\mathrm{EM}}\alpha_{\mathrm{EM}}\right]. \qquad (201)$$

Using an expansion of the form (17), it was concluded that

$$\eta_{AB} = \kappa^2(\phi_0 - \phi_m)^2\left[C_B\Delta\left(\frac{B}{M}\right) + C_D\Delta\left(\frac{D}{M}\right) + C_E\Delta\left(\frac{E}{M}\right)\right] \qquad (202)$$

with $B = N + Z$, $D = N - Z$ and $E = Z(Z-1)/(N+Z)^{1/3}$ and where the value of the parameters C_i are model-dependent.

It follows from this model that:

- The PPN parameters, the time variation of α and G today and the violation of the university of free-fall all scale as $\Delta\phi_0^2$.

- The field is driven toward ϕ_m during the cosmological evolution, a point at which the scalar field decouples from the matter field. The mechanism is usually called *the least coupling principle*.

- Once the dynamics for the scalar field is solved, $\Delta\phi_0$ can be related to $\Delta\phi_i$ at the end of inflation. Interestingly, this quantity can be expressed in terms of amplitude of the density contrast at the end of inflation, that is to the energy scale of inflation.

- The numerical estimations [135] indicate that $\eta_{U,H} \sim -5.4 \times 10^{-5}(\gamma^{\mathrm{PPN}} - 1)$ showing that in such a class of models, the constraint on $\eta \sim 10^{-13}$ implies $1 - \gamma^{\mathrm{PPN}} \sim 2 \times 10^{-9}$, which is a better constraint that the one obtained directly.

This model was extended [133] to the case where the coupling functions have a smooth finite limit for infinite value of the bare string coupling, so that $B_i = C_i + \mathcal{O}(\mathrm{e}^{-\phi})$, folling [229]. The dilaton runs away toward its attractor at infinity during a stage of inflation. The late time dynamics of the scalar field is similar as in quintessence models, so that the model can also explain the late time acceleration of the cosmic expansion. The amplitude of residual dilaton interaction is related to the amplitude of the primordial density fluctuations and it induces a variation of the fundamental constants, provided it couples to dark matter or dark energy. It is concluded that, in this framework, the largest allowed variation of α_{EM} is of order 2×10^{-6}, which is reached for a violation of the universality of free fall of order 10^{-12} and it was established that

$$\left.\frac{\dot{\alpha}_{\mathrm{EM}}}{\alpha_{\mathrm{EM}}}\right|_0 \sim \pm 10^{-16}\sqrt{1 + q_0 - \frac{3}{2}\Omega_{\mathrm{m0}}}\sqrt{10^{12}\eta}\ \mathrm{yr}^{-1}, \tag{203}$$

where the first square-root arises from the computation of $\dot{\phi}_0$. The formalism was also used to discuss the time variation of α_{EM} and μ [97].

The coupling of the dilaton to the standard model fields was further investigated in [122, 121]. Assuming that the heavy quarks and weak gauge bosons have been integrated out and that the dilaton theory has been matched to the light fields below the scale of the heavy quarks, the coupling of the dilaton has been parameterized by 5 parameters: d_e and d_g for the couplings to the electromagnetic and gluonic field-strength terms, and d_{m_e}, d_{m_u} and d_{m_d} for the couplings to the fermionic mass terms so that the interaction Lagrangian is reduces to a linear coupling (e.g., $\propto d_e\phi F^2$ for the coupling to electromagnetism etc.) It follows that $\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}} = d_e\kappa\phi$ for the fine structure constant, $\Delta\Lambda_{\mathrm{QCD}}/\Lambda_{\mathrm{QCD}} = d_d\kappa\phi$ for the strong sector and $\Delta m_i/m_i = d_{m_i}\kappa\phi$ for the masses of the fermions. These parameters can be constrained by the test of the equivalence principle in the solar system [see Section 6.3].

In these two string-inspired scenarios, the amplitude of the variation of the constants is related to the one of the density fluctuations during inflation and the cosmological evolution.

5.4.2   The Chameleon mechanism

A central property of the least coupling principle, that is at the heart of the former models, is that all coupling functions have the same minimum so that the effective potential entering the Klein–Gordon equation for the dilaton has a well-defined minimum.

It was realized [287] that if the dilaton has a coupling $A^2(\phi)$ to matter while evolving in a potential $V(\phi)$ the source of the Klein–Gordon equation (168) has a an effective potential

$$V_{\mathrm{eff}} = V(\phi) + A^2(\phi)\rho.$$

In the case where V is a decreasing function of ϕ, e.g., a runaway potential, and the coupling is an increasing function, e.g., $A^2 = \exp\beta\phi/M_{\mathrm{P}}$, the effective potential has a minimum the value of which depends on the matter local density ρ (see also [186]). Thus, the field is expected to be massive on Earth where the density is high and light in space in the solar system. It follows that the experiment on the universality of free fall in space may detect violations of the universality of free fall larger than the bounds derived by laboratory experiments [288, 498]. It follows (1) that the constraints on the time variation of the constants today can be relaxed if such a mechanism is at work and (2) that is the constants depend on the local value of the chameleon field, their value will be environment dependent and will be different on Earth and in space.

The cosmological variation of α_{EM} in such model was investigated in [70, 71]. Models based on the Lagrangian (209) and exhibiting the chameleon mechanism were investigated in [398].

The possible shift in the value of μ in the Milky Way (see Section 6.1.3) was related [323, 324, 322] to the model of [398] to conclude that such a shift was compatible with this model.

5.4.3   Bekenstein and related models

Bekenstein [39, 40] introduced a theoretical framework in which only the electromagnetic sector was modified by the introduction of a dimensionless scalar field ϵ so that all electric charges vary in unison $e_i = e_{0i}\epsilon(x^\alpha)$ so that only $\alpha_{\rm EM}$ is assumed to possibly vary.

To avoid the arbitrariness in the definition of ϵ, which can be rescaled by a constant factor while e_{0i} is inversely rescales, it was postulated that the dynamics of ϵ be invariant under global rescaling so that its action should be of the form

$$S_\epsilon = -\frac{\hbar c}{2l^2} \int \frac{g^{\mu\nu}\partial_\mu\epsilon\partial_\nu\epsilon}{\epsilon^2}\sqrt{-g}{\rm d}^4x, \qquad (204)$$

l being a constant length scale. Then, ϵ is assumed to enter all electromagnetic interaction via $e_i A_\mu \to e_{0i}\epsilon A_\mu$ where A_μ is the usual electromagnetic potential and the gauge invariance is then preserved only if $\epsilon A_\mu \to \epsilon A_\mu + \lambda_{,\mu}$ for any scalar function λ. It follows that the the action for the electromagnetic sector is the standard Maxwell action

$$S_\epsilon = -\frac{1}{16\pi} \int F^{\mu\nu}F_{\mu\nu}\sqrt{-g}{\rm d}^4x, \qquad (205)$$

for the generalized Faraday tensor

$$F_{\mu\nu} = \frac{1}{\epsilon}\left[(\epsilon A_\nu)_{,\mu} - (\epsilon A_\mu)_{,\nu}.\right] \qquad (206)$$

To finish the gravitational sector is assumed to be described by the standard Einstein–Hilbert action. Finally, the matter action for point particles of mass m takes the form $S_m = \sum \int[-mc^2 + (e/c)u^\mu A_\mu]\gamma^{-1}\delta^3(x^i - x^i(\tau)){\rm d}^4\mathbf{x}$ where γ is the Lorentz factor and τ the proper time. Note that the Maxwell equation becomes

$$\nabla_\mu\left(\epsilon^{-1}F^{\mu\nu}\right) = 4\pi j^\nu, \qquad (207)$$

which is the same as for electromagnetism in a material medium with dielectric constant ϵ^{-2} and permeability ϵ^2 (this was the original description proposed by Fierz [195] and Lichnerowicz [332]; see also Dicke [152]).

It was proposed [445] to rewrite this theory by introducing the two fields

$$a_\mu \equiv \epsilon A_\mu, \qquad \psi \equiv \ln\epsilon$$

so that the theory takes the form

$$S = \frac{c^3}{16\pi g}\int R\sqrt{-g}{\rm d}^4x - \frac{1}{16\pi}\int {\rm e}^{-2\psi}f^{\mu\nu}f_{\mu\nu}\sqrt{-g}{\rm d}^4x - \frac{1}{8\pi\kappa^2}\int(\partial_\mu\psi)^2\sqrt{-g}{\rm d}^4x \qquad (208)$$

with $\kappa = l/(4\pi\hbar c)$ and $f_{\mu\nu}$ the Faraday tensor associated with a_μ. The model was further extended to include a potential for ψ [32] and to include the electroweak theory [461].

As discussed previously, this class of models predict a violation of the universality of free fall and, from Equation (14), it is expected that the anomalous acceleration is given by $\delta\mathbf{a} = -M^{-1}(\partial E_{\rm EM}/\partial\epsilon)\nabla\epsilon$.

From the confrontation of the local and cosmological constraints on the variation of ϵ Bekenstein [39] concluded, given his assumptions on the couplings, that $\alpha_{\rm EM}$ "*is a parameter, not a dynamical variable*" (see, however, [40] and then [301]). This problem was recently bypassed by Olive and Pospelov [397] who generalized the model to allow additional coupling of a scalar field $\epsilon^{-2} = B_F(\phi)$ to non-baryonic dark matter (as first proposed in [126]) and cosmological constant, arguing that in supersymmetric dark matter, it is natural to expect that ϕ would couple more

strongly to dark matter than to baryon. For instance, supersymmetrizing Bekenstein model, ϕ will get a coupling to the kinetic term of the gaugino of the form $M_*^{-1}\phi\bar{\chi}\partial\chi$. Assuming that the gaugino is a large fraction of the stable lightest supersymmetric particle, the coupling to dark matter would then be of order $10^3 - 10^4$ times larger. Such a factor could almost reconcile the constraint arising from the test of the universality of free fall with the order of magnitude of the cosmological variation. This generalization of the Bekenstein model relies on an action of the form

$$S = \frac{1}{2}M_4^2 \int R\sqrt{-g}\mathrm{d}^4\mathbf{x} - \int \left[\frac{1}{2}M_*^2\partial_\mu\phi\partial^\mu\phi + \frac{1}{4}B_F(\phi)F_{\mu\nu}F^{\mu\nu}\right]\sqrt{-g}\mathrm{d}^4\mathbf{x} \qquad (209)$$

$$- \int \left\{\sum \bar{N}_i[i\not{D} - m_i B_{N_i}(\phi)]N_i + \frac{1}{2}\bar{\chi}\partial\chi + M_4^2 B_\Lambda(\phi)\Lambda + \frac{1}{2}M_\chi B_\chi(\phi)\chi^T\chi\right\}\sqrt{-g}\mathrm{d}^4\mathbf{x}$$

where the sum is over proton $[\not{D} = \gamma^\mu(\partial_\mu - ie_0 A_\mu)]$ and neutron $[\not{D} = \gamma^\mu\partial_\mu]$. The functions B can be expanded (since one focuses on small variations of the fine-structure constant and thus of ϕ) as $B_X = 1 + \zeta_X\phi + \xi_X\phi^2/2$. It follows that $\alpha_{\mathrm{EM}}(\phi) = e_0^2/4\pi B_F(\phi)$ so that $\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}} = \zeta_F\phi + (\xi_F - 2\zeta_F^2)\phi^2/2$. This framework extends the analysis of [39] to a 4-dimensional parameter space $(M_*, \zeta_F, \zeta_m, \zeta_\Lambda)$. It contains the Bekenstein model ($\zeta_F = -2$, $\zeta_\Lambda = 0$, $\zeta_m \sim 10^{-4}\xi_F$), a Jordan–Brans–Dicke model ($\zeta_F = 0$, $\zeta_\Lambda = -2\sqrt{2/2\omega + 3}$, $\zeta_m = -1/\sqrt{4\omega + 6}$), a string-like model ($\zeta_F = -\sqrt{2}$, $\zeta_\Lambda = \sqrt{2}$, $\zeta_m = \sqrt{2}/2$) so that $\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}} = 3$) and a supersymmetrized Bekenstein model ($\zeta_F = -2$, $\zeta_\chi = -2$, $\zeta_m = \zeta_\chi$ so that $\Delta\alpha_{\mathrm{EM}}/\alpha_{\mathrm{EM}} \sim 5/\omega$). In all the models, the universality of free fall sets a strong constraint on $\zeta_F/\sqrt{\omega}$ (with $\omega \equiv M_*/2M_4^2$) and the authors showed that a small set of models may be compatible with a variation of α_{EM} from quasar data while being compatiblewith the equivalence principle tests. A similar analysis [347] concluded that such models can reproduce the variation of α_{EM} from quasars while being compatible with Oklo and meteorite data. Note that under this form, the effective theory is very similar to the one detailed in Section 5.4.2.

This theory was also used [41] to study the spacetime structure around charged black-hole, which corresponds to an extension of dilatonic charged black hole. It was concluded that a cosmological growth of α_{EM} would decrease the black-hole entropy but with half the rate expected from the earlier analysis [139, 339].

5.4.4   Other ideas

Let us mention without details other theoretical models, which can accommodate varying constants:

- Models involving a late time phase transition in the electromagnetic sector [87, 10];

- Braneworld models [336, 8, 73, 331, 403] or extra-dimensions [477];

- Model with pseudo-scalar couplings [203];

- Growing neutrino models [9, 533] in which the neutrino masses are a function of a scalar field, that is also responsible for the late time acceleration of the universe. In these models the neutrinos freeze the evolution of the scalar field when they become non-relativistic while its evolution is similar as in quintessence when the neutrinos are ultra-relativistic;

- Models based on discrete quantum gravity [223] or on loop quantum gravity in which the Barbero–Immirzi parameter controls the minimum eigenvalue of the area operator and could be promoted to a field, leading to a classical coupling of Einstein's gravity with a scalar-field stress-energy tensor [354, 483]

- "varying speed of light" models for which we refer to the review [341] and our previous analysis [183] for a critical view;

- Quintessence models with a non-minimal coupling of the quintessence field [20, 11, 96, 112, 162, 217, 315, 314, 389, 347, 404, 531] [see discussion Section 2.2.3];

- Mass varying neutrinos [9, 533];

- Holographic dark energy models with non-minimal couplings [235]

6   Spatial Variations

The constraints on the variation of the fundamental constant that we have described so far are mainly related to their cosmological evolution so that, given the Copernican principle, they reduce to constrains on the time variation of the fundamental constants. Indeed, spatial variations can also occur. They may be used to set constraints in two regimes:

- On cosmological scales, the fields dictating the variation of the constants have fluctuations that can let their imprint in some cosmological observables.

- On local scales (e.g., our solar system or the Milky Way) the fields at the origin of the variation of the constants are sourced by the local matter distribution so that one expect that the constants are not homogeneous on these scales.

6.1   Local scales

In order to determine the profile of the constant in the solar system, let us assume that their value is dictated by the value of a scalar field. As in Section 5.4.1, we can assume that at lowest order the profile of the scalar field will be obtained from the scalar-tensor theory, taking into account that all masses scale as $\Lambda_{\mathrm{QCD}}(\phi_*)$ where ϕ_* is the value of the field in the Einstein frame.

6.1.1   Generalities

We restrict to the weakly self-gravitating ($V_*/c^2 \ll 1$) and slow moving ($T^{01} \ll T^{00}$) localized material systems and follow [124]. Using harmonic coordinates, defined with respect to the metric g_*, the Einstein frame metric can be expanded as

$$g_{00}^* = -\exp\left(-2\frac{V_*}{c^2}\right) + \mathcal{O}(c^{-6}), \qquad g_{0i}^* = -\frac{4}{c^3}V_i^* + \mathcal{O}(c^{-5}), \qquad g_{ij}^* = -\exp\left(2\frac{V_*}{c^2}\right)\delta_{ij} + \mathcal{O}(c^{-6}),$$

so that Eqs. (162–163) take the form

$$\Box_* V_* = -4\pi G_* \sigma_* + \mathcal{O}(c^{-4}), \quad \Box_* V_*^i = -4\pi G_* \sigma_*^i + \mathcal{O}(c^{-2}), \quad \Box_* \phi_* = -4\pi G_* \frac{S}{c^2} + \mathcal{O}(c^{-6}) \quad (210)$$

where $\Box_*$ is the flat d'Alembertian and where the scalar field has been assumed to be light so that one can neglect its potential. The source terms can be expressed in terms of the matter stress-energy tensor in the Einstein frame as

$$\sigma_* c^2 = T_*^{00} + T_*^{ii}, \qquad \sigma_*^i = T_*^{0i}, \qquad S c^2 = -\alpha(\phi_*)(T_*^{00} - T_*^{ii}).$$

Restricting to the static case with a single massive point source, the only non-vanishing source terms are $\sigma_*(\mathbf{r}) = M_* \delta^3(\mathbf{r}_*)$ and $S(\mathbf{r}) = -\alpha(\phi_*) M_* \delta^3(\mathbf{r}_*)$ so that the set of equations reduces to two Poisson equations

$$\Delta_* V_* = -4\pi G_* M_* \delta^3(\mathbf{r}_*) + \mathcal{O}(c^{-4}), \qquad \Delta_* \phi_* = 4\pi \frac{G_* M_*}{c^2}\delta^3(\mathbf{r}_*) + \mathcal{O}(c^{-6}). \qquad (211)$$

This set of equations can be solved by means of the retarded Green function. It follows that the Einstein frame gravitational potential is $V_*(r_*) = G_* M_*/r_*$. The equation for ϕ_* can be solved iteratively, since at lowest order in G_*/c^2 it has solution

$$\phi_* = \phi_1(r_*) \equiv \phi_0 - \frac{\alpha_0}{c^2}V_*(r_*).$$

This can be used to determine the Jordan frame metric and the variation of the scalar field in function of the Jordan frame coordinates. It follows that at lowest order the Newton potential and the scalar field are given by

$$\Phi_N = \frac{GM}{r}, \qquad \phi_* = \phi_1(r) \equiv \phi_0 - \alpha_0 \frac{\Phi_N(r)}{c^2}, \tag{212}$$

where we have neglected the corrections $-\alpha(\phi)(\phi - \phi_0)$ for the gravitational potential, which, given the solar system constraints on α_0, is a good approximation.

Now let us consider any constant α_i function of ϕ. Thus, its profile is given by $\alpha_i(r) = \alpha_i(\phi_0) - \alpha_0 \alpha_i'(\phi_0) \Phi_N(r)/c^2$ so that

$$\frac{\Delta \alpha_i}{\alpha_i}(r) = -s_i(\phi_0) \alpha_0 \frac{\Phi_N(r)}{c^2} \tag{213}$$

where $s_i(\phi_0)$ is the sensitivity of the constant α_i to a variation of the scalar field, $s_i \equiv \mathrm{d} \ln \alpha_i / \mathrm{d} \phi$. For laboratory in orbit on an elliptic trajectory,

$$r = \frac{a(1 - e^2)}{1 + e \cos \psi}, \qquad \cos \psi = \frac{\cos E - e}{1 - e \cos E}, \qquad t = \sqrt{\frac{a^3}{GM}}(E - e \sin E)$$

where a is the semi-major axis, e the eccentricity and ψ the true anomaly. It follows that

$$\frac{\Delta \alpha_i}{\alpha_i}(a, \psi) = -s_0 \alpha_0 \frac{GM}{ac^2} - s_0 \alpha_0 \frac{GM}{ac^2} e \cos \psi + \mathcal{O}(e^2).$$

The first term represents the variation of the mean value of the constant on the orbit compared with its cosmological value. This shows that local terrestrial and solar system experiments do measure the effects of the cosmological variation of the constants [124, 461, 460, 462]. The second term is a seasonal modulation and it is usually parameterized [209] as

$$\left.\frac{\Delta \alpha_i}{\alpha_i}\right|_{\text{seasonal}} = k_i \frac{\Delta \Phi_N}{c^2}, \tag{214}$$

defining the parameters k_i.

6.1.2   Solar system scales

The parameters k_i can be constrained from laboratory measurements on Earth. Since $e \simeq 0.0167$ for the Earth orbit, the signal should have a peak-to-peak amplitude of $2\,GMe/ac^2 \sim 3.3 \times 10^{-10}$ on a period of 1 year. This shows that the order of magnitude of the constraints will be roughly of $10^{-16}/10^{-10} \sim 10^{-6}$ since atomic clocks reach an accuracy of the order of 10^{-16}. The data of [214] and [37] lead respectively to the two constraints [209]

$$k_{\alpha_{\text{EM}}} + 0.17 k_e = (-3.5 \pm 6) \times 10^{-7}, \qquad |k_{\alpha_{\text{EM}}} + 0.13 k_e| < 2.5 \times 10^{-5}, \tag{215}$$

for α_{EM} and $m_{\text{e}}/\Lambda_{\text{QCD}}$ respectively. The atomic dysprosium experiment [100] allowed to set the constraint [193]

$$k_{\alpha_{\text{EM}}} = (-8.7 \pm 6.6) \times 10{-6}, \tag{216}$$

which, combined with the previous constraints, allows to conclude that

$$k_e = (4.9 \pm 3.9) \times 10^{-5}, \qquad k_q = (6.6 \pm 5.2) \times 10^{-5}, \tag{217}$$

for $m_\mathrm{e}/\Lambda_\mathrm{QCD}$ and $m_\mathrm{q}/\Lambda_\mathrm{QCD}$ respectively. [61], using the comparison of cesium and a strontium clocks derived that

$$k_{\alpha_\mathrm{EM}} + 0.36 k_e = (1.8 \pm 3.2) \times 10^{-5}, \tag{218}$$

which, combined with measurement of H-maser [17], allow one to set the three constraints as

$$k_{\alpha_\mathrm{EM}} = (2.5 \pm 3.1) \times 10^{-6}, \qquad k_\mu = (-1.3 \pm 1.7) \times 10^{-5}, \qquad k_q = (-1.9 \pm 2.7) \times 10^{-5}. \tag{219}$$

[34, 463] reanalyzed the data by [408] to conclude that $k_{\alpha_\mathrm{EM}} + 0.51 k_\mu = (7.1 \pm 3.4) \times 10^{-6}$. Combined with the constraint (218), it led to

$$k_\mu = (3.9 \pm 3.1) \times 10^{-6}, \qquad k_q = (0.1 \pm 1.4) \times 10^{-5}. \tag{220}$$

[34] also used the data of [440] to conclude

$$k_{\alpha_\mathrm{EM}} = (-5.4 \pm 5.1) \times 10^{-8}. \tag{221}$$

All these constraints use the sensitivity coefficients computed in [14, 210]. We refer to [265] as an unexplained seasonal variation that demonstrated the difficulty to interpret phenomena.

Such bounds can be improved by comparing clocks on Earth and onboard of satellites [209, 444, 343] while the observation of atomic spectra near the Sun can lead to an accuracy of order unity [209]. A space mission with atomic clocks onboard and sent to the Sun could reach an accuracy of 10^{-8} [343, 547].

6.1.3   Milky Way

An attempt [323, 358] to constrain k_μ from emission lines due to ammonia in interstellar clouds of the Milky Way led to the conclusion that $k_\mu \sim 1$, by considering different transitions in different environments. This is in contradiction with the local constraint (219). This may result from rest frequency uncertainties or it would require that a mechanism such as chameleon is at work (see Section 5.4.2) in order to be compatible with local constraints. The analysis was based on an ammonia spectra atlas of 193 dense protostellar and prestellar cores of low masses in the Perseus molecular cloud, comparison of N_2H^+ and N_2D^+ in the dark cloud L183.

A second analysis [324] using high resolution spectral observations of molecular core in lines of NH_3, HC_3N and N_2H^+ with 3 radio-telescopes showed that $|\Delta\mu/\mu| < 3 \times 10^{-8}$ between the cloud environment and the local laboratory environment. However, an offset was measured that could be interpreted as a variation of μ of amplitude $\Delta\bar{\mu}/\bar{\mu} = (2.2 \pm 0.4_\mathrm{stat} \pm 0.3_\mathrm{sys}) \times 10^{-8}$. A second analysis [322] map four molecular cores L1498, L1512, L1517, and L1400K selected from the previous sample in order to estimate systematic effects due to possible velocity gradients. The measured velocity offset, once expressed in terms of $\Delta\bar{\mu}$, gives $\Delta\bar{\mu} = (26 \pm 1_\mathrm{stat} \pm 3_\mathrm{sys}) \times 10^{-9}$.

A similar analysis [326] based on the fine-structure transitions in atomic carbon C I and low-laying rotational transitions in ^{13}CO probed the spatial variation of $F = \alpha_\mathrm{EM}^2 \mu$ over the galaxy. It concluded that

$$|\Delta F'/F'| < 3.7 \times 10^{-7} \tag{222}$$

between high (terrestrial) and low (interstellar) densities of baryonic matter. Combined with the previous constraint on μ it would imply that $|\Delta\alpha_\mathrm{EM}/\alpha_\mathrm{EM}| < 2 \times 10^{-7}$. This was updated [319] to $|\Delta F'/F'| < 2.3 \times 10^{-7}$ so that $|\Delta\alpha_\mathrm{EM}/\alpha_\mathrm{EM}| < 1.1 \times 10^{-7}$.

Since extragalactic gas clouds have densities similar to those in the interstellar medium, these bounds give an upper bound on a hypothetic chameleon effect, which are much below the constraints obtained on time variations from QSO absorption spectra.

6.2 Cosmological scales

During inflation, any light scalar field develop super-Hubble fluctuations of quantum origin, with an almost scale invariant power spectrum (see chapter 8 of [409]). It follows that if the fundamental constants depend on such a field, their value must fluctuate on cosmological scales and have a non-vanishing correlation function. More important these fluctuations can be correlated with the metric perturbations.

In such a case, the fine-structure constant will behave as $\alpha_{\rm EM} = \alpha_{\rm EM}(t) + \delta\alpha_{\rm EM}(\mathbf{x}, t)$, the fluctuations being a stochastic variable. As we have seen earlier, $\alpha_{\rm EM}$ enters the dynamics of recombination, which would then become patchy. This has several consequences for the CMB anisotropies. In particular, similarly to weak gravitational lensing, it will modify the mean power spectra (this is a negligible effect) and induce a curl component (B mode) to the polarization [466]. Such spatial fluctuations also induce non-Gaussian temperature and polarization correlations in the CMB [466, 417]. Such correlations have not allowed to set observational constraints yet but they need to be included foe consistency, see e.g., the example of CMB computation in scalar-tensor theories [435]. The effect on large the scale structure was also studied in [30, 363] and the Keck/HIRES QSO absorption spectra showed [377] that the correlation function of the fine-structure constant is consistent on scales ranging between 0.2 and 13 Gpc.

Recently, it has been claimed [50, 523] that the fine structure constant may have a dipolar variation that would explain consistently the data from the Southern and Northern hemispheres (see Section 3.4.3). Let assume a constant, X say, depend on the local value of a dynamical scalar field ϕ. The value of X at the observation point is compared to its value here and today,

$$\Delta X/X_0 \equiv X(\phi)/X(\phi_0) - 1.$$

Decomposing the scalar field as $\phi = \phi_0 + \Delta\phi$, one gets that $\Delta X/X_0 = s_X(\phi)\Delta\phi$, with s_X defined in Equation (233). Now the scalar field can be decomposed into a background and perturbations as $\phi = \bar{\phi}(t) + \delta\phi(\mathbf{x}, t)$ where the background value depends only on t because of the Copernican hypothesis. It follows that

$$\frac{\Delta X(\mathbf{x}, t)}{X_0} = s_X(\bar{\phi})[\bar{\phi}(t) - \phi_0] + \{s_X(\bar{\phi}) + s_X'(\bar{\phi})[\bar{\phi}(t) - \phi_0]\}\delta\phi(\mathbf{x}, t)$$

$$\equiv s_X(\bar{\phi})\Delta\bar{\phi} + \mathcal{S}_X(\bar{\phi})\delta\phi(\mathbf{x}, t). \tag{223}$$

The first term of the r.h.s. depends only on time while the second is space-time dependent. It is also expected that the second term in the curly brackets is negligible with respect to the first, i.e., $\mathcal{S}_X(\bar{\phi}) \sim s_X(\bar{\phi})$. It follows that one needs $\delta\phi(\mathbf{x}, t)$ not to be small compared to the background evolution term $\Delta\bar{\phi}$ for the spatial dependence to dominate over the large scale time dependence. This can be achieved for instance if ϕ is a seed field whose mean value is frozen. Because of statistical isotropy, and in the same way as for CMB anisotropies (see, e.g., [409]), one can express the equal-time angular power spectrum of $\Delta X/X_0$ for two events on our past lightcone as

$$\left\langle \frac{\Delta X(\mathbf{n}_1, r, t)}{X_0} \frac{\Delta X(\mathbf{n}_2, r, t)}{X_0} \right\rangle = \sum_\ell \frac{2\ell + 1}{4\pi} C_\ell^{(XX)}(z) P_\ell(\mathbf{n}_1 \cdot \mathbf{n}_2). \tag{224}$$

If $\delta\phi$ is a stochastic field characterized by its power spectrum, $\langle \delta\phi(\mathbf{k}_1, t)\delta\phi(\mathbf{k}_2, t)\rangle = P_\phi(k, t)\delta(\mathbf{k}_1 + \mathbf{k}_2)$ in Fourier space, then

$$C_\ell^{(XX)}(z) = \frac{2}{\pi}\mathcal{S}_X^2[\bar{\phi}(z)] \int P_\phi(k, z) j_\ell[k(\eta_0 - \eta)] k^2 \mathrm{d}k, \tag{225}$$

j_ℓ being a spherical Bessel function. For instance, if $P_\phi \propto k^{n_s - 1}$ where n_s is a spectral index, $n_s = 1$ corresponding to scale invariance, one gets that $\ell(\ell + 1)C_\ell^{(XX)} \propto \ell^{n_s - 1}$ on large angular scales.

The comparison of the amplitude of the angular correlation and the isotropic (time) variation is model-dependent and has not yet been investigated. It was suggested that a spatial variation of $\alpha_{\rm EM}$ would induce a dipolar modulation of CMB anisotropies [362], but at a level incompatible with existing constraints [424].

This has lead to the idea [396] of the existence of a low energy domain wall produced in the spontaneous symmetry breaking involving a dilaton-like scalar field coupled to electromagnetism. Domains on either side of the wall exhibit slight differences in their respective values of $\alpha_{\rm EM}$. If such a wall is present within our Hubble volume, absorption spectra at large redshifts may or may not provide a variation in $\alpha_{\rm EM}$ relative to the terrestrial value, depending on our relative position with respect to the wall.

Another possibility would be that the Copernican principle is not fully satisfied, such as in various void models. Then the background value of ϕ would depend, e.g., on r and t for a spherically symmetric spacetime (such as a Lemaître–Tolman–Bondi spacetime). This could give rise to a dipolar modulation of the constant if the observer (us) is not located at the center of the universe. Note, however, that such a cosmological dipole would also reflect itself, e.g., on CMB anisotropies. Similar possibilities are also offered within the chameleon mechanism where the value of the scalar field depends on the local matter density (see Section 5.4.2).

More speculative, is the effect that such fluctuations can have during preheating after inflation since the decay rate of the inflaton in particles may fluctuate on large scales [293, 294].

6.3 Implication for the universality of free fall

As we have seen in the previous sections, the tests of the universality of free fall is central in containing the model involving variations of the fundamental constants.

From Equation (14), the amplitude of the violation of the universality of free fall is given by η_{AB}, which takes the form

$$\eta_{AB} = \frac{1}{g_N} \sum_i |f_{Ai} - f_{Bi}| \, |\nabla \alpha_i|.$$

In the case in which the variation of the constants arises from the same scalar field, the analysis of Section 6.1 implies that $\nabla \alpha_i$ can be related to the gravitational potential by $|\nabla \alpha_i| = \alpha_i s_i(\phi) \alpha_{\rm ext} g_N$ so that

$$\eta_{AB} = \sum_i |f_{Ai} - f_{Bi}| \, s_i(\phi) \alpha_i \alpha_{\rm ext} = \sum_i |\lambda_{Ai} - \lambda_{Bi}| \, s_i(\phi) \alpha_{\rm ext}. \tag{226}$$

This can be expressed in terms of the sensitivity coefficient k_i defined in Equation (214) as

$$\eta_{AB} = \sum_i |\lambda_{Ai} - \lambda_{Bi}| \, k_i, \tag{227}$$

since $|\nabla \alpha_i| = \alpha_i k_i g_N$. This shows that each experiment will yield a constraint on a linear combination of the coefficients k_i so that one requires at least as many independent pairs of test bodies as the number of constants to be constrained.

While the couplings to mass number, lepton number and the electromagnetic binding energy have been considered [118] [see the example of Section 5.4.1] the coupling to quark masses remains a difficult issue. In particular, the whole difficulty lies in the determination of the coefficients λ_{ai} [see Section 5.3.2]. In the formalism developed in [122, 121], see Section 5.4.1, one can relate the expected deviation from the universality of free fall to the 5 parameters d and get constraints on $D_{\hat{m}} \equiv d_g^*(d_{\hat{m}} - d_g)$ and $D_e \equiv d_g^* d_e$ where $d_g^* \equiv d_g + 0.093(d_{\hat{m}} - d_g) + 0.00027 d_e$. For instance, the Be-Ti EötWash experiment and LRR experiment respectively imply

$$|D_{\hat{m}} + 0.22 D_e| < 5.1 \times 10^{-11}, \qquad |D_{\hat{m}} + 0.28 D_e| < 24.6 \times 10^{-11}.$$

This shows that while the Lunar experiment has a slightly better differential-acceleration sensitivity, the laboratory-based test is more sensitive to the dilaton coefficients because of a greater difference in the dilaton charges of the materials used, and of the fact that only one-third of the Earth mass is made of a different material.

The link between the time variation of fundamental constants and the violation of the universality of free fall have been discussed by Bekenstein [39] in the framework described in Section 5.4.2 and by Damour–Polyakov [135, 136] in the general framework described in Section 5.4.1. In all these models, the two effects are triggered by a scalar field. It evolves according to a Klein–Gordon equation ($\ddot{\phi} + 3H\dot{\phi} + m^2\phi + \ldots = 0$), which implies that ϕ is damped as $\dot{\phi} \propto a^{-3}$ if its mass is much smaller than the Hubble scale. Thus, in order to be varying during the last Hubble time, ϕ has to be very light with typical mass $m \sim H_0 \sim 10^{-33}$ eV. As a consequence, ϕ has to be very weakly coupled to the standard model fields to avoid a violation of the universality of free fall.

This link was revisited in [96, 166, 532] in which the dependence of α_{EM} on the scalar field responsible for its variation is expanded as

$$\alpha_{EM} = \alpha_{EM}(0) + \lambda \frac{\phi}{M_4} + \mathcal{O}\left(\frac{\phi^2}{M_4^2}\right). \tag{228}$$

The cosmological observation from QSO spectra implies that $\lambda \Delta \phi / M_4 \sim 10^{-7}$ at best during the last Hubble time. Concentrating only on the electromagnetic binding energy contribution to the proton and of the neutron masses, it was concluded that a test body composed of n_n neutrons and n_p protons will be characterized by a sensitivity $\lambda(\nu_p B_p + \nu_n B_n)/m_N$ where ν_n (resp. ν_p) is the ratio of neutrons (resp. protons) and where it has been assumed that $m_n \sim m_p \sim m_N$. Assuming[13] that $\nu_{n,p}^{Earth} \sim 1/2$ and using that the compactness of the Moon-Earth system $\partial \ln(m_{Earth}/m_{Moon})/\partial \ln \alpha_{EM} \sim 10^{-3}$, one gets $\eta_{12} \sim 10^{-3}\lambda^2$. Dvali and Zaldarriaga [166] obtained the same result by considering that $\Delta \nu_{n,p} \sim 6 \times 10^{-2} - 10^{-1}$. This implies that $\lambda < 10^{-5}$, which is compatible with the variation of α_{EM} if $\Delta \phi / M_4 > 10^{-2}$ during the last Hubble period. From the cosmology one can deduce that $(\Delta \phi / M_4)^2 \sim (\rho_\phi + P_\phi)/\rho_{total}$. If ϕ dominates the matter content of the universe, ρ_{total}, then $\Delta \phi \sim M_4$ so that $\lambda \sim 10^{-7}$ whereas if it is sub-dominant $\Delta \phi \ll M_4$ and $\lambda \gg 10^{-7}$. In conclusion $10^{-7} < \lambda < 10^{-5}$. This makes explicit the tuning of the parameter λ. Indeed, an important underlying approximation is that the ϕ-dependence arises only from the electromagnetic self-energy. This analysis was extended in [143] who included explicitly the electron and related the violation of the universality of free fall to the variation of μ.

In a similar analysis [532], the scalar field is responsible for both a variation of α_{EM} and for the acceleration of the universe. Assuming its equation of state is $w_h \neq 1$, one can express its time variation (as long as it has a standard kinetic term) as

$$\dot{\phi} = H \sqrt{3\Omega_\phi(1 + w_h)}.$$

It follows that the expected violation of the universality of free fall is related to the time variation of α_{EM} today by

$$\eta = -1.75 \times 10^{-2} \left(\frac{\partial \ln \alpha_{EM}}{\partial z}\right)^2_{z=0} \frac{(1 + \tilde{Q})\Delta \frac{Z}{Z+N}}{\Omega_\phi^{(0)}(1 + w_h^{(0)})},$$

where $\tilde{Q}$ is a parameter taking into account the influence of the mass ratios. Again, this shows that in the worse case in which the Oklo bound is saturated (so that $\partial \ln \alpha_{EM}/\partial z \sim 10^{-6}$), one requires $1 + w_h^{(0)} \gtrsim 10^{-2}$ for $\eta \lesssim 10^{-13}$, hence providing a string bond between the dark energy equation of state and the violation of the universality of free fall. This was extended in [149] in terms of the phenomenological model of unification presented in Section 5.3.1. In the case of the

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[13] For copper $\nu_p = 0.456$, for uranium $\nu_p = 0.385$ and for lead $\nu_p = 0.397$.

string dilaton and runaway dilaton models, one reaches a similar conclusion [see Equation (203) in Section 5.4.1]. A similar result [348] was obtained in the case of pure scalar-tensor theory, relating the equation of state to the post-Newtonian parameters. In all these models, the link between the local constraints and the cosmological constraints arise from the fact that local experiments constrain the upper value of $\dot{\phi}_0$, which quantify both the deviation of its equation of state from -1 and the variation of the constants. It was conjectured that most realistic quintessence models suffer from such a problem [68].

One question concerns the most sensitive probes of the equivalence principle. This was investigated in [144] in which the coefficients λ_{Ai} are estimated using the model (189). It was concluded that they are $2-3$ orders of magnitude over cosmic clock bounds. However, [148] concluded that the most sensitive probe depends on the unification relation that exist between the different couplings of the standard model. [463] concluded similarly that the universality of free fall is more constraining that the seasonal variations. The comparison with QSO spectra is more difficult since it involves the dynamics of the field between $z \sim 1$ and today. To finish, let us stress that these results may be changed significantly if a chameleon mechanism is at work.

7   Why Are The Constants Just So?

The numerical values of the fundamental constants are not determined by the laws of nature in which they appear. One can wonder why they have the values we observe. In particular, as pointed out by many authors (see below), the constants of nature seem to be fine tuned [317]. Many physicists take this fine-tuning to be an explanandum that cries for an explanans, hence following Hoyle [258] who wrote that "one must at least have a modicum of curiosity about the strange dimensionless numbers that appear in physics."

7.1   Universe and multiverse approaches

Two possible lines of explanation are usually envisioned: a *design or consistency hypothesis* and an *ensemble hypothesis*, that are indeed not incompatible together. The first hypothesis includes the possibility that all the dimensionless parameters in the "final" physical theory will be fixed by a condition of consistency or an external cause. In the ensemble hypothesis, the universe we observe is only a small part of the totality of physical existence, usually called the multiverse. This structure needs not be fine-tuned and shall be sufficiently large and variegated so that it can contain as a proper part a universe like the one we observe the fine-tuning of which is then explained by an *observation selection effect* [64].

These two possibilities send us back to the large number hypothesis by Dirac [155] that has been used as an early motivation to investigate theories with varying constants. The main concern was the existence of some large ratios between some combinations of constants. As we have seen in Section 5.3.1, the running of coupling constants with energy, dimensional transmutation or relations such as Equation (185) have opened a way to a rational explanation of very small (or very large) dimensional numbers. This follows the ideas developed by Eddington [178, 179] aiming at deriving the values of the constants from consistency relations, e.g., he proposed to link the fine-structure constant to some algebraic structure of spacetime. Dicke [151] pointed out another possible explanation to the origin of Dirac large numbers: the density of the universe is determined by its age, this age being related to the time needed to form galaxies, stars, heavy nuclei.... This led Carter [82] to argue that these numerical coincidence should not be a surprise and that conventional physics and cosmology could have been used to predict them, at the expense of using the anthropic principle.

The idea of such a structure called the *multiverse* has attracted a lot of attention in the past years and we refer to [79] for a more exhaustive account of this debate. While many versions of what such a multiverse could be, one of them finds its root in string theory. In 2000, it was realized [66] that vast numbers of discrete choices, called flux vacua, can be obtained in compactifying superstring theory. The number of possibilities is estimated to range between 10^{100} and 10^{500}, or maybe more. No principle is yet known to fix which of these vacua is chosen. Eternal inflation offers a possibility to populate these vacua and to generate an infinite number of regions in which the parameters, initial conditions but also the laws of nature or the number of spacetime dimensions can vary from one universe to another, hence being completely contingent. It was later suggested by Susskind [482] that the anthropic principle may actually constrain our possible locations in this vast string landscape. This is a shift from the early hopes [270] that M-theory may conceivably predict all the fundamental constants uniquely.

Indeed such a possibility radically changes the way we approach the question of the relation of these parameters to the underlying fundamental theory since we now expect them to be distributed randomly in some range. Among this range of parameters lies a subset, that we shall call the *anthropic range*, which allow for universe to support the existence of observers. This range can be determined by asking ourselves how the world would change if the values of the constants were changed, hence doing *counterfactual cosmology*. However, this is very restrictive since the

mathematical form of the law of physics managed as well and we are restricting to a local analysis in the neighborhood of our observed universe. The determination of the anthropic region is not a prediction but just a characterization of the sensitivity of "our" universe to a change of the fundamental constants *ceteris paribus*. Once this range is determined, one can ask the general question of quantifying the probability that we observe a universe as ours, hence providing a probabilistic prediction. This involves the use of the anthropic principle, which expresses the fact that we observe are not just observations but observations made by us, and requires us to state what an observer actually is [383].

7.2 Fine-tunings and determination of the anthropic range

As we have discussed in the previous sections, the outcome of many physical processes are strongly dependent on the value of the fundamental constants. One can always ask the scientific question of what would change in the world around us if the values of some constants were changed, hence doing some counterfactual cosmology in order to determine the range within which the universe would have developed complex physics and chemistry, what is usually thought to be a prerequisit for the emergence of complexity and life (we emphasize the difficulty of this exercise when it goes beyond small and local deviations from our observed universe and physics, see, e.g., [245] for a possibly life supporting universe without weak interaction). In doing so, one should consider the fundamental parameters entering our physical theory but also the cosmological parameters.

First there are several constraints that the fundamental parameters listed in Table 1 have to satisfy in order for the universe to allow for complex physics and chemistry. Let us stress, in a non-limiting way, some examples.

- It has been noted that the stability of the proton requires $m_d - m_u \gtrsim \alpha_{EM}^{3/2} m_p$. The anthropic bounds on m_d, m_u and m_e (or on the Higgs vev) arising from the existence of nuclei, the di-neutron and the di-proton cannot form a bound state, the deuterium is stable have been investigated in many works [5, 6, 120, 145, 160, 161, 252, 254], even allowing for nuclei made of more than 2 baryon species [264]. Typically, the existence of nuclei imposes that $m_d + m_u$ and v cannot vary by more that 60% from their observed value in our universe.

- If the difference of the neutron and proton masses where less that about 1 MeV, the neutron would become stable and hydrogen would be unstable [442, 253] so that helium would have been the most abundant at the end of BBN so that the whole history of the formation and burning of stars would have been different. It can be deduced that [252] one needs $m_d - m_u - m_e \gtrsim 1.2$ MeV so that the universe does not become all neutrons; $m_d - m_u + m_e \lesssim 3.4$ MeV for the *pp* reaction to be exothermic and $m_e > 0$ leading to a finite domain.

- A coincidence emerges from the existence of stars with convective and radiative envelopes, since it requires [80] that $\alpha_G \sim \alpha_{EM}^{20}$. It arises from the fact that the typical mass that separates these two behavior is roughly $\alpha_G^{-2} \alpha_{EM}^{10} m_p$ while the masses of star span a few decades around $\alpha_G^{-3} m_p$. Both stars seem to be needed since only radiative stars can lead to supernovae, required to disseminate heavy elements, while only convective stars may generate winds in their early phase, which may be associated with formation of rocky planets. This relation while being satisfied numerically in our universe cannot be explained from fundamental principles.

- Similarly, it seems that for neutrinos to eject the envelope of a star in a supernovae explosion, one requires [80] $\alpha_G \sim \alpha_W^4$.

- As we discussed in Section 3.5, the production of carbon seems to imply that the relative strength of the nuclear to electromagnetic interaction must be tuned typically at the 0.1% level.

Other coincidences involve also the physical properties, not only of the physical theories, but also of our universe, i.e., the cosmological parameters summarized in Table 4. Let us remind some examples.

- The total density parameter Ω must lie within an order of magnitude of unity. If it were much larger the universe will have re-collapsed rapidly, on a time scale much shorter that the main-sequence star lifetime. If it were to small, density fluctuations would have frozen before galaxies could form. Typically one expects $0.1 < \Omega_0 < 10$. Indeed, most inflationary scenarios lead to $\Omega_0 \sim 1$ so that this may not be anthropically determined but in that case inflation should last sufficiently long so that this could lead to a fine tuning on the parameters of the inflationary potential.

- The cosmological constant was probably the first one to be questioned in an anthropical way [527]. Weinberg noted that if Λ is too large, the universe will start accelerating before structures had time to form. Assuming that it does not dominate the matter content of the universe before the redshift z_* at which earliest are formed, one concludes that $\rho_V = \Lambda/8\pi G < (+z_*)\rho_{\mathrm{mat0}}$. Weinberg [527] estimated $z_* \sim 4.5$ and concluded that "if it is the anthropic principle that accounts for the smallness of the cosmological constant, then we would expect the vacuum energy density $\rho_v \sim (10 - 100)\rho_{\mathrm{mat0}}$ because there is no anthropic reason for it to be smaller". Indeed, the observations indicate $\rho_v \sim 2\rho_{\mathrm{mat0}}$

- Tegmark and Rees [486] have pointed out that the amplitude of the initial density perturbation, Q enters into the calculation and determined the anthropic region in the plane (Λ, Q). This demonstrates the importance of determining the parameters to include in the analysis.

- Different time scales of different origin seem to be comparable: the radiative cooling, galactic halo virialization, time of cosmological constant dominance, the age of the universe today. These coincidence were interpreted as an anthropic sign [65].

These are just a series of examples. For a multi-parameter study of the anthropic bound, we refer, e.g., to [485] and to [243] for a general anthropic investigation of the standard model parameters.

7.3   Anthropic predictions

The determination of the anthropic region for a set of parameters is in no way a prediction but simply a characterization of our understanding of a physical phenomenon P that we think is important for the emergence of observers. It reflects that, the condition C stating that the constants are in some interval, $C \implies P$, is equivalent to $!P \implies !C$.

The anthropic principle [82] states that "what we can expect to observe must be restricted by the conditions necessary for our presence as observers". It has received many interpretations among which the *weak anthropic principle* stating that "we must be prepared to take account of the fact that our location in the universe in necessarily privileged to the extent of being compatible with our existence as observers", which is a restriction of the Copernican principle often used in cosmology, and the *strong anthropic principle* according to which "the universe (and hence the fundamental parameters on which it depends) must be such as to admit the creation of observers within it at some stage." (see [35] for further discussions and a large bibliography on the subject).

One can then try to determine the probability that an observer measure the value x of the constant X (that is a random variable fluctuating in the multiverse and the density of observers depend on the local value of X). According to Bayes theorem,

$$P(X = x|\mathrm{obs}) \propto P(\mathrm{obs}|X = x)P(X = x).$$

$P(X = x)$ is the prior distribution, which is related to the volume of those parts of the universe in which $X = x$ at dx. $P(\text{obs}|X = x)$ is proportional to the density of observers that are going to evolve when $X = x$. $P(X = x|\text{obs})$ then gives the probability that a randomly selected observer is located in a region where $X = x \pm \text{d}x$. It is usually rewritten as [519]

$$P(x)\text{d}x = n_{\text{obs}}(x)P_{\text{prior}}\text{d}x.$$

This higlights the difficulty in making a prediction. First, one has no idea of how to compute $n_{\text{obs}}(x)$. When restricting to the cosmological constant, one can argue [519] that Λ does not affect microphysics and chemistry and then estimate $n_{\text{obs}}(x)$ by the fraction of matter clustered in giant galaxies and that can be computed from a model of structure formation. This may not be a good approximation when other constants are allowed to vary and it needs to be defined properly. Second, P_{prior} requires an explicit model of multiverse that would generate sub-universes with different values x_i (continuous or discrete) for x. A general argument [528] states that if the range over which X varies in the multiverse is large compared to the anthropic region $X \in [X_{\text{min}}, X_{\text{max}}]$ one can postulate that P_{prior} is flat on $[X_{\text{min}}, X_{\text{max}}]$. Indeed, such a statement requires a measure in the space of the constants (or of the theories) that are allowed to vary. This is a strong hypothesis, which is difficult to control. In particular if P_{prior} peaks outside of the anthropic domain, it would predict that the constants should lie on the boundary of the anthropic domain [443]. It also requires that there are sufficiently enough values of x_i in the anthropic domain, i.e., $\delta x_i \ll X_{\text{max}} - X_{\text{min}}$. Garriga and Vilenkin [228] stressed that the hypothesis of a flat P_{prior} for the cosmological constant may not hold in various Higgs models, and that the weight can lower the mean viable value. To finish, one want to consider $P(x)$ as the probability that a random observer measures the value x. This relies on the fact that we are a typical observer and we are implicitly making a self sampling hypothesis. It requires to state in which class of observers we are supposed to be typical (and the final result may depend on this choice [383]) and this hypothesis leads to conclusions such as the doomsday argument that have be debated actively [64, 383].

This approach to the understanding of the observed values of the fundamental constants (but also of the initial conditions of our universe) by resorting to the actual existence of a multiverse populated by a different "low-energy" theory of some "mother" microscopic theory allows us to explain the observed fine-tuning by an observational selection effect. It also sets a limit to the Copernican principle stating that we do not live in a particular position in space since we have to live in a region of the multiverse where the constants are inside the anthropic bound. Such an approach is indeed not widely accepted and has been criticized in many ways [7, 182, 480, 402, 479, 511, 475].

Among the issues to be answered before such an approach becomes more rigorous, let us note: (1) what is the shape of the string landscape; (2) what constants should we scan. It is indeed important to distinguish the parameters that are actually fine-tuned in order to determine those that we should hope to explain in this way [537, 538]. Here theoretical physics is indeed important since it should determine which of the numerical coincidences are coincidences and which are expected for some unification or symmetry reasons; (3) how is the landscape populated; (4) what is the measure to be used in order and what is the correct way to compute anthropically-conditioned probabilities.

While considered as not following the standard scientific approach, this is the only existing window on some understanding of the value of the fundamental constants.

8 Conclusions

The study of fundamental constants has witnessed tremendous progresses in the past years. In a decade, the constraints on their possible space and time variations have flourished. They have reached higher precision and new systems, involving different combinations of constants and located at different redshifts, have been considered. This has improved our knowledge on the equivalence principle and allowed to test it on astrophysical and cosmological scales. We have reviewed them in Section 3 and Section 4. We have emphasized the experimental observational progresses expected in the coming years such as the E-ELT, radio observations, atomic clocks in space, or the use of gravitational waves.

From a theoretical point of view, we have described in Section 5 the high-energy models that predict such variation, as well as the link with the origin of the acceleration of the universe. In all these cases, a spacetime varying fundamental constant reflects the existence of an almost massless field that couples to matter. This will be at the origin of a violation of the universality of free fall and thus of utmost importance for our understanding of gravity and of the domain of validity of general relativity. Huge progress has been made in the understanding of the coupled variation of different constants. While more model-dependent, this allows one to set stronger constraints and eventually to open an observational window on unification mechanisms.

To finish, we have discussed in Section 7 the ideas that try to understand the value of the fundamental constant. While considered as borderline with respect to the standard physical approach, it reveals the necessity of considering a universe larger than our own, and called the multiverse. It will also give us a hint on our location in this structure in the sense that the anthropic principle limits the Copernican principle at the basis of most cosmological models. We have stressed the limitations of this approach and the ongoing debate on the possibility to make it predictive.

To conclude, the puzzle about the large numbers pointed out by Dirac has led to a better understanding of the fundamental constants and of their roles in the laws of physics. They are now part of the general tests of general relativity, as well as a breadcrumbs to understand the origin of the acceleration of the universe and to more speculative structures, such as a multiverse structure, and possibly a window on string theory.

9 Acknowledgments

A Notations

A.1 Constants

The notations and numerical values of the constants used in this review are summarized in Table 1 and Table 2.

A.2 Sensitivity coefficients

The text introduces several sensitivity coefficients. We recall their definition here.

- Given an observable O, the value of which depends on a set of primary parameters G_k, the sensitivity of the measured value of O to these parameters is

$$\frac{\mathrm{d}\ln O}{\mathrm{d}\ln G_k} = c_k. \tag{229}$$

  The value of the quantities c_k requires a physical description of the system.

- the parameters G_k can be related to a set of fundamental constant α_i and we define

$$\frac{\mathrm{d}\ln G_k}{\mathrm{d}\ln \alpha_i} = d_{ki}. \tag{230}$$

  The computation of the coefficients d_{ki} requires one to specify the theoretical framework and depends heavily on our knowledge of nuclear physics and the assumptions on unification.

- A particular sets of parameters d_{ki} has been singled out for the sensitivity of the mass of a body A to a variation of the fundamental constants

$$\frac{\mathrm{d}\ln m_A}{\mathrm{d}\alpha_i} = f_{Ai}. \tag{231}$$

  One also introduces

$$\frac{\mathrm{d}\ln m_A}{\mathrm{d}\ln \alpha_i} = \lambda_{Ai} \tag{232}$$

  so that

$$\lambda_{Ai} = \alpha_i f_{Ai}.$$

- In models where the variation of the fundamental constants are induced by the variation of a scalar field with define

$$\frac{\mathrm{d}\ln \alpha_i}{\mathrm{d}\phi} = s_i(\phi). \tag{233}$$

- In class of models the variation of the constants can be related to the gravitational potential by

$$\frac{\mathrm{d}\ln \alpha_i}{\mathrm{d}\Phi_N} = k_i. \tag{234}$$

A.3   Background cosmological spacetime

We consider that the spacetime is describe by a manifold $\mathcal{M}$ with metric $g_{\mu\nu}$ with signature $(-,+,+,+)$. In the case of a Minkowsky spacetime $g_{\mu\nu} = \eta_{\mu\nu}$.

In the cosmological context, we will describe the universe by a Friedmann–Lemaître spacetime with metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j \tag{235}$$

where t is the cosmic time, a the scale factor and γ_{ij} the metric on the constant time hypersurfaces. The Hubble function is defined as $H \equiv \dot{a}/a$. We also define the redshift by the relation $1+z = a_0/a$, with a_0 the scale factor evaluated today.

The evolution of the scale factor is dictated by the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3}, \tag{236}$$

where $\rho = {}_i\rho_i$ is the total energy density of the matter components in the universe. Assuming the species i has a constant equation of state $w_i = P_i/\rho_i$, each component evolves as $\rho_i = \rho_{i0}(1+z)^{2(1+w_i)}$. The Friedmann equation can then be rewritten as

$$\frac{H^2}{H_0^2} = \sum \Omega_i(1+z)^{3(1+w_i)} + \Omega_K(1+z)^2 + \Omega_\Lambda, \tag{237}$$

with the density parameters defined by

$$\Omega_i \equiv \frac{8\pi G\rho_{i0}}{3H_0^2}, \qquad \Omega_i \equiv -\frac{K}{3H_0^2}, \qquad \Omega_\Lambda \equiv \frac{\Lambda}{3H_0^2}. \tag{238}$$

They clearly satisfy $\sum \Omega_i + \Omega_K + \Omega_\Lambda = 1$.

Concerning the properties of the cosmological spacetime, I follow the notations and results of [409].

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