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The space–time references of BeiDou Navigation Satellite System

Chunhao Han[1,2], Li Liu[2*], Zhiwu Cai[2] and Yuting Lin[2]

Abstract

The BeiDou Navigation Satellite System (BDS) is essentially a precise time measurement and time synchronization system for a large-scale space near the Earth. General relativity is the basic theoretical framework for the information processing in the master control station of BDS. Having introduced the basic conceptions of relativistic space–time reference systems, the space–time references of BDS are analyzed and the function and acquisition method of the Earth Orientation Parameters (EOP) are briefly discussed. The basic space reference of BDS is BeiDou Coordinate System (BDCS), and the time standard is the BDS Time (BDT). BDCS and BDT are the realizations of the Geocentric Terrestrial Reference System (GTRS) and the Terrestrial Time (TT) for BDS, respectively. The station coordinates in the BDCS are consistent with those in International Terrestrial Reference Frame (ITRF)2014 at the cm–level and the difference in scale is about 1.1×10^{-8}. The time deviation of BDT relative to International Atomic Time (TAI) is less than 50 ns and the frequency deviation is less than 2×10^{-14}. The Geocentric Celestial Reference System (GCRS) and the solar Barycentric Celestial Reference System (BCRS) are also involved in the operation of BDS. The observation models for time synchronization and precise orbit determination are established within the GCRS framework. The coordinate transformation between BDCS and GCRS is consistent with the International Earth Rotation and Reference Systems Service (IERS). In the autonomous operation mode without the support of the ground master control station, Earth Orientation Parameters (EOP) is obtained by means of long-term prediction and on-board observation. The observation models for the on-board astrometry should be established within the BCRS framework.

Keywords: BDS, BDT, Space–time references, BeiDou coordinate system, Earth orientation parameters

Introduction

Since July 31, 2020, BeiDou-3 Navigation Satellite System (BDS-3) has been officially operational. As a major national infrastructure, BDS-3 provides global positioning, navigation, and timing, as well as Global Short Message Communication (GSMC) and Search and Rescue (SAR) services. It also provides Satellite-Based Augmentation Service (SBAS), Precise Point Positioning (PPP), Regional Short Message Communication (RSMC) and other services in and around China (Yang et al., 2018; Guo et al., 2019).

Precise orbit determination and time synchronization are conducted with different methods, but the basic observation data are the same, i.e., pseudo-ranges at the ground monitor stations. The joint calculation method of satellite orbit and satellite clock offset is adopted by Global Positioning System (GPS) and Galileo Navigation Satellite System (Galileo), which will result in a strong correlation between the derived satellite orbit and satellite clock offset (Demetrios et al., 2008). There is no doubt that the longer the satellite's tracking arc and the higher the frequency stability of the on-board clock, the more accurate the results of the orbit and the clock offset will be. Considering the limited deployment range of monitor stations and the non-ideal stability of the satellite clocks during the initial construction of BeiDou Navigation Satellite (Regional) System (BDS-2) (the daily stability was at 1×10^{-13} level), to prevent the mutual error

*Correspondence: lliu@shao.ac.cn
[2] Beijing Satellite Navigation Center, Beijing 100094, China
Full list of author information is available at the end of the article

pollution between the satellite clock and the satellite orbit, BDS-2 adopted a time synchronization technology which is completely different from that of GPS, GLObal NAvigation Satellite System (GLONASS) and Galileo (Liu et al., 2009; Domnin et al., 2009). Two-Way Time and Frequency Transfer (TWTFT) is used between the satellites and ground stations, including the master station and the time synchronization/upload stations. The results of the long-term operation of BDS-2 show that the time synchronization is very successful with the uncertainty of satellite-to-ground time comparison better than 1 ns, which effectively improves the orbit and time synchronization accuracy of the satellites (Liu et al., 2009; Han et al., 2013; Guo et al., 2018). For this reason, the BDS-3 continues to use this time synchronization technique, and further adds inter-satellite time comparison links to directly measure the distances and relative clock offsets between the satellites (Liu et al., 2019; Chen et al. 2016). The distinctive time synchronization technique is one of the important innovations of BeiDou Navigation Satellite System (BDS).

BDS is essentially a system of precision time measurement and time synchronization in a large-scale space near the Earth. At present, with the daily frequency stability of the BDS satellite atomic clock at the level of 1×10^{-15}, the time synchronization uncertainty is required to reach 1 ns or even higher, and the accuracy of the space–time measurement and observation model is required to reach the order of picosecond (Han and Cai 2019). The constellation of BDS is composed of Geostationary Earth Orbit (GEO), Inclined Geo-Synchronous Orbit (IGSO) and Medium Earth Orbit (MEO) satellites, covering a space range of 40,000 km near the Earth (Tan, 2017). Although it is in the scope of the Earth's gravitational field, the influence of the sun and moon gravitational fields on the satellite orbit and satellite clock cannot be ignored (Petit and Wolf, 1994; Wolf and Petit, 1995; Klioner, 2003). Under such a large scale of space and high-precision index requirements, the space–time theory of Euclidean geometry and Newtonian mechanics is difficult to meet the needs of high-precision measurement modeling (the accuracy of Newtonian mechanics is equivalent to the level of 1×10^{-8}). Relativity is the theoretical basis for precise orbit determination, time synchronization, and measurement modeling of BDS (Han, 2002; Han and Cai, 2019).

As a typical realization of the basic theory of relativistic space–time reference system, the definition and realization of BDS space–time reference system will be discussed in depth below. At the same time, in view of the autonomous operation of navigation satellite constellation, the processing method of Earth Orientation Parameter (EOP) will also be briefly analyzed.

Basic concepts of the space–time reference system

Newton's time and space reference system

Under the framework of Newtonian mechanics, time and space are independent of each other. Time passes evenly and space is uniform everywhere which satisfies Euclidean geometry. Under this assumption, the space–time reference system can be simply understood as a Three-Dimensional (3D) space frame and a clock carried by an observer. The former is used as a space reference for direction and distance, and the later as a time reference. A space frame is somewhat alike an electronic total station that can measure distance and direction. Due to the flatness of space, the measurement range of the space frame has no limits theoretically. Therefore, the space–time reference system can be defined by a time scale and a 3D space reference system. A time scale is composed of a starting point and the time unit, which has nothing to do with the choice of space position and coordinates. A space reference system is defined by its origin, orientation of axes and scale or unit of distance. For convenience, it is usually required that the three axes are orthogonal to each other and have the same coordinate unit. There is a special kind of space reference systems in Newtonian mechanics, called inertial reference systems, in which Newton's law of inertia are satisfied. The laws of physics have the same form of expression in all inertial reference systems, which are called Galileo's principle of relativity.

Practically, a complete space reference system needs to define not only a reference frame, but also some geometric parameters and related physical field parameters, such as the reference ellipsoid, the geocentric gravitational constant and gravitational model in the terrestrial reference system, etc..

The concepts of relativistic space–time reference system

Compared with Newtonian mechanics, the concepts of relativistic space–time reference system are much more complicated. First, the physical time and space are not absolutely independent. They are interrelated and cannot be completely separated. In fact, whether the objects in the universe are moving or static, whether the moving speed is large or small, and whether the motion path is a straight or a curve are all determined by the observer or the frame of reference. The space point of one observer may be a line in the eyes of another. What one sees is a straight line may be a curve seen by another. The spatial points, straight lines, curves, even planes and curved surfaces in Euclidean geometry do not have complete objectivity. The distance between two different events in the universe is closely related to the observer or the frame of reference. For different spatial points, the observer's definition of simultaneity is related to distance, and the relativity of distance will inevitably lead to the correlation between time and space. For any two events that occur in the universe, the time intervals and spatial distances measured by observers moving relatively are different, even if they use same clocks and rulers. Therefore, the theory of relativity considers time and space as a whole and supposes that the space-time interval between two events is an invariant quantity that has nothing to do with the observers (Han, 2017). Under this assumption, the coordinate relationship between the two relative moving inertial reference systems no longer satisfies the classic Galileo transformation, but the Lorentz transformation. All the laws of physics remain unchanged under the Lorentz transformation, which is called the principle of special relativity.

Secondly, space–time has non-uniformity or non-Euclidean characteristics. There is a universal interaction among all objects in the universe. The uneven distribution of matter will inevitably lead to the unevenness of the space–time gravitational field, and hence there is no objective straight line in a large-scale space. We know that the basic postulate of Euclidean geometry is straight line, so Euclidean geometry does not hold in a large-scale space. The general theory of relativity considers that space-time including the gravitational field is a curved four-dimensional pseudo-Riemann Space. Therefore, it is impossible to construct a Cartesian coordinate system with a large-scale spatial coverage. Space–time is the basic form of material existence. The vacuum or space without matter is only the result of artificial abstraction, and space–time itself does not have the characteristics of straight or bending. Fundamentally, the curvature of space–time is just the result of gravitation geometrization in general relativity. Therefore, it is easier to be understood if saying that space-time is inhomogeneous rather than that curved (Han, 2017).

The inhomogeneity of space–time also leads to no ideal inertial space in our universe. Both inertia and gravitation are the result of the interaction of substance in the universe. It is impossible to separate them completely. In studying dynamic and kinematic problems, we cannot take all the celestial bodies into account. An effective way is to separate them into two groups, i.e., the near celestial bodies and the far distant ones. The effect of the former is called gravitation, and that of the later is the inertia, which is the so-called Mach principle. Therefore, both gravitational field and inertial space are relative. Inertial space is not only local but also approximate. The spatial scope of application of the inertia depends on remoteness of the celestial bodies that forms the inertial effect, as well as our requirements for the accuracy of space–time measurement.

The benchmark of space–time metric in relativity is essentially the light (Han, 1997). For two determined events, the time intervals measured by observers at different spatial positions are different, even they are relatively static and carry the same ideal atomic clocks which are always consistent with the SI second. Therefore, we believe that the gravitational field will change the frequency or clock speed of an atomic clock. Due to the local flatness of space, the concept of space frame used in Newtonian mechanics is still applicable, but the difference is that it can only be used locally by the observer and cannot extend outward infinitely.

A relativistic space-time reference system consists of a Four-Dimensional (4D) coordinate system and the corresponding metric coefficients. It maps the space-time points, one by one, to the Minkowski Space in which one dimension is imaginary and other three are real. Therefore every event occurred in the universe has a set of clear and unique space–time coordinates, while the trajectory of any object and the measurement characteristics of space–time are determined by the space–time metric. The space–time metric is a second-order symmetric tensor field, which is determined by the matter distribution of space–time and satisfies the Einstein field equation. The metric tensor has 10 independent coordinate components, which are the so called metric coefficients. Obviously, the metric coefficients are the functions of space–time points and closely related to the basis vectors of coordinates. Different basis vectors lead to different metric coefficients. The Einstein field equation has 6 independent nonlinear equations. To solve for 10 metric coefficients, 4 coordinate conditions are required. In the theory of relativity, the coordinate conditions can be arbitrarily selected. Then the space–time coordinates in general relativity have no clear physical or geometric meaning, but arbitrariness and equivalence. Therefore, the coordinates of a space–time point in the gravitational field depend not only on the space–time reference frame located at the coordinate origin, but also on the space–time metric or the coordinate conditions.

Local inertial reference system

According to the principle of equivalence of relativity, for any mass point as an observer that moves freely in space–time, an inertial space reference frame or a local inertial reference system that is applicable to the observer local space–time can be constructed. The local inertial reference system, which may also be called local Lorentz reference frame, meets the following basic conditions:

- The coordinate origin is a mass point freely moving in the space–time.
- The time reference is the reading of the atomic clock coming with the origin.
- The space axes or the coordinate base vectors do not rotate relative to inertial gyroscopes.

Note that the reason why inertial gyroscopes are used here instead of distant celestial bodies to define the non-rotating characteristics of inertial space is that the influences of non-far-distant celestial bodies need to be taken into account for the local inertial space. The spin of a gyro will undergo a so-called de Sitter precession relative to the far-distant celestial bodies, which is also named as geodesic precession.

There is no doubt that the gravitation forces acting on the particles near the origin are almost the same, and free particles move in a uniform form in the eyes of the observer located at the origin. Therefore, we need no gravitation but inertia to describe the particle motions in the local space, in which the tidal forces generated by external celestial bodies can be ignored. However, the local Lorentz reference frame satisfies the Newtonian inertia condition is just a differential approximation in mathematics, and its spatial application range is very limited. Due to the non-uniform nature of the gravitational field, there is no true Newtonian inertial space in the real large-scale space–time.

For a curved space, if the coordinate basis vectors $\{e_\alpha\}$ of a coordinate system $\{x^\alpha\}$ are orthogonal to a certain space-time point $P\left(x_A^\lambda\right)$, and its affine connection coefficients are zero, then the reference frame formed by the coordinate basis vectors of the point is a Lorenz reference frame. Therefore, it forms a local inertial coordinate system nearby. The basic conditions can be expressed as:

$$\begin{cases} g_{\mu\nu}\left(x_A^\lambda\right) = \eta_{\mu\nu} \\ \Gamma^\mu_{\alpha\beta}\left(x_A^\lambda\right) = 0 \end{cases} \tag{1}$$

where $g_{\mu\nu}\left(x_A^\lambda\right)$ are the common metric coefficients, $\eta_{\mu\nu}$ are the Minkowski ones, $\Gamma^\mu_{\alpha\beta}\left(x_A^\lambda\right)$ are the affine coefficients of connection or expressed in vectors:

$$\begin{cases} \boldsymbol{e}_\mu\left(x_A^\lambda\right) \cdot \boldsymbol{e}_\nu\left(x_A^\lambda\right) = \eta_{\mu\nu} \\ \boldsymbol{\Gamma}_{\mu\nu}\left(x_A^\lambda\right) = 0 \end{cases} \tag{2}$$

The first equation of Eq. (1) or Eq. (2) is is the orthogonal normalization condition of the coordinate basis vectors. Although orthogonal normalization is not a necessary condition for the inertial reference system, the Cartesian coordinate system has natural application advantages. Therefore, when establishing a reference system, we always hope that the coordinate bases can meet the condition of orthogonal normalization. The second condition equation is the core of the local inertial system, which requires the coordinate basis vectors to satisfy the characteristics of parallel movement at the origin. In other words, the time axis of the local inertial system is a time-like geodesic, and the space coordinate axes near the origin are space-like geodesic lines.

The coordinates that satisfy the geodesic condition are called Fermi coordinates, so the local inertial system is an orthogonal Fermi coordinate system or Fermi normal coordinates. Due to the in-homogeneity of space–time, the local inertial system of a space–time point is limited not only in space, but also in time. In practice, we often need a local inertial system that is not limited in time, such as a local inertial system centered at a spacecraft. For such a local reference system, the scope of adaptation is not a sphere but a pipeline in the 4D space–time. This unrestricted local inertial system in time is the local Lorentz reference frame for a free observer.

It is very convenient to use the observer's local inertial system to express the events that occur in the space near the observer. But in most cases, a global coordinate system must be used which covers the entire space–time range to describe the movement of substance in a large-scale space. Therefore, it is often necessary to give the transformation relationship between the local inertial system and the global coordinate system.

According to Eqs. (1) or (2), the relationship between the local inertial system $\{x'^\alpha\}$ and the global coordinate system $\{x^\alpha\}$ can be expressed as:

$$x^\mu = x_A^\mu\left(t'\right) + e_j^\mu x'^j - \frac{1}{2}\Gamma^\mu_{\lambda\gamma}\left(x_A^\kappa\right) e_j^\lambda e_k^\gamma x'^j x'^k + \cdots \tag{3}$$

or

$$\begin{cases} t = \int e_0^0 \mathrm{d}t' + \frac{1}{c} e_j^0 x'^j - \frac{1}{2c}\Gamma^0_{\lambda\gamma}\left(x_A^\kappa\right) e_k^\lambda e_j^\gamma x'^j x'^k + \cdots \\ x^i = x_A^i\left(t'\right) + e_j^i x'^j - \frac{1}{2}\Gamma^i_{\lambda\gamma}\left(x_A^\kappa\right) e_k^\lambda e_j^\gamma x'^j x'^k + \cdots \end{cases} \tag{4}$$

It can be seen from the coordinate relationship Eq. (4) that the relationship is nonlinear between the local inertial system and the global coordinate system. Since the coordinate relationship is developed approximately by using the Taylor series of $1/c$, the applicable scope of the local inertial system is determined by the convergence of the series.

The barycentric celestial reference system

In astronomy, the mass center of the celestial body or system under study is generally chosen as the origin of the space–time reference system, and its coordinate axes are required to have spatial non-rotating characteristics. The so-called non-rotating has two meanings. One is that the space coordinate axes have no spatial rotation relative to a far-distant celestial body such as the extragalactic radio sources, which is named as the kinematical non-rotating, and the other is that they are relative to gyro or the inertial space and named as the dynamical non-rotating (Han, 1997).

For an isolated celestial system, the kinematic non-rotation and the dynamic non-rotation are equivalent. However, for non-isolated systems, due to the influence of local substance on space–time there will be a slight difference between them. For example, there is a very slow spatial rotation between the geocentric local inertial frame and the geocentric kinematic non-rotating reference frame, which is about 1.92 arc seconds per hundred years and named as geodesic precession. Because of the small dynamical effect on the motion of objects, it can be ignored in the usual cases.

In modern astrometry and space geodesy, there are three most important space–time reference systems, i.e., the Barycentric Celestial Reference System (BCRS), the Geocentric Celestial Reference System (GCRS) and the Geocentric Terrestrial Reference System (GTRS). The origin of BCRS is the mass center of the Solar System, which takes into account the distributions of all the masses of the Sun and the planets, and the coordinate axes are required to have no spatial rotation relative to far-distant celestial bodies. It is mainly used to study the orbital motion of celestial bodies of the solar system and the observation modeling of distant celestial bodies. The coordinate origin of GCRS is defined at the center of Earth's mass and the spatial axes have no spatial rotation relative to BCRS. GCRS is mainly used to study the rotation of the Earth and the orbital motion of artificial Earth satellites. The coordinate origin of GTRS is the same as GCRS, but the space coordinate axes are fixed to the Earth and rotate daily with it, which is mainly used to describe the locations of ground stations and various geophysical phenomena.

Obviously, there exists arbitrariness in the definition and implementation of these reference systems. If there were no standards, the results of observation or research given by different teams could not be compared, communicated or shared. To this end, international organizations, such as the International Astronomical Union (IAU), the International Union of Geodesy and Geophysics (IUGG), and the International Bureau of Weights and Measures (BIPM) have long term commitments in the definition, implementation, and coordination of the recommendations for the space–time reference system and the related physical constants.

According to IAU2000 resolution B1.3, both the BCRS and the GCRS are required to meet the harmonic conditions (Petit, 2000; Soffel et al., 2003). The BCRS space–time metric form can be expressed as:

$$\begin{cases} g_{00} = -\left(1 - \dfrac{2w}{c^2} + \dfrac{2w^2}{c^4}\right) + O(c^{-6}) \\ g_{0i} = -\dfrac{4w^i}{c^3} + O(c^{-5}) \\ g_{ij} = \delta_{ij}\left(1 + \dfrac{2w}{c^2}\right) + O(c^{-4}) \end{cases} \tag{5}$$

where δij is Kronecker delta, w and w^i are the Newtonian and vector potentials of the gravitational field respectively. Where potential functions

$$\begin{cases} w(t,x^j) = G\displaystyle\int \frac{\sigma(t,\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\mathrm{d}^3x' + \frac{G}{2c^2}\int \frac{\partial^2}{\partial t^2}\sigma(t,\boldsymbol{x}')|\boldsymbol{x}-\boldsymbol{x}'|\mathrm{d}^3x' \\ w^{\mathrm{i}}(t,x^j) = G\displaystyle\int \frac{\sigma^i(t,\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\mathrm{d}^3x' \end{cases} \tag{6}$$

here $t=$ TCB, called barycentric coordinate time, and σ and σ^i denote mass density and flow density respectively, G is the gravitational constant. Obviously, the potential functions of the metric are zero at infinity (Deng, 2012).

The BCRS can be regarded as a very good inertial reference system in dynamics. The stars outside the solar system are very far away, and the tidal effect generated by them is negligible in the solar system. Therefore, it is not difficult to imagine that if the observer was at the barycenter of the solar system and moved together without rotation with respect to the far-distant celestial bodies, he would have a very flat space, apart from the interaction of the celestial bodies in the solar system, and the time given by the carried atomic clock would be also very uniform. Thus, we can regard the solar system as an isolated system and the BCRS as a space–time reference system with good inertia characteristics and orthogonal coordinates. The interaction among the celestial bodies in the solar system is expressed by the space–time metric determined by the coordinates.

Though IAU2000 Resolution B1.3 has given the form of space–time metric for BCRS, the orientation of the spatial coordinate axes are not given. For this reason, IAU2006 Resolution B2 further clarifies that for all practical applications, unless otherwise stated, the BCRS is assumed to be oriented according to the ICRS axes.

ICRS is the International Celestial Reference System, which is a realization of BCRS, including the International Celestial Reference Frame (ICRF) and related standards, constants, and models. ICRF realizes an ideal reference system by precise equatorial coordinates of extragalactic radio sources observed with Very Long Baseline Interferometry (VLBI). It is established and maintained by the International Earth Rotation and Reference Systems Service (IERS). IERS was jointly established by IAU and IUGG in 1987. Its basic mission is to provide Earth rotation, space reference systems and related data and standard services for astronomy, geodesy, and geophysics. The establishment and maintenance of the time scale is the responsibility of BIPM.

The geocentric celestial reference system

Due to the orbital motion of the Earth's center of mass relative to the solar barycenter and the influence of tidal forces caused by other celestial bodies of the solar system, the Earth cannot be regarded as an isolated body. The geocentric reference system is very complicated in conception and definition. Simply, the coordinate origin of GCRS is defined at the mass center of the Earth, and the coordinate axes near the Earth are orthogonal to each other with no spatial rotation relative to the coordinate axes of BCRS. The coordinate axes of GCRS are essentially defined by the coordinate relationship between GCRS and BCRS, which are only locally straight and orthogonal. Therefore, if BCRS were viewed as straight line coordinates, GCRS would be curvilinear coordinates.

According to IAU2000 Resolution B1.3, the spatial coordinate axes of GCRS are consistent with the spatial orientation of BCRS. The metric of GCRS is required to take the same form as the barycentric one:

$$\begin{cases} G_{00} = -\left(1 - \frac{2W}{c^2} + \frac{2W^2}{c^4}\right) + O(c^{-6}) \\ G_{0i} = -\frac{4W^i}{c^3} + O\left(c^{-5}\right) \\ G_{ij} = \delta_{ij}\left(1 + \frac{2W}{c^2}\right) + O\left(c^{-4}\right) \end{cases} \tag{7}$$

$W = W^0$ is the scalar potential, which is the sum of the earth's gravitational potential and the tide forces of the Sun and other external celestial bodies. Where the potential functions:

$$W^{\mu}(T, X^k) = W^{\mu}_{\mathrm{E}}(T, X^k) + W^{\mu}_{\mathrm{ext}}(T, X^k) \tag{8}$$

$W^{\mu}_{\mathrm{E}}, W^{\mu}_{\mathrm{ext}}$ represent the geocentric potentials and the external potentials. In the outer space of the Earth, the geocentric potentials can be expressed as:

$$\begin{cases} W_{\mathrm{E}}(T, X^k) = \frac{GM_{\mathrm{E}}}{R}\left\{1 + \sum_{l=2}^{\infty}\sum_{m=0}^{l}\left(\frac{a_{\mathrm{E}}}{R}\right)^l P_{lm}(\cos\theta)[C_{lm}\cos(m\phi) + S_{lm}\sin(m\phi)]\right\} \\ W^i_{\mathrm{E}}(T, X^k) = \frac{G}{2R^3}\varepsilon_{ijk}S^j_{\mathrm{E}}X^k \end{cases} \tag{9}$$

Here $T \equiv \mathrm{TCG}$, called Geocentric Coordinate Time (TCG), M_{E} denotes the mass of the Earth, R is the radius of the earth, $P_{lm}(\cos\theta)$ is Legendre expansion, S^i_{E} the angular momentum of Earth rotation, a_{E} the semi-major axis of the Earth's equator, and (C_{lm}, S_{lm}) the geocentric gravitational potential coefficients.

IAU2006 Resolution B2 clearly stated that GCRS orientation is derived from ICRS-oriented BCRS. According to the Eq. (4), if the geodesic precession is ignored, under the post-Newtonian approximation, the coordinate relationship between GCRS and BCRS can be expressed as follows:

$$\begin{cases} t \equiv T + \int(\gamma_{\mathrm{E}} - 1)\mathrm{d}T + e^0_j X^j/c \\ x^j \equiv x^i_{\mathrm{E}}(T) + e^i_j X^j + \left[\frac{1}{2}a^i_{\mathrm{E}}X^k X^k - a^k_{\mathrm{E}}X^i X^k\right] \Big/ c^2 \\ t \equiv T + \int(\gamma_{\mathrm{E}} - 1)\mathrm{d}T + e \end{cases} \tag{10}$$

where

$$\begin{cases} \gamma_{\mathrm{E}} \equiv e^0_0 = \left[1 - \frac{1}{c^2}(2\overline{w}_{\mathrm{E}} + v^2_{\mathrm{E}}) + \frac{1}{c^4}\left(2\overline{w}^2_{\mathrm{E}} + 8\overline{w}^j_{\mathrm{E}}v^j_{\mathrm{E}} - 2\overline{w}_{\mathrm{E}}v^2_{\mathrm{E}}\right)\right]^{\frac{1}{2}} \\ = 1 + \frac{1}{c^2}\left(\overline{w}_{\mathrm{E}} + \frac{1}{2}v^2_{\mathrm{E}}\right) + \frac{1}{c^4}\left(\frac{1}{2}\overline{w}^2_{\mathrm{E}} + \frac{5}{2}\overline{w}_{\mathrm{E}}v^2_{\mathrm{E}} + \frac{3}{8}v^4_{\mathrm{E}} - 4\overline{w}^j_{\mathrm{E}}v^j_{\mathrm{E}}\right) + O\left(c^{-6}\right) \\ e^i_0 = \gamma_{\mathrm{E}}v^j_{\mathrm{E}}/c \\ e^0_j = \gamma_{\mathrm{E}}v^j_{\mathrm{E}}/c + \frac{1}{c^3}\left(2\overline{w}_{\mathrm{E}}v^j_{\mathrm{E}} - 4\overline{w}^j_{\mathrm{E}}\right) + O\left(c^{-5}\right) \\ e^i_j = \delta_{ij}\left(1 - \frac{1}{c^2}\overline{w}_{\mathrm{E}}\right) + \frac{1}{2c^2}v^i_{\mathrm{E}}v^j_{\mathrm{E}} + O\left(c^{-4}\right) \end{cases} \tag{11}$$

and $x^i_{\mathrm{E}}, v^i_{\mathrm{E}}, a^i_{\mathrm{E}}$ are respectively the position, velocity, and acceleration of the center of the Earth in BCRS, $\overline{w}_{\mathrm{E}}, \overline{w}^i_{\mathrm{E}}$ the scalar potential and the vector potential at the center of the Earth. It can be seen from the coordinate relationships Eqs. (4) and (10) that if the coordinates in BCRS are considered as Euclidean linear coordinates, the coordinates of GCRS are curvilinear coordinates. Moreover, the scope of application of GCRS is limited to the local space near the Earth, which is much smaller than the space range of the Earth-Moon system. Since the tidal potentials of the space–time metric in GCRS are from the BCRS, the definition of GCRS is conceptually derived from BCRS.

Geocentric terrestrial reference system

The definition of GTRS is given by IAU2000 Resolution B1.3. It is a space coordinate system whose origin is at the center of the Earth and makes diurnal apparent motion with the Earth. In this reference system, the coordinates of points on the solid surface of the Earth remain almost unchanged except of the small changes caused by geophysical effects. IUGG2007 Resolution 2 clarified that GTRS was a geocentric space–time coordinate system under the framework of the theory of relativity. The coordinate transformation between GTRS and GCRS is realized through a space rotation determined with EOP.

The International Terrestrial Reference System (ITRS) maintained by IERS is an implementation of GTRS, which constitutes a set of prescriptions and conventions together with the modeling required to define the origin, scale, orientation, and time evolution. The system is realized as the International Terrestrial Reference Frame (ITRF) based upon the estimated coordinates and velocities of a set of stations observed by VLBI, Lunar Laser Ranging (LLR), GPS, Satellite Laser Ranging (SLR), and Doppler Orbitography and Radiopositioning Integrated by Satellite (DORIS). According to IERS, ITRS meet the following:

- The origin is at the mass center of the Earth, including oceans and atmosphere.
- The unit of length (scale) is the meter (SI).
- The orientation is initially given by the Bureau International de I'Heure(BIH) orientation at 1984.0.
- The time evolution of the orientation is ensured by using a no-net-rotation condition with regard to horizontal tectonic motions over the whole Earth.

Up to now, there are 12 ITRF realizations: ITRF89, ITRF90, ITRF91, ITRF92, ITRF93, ITRF94, ITRF96, ITRF97, ITRF2000, ITRF2005, ITRF2008, ITRF2014 (https://www.iers.org/).

The space–time references of BDS

A Global Navigation Satellite System (GNSS) involves a large space–time range near the Earth. To achieve high-precision Positioning, Navigation, and Timing (PNT), the unified and high-precision space–time reference systems must be established. The construction of observation models for the precise satellite orbit determination, time synchronization, and other information process should be performed within a framework of the same space–time benchmark.

Conceptually, two space-time reference systems are mainly involved in BDS in the operation mode supported by the master control station and monitor stations. One is GCRS which is a unified space-time reference for its precise orbit determination, time synchronization, and observation information processing, and the other is GTRS which is the reference of the Earth's surface and geostationary space and mainly used to express the positions of the users and the ground stations.

The observation modeling of satellite precise orbit determination and time synchronization in BDS are all based on GCRS. Since the positions of satellites and ground stations are separately expressed in two spatial reference systems, GCRS and GTRS, the coordinate transformation between them is required. The relationship can be written as follows:

$$[\text{GCRS}]=\boldsymbol{Q}(t)\boldsymbol{R}(t)\boldsymbol{W}(t)[\text{GTRS}] \tag{12}$$

Here, $\boldsymbol{Q}(t), \boldsymbol{R}(t), \boldsymbol{W}(t)$ are respectively the matrices of precession and nutation, the Earth rotation, and the polar motion, and.

$$\boldsymbol{Q}(t) \equiv \boldsymbol{R}_3(-E)\boldsymbol{R}_2(-d)\boldsymbol{R}_3(E+s) \tag{13}$$

$$R(t) \equiv R_3(\omega_{\text{E}}) \tag{14}$$

$$\boldsymbol{W}(t) \equiv \boldsymbol{R}_3(-s')\boldsymbol{R}_2(x_p)\boldsymbol{R}_1(y_p) \tag{15}$$

Where R_1, R_2, and R_3 represent the rotation matrices about the X, Y, and Z axes respectively. The acquisition and calculation methods of each parameter were given in IERS 2010 Technical Note (Petit and Luzum, 2010).

The geocentric terrestrial reference system used in BDS is called BeiDou Coordinate System (BDCS) and its definition is consistent with ITRS (Wu, 2018). The origin of the coordinates is at the mass center of the Earth, and the direction and scale of the coordinate axes are the same as ITRF. Table 1 shows the reference ellipsoid and the Earth's gravitational field constants defined in BDCS, including the semi-major axis and the flatness of the reference ellipsoid, the geocentric gravitational constant and the angular velocity of the Earth rotation.

The latest implementation of BDCS is done by using more than 100 globally distributed ground stations as

Table 1 The reference ellipsoid and geocentric gravitational constant adopted in BDCS

Semi-major axis	$a = 6{,}378{,}137.0$ m
Geocentric gravitational constant (Mass of Earth's atmosphere included)	$\mu = 3.986{,}004{,}418 \times 10^{14}$ m^3/s^2
Factor of ellipsoid flatness	$f = 1/298.257222101$
Mean angular velocity of the earth rotation	$\dot{\Omega}_e = 7.292115 \times 10^{-5}$ rad/s

reference frame points (Liu, 2019). About 10 monitoring stations, 3 International GNSS Service (IGS) stations in China and some other IGS stations around the world are involved in the BDCS. GNSS observations are mainly used at present, and other observations, such as VLBI, SLR, may also be used. The frame points are monitored continuously and their coordinates are adjusted together with IGS stations. The coordinates and velocities of frame points will be determined regularly.

The coordinates are in accordance with ITRF2014 at centimeter level. The scale difference is about 1.1×10^{-8}. Table 2 shows the transformation parameters between the two coordinate systems (Liu, 2018), where mas is the milliarcsecond and ppb is one part per 1,000,000,000 (10^9) parts.

BDCS is a realization of GTRS, and its relationship with GCRS satisfies Eq. (12). At present, the implementation accuracy of BDCS can only reach the magnitude of centimeters. In the future, in order to meet the application needs with higher accuracy, we must realize BDCS with millimeter accuracy, improve the accuracy of observations, use multi-source data, and continuously improve the accuracy of reference frame points and EOP. EOP is indispensable for the precise orbit determination and prediction of BDS satellites and precise time synchronization. However, the dynamical modeling error, the so-called Coriolis force of non-inertial reference frame caused by the error of EOP, is not significant in the operation mode supported by MCS. Although the orbit is given in the GCRS, the positions of navigation satellites are essentially determined by the ground monitor stations. Usually, the orbit determination arc is not long, about a few days, the systematic influence of the EOP error on the coordinates can be ignored because the orbit parameters broadcast by the navigation satellite are transformed back to the BDCS through almost the same coordinate transformation. Essentially, in this operation mode the process of positioning and timing is determining the space–time coordinates of an unknown station or user by the known ground stations. High accuracy of EOP is not needed, and milli-arcsecond level is enough.

In the autonomous operation mode where the system loses the support of MCS, there are two ways to obtain the EOP: one is the long-term forecasting with the accuracy depending on the length of the forecast time; and the other is the autonomous solution on the satellite. The EOP is essentially three Euler angles between GCRS and GTRS. To solve for EOP the observations that connect two reference systems are needed. The satellites need to observe celestial bodies, which reflect the orientation of GCRS, and the ground stations, called anchor stations, which reflect the orientation of GTRS. In this case, the solution accuracy of EOP depends on not only the astronomical observations but also the number and distribution of anchor stations. Similar to the mode with the supports of MCS, the ground anchor stations have great impacts on the accuracy of the user's positioning and timing.

It should be noted that the observation model of EOP in the autonomous operation mode must be established within the BCRS framework, considering that celestial bodies are very far away and the scope of application of GCRS is limited to the vicinity of the Earth.

BDS time

The time reference for BDS Time (BDT) synchronization, precise orbit determination and system operation is BDT. Similar to GPS Time (GPST), BDT is different from Coordinated Universal Time (UTC). It is a continuous time scale without leap seconds. In the BDS Radio Navigation Satellite Service (RNSS), BDT is counted in Week Number (WN) and Seconds of Week (SOW). The repetition period of WN in BDS is 8 times larger than that of GPS, and the maximum of WN does not exceed 8192. In the BDS Radio Determination Satellite Service (RDSS), BDT is counted in Year Number (YN) and Minutes of Year (MOY). The zero point of BDT is UTC 00:00:00 on January 1, 2006. At this moment WN and SOW are equal to zero, YN is 6 and MOY is 480. Compared with the system times of GPS and Galileo, the choice of the zero point of BDT is not only for saving bytes, but also for being easier to remember. It is the beginning of the year (Han et al., 2011).

Conceptually, BDT is an implementation of Terrestrial Time (TT). According to IAU2000 Resolution B1.9, the following relationship holds between TT and TCG (Han, 2017; Soffel et al., 2003):

Table 2 Seven parameters of conversion between BDCS and ITRF2014

Item	T_x (mm)	T_y (mm)	T_z (mm)	R_x in mas	R_y in mas	R_z in mas	Scale factor in ppb
Value	−0.37	1.12	−0.55	0.01	−0.02	0.05	0.011
RMS	0.74	0.74	0.74	0.03	0.03	0.04	0.012

$$\begin{cases} \dfrac{\mathrm{dTT}}{\mathrm{dTCG}} \equiv 1 - L_G \\ \mathrm{TCG} - \mathrm{TT} = \dfrac{L_G}{1 - L_G}(\mathrm{JD}_{\mathrm{TT}} - T_0) \times 86400 \end{cases} \tag{16}$$

where $\mathrm{JD}_{\mathrm{TT}}$ is the Julian Days at the time of TT, and

$$\begin{cases} L_G \equiv 6.969290134 \times 10^{-10} \\ T_0 \equiv 2443144.5003725 \end{cases} \tag{17}$$

The starting point of TT is International Atomic Time (TAI) 00:00:00 of January 1, 1977. T_0 is the corresponding number of Julian days. At this moment, TT is different from TAI by 0.0003725d = 32.184s, so the relationship between TAI and TT can be written as:

$$\mathrm{TT} = \mathrm{TAI} + 32.184s \tag{18}$$

And then the relationships between BDT, TAI and TT can be approximately expressed as:

$$\begin{cases} \mathrm{BDT} = \mathrm{TAI} + \Delta \mathrm{BDT}_{\mathrm{PPS}} - 33s \\ \mathrm{TT} = \mathrm{BDT} + \Delta \mathrm{BDT}_{\mathrm{PPS}} + 65.184s \end{cases} \tag{19}$$

Here $\Delta\mathrm{BDT}_{\mathrm{PPS}}$ is the time deviation of the second pulse of BDT relative to TAI or UTC, expressed as:

$$\Delta \mathrm{BDT}_{\mathrm{PPS}} = \mathrm{BDT}_{\mathrm{PPS}} - \mathrm{TAI}_{\mathrm{PPS}} \tag{20}$$

BDT is realized in the form of composed clock by an ensemble of atomic clocks of the master control station, and aligned with UTC through the National Time Service Center (NTSC) of the Chinese Academy of Sciences and China National Institute of Metrology (NIM). The clock ensemble consists of several active hydrogen maser and cesium atomic clocks. A new time scale algorithm derived from ALGOS algorithm is adopted. The time differences between BDT and UTC(k)/UTC are monitored continuously by the links of GNSS common view and TWTFT. When it is necessary, frequency adjustment will be introduced into BDT to keep the consistency between BDT and UTC. Under normal circumstances, the frequency deviation of BDT relative to TAI or UTC is less than 2×10^{-14}, and the absolute time deviation does not exceed 50 ns (Zhang and Cai, 2018). Therefore, the difference between BDT and TT is negligible when being used as time reference. The time offset between BDT and UTC is shown in figure (Fig. 1).

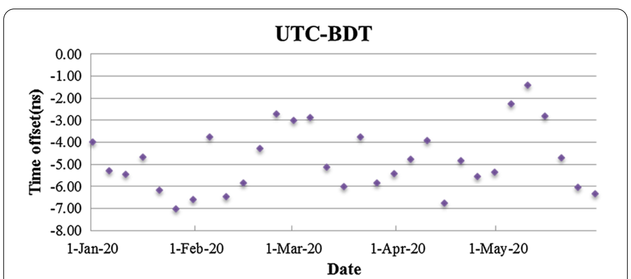

Fig. 1 the time offset between BDT and UTC

Conclusions

BDS adopts a technique for time synchronization completely different from GPS, GLONASS, and Galileo. TWTFT technology is used in BDS to directly measure the clock offsets between satellites and ground stations. Therefore, BDS is essentially a precision time measurement and time synchronization system for a large-scale space near the Earth. General relativity is the basic theoretical framework for the data processing in BDS. The spatial reference of BDS is BDCS, and the time reference is BDT. BDCS and BDT are the realizations of GTRS and TT in BDS, respectively. The BDS ground station coordinates are consistent with ITRF2014 at the centimeter level. The scale difference is approximately 1.1×10^{-8}. BDT is maintained by the master control station of BDS. The frequency deviation of BDT is less than 2×10^{-14}, and the time offset is less than 50 ns relative to TAI or UTC.

In addition to BDCS and BDT, GCRS and BCRS are also involved in the operation of BDS. The observation model of time synchronization and precise orbit determination is established within the GCRS framework. The coordinate transformation between BDCS and GCRS is consistent with IERS. In the autonomous operation mode without the support of ground master control station, EOP is obtained by means of long-term prediction and on-board observation. Observation models for on-board astrometry should be established within the BCRS framework.

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Authors' contribution
HC carried out the space–time reference system studies. LL participated in the space–time references of BDS and drafted the manuscript. CZ carried out the Local inertial reference system studies. LY participated in the BDT time system studies and helped to draft the manuscript. All authors read and approved the final manuscript.

Authors' information
Han Chunhao graduated with a Ph.D. in Astronomy from Nanjing University in 1990. He is a researcher at Beijing Satellite Navigation Center, and concurrently a professor at the University of Information Engineering and Shanghai Astronomical Observatory. Mainly engaged in the research of satellite navigation system, time and frequency control and relativistic space–time reference system. E-mail: hanchunhao@shao.ac.cn.

Liu Li is a senior engineer at Beijing Satellite Navigation Center, mainly engaged in data processing such as precise orbit determination and time synchronization of satellite navigation. E-mail: lliu@shao.ac.cn.

Cai Zhiwu is currently a senior engineer at Beijing Satellite Navigation Center. He engaged in research on satellite navigation technique and time scale theory. E-mail: zhiwucai@foxmail.com.

Lin Yuting is currently a senior engineer at Beijing Satellite Navigation Center. He obtained his Ph.D. from the University of Information Engineering in 2009. His current research focuses mainly on time keeping, satellite timing and precise time and frequency transfer technologies. E-mail: lyt1108@163.com.

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Author details
[1]Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, China. [2]Beijing Satellite Navigation Center, Beijing 100094, China.

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