Dmitrenko and Selivanov
Advances in Aerodynamics (2022) 4:40
https://doi.org/10.1186/s42774-022-00130-0

Advances in Aerodynamics

RESEARCH Open Access

An estimation of the velocity profile for the laminar-turbulent transition in the plane jet on the basis of the theory of stochastic equations and equivalence of measures

Artur V. Dmitrenko* and A. S. Selivanov

This article is dedicated to the memory of Academician N. A. Anfimov.

*Correspondence:
ammsv@yandex.ru
Department of Thermal Engineering, Russian University of Transport (MIIT), Obraztsova Street 9, Moscow 127994, Russian Federation

Abstract

The theory of stochastic equations and the theory of equivalence of measures previously applied to flows in the boundary layer and in the pipe are considered to calculate the velocity profile of the flat jet. This theory previously made it possible to determine the critical Reynolds number and the critical point for the flow of the plane jet. Here based on these results the analytical dependence for the index of the velocity profile is derived. Velocity profiles are calculated for a laminar-turbulent transition in the jet. This formula reliably reflects an increase of the energy transferred from a deterministic state to a random one with an increase of the index of the velocity profile. Results show satisfactory agreement with the known experimental data for the velocity profile of the flat jet. Using obtained results it is possible to determine the location of technical devices for laminarization of the flow in the jet. This is important both for reducing friction in the flow around aerodynamic vehicles and for maintaining the jet profile if it is necessary to ensure the stability of the flow characteristics. Also the obtained relations can be useful for researching of the processes in combustion chambers, in the case of welding and in other technical devices.

Keywords: Stochastic equations, Equivalence of measures, Turbulence, Critical point, Reynolds number, The velocity profile, The flat jet

1 Introduction

The ideas of the theory of the onset of turbulence [1–13] both theoretical and numerical [14–23], including also the statistic and DNS modeling [2, 24–32], provide the necessity for determination of the stochastic equations and the regularity of equivalence of measures. Investigations of the onset of turbulence based on the theories of stochastic equations and the equivalence of measures between deterministic and random motion were carried out mainly for flows in shear flows in a circular tube [33–43], on a flat smooth plate [44–55], as well as for motion near a rotating disk [56] and for motion between rotating coaxial cylinders [57]. The last research was done for laminar–turbulent transition on a flat plate [58, 59]. All of these publications refer to shear flows in the presence of a solid surface (wall). An article related to forced shear flow in the absence of a wall determined the critical Reynolds number and the critical point for a plane jet flow [60]. In this regard, it seems important to present a procedure for deriving the velocity profile parameters for a plane jet flow during the transition from laminar to turbulent motion. The problem of the transition of laminar flow in a jet to turbulent is discussed in articles [60–68]. Note that in accordance to the opinions of works [61, 64], the problem of the transition to turbulence in the jet does not cause such a keen interest because the transition has already happened when critical Reynolds numbers equal to 30. In this case, the Reynolds number is made up of the entire width of a flat slit. In the case where the Reynolds number is made up using half the width of a flat slit, it equals 5–10. In this article, the procedure of calculation of parameters velocity profile of flat jet is considered on the basis of the theory of stochastic equations and the theory of equivalence of measures. The analytical dependence for the index of the velocity profile has the right and the left sides. The right side of the equation includes turbulent Reynolds numbers. The left side of the equation, which includes the desired profile index, is determined by the parameters of the averaged motion values and Reynolds number. As a result, having calculated the left side of the formula from the averaged values of the motion at a fixed velocity profile index, we obtain the result and compare it with the right side of the equation, which is determined by the experimental turbulent Reynolds number. If both parts of the equation agree satisfactorily, we compare the given speed profile index with its values from the known experimental range.

2 Conservation stochastic equations

Conservation stochastic equations were derived in [33–43]:
the equation of continuity

$$\frac{d(\rho)_{col_{st}}}{d\tau} = -\frac{(\rho)_{st}}{\tau_{cor}} - \frac{d(\rho)_{st}}{d\tau}, \tag{1}$$

the momentum equation

$$\frac{d(\rho u_i)_{col_{st}}}{d\tau} = div\left(\tau_{i,j}\right)_{col_{st}} + div\left(\tau_{i,j}\right)_{st} - \frac{(\rho u_i)_{st}}{\tau_{cor}} - \frac{d(\rho u_i)_{st}}{d\tau} + F_{col_{st}} + F_{st}, \tag{2}$$

and the energy equation

$$\frac{dE_{col_{st}}}{d\tau} = div\left(\lambda\frac{\partial T}{\partial x_j} + u_i\tau_{i,j}\right)_{col_{st}} + div\left(\lambda\frac{\partial T}{\partial x_j} + u_i\tau_{i,j}\right)_{st} - \left(\frac{E_{st}}{\tau_{cor}}\right) - \left(\frac{dE_{st}}{d\tau}\right) + (u_iF)_{col_{st}} + (u_iF)_{st}. \tag{3}$$

Here, $E, \rho, \overrightarrow{U}, u_i, u_j, u_l, \mu, \tau, \tau_{i,j}$ are the energy, the density; the velocity vector; the velocity components in directions x_i, x_j, x_l (i, j, l=1, 2, 3); the dynamic viscosity; the time; and the stress tensor $\tau_{i,j}=P+\sigma_{i,j}$, $\sigma_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \delta_{ij}\left(\xi - \frac{2}{3}\mu\right)\frac{\partial u_l}{\partial x_l}$, $\tau_{cor} = \frac{L}{\left((E_{st})_{U,P}/\rho\right)^{1/2}}$, $\delta_{ij}=1$ if $i=j$, $\delta_{ij}=0$ for $i \neq j$. P is the pressure of liquid or gas; λ is the thermal conductivity; c_p and c_v are the specific heat at constant pressure and volume, respectively; F is the external force. Further, $L=L_{U,\,P}=L_U$ is the scale of turbulence. Indexes $(_{U,\,P})$ and $(_U)$ refer to the velocity field. L_y on $x_2=y$, or L_x, $x_1=x$. Here, x_1 and x_2 are coordinates along and normal to the axis of the jet. Index "col_{st}" refers to components, which are actually the deterministic. Index "*st*" refers to component, which are actually the stochastic. Then for the non-isothermal motion of the medium, using the definition of equivalency measures between deterministic and random process in the critical point, the sets of stochastic equations of energy, momentum, and mass are defined for the next space-time areas: 1) the beginning of the generation (index 1,0 or 1); 2) generation (index 1,1); 3) diffusion (1,1,1) and 4) the dissipation of the turbulent fields. These results provide an opportunity to introduce the concept of the correlator, which is defined for the potential physical quantities and combinations (N, M). This correlator, in its structure, will determine the possible range of motion in space depending on the different combinations of (M, N) and the corresponding values that determine the correlation interval of space-time. According to [2, 32–50], the correlator in space-time is

$$m_i \to m_c; r_i \overset{\lim}{\to} r_c; \Delta\tau_i \to \tau_c \left(D_{N,M}(m_i; r_i; \Delta\tau_i)\right) = 0, \tag{4}$$

$$D_{N,M}(m_c; r_c; \tau_c) = \sum_i {}_{m_i \xrightarrow{\lim} m_c r_i \xrightarrow{\lim} rc \Delta\tau_i \xrightarrow{\lim} \tau_c} \left\{ m\left(T^M Z^* \cap T^N Y^*\right) - R_{1_{T^M Z^* T^N Y^*}} m\left(T^M Z^*\right) \right\}. \tag{5}$$

Subscript j denotes the parameters m_{cj} ($j=3$ means mass, momentum, and energy). For the case of the binary intersections, it was written that $X = Y + Z + W$. Here subscripts *cr* or *c* refer to critical point $r(x_{cr}, \tau_{cr})$ or r_c: the space-time point of the beginning of the interaction between the deterministic field and random field that leads to turbulence. In addition, subsets Y, Z, W are called extended in X. For the transfer of the substantial quantity Φ (mass (density ρ), momentum ($\rho\boldsymbol{U}$), energy (E)) of the deterministic (laminar) motion into a random (turbulent) one, for domain 1 of the start of turbulence generation, pair (N, M) = (1, 0), with the equivalence of measures being written as $(d\Phi_{col_{st}})_{1,0} = -R_{1,0}(\Phi_{st})$ and $\left(\frac{d(\Phi)_{col_{st}}}{d\tau}\right)_{1,0} = -R_{1,0}\left(\frac{\Phi_{st}}{\tau_{cor}}\right)$. Applying correlation $D_{N,M}(r_c; m_{ci}; \tau_c) = D_{1,1}(r_c; m_{ci}, \tau_c)$ derived in [2, 32–42], the equivalence relation for pair (N, M) = (1, 1) was defined as $(d\Phi_{col_{st}})_{1,1} = -R_{1,1}(d\Phi_{st})$, $\left(\frac{d(\Phi)_{col_{st}}}{d\tau}\right)_{1,1} = -R_{1,1}\left(\frac{d\Phi_{st}}{d\tau}\right)$, where $R_{1,0}$ and $R_{1,1}$ are fractal coefficients, $\Phi_{col_{st}}$ is the part of the field of Φ, notably, its deterministic component (subscript col_{st}) is the stochastic component of the measure, which is zero; Φ_{st} is the part of Φ, notably, the proper stochastic component (subscript *st*).

3 Critical Reynolds number and critical point

In article [58] it was shown that the critical Reynolds number and critical point for the plane jet were determined using the set of stochastic Eqs. (1)–(3) for the area 1), which, referring the pair (N, M) = (1, 0) is:

$$\left(\frac{d(\rho)col_{st}}{d\tau}\right)_{1,0} = -\frac{\rho_{st}}{\tau_{cor}};$$

$$\begin{cases} \left(\frac{d\left(\rho \vec{U}\right)_{col_{st}}}{d\tau} \right)_{1,0} = -\left(\frac{\left(\rho \vec{U}\right)_{st}}{\tau_{cor}} \right); \\ div\left(\tau_{i,j}\right)_{col_{st0}} = \frac{\left(\rho \vec{U}\right)_{st}}{\tau_{cor}}. \end{cases} \tag{6}$$

$$\begin{cases} \left(\frac{d(E)_{col_{st}}}{d\tau} \right)_{1,0} = -\left(\frac{(E)_{st}}{\tau_{cor}} \right)_{1,0}; \\ div\left(\lambda \frac{\partial T}{\partial x_j} + u_i \tau_{i,j}\right)_{col_{st0}} = \left(\frac{(E)_{st}}{\tau_{cor}} \right)_{1,0}. \end{cases}$$

So, the formula for critical Reynolds number was written as

$$(Re)_{cr} = 21.1\left(\frac{U_m}{\sqrt{E_{st}/\rho}} \right)^{0.568} \left(\frac{L}{x_2} \right)^{0.973} \left(\frac{h}{L} \right)^{0.243} \tag{7}$$

and in according with data [52–57], the theoretical value for critical Reynolds number is

$$(Re)_{cr} = 7.6 \div 25. \tag{8}$$

U_m is the average velocity on the jet axis, h the width of the initial section of the jet. The definition of the value of the critical point is found from the equation $\int\limits_{-\Delta V|2}^{+\Delta V|2} d\left(E_{col_{st}}\right)_{1,0} = \int\limits_X dE_{st}$. Here E_{st} is the stochastic energy component in the space X with the measure $m(E_{st}) < \infty$ and $E_{st} = E_{st}\left(\vec{x_i}, \tau_i, m_i\right) < \infty$. According to the main results of the ergodic theory we have $\int\limits_X dE_{st} = \frac{1}{\Delta V} \int\limits_V E_{st} \delta((\Delta V)_{critic} - \Delta V) dV = \frac{1}{\tau^0_{cor}} \int\limits_\tau E_{st} \delta\left(\tau^0_{cor} - \tau\right) d\tau = (E_{st})_{critic.}$ $(E_{st})_{critic}$ is the stochastic energy at the critical point. Then it is possible to write as $\int\limits_X dE_{st} = \frac{1}{L} \int\limits_L E_{st} \delta\left(\left(x_i\right)_{critic} - x_i\right) dL = \frac{1}{\tau^0_{cor}} \int\limits_\tau E_{st} \delta\left(\tau^0_{cor} - \tau\right) d\tau = \left(E_{st}\right)_{critic}$, here L is the scale of disturbance. So using these formulas, the equation for the critical point in the plane jet was determined [58]:

$$\left(\frac{x_2}{x_1} \right)_{cr} = (1.75)^{-10/9} \cdot \left(\frac{U_m}{\sqrt{E_{st}/\rho}} \right)^{-4/3} \left(\frac{L}{x_2} \right)^{-10/9} \left(\frac{h}{L} \right)^{-10/37}, \tag{9}$$

and in according with data [52–58], the theoretical value for critical point $\left(\frac{x_2}{x_1} \right)_{cr}$ is

$$\left(\frac{x_2}{x_1} \right)_{cr} = 0.025. \tag{10}$$

4 The velocity profile characteristics of the flat jet

The set of stochastic Eqs. (1)–(3) for the area 2), referring the pair (N,M) = (1,1) is:

$$\left(\frac{d(\rho)_{col_{st}}}{d\tau} \right)_{1,1} = -\frac{d(\rho)_{st}}{d\tau};$$

$$\begin{cases} \left(\frac{d\left(\rho \vec{U}\right)_{col_{st}}}{d\tau} \right)_{1,1} = -\left(\frac{d\left(\rho \vec{U}\right)_{st}}{d\tau} \right); \\ div\left(\tau_{i,j}\right)_{col_{st1}} = \frac{d\left(\rho \vec{U}\right)_{st}}{d\tau}. \end{cases} \tag{11}$$

$$\begin{cases} \left(\frac{d(E)_{col_{st}}}{d\tau} \right)_{1,1} = -\left(\frac{d(E)_{st}}{d\tau} \right)_{1,1}; \\ div\left(\lambda \frac{\partial T}{\partial x_j} + u_i \tau_{i,j} \right)_{col_{st1}} = \left(\frac{d(E)_{st}}{d\tau} \right)_{1,1}. \end{cases}$$

Then, taking into account the definition of the velocity u_1 of laminar motion in a plane jet [61, 64].

$$u_1 = 0.4543 \left(\frac{\left(U_m^2 h\right)^2}{\nu \cdot x_1} \right)^{1/3} \left(1 - (th(\xi))^2 \right). \tag{12}$$

Taking into account that

$$\xi = 0.2752 \left(\frac{\left(U_m^2 h\right)}{\nu^2} \right)^{1/3} \frac{x_2}{x_1^{2/3}} \tag{13}$$

and introducing the relation

$$\frac{th(\xi)}{ch(\xi)} = \frac{\xi - \frac{\xi^3}{3} + \frac{2\xi^5}{15} \cdots}{1 + \frac{\xi^2}{2!} + \frac{\xi^4}{4!} \cdots} = f(\xi), \tag{14}$$

the derivative $\frac{du_1}{dx_1}$ is

$$\frac{du_1}{dx_1} = -\frac{1}{3} 0.4543 \left(\frac{\left(U_m^2 h\right)^2}{\nu \cdot x_1} \right)^{1/3} \frac{1}{x_1} - 0.4543 \left(\frac{\left(U_m^2 h\right)^2}{\nu \cdot x_1} \right)^{1/3} \cdot 2f(\xi) \frac{d\xi}{dx_1}, \tag{15}$$

$$\frac{d\xi}{dx_1} = \left[0.2752 \left(\frac{\left(U_m^2 h\right)}{\nu^2} \right)^{1/3} \frac{x_2}{x_1^{2/3}} \right] \cdot \left(-\frac{2}{3} \frac{1}{x_1} \right) = -\frac{2}{3} \frac{1}{x_1} \cdot \xi, \tag{16}$$

$$\frac{du_1}{dx_1} = -\frac{1}{3} 0.4543 \left(\frac{\left(U_m^2 h\right)^2}{\nu \cdot x_1} \right)^{1/3} \frac{1}{x_1} - 0.4543 \left(\frac{\left(U_m^2 h\right)^2}{\nu \cdot x_1} \right)^{1/3} \cdot 2f(\xi) \left(-\frac{2}{3} \frac{1}{x_1} \cdot \xi \right) = 0.4543 \left(\frac{\left(U_m^2 h\right)^2}{\nu \cdot x_1} \right)^{1/3} \frac{1}{x_1} \left(-\frac{1}{3} + \frac{4}{3} \cdot \xi^2 \right), \tag{17}$$

$$\left(\frac{du_1}{dx_1} \right)^2 = \left[0.4543 \left(\frac{\left(U_m^2 h\right)^2}{\nu \cdot x_1} \right)^{1/3} \frac{1}{x_1} \left(-\frac{1}{3} + \frac{4}{3} \cdot \xi^2 \right) \right]^2. \tag{18}$$

In case of small values of ξ for laminar flow we have next formulas:

$$\left(\frac{du_1}{dx_1}\right)^2 = \left[0.4543\left(\frac{\left(U_m^2 h\right)^2}{\nu\cdot x_1}\right)^{1/3}\frac{1}{x_1}\left(-\frac{1}{3}\right)\right]^2 = \frac{1}{9}\left[0.4543\left(\frac{\left(U_m^2 h\right)^2}{\nu\cdot x_1}\right)^{1/3}\frac{1}{x_1}\right]^2, \tag{19}$$

$$\left(\frac{du_1}{dx_1}\right)^2 = \left[\frac{1}{9}0.4543^2\left(\frac{\left(U_m^2 h\right)^2}{\nu\cdot x_1}\right)^{1/3}\left(\frac{\left(U_m^2 h\right)^2}{\nu\cdot x_1}\right)^{1/3}\frac{1}{x_1^2}\right] = \left[\frac{1}{9}0.4543^2\left(\frac{\left(U_m^8 h^4\right)}{x_1^2\nu^2}\right)^{1/3}\frac{1}{x_1^2}\right], \tag{20}$$

$$\left(\frac{du_1}{dx_1}\right)^2 = \left[\frac{1}{9}0.4543^2\left(\frac{\left(U_m^8 h^4\right)}{x_1^2\nu^2}\right)^{1/3}\frac{1}{x_1^2}\right] = \left[\frac{1}{9}0.4543^2 U_m^2\left(\frac{h^2}{x_1^2}\right)^{1/3}\left(\frac{\left(U_m^2 h^2\right)}{\nu^2}\right)^{1/3}\frac{1}{x_1^2}\right]. \tag{21}$$

So finally we have

$$\mu\left(\frac{du_1}{dx_1}\right)^2 = \rho\nu\left[\frac{1}{9}0.4543^2 U_m^2\left(\frac{h}{x_1}\right)^{2/3}(Re)^{2/3}\frac{1}{x_1^2}\right]. \tag{22}$$

For the flow region of the beginning of turbulence initiation, we assume the same dependence, but with a different degree "n". As is known, if the velocity u_1 in laminar motion is ~ $(x_1)^{-1/3}$, then for a developed turbulent region $u_1 \sim (x_1)^{-1/n}$ and "$n = 2$". Thus, for the transition region the values of degree "n" are in the interval $2 \le n \le 3$. Next, we write down the relation determined earlier from the equivalence of measures for the region (1, 1) - the generation of turbulence [33–58]:

$$\frac{\left[\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2\Delta\tau\right]_{stoch}}{\left[\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2\Delta\tau\right]_{lamin}} = \left|Re_{st} - \frac{1}{Re_{st}}\right|. \tag{23}$$

As it was shown in the papers [33–58], the Eq. (23) determines the relative increase of the energy transferred from the deterministic state to the random one. Subscript "lamin" refers to the laminar flow, and subscript "stoch" refers to the non-laminar flow. Taking into account that for transition regime formula (22) may be written as $\mu\left(\frac{du_1}{dx_1}\right)^2 = \rho\nu\left[\frac{1}{n^2}0.4543^2\left(\frac{U_m}{x_1}\right)^2\left(\frac{h}{x_1}\right)^{2/n}(Re)^{2/n}\right]$. Then in accordance with [44–49, 60], the Eq. (23) is

$$\frac{\left[\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{stoch}\right]\Delta\tau_{stoch}}{\left[\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{lamin}\right]\Delta\tau_l} = \frac{9}{n^2}\left[(Re)\left(\frac{h}{L_2}\right)\left(\frac{L_2}{x_1}\right)\right]^{(2/n)-2/3}\frac{\Delta\tau_{stoch}}{\Delta\tau_l}. \tag{24}$$

Easy to see that for laminar flow $n=3$, the right side of Eq. (24) is equal to 1. Let us determine the value of the left side of the Eq. (23) (or the right side of Eq. 24) for various values of the indicator "n" and then compare with the calculated value of the right side of the Eq. (23). For the left side of the equation, if $n=2.5$, we have $K_\tau = \frac{\Delta\tau_{stoch}}{\Delta\tau_l} \approx 0.8 \div 1.2$

[56–58] and in accordance with [61–66], $\left(\frac{h}{L_2}\right)\cdot\left(\frac{L_2}{x_1}\right) = (2.5 \div 14.5)\cdot(0.01 \div 0.0157) = 0.025 \div 0.22765$. As can be seen, the experimental spread of values refers to the magnitude of the turbulence scale $\left(\frac{h}{L_2}\right)$. Therefore, the main attention will be focused on the correspondence of the calculated values of the right side of Eq. (24) to the left side of this equation. Then it can be written that

$$\frac{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{stoch}}{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{lamin}}K_\tau = \frac{9}{6.25}\cdot\left[\left(\frac{h}{L_2}\right)\cdot\left(\frac{L_2}{x_1}\right)\right]^{0.1333}\cdot(Re)^{0.1333}\cdot K_\tau = 1.44\cdot[0.025 \div 0.22765]^{0.1333}\cdot(7.25 \div 25)^{0.1333}\cdot(0.8 \div 1.2), \tag{25}$$

$$\frac{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{stoch}}{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{lamin}}K_\tau = (1.44)\cdot(0.61 \div 0.821)\cdot Re^{0.133}\cdot(0.8 \div 1.2) = (0.878 \div 1.18)\cdot(1.3 \div 1.53)\cdot(0.8 \div 1.2). \tag{26}$$

So for $n = 2.5$, the left side of the equation is

$$\frac{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{stoch}}{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{lamin}}K_\tau = (0.913 \div 1.62) \div (1.235 \div 2.18) = 1.5. \tag{27}$$

Then for $n = 2.0$ and the same initial data for the critical Reynolds number $(Re)_{cr} \sim 7.25 \div 25$, $K_\tau = \frac{\Delta\tau_{stoch}}{\Delta\tau_l} \approx 0.8 \div 1.2$ [54–58] and $\left(\frac{h}{L_2}\right)\cdot\left(\frac{L_2}{x_1}\right) = (2.5 \div 14.5)\cdot(0.01 \div 0.0157) = 0.025 \div 0.22765$ [61–67], the next equation can be written as

$$\frac{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{stoch}}{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{lamin}}K_\tau = \frac{9}{4}\cdot\left[\left(\frac{h}{L_2}\right)\cdot\left(\frac{L_2}{x_1}\right)\right]^{1/3}\cdot(Re)^{1/3}\cdot K_\tau = |Re_{st} - (1/Re_{st})|. \tag{28}$$

Substituting the numerical values, it is possible to find that

$$\frac{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{stoch}}{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{lamin}}\cdot K_\tau = \frac{9}{4}\cdot\left[\left(\frac{h}{L_2}\right)\cdot\left(\frac{L_2}{x_1}\right)\right]^{1/3}\cdot(Re)^{1/3}\cdot K_\tau = 2.25\cdot[0.025 \div 0.22765]^{1/3}\cdot(7.25 \div 25)^{1/3}\cdot(0.8 \div 1.2). \tag{29}$$

So for $n = 2.0$, the left side of the equation is

$$\frac{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{stoch}}{\rho\nu\left(\frac{\partial u_1}{\partial x_2}\right)^2_{lamin}}K_\tau = 2.25\cdot(0.292 \div 0.608)\cdot(0.8 \div 1.2)\cdot(1.94 \div 2.924) = (1.02 \div 2.3) \div (2.12 \div 4.81) \approx 2.5. \tag{30}$$

Then determine the right side of Eq. (23). Within the framework of the available data in the case of the origin of turbulence, experimental values of turbulent Reynolds number are $Re_{st} \sim 0.4 \div 0.5$ [61–67]. In the case of the developed turbulence, according to [2], experimental values of turbulent Reynolds number are $Re_{st} \sim 2 \div 4$. So the right side of Eq. (23) for $n = 3.0 \div 2.5$ has value

$$|Re_{st}\text{-}1/Re_{st}| = |(0.5 \div 0.4)\text{-}1/(0.5 \div 0.4)| \approx (1.5 \div 2.1) \approx 1.7,$$

and for $n = 2.0$ the value is

$$|Re_{st}\text{-}1/Re_{st}| = |(2 \div 4)\text{-}1/(2 \div 4)| \approx (1.5 \div 3.8) \approx 2.7. \quad (31)$$

Thus, the estimate of the profile index is satisfactorily determined by the obtained relation $n = 2.0$, corresponding to the turbulent flow value of index 'n' in empirical formula $u_1 \sim (x_1)^{-1/2}$ [61, 64]. It should be noted that the main last studies [61–119] do not contain information on analytical solutions for the jet profile in the laminar-turbulent transition regime.

5 Conclusions

On the basis of the theory of stochastic equations and the theory of equivalence of measures, the velocity profile of a plane jet is considered. In accordance with these theories, the analytical dependence of the velocity profile of flat jet is derived for the transition flow from the laminar flow to turbulent motion. It should be noted that the calculated values obtained from the formulas for the transition from laminar movement to turbulent flow in a plane jet are different to the calculated values for the velocity profile in the boundary layer on the flat plate and in the pipe, in the case of a smooth wall [33–43, 49–52]. This fact agrees with the well-known data [61, 64]. It is also seen that the analytical formulas (27) and (30) reliably reflect an increase of the transferred energy from a deterministic state to a random one with an increase of the index $(1/n)$. It is shown that the new equation reflects the experimental fact of changing the dependence for the longitudinal velocity u_1 during laminar motion from the coordinate along the axis of the jet $u_1 \sim (x_1)^{-1/3}$ $(n = 3)$ into a dependence for the developed turbulent region $u_1 \sim (x_1)^{-1/2}$ $(n = 2)$. The calculations carried out using new formulas (24)–(31) showed satisfactory agreement with the known experimental values for parameters of the velocity profile of flat jet.

The practical significance of the theoretical determination of velocity profiles in a jet at a laminar-turbulent transition from the values of the Reynolds number and vice versa is essential for a number of processes taking place, in particular, in combustion chambers [30] and in the case of welding [43, 120]. Also, knowing the initial parameters of the disturbance, it is possible, depending on the distance x_1, to determine the amount of energy transferred into random motion. This makes it possible to determine the location of technical devices for reducing friction [58, 59] in the flow around aerodynamic vehicles and for maintaining the jet profile if it is necessary to ensure the stability of the flow characteristics.

Acknowledgements
Not applicable.

Authors' contributions
Dmitrenko A.V. realized the derivation of formulas, calculations, formulation of conclusions. Selivanov A.S. conducted a search for experimental data and participated in the calculations and drawing conclusions. The authors read and approved the final manuscript.

Funding
This work was supported by the program "PRIORITET-2030".

Availability of data and materials
The datasets analyzed and used during the current study are available from the corresponding author as mentioned in references.

Declarations

Competing interests

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

Received: 15 September 2022 Accepted: 20 October 2022
Published online: 27 December 2022

References

1. Landau LD (1944) On the problem of turbulence. Dokl Akad Nauk SSSR 44:339–343. (in Russian)
2. Landau LD, Lifshitz EM (1959) Fluid mechanics. Pergamon Press, Oxford
3. Lorenz EN (1963) Deterministic nonperiodic flow. J Atmos Sci 20:130–141. https://doi.org/10.1175/1520-0469
4. Feigenbaum MJ (1980) The transition to aperiodic behavior in turbulent systems. Commun Math Phys 77:65–86
5. Ruelle D, Takens F (1971) On the nature of turbulence. Commun Math Phys 20:167–192. https://doi.org/10.1007/bf01646553
6. Kolmogorov AN (1941) Dissipation of energy in locally isotropic turbulence. Dokl Akad Nauk SSSR 32(1):16–18
7. Kolmogorov AN (1958) A new metric invariant of transient dynamical systems and automorphisms in Lebesgue spaces. Dokl Akad Nauk SSSR 119(5):861–864
8. Kolmogorov AN (1959) Entropy per unit time as a metric invariant of automorphisms. Dokl Akad Nauk SSSR 124(4):754–755
9. Kolmogorov AN (2004) Mathematical models of turbulent motion of an incompressible viscous fluid. Usp Mat Nauk 59(1):5–10
10. Struminskii VV (1989) The onset of turbulence. Dokl Akad Nauk SSSR 307(3):564–567
11. Klimontovich YL (1989) Problems in the statistical theory of open systems: Criteria for the relative degree of order in self-organization processes. Sov Phys Usp 32(5):416–433. https://doi.org/10.1070/PU1989v032n05ABEH002717
12. Samarskii AA, Mazhukin VI, Matus PP et al (1997) L2-conservative schemes for Korteweg-de Vries equation. Dokl Akad Nauk 357(4):458–461
13. Haller G (1999) Chaos near resonance. Springer, New York. https://doi.org/10.1007/978-1-4612-1508-0
14. Orszag SA, Kells LC (1980) Transition to turbulence in plane Poiseuille and plane Couette flow. J Fluid Mech 96(1):159–205. https://doi.org/10.1017/s0022112080002066/
15. Ladyzhenskaya OA (1975) A dynamical system generated by the Navier-Stokes equations. J Sov Math 3:458–479
16. Vishik MI, Komech AI (1983) Kolmogorov equations corresponding to a two-dimensional stochastic Navier-Stokes system. Tr Mosk Mat Obs 46:3–43
17. Packard NH, Crutchfield JP, Farmer JD, Shaw RS (1980) Geometry from a time series. Phys Rev Lett 45(9):712–716
18. Malraison B, Atten P, Berge P et al (1983) Dimension of strange attractors: an experimental determination for the chaotic regime of two convective systems. J Phys Lett 44(22):897–902. https://doi.org/10.1051/jphyslet:019830044022089700
19. Dmitrenko AV (2002) Calculation of the boundary layer of a two-phase medium. High Temp 40(5):706–715. https://doi.org/10.1023/A:1020436720213
20. Dmitrenko AV (2000) Heat and mass transfer and friction in injection to a supersonic region of the Laval nozzle. Heat Trans Res 31(6–8):338–399. https://doi.org/10.1615/HeatTransRes.v31.i6-8.30
21. Grassberger P, Procaccia I (1984) Dimensions and entropies of strange attractors from a fluctuating dynamics approach. Phys D Nonlinear Phenom 13(1–2):34–54. https://doi.org/10.1016/0167-2789(84)90269-0
22. Kozlov VV, Rabinovich MI, Ramazanov MP et al (1987) Correlation dimension of the flow and spatial development of dynamical chaos in a boundary layer. Phys Lett A 128(9):479–482
23. Brandstäter A, Swift J, Swinney HL et al (1983) Low-dimensional chaos in hydrodynamic system. Phys Rev Lett 51(16):1442–1446
24. Sreenivasan KR (1991) Fractals and multifractals in fluid turbulence. Annu Rev Fluid Mech 23:539–604
25. Priymak VG (2013) Splitting dynamics of coherent structures in a transitional round-pipe flow. Dokl Phys 58(10):457–465
26. Newton PK (2016) The fate of random initial vorticity distributions for two-dimensional Euler equations on a sphere. J Fluid Mech 786:1–4
27. Dmitrenko AV (2008) Fundamentals of heat and mass transfer and hydrodynamic of single-phase and two-phase media. Criterion, integral, statistical and DNS methods, vol 398. Galleya Print, Moscow
28. Dmitrenko AV (1997) Film cooling in nozzles with the large geometric expansion using method of integral relations and second moment closure model for turbulence. Paper presented at the 33rd joint propulsion conference and exhibit, Seattle, 6-9 July 1997. https://doi.org/10.2514/6.1997-2911
29. Dmitrenko AV (2017) Estimation of the critical Rayleigh number as a function of the initial turbulence of the boundary layer at a vertical heated plate. Heat Trans Res 48(13):1195–1202. https://doi.org/10.1615/HeatTransRes.2017018750
30. Dmitrenko AV (1998) Heat and mass transfer in combustion chamber using a second-moment turbulence closure including an influence coefficient of the density fluctuation in film cooling conditions. Paper presented at the 34th AIAA/ASME/SAE/ASEE joint propulsion conference and exhibit, Cleveland, 13-15 July 1998. https://doi.org/10.2514/6.1998-3444
31. Dmitrenko AV (1993) Nonselfsimilarity of a boundary-layer flow of a high-temperature gas in a Laval nozzle. Aviats Tekh 1:39–42

32. Dmitrenko AV (1986) Computational investigations of a turbulent thermal boundary layer in the presence of external flow pulsations. In: Proceedings of the 11th conference on young scientists, Moscow, Physicotechnical Institute, part 2, p 48–52. Deposited at VINITI 08.08.86, no. 5698-B8
33. Dmitrenko AV (2007) Calculation of pressure pulsations for a turbulent heterogeneous medium. Dokl Phys 52(7):384–387. https://doi.org/10.1134/s1028335807120166
34. Dmitrenko AV (2013) Equivalence of measures and stochastic equations for turbulent flows. Dokl Phys 58(6):228–235. https://doi.org/10.1134/s1028335813060098
35. Dmitrenko AV (2014) Some analytical results of the theory of equivalence measures and stochastic theory of turbulence for non-isothermal flows. Adv Stud Theor Phys 8(25):1101–1111. https://doi.org/10.12988/astp.2014.49131
36. Dmitrenko AV (2016) Determination of critical Reynolds numbers for nonisothermal flows by using the stochastic theories of turbulence and equivalent measures. Heat Trans Res 47(1):41–48. https://doi.org/10.1615/HeatTransRes.2015014191
37. Dmitrenko AV (2016) The theory of equivalence measures and stochastic theory of turbulence for non-isothermal flow on the flat plate. Int J Fluid Mech Res 43(2):182–187. https://doi.org/10.1615/InterJFluidMechRes.v43.i2.60
38. Dmitrenko AV (2015) Analytical estimation of velocity and temperature fields in a circular pipe on the basis of stochastic equations and equivalence of measures. J Eng Phys Thermophy 88(6):1569–1576. https://doi.org/10.1007/s10891-015-1344-x
39. Dmitrenko AV (2016) An estimation of turbulent vector fields, spectral and correlation functions depending on initial turbulence based on stochastic equations. The Landau fractal equation. Int J Fluid Mech Res 43(3):271–280. https://doi.org/10.1615/InterJFluidMechRes.v43.i3.60
40. Dmitrenko AV (2017) Stochastic equations for continuum and determination of hydraulic drag coefficients for smooth flat plate and smooth round tube with taking into account intensity and scale of turbulent flow. Continuum Mech Thermodyn 29(1):1–9. https://doi.org/10.1007/s00161-016-0514-1
41. Dmitrenko AV (2017) Analytical determination of the heat transfer coefficient for gas, liquid and liquid metal flows in the tube based on stochastic equations and equivalence of measures for continuum. Continuum Mech Thermodyn 29(6):1197–1205. https://doi.org/10.1007/s00161-017-0566-x
42. Dmitrenko AV (2017) Determination of the coefficients of heat transfer and friction in supercritical-pressure nuclear reactors with account of the intensity and scale of flow turbulence on the basis of the theory of stochastic equations and equivalence of measures. J Eng Phys Thermophy 90(6):1288–1294. https://doi.org/10.1007/s10891-017-1685-8
43. Dmitrenko AV (2018) Results of investigations of non-isothermal turbulent flows based on stochastic equations of the continuum and equivalence of measures. J Phys Conf Ser 1009:012017. https://doi.org/10.1088/1742-6596/1009/1/012017
44. Dmitrenko AV (2018) The stochastic theory of the turbulence. IOP Conf Ser Mater Sci Eng 468:012021. https://doi.org/10.1088/1757-899X/468/1/01202
45. Dmitrenko AV (2019) Determination of the correlation dimension of an attractor in a pipe based on the theory of stochastic equations and equivalence of measures. J Phys Conf Ser 1250:012001. https://doi.org/10.1088/1742-6596/1250/1/012001
46. Dmitrenko AV (2019) The construction of the portrait of the correlation dimension of an attractor in the boundary layer of Earth's atmosphere. J Phys Conf Ser 1301:012006. https://doi.org/10.1088/1742-6596/1301/1/012006
47. Dmitrenko AV (2019) The theoretical solution for the Reynolds analogy based on the stochastic theory of turbulence. JP J Heat and Mass Transf 18(2):463–476. https://doi.org/10.17654/HM018020463
48. Dmitrenko AV (2020) The correlation dimension of an attractor determined on the base of the theory of equivalence of measures and stochastic equations for continuum. Continuum Mech Thermodyn 32(2):63–74. https://doi.org/10.1007/s00161-019-00784-0
49. Dmitrenko AV (2020) The possibility of using low-potential heat based on the organic Rankine cycle and determination of hydraulic characteristics of industrial units based on the theory of stochastic equations and equivalence of measures. JP J Heat Mass Transf 21(1):125–132. https://doi.org/10.17654/HM021010125
50. Dmitrenko AV (2020) Some aspects of the formation of the spectrum of atmospheric turbulence. JP J Heat Mass Transf 19(1):201–208
51. Dmitrenko AV (2020) The uncertainty relation in the turbulent continuous medium. Continuum Mech Thermodyn 32(1):161–171. https://doi.org/10.1007/s00161-019-00792-0
52. Dmitrenko AV (2020) Formation of a turbulence spectrum in the inertial interval on the basis of the theory of stochastic equations and equivalence of measures. J Eng Phys Thermophy 93(1):122–127. https://doi.org/10.1007/s10891-020-02098-4
53. Dmitrenko AV (2020) The spectrum of the turbulence based on theory of stochastic equations and equivalence of measures. J Phys Conf Ser 1705:012021. https://doi.org/10.1088/1742-6596/1705/1/012021
54. Dmitrenko AV (2021) Reynolds analogy based on the theory of stochastic equations and equivalence of measures. J Eng Phys Thermophy 94:186–193. https://doi.org/10.1007/s10891-021-02296-8
55. Dmitrenko AV (2021) Theoretical solutions for spectral function of the turbulent medium based on the stochastic equations and equivalence of measures. Continuum Mech Thermodyn 33:603–610. https://doi.org/10.1007/s00161-020-00890-4
56. Dmitrenko AV (2021) Determination of critical Reynolds number for the flow near a rotating disk on the basis of the theory of stochastic equations and equivalence of measures. Fluids 6(1):5. https://doi.org/10.3390/fluids6010005
57. Dmitrenko AV (2021) Analytical estimates of critical Taylor number for motion between rotating coaxial cylinders based on theory of stochastic equations and equivalence of measures. Fluids 6(9):306. https://doi.org/10.3390/fluids6090306
58. Dmitrenko AV (2022) Prediction of laminar–turbulent transition on flat plate on the basis of stochastic theory of turbulence and equivalence of measures. Continuum Mech Thermodyn 34:601–615. https://doi.org/10.1007/s00161-021-01078-0

59. Dmitrenko AV (2022) Theoretical calculation of laminar–turbulent transition in the round tube on the basis of stochastic theory of turbulence and equivalence of measures. Continuum Mech Thermodyn 34:1375–1392. https://doi.org/10.1007/s00161-022-01125-4
60. Dmitrenko AV (2020) Determination of critical Reynolds number in the jet based on the theory of stochastic equations and equivalence of measures. J Phys Conf Ser 1705:012015. https://doi.org/10.1088/1742-6596/1705/1/012015
61. Davidson PA (2004) Turbulence: an introduction for scientists and engineers, 1st edn. Oxford University Press, New York, p 678
62. Schlichting H (1968) Boundary-layer theory, 6th edn. McGraw-Hill Book Co., Inc., New York, p 747
63. Hinze JO (1975) Turbulence. McGraw-Hill Book Co., Inc., New York, p 790
64. Abramovich GN (1963) The theory of turbulent jets. MIT Press, Cambridge, p 671
65. Corrsin S (1943) Investigation of flow in an axially symmetrical heated jet of air. NACA Wartime Rep NACA-ACR-3L23
66. Corrsin S, Uberoi MS (1949) Further experiments on the flow and heat transfer in a heated turbulent air jet. NACA Tech Note NACA-TN-1865
67. Wygnanski IJ, Champagne FH (1973) On transition in a pipe. Part 1. The origin of puffs and slugs and the flow in a turbulent slug. J Fluid Mech 59(2):281–335
68. Wygnanski I, Sokolov M, Friedman D (1975) On transition in a pipe. Part 2. The equilibrium puff. J Fluid Mech 69(2):283–304
69. Risso F, Fabre J (1997) Diffusive turbulence in a confined jet experiment. J Fluid Mech 337:233–261
70. Karimipanah T (1996) Turbulent jets in confined spaces. Dissertation, Royal Institute of Technology, Sweden
71. Arndt REA, Long DF, Glauser MN (1997) The proper orthogonal decomposition of pressure fluctuations surrounding a turbulent jet. J Fluid Mech 340:1–33
72. Citriniti JH, George WK (2000) Reconstruction of the global velocity field in the axisymmetric mixing layer utilizing the proper orthogonal decomposition. J Fluid Mech 418:137–166
73. Tam CKW, Auriault L (1999) Jet mixing noise from fine-scale turbulence. AIAA J 37(2):145–153
74. Mataoui A, Schiestel R, Salem A (2001) Flow regimes of a turbulent plane jet into a rectangular cavity: experimental approach and numerical modeling. Flow Turbul Combust 67:267–304
75. Mataoui A, Schiestel R, Salem A (2002) Oscillatory phenomena of a turbulent plane jet flowing inside a rectangular cavity. In: Rahman M, Verhoeven R, Brebbia CA (eds) Advances in fluid mechanics IV. WIT Press, Southampton, p 151–162
76. Mataoui A, Schiestel R, Salem A (2003) Study of the oscillatory regime of a turbulent plane jet impinging in a rectangular cavity. Appl Math Model 27(2):89–114
77. Saiyed NH, Mikkelsen KL, Bridges JE (2003) Acoustics and thrust of quiet separate-flow high-bypass-ratio nozzles. AIAA J 41(3):372–378
78. Alkislar MB, Krothapalli A, Choutapalli I et al (2005) Structure of supersonic twin jets. AIAA J 43(11):2309–2318
79. Kompenhans J, Arnott A, Agos A et al (2002) Application of PIV for the investigation of high speed flow fields. In: West East High Speed Flow Field. Artes Gráficas Torres S.A., Barcelona, p 39–52
80. Knob M, Safarik P, Uruba V et al (2005) The effect of the side walls on a two-dimensional impinging jet. Paper presented at the 16th international symposium on transport phenomena, Prague, 2005. http://fluids.fs.cvut.cz/akce/konference/istp_2005/full/168.pdf
81. Suzuki T, Colonius T (2006) Instability waves in a subsonic round jet detected using a near-field phased microphone array. J Fluid Mech 565:197–226
82. Kopiev V, Zaitsev MY, Chernyshev SA et al (2007) Vortex ring input in subsonic jet noise. Int J Aeroacoust 6(4):375–405
83. Jordan P, Gervais Y (2008) Subsonic jet aeroacoustics: associating experiment, modelling and simulation. Exp Fluids 44:1–21
84. Tam CKW, Viswanathan K, Ahuja KK et al (2008) The sources of jet noise: experimental evidence. J Fluid Mech 615:253–292
85. Guadalfajara M (2009) Interference between supply jet and room surfaces in ventilated room - a model study. Thesis, University of Gävle, Sweden
86. Gudmundsson K (2010) Instability wave models of turbulent jets from round and serrated nozzles. Dissertation, California Institute of Technology
87. Yarygin V, Gerasimov Y, Krylov A et al (2011) Experimental study of the International Space Station contamination by its orientation thrusters jets. Microgravity Sci Technol 23(Suppl 1):15–23
88. Shimshi E, Ben-Dor G, Levy A (2011) Viscous simulation of shock reflection hysteresis in ideal and tapered overexpanded planar nozzles. Shock Waves 21(3):205–214. https://doi.org/10.1007/s00193-011-0325-z
89. van Hooff T, Blocken B, Defraeye T et al (2012) PIV measurements of plane wall jet in a confined space at transitional slot Reynolds numbers. Exp Fluids 53:499–517
90. Gvozdeva LG, Gavrenkov SA (2013) Influence of the adiabatic index on switching between different types of shock wave reflection in a steady supersonic gas flow. Tech Phys 58(8):1238–1241. https://doi.org/10.1134/S1063784213080148
91. Gvozdeva LG, Gavrenkov SA (2012) Formation of triple shock configurations with negative reflection angle in steady flows. Tech Phys Lett 38(4):372–374. https://doi.org/10.1134/S1063785012040232
92. Cavalieri AVG, Rodríguez D, Jordan P et al (2013) Wavepackets in the velocity field of turbulent jets. J Fluid Mech 730:559–592
93. Jordan P, Colonius T (2013) Wave packets and turbulent jet noise. Annu Rev Fluid Mech 45:173–195
94. Gvozdeva LG, Gavrenkov SA (2015) A new configuration of irregular reflection of shock waves. Prog Flight Phys 7:437–452. https://doi.org/10.1051/eucass/201507437
95. Hadjadj A, Perrot Y, Verma S (2015) Numerical study of shock/boundary layer interaction in supersonic overexpanded nozzles. Aerosp Sci Technol 42:158–168. https://doi.org/10.1016/j.ast.2015.01.010

96. Benderskii LA, Krasheninnikov SY (2016) Investigation of noise generation by turbulent jets on the basis of numerical simulation of unsteady flow in mixing layers. Fluid Dyn 51(4):568–580
97. Kopiev VF, Chernyshev SA (2012) New correlation model for the cascade of turbulent pulsations as a noise source in jets. Acoust Phys 58(4):442–456
98. Lyu B, Dowling AP, Naqavi I (2017) Prediction of installed jet noise. J Fluid Mech 811:234–268
99. Bychkov OP, Faranosov GA (2018) An experimental study and theoretical simulation of jet-wing interaction noise. Acoust Phys 64(4):437–452
100. Belyaev IV, Bychkov OP, Zaitsev MY et al (2018) Development of the strategy of active control of instability waves in unexcited turbulent jets. Fluid Dyn 53(3):347–360
101. Krasheninnikov SY, Mironov AK, Benderskii LA (2018) Analysis of noise generation by turbulent jets from consideration of their near acoustical field. Acoust Phys 64(6):718–730
102. Xu HY, Xing SL, Ye ZY (2015) Numerical study of the S809 airfoil aerodynamic performance using a co-flow jet active control concept. J Renew Sustain Ener 7(2):023131
103. Xu HY, Qiao CL, Ye ZY (2016) Dynamic stall control on the wind turbine airfoil via a co-flow jet. Energies 9(6):429
104. Yang YC, Zha GC (2018) Super lift coefficient of co-flow jet circular cylinder. Paper presented at the 2018 AIAA aerospace sciences meeting, Kissimmee, 8-12 January 2018
105. Zhang JH, Xu KW, Yang YC et al (2018) Aircraft control surfaces using co-flow jet active flow control airfoil. Paper presented at the 2018 applied aerodynamics conference, Atlanta, 25-29 June 2018
106. Yang XD, Jiang WR, Zhang SL (2019) Analysis of co-flow jet effect on dynamic stall characteristics applying to rotor airfoils. IOP Conf Ser Mater Sci Eng 491:012010
107. Bychkov O, Faranosov G, Kopiev V et al (2019) The modelling of jet-plate interaction noise in the presence of co-flow. Paper presented at the 25th AIAA/CEAS aeroacoustics conference, Delft, 20-23 May 2019
108. Xu KW, Zhang JH, Zha GC (2019) Drag minimization of co-flow jet control surfaces at cruise conditions. Paper presented at the AIAA Scitech 2019 forum, San Diego, 7-11 January 2019
109. Yang YC, Fernandez M, Zha GC (2018) Improved delayed detached eddy simulation of super-lift coefficient of subsonic co-flow jet flow control airfoil. Paper presented at the 2018 AIAA aerospace sciences meeting, Kissimmee, 8-12 January 2018
110. Lemanov V, Lukashov V, Sharov K et al (2019) Turbulent spots in the flame of a diffusion torch. J Phys Conf Ser 1382:012058. https://doi.org/10.1088/1742-6596/1382/1/012058
111. Dubnishchev YN, Arbuzov VA, Lukashov VV et al (2019) Optical Hilbert diagnostics of hydrogen jet burning. Optoelectron Instrum Data Process 55(1):16–19. https://doi.org/10.3103/S8756699019010035
112. Soares LFM, Cavalieri AVG, Kopiev V et al (2020) Flight effects on turbulent-jet wave packets. AIAA J 58(9):3877–3888
113. Baykov ND, Petrov AG (2020) Collapse of capillary-gravitational waves and the generation of cumulative jets. Fluid Dyn 55:953–964
114. Lemanov VV, Lukashov VV, Sharov KA (2020) Transition to turbulence through intermittence in inert and reacting jets. Fluid Dyn 55:768–777
115. Alekseenko SV, Bilsky AV, Dulin VM et al (2007) Experimental study of an impinging jet with different swirl rates. Int J Heat Fluid Flow 28(6):1340–1359. https://doi.org/10.1016/j.ijheatfluidflow.2007.05.011
116. Ball CG, Fellouah H, Pollard A (2012) The flow field in turbulent round free jets. Prog Aerosp Sci 50:1–26. https://doi.org/10.1016/j.paerosci.2011.10.002
117. Zaiko YS, Reshmin AI, Teplovodskii SK et al (2018) Investigation of submerged jets with an extended initial laminar region. Fluid Dyn 53(1):95–104. https://doi.org/10.1134/S0015462818010184
118. Bychkov OP, Faranosov GA (2021) On the relationship between velocity and pressure fluctuations at the axis of a turbulent jet and in its near field. Fluid Dyn 56:481–491
119. Zhang YZ, Xu HY, Chu YW et al (2021) Two-dimensional numerical study of the pulsed co-flow jet. Fluid Dyn 56(3):361–370
120. Dudoladov SO, Larionov NV (2021) The condition for application of the Crocco integral in the mathematical description of a laser welding plasma plume. St Petersburg Polytechnic Univ J Phys Math 14(3):60–75

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.