Financial Innovation

RESEARCH

Open Access

Optimal reduction and equilibrium carbon allowance price for the thermal power industry under China's peak carbon emissions target

Jiaojiao Sun and Feng Dong*

*Correspondence:
dongfeng2008@126.com

School of Economics and Management, China University of Mining and Technology, Xuzhou 221116, People's Republic of China

Abstract

As the largest source of carbon emissions in China, the thermal power industry is the only emission-controlled industry in the first national carbon market compliance cycle. Its conversion to clean-energy generation technologies is also an important means of reducing CO_2 emissions and achieving the carbon peak and carbon neutral commitments. This study used fractional Brownian motion to describe the energy-switching cost and constructed a stochastic optimization model on carbon allowance (CA) trading volume and emission-reduction strategy during compliance period with the Hurst exponent and volatility coefficient in the model estimated. We defined the optimal compliance cost of thermal power enterprises as the form of the unique solution of the Hamilton–Jacobi–Bellman equation by combining the dynamic optimization principle and the fractional Itô's formula. In this manner, we obtained the models for optimal emission reduction and equilibrium CA price. Our numerical analysis revealed that, within a compliance period of 2021–2030, the optimal reductions and desired equilibrium prices of CAs changed concurrently, with an increasing trend annually in different peak-year scenarios. Furthermore, sensitivity analysis revealed that the energy price indirectly affected the equilibrium CA price by influencing the Hurst exponent, the depreciation rate positively impacted the CA price, and increasing the initial CA reduced the optimal reduction and the CA price. Our findings can be used to develop optimal emission-reduction strategies for thermal power enterprises and carbon pricing in the carbon market.

Keywords: Carbon peak, Fractional Brownian motion, Optimal control, Carbon allowance price

Introduction

Climate change is a pressing global concern. China is the world's leading emitter of CO_2 (Zhu et al. 2018), with the power sector accounting for 42.5% of total energy emissions in 2020 (IEA 2021). In particular, the thermal power sector is a major source of emissions (Sun and Dong 2022; Pan and Dong 2022a). As a result, it will be critical in achieving China's carbon peak and carbon neutrality targets. The "World Energy Outlook 2017: China Special Report" by the International Energy Agency estimated that China's share of coal-fired power generation will fall below 40% by 2040, whereas gas-fired power generation will remain within 10% (IEA 2017). Meanwhile, in 2021, the Global Energy Internet Development Cooperation Organization (GEIDCO) hosted a "China Carbon Neutrality Workshop" and published a research report titled "Energy and power development plan for 2030 and outlook for 2060" ("Outlook" hereinafter). According to the report, China's electricity consumption will grow at a 4.2% annual rate from 2020 to 2025, a 3% annual growth rate from 2025 to 2030, and a declining average annual growth rate of 2% from 2030 to 2050 (GEIDCO 2021).

Switching from coal to natural gas is currently a key strategy for reducing emissions in end-use power generations (Li et al. 2021; Pan and Dong 2022b). As natural gas burns more cleanly than coal, converting coal to natural gas improves environmental quality on global and local scales by lowering greenhouse gas emissions and airborne pollutant emissions (Rivera and Loveridge 2022). In the UK, the share of coal power fell by 75–90% in just 4 years (2012–2016) as a result of energy switching and the retirement of coal-fired power plants; this drove the UK's power sector to achieve its largest annual CO_2 emissions reduction of 25 million tons in 2016 (Wilson and Staffell 2018). Natural-gas power generation in China has grown rapidly since the 13th Five-Year Plan, with installed capacity increasing from approximately 66.37 million kW in 2015 to more than 100 million kW in January 2021,[1] making it one of the most effective technologies for peak shaving in the power supply system (Yu et al. 2022). As a clean, low-carbon alternative, natural gas is a transitional resource in the decoalization process because of its flexible and efficient output characteristics when used to generate electricity (Yu et al. 2021).

Carbon markets, as a cost-effective means of addressing climate change, have the potential to reduce emissions at the lowest possible cost (Zhu et al. 2018). Characterized by a "fixed total amount and variable price," the carbon market is conducive to achieving China's carbon peak and neutrality targets (Shen 2021; Zhang and Dong 2023). China launched its first carbon-trading market in Shenzhen in 2013, which was initially proposed in 2011, followed by trading pilots in Shanghai, Beijing, Guangdong, Tianjin, Hubei, Chongqing, and Fujian. All of these pilots are managed by local environmental protection departments, with the goal of reducing carbon emissions in the pilot provinces and cities (Zhang et al. 2020). Subsequently, a national carbon market was launched in 2017 and officially opened for trading on July 16, 2021, with a batch of 2225 thermal power companies (Guo and Feng 2021). With the continuous advancement of the carbon emissions reduction processes, China's carbon allowance (CA) trading market has matured, progressing from scattered pilots to nationwide implementation. Carbon trading has thus emerged as an important mechanism for reducing the emission-abatement costs of enterprises and, ultimately, mitigating climate change in China (Weng and Xu 2018; Jin et al. 2019; Dong et al. 2022a).

Therefore, against the backdrop of the Chinese carbon market's launch, with thermal power companies as the primary participants, this study focuses on the following: how the carbon reductions achieved by energy conversion from coal to natural gas and carbon trading volume can be rationalized to minimize total emission-reduction costs; how the share of energy generation for thermal power companies can be determined in

[1] The data are sourced from China Electricity Council.

accordance with the carbon-peak target; and what factors affect the CA price trends in carbon trading. To answer these questions, this study combines stochastic analysis in fractal markets and optimal control theory to investigate the optimal emission reduction of thermal power companies and the equilibrium price in the carbon market. Our findings could serve as a reference for setting reasonable prices in the carbon market, as well as a theoretical foundation for carbon-reduction decisions in the thermal power industry.

The remainder of this study is organized as follows. The following section presents and summarizes the literature review on related topics. Then, the optimization model is built in the section "Model construction." Subsequently, the solution to the optimal model is presented. The "Parameter estimation for the stochastic model" then estimates the parameters of the stochastic model proposed in the previous section. Then, the numerical simulation and results are described. Finally, the last section concludes, provides some policy suggestions, and discusses some limitations and future research ideas.

Literature review

Thermal power companies can freely switch generation fuel inputs between coal and natural gas to produce the same amount of electricity based on relative energy prices (Lu et al. 2021). This type of hybrid energy generation can effectively reduce carbon emissions while also being more cost-effective than a single-energy abatement strategy. Because it represents the carbon price's threshold value, the switching price from coal to natural gas can be considered a shadow price (Chevallier and Goutte 2017). Power plants profit from switching from coal to gas above that price, and profit from switching from coal to gas below that price. The switching price is more sensitive to changes in natural gas prices than to changes in coal prices (Kanen 2006). Factors, usually influenced by supply and demand in the energy market, such as coal and natural gas prices and carbon prices, affect the energy conversion and energy-switching prices. The energy market is primarily influenced by micro and macroeconomic factors such as politics, weather, and economic growth. Therefore, energy-switching price fluctuates stochastically, and many researchers have used stochastic fluctuation models to depict the dynamics of this price. In terms of stochastic process form, the standard Brownian motion process captures the aforementioned uncertainty factors more simply than other stochastic models. The models expanded on the Brownian motion process include the Lévy-driven jump (Chevallier and Goutte 2017), regime-switching, Lévy-driven Ornstein–Uhlenbeck, and inhomogeneous Brownian motion processes (Arrigoni 2019; Lu et al. 2021). Given the data analysis of historical energy-switching prices, this study used fractional Brownian motion to model the prices' "leptokurtosis and fat-tail" characteristics.

Among various energy-saving and emission-reduction initiatives, the establishment of China's carbon-trading market is a significant step toward green, low-carbon development (Brutschin and Fleig 2016; Tan and Wang 2017a; Dong et al. 2022b). Carbon credit pricing is an important regulatory initiative in the fight against climate change (Steinebach et al. 2020; Zhang et al. 2021a). Energy prices, weather conditions, and macro-risk factors all have a significant impact on the CA price (Tang et al. 2020; Batten et al. 2021). Carbon trading prices can thus fluctuate stochastically. Duan et al. (2021) found that energy prices influenced the carbon-trading price in phase III of the European Union

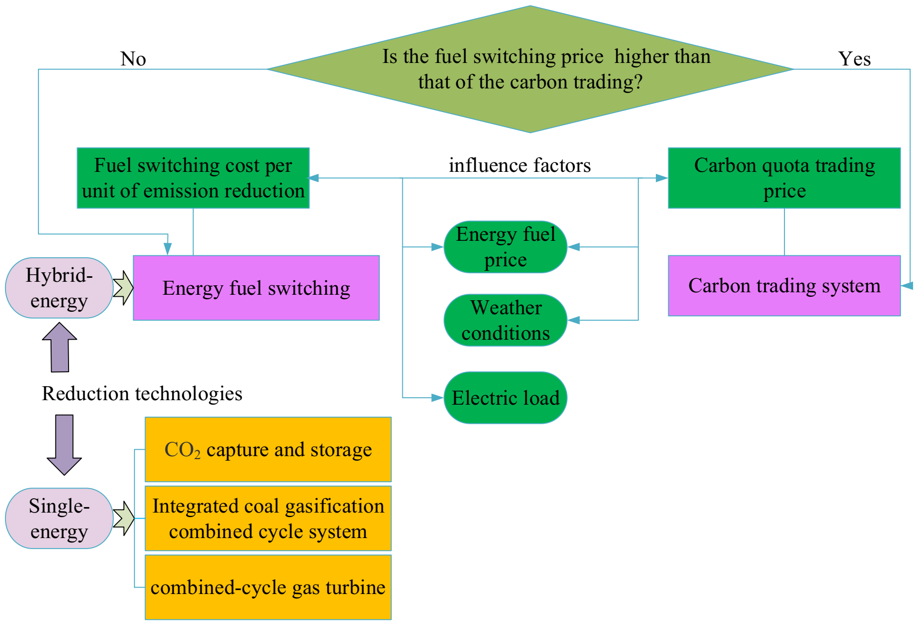

Fig. 1 Carbon-reduction technology for thermal power enterprises and carbon trading system

(EU) Emission Trading Scheme (ETS). The relative costs of coal and natural gas increase the volatility of carbon-trading prices because energy producers can switch between energy resources (Benz and Trück 2009); natural gas reduces carbon emissions, with the demand for permitted emissions decreasing by more than 50%. Moreover, extreme weather events, such as cold winters, can increase energy demand for heating, resulting in an increase in CA demand among energy producers and an increase in carbon prices (Batten et al. 2021). Carbon prices, as measured by EU CA futures prices, have only a tenuous relationship with macroeconomic risk factors. The primary factor influencing changes in carbon futures prices is electricity producers' fuel-switching behavior (Chevallier 2009; Tan and Wang 2017b). Therefore, this study investigates the factors influencing carbon trading equilibrium prices under energy-switching behavior in terms of minimizing total emission-reduction costs in the thermal power industry. Its goal is to establish the equilibrium guidance prices for carbon-emission allowance. Figure 1 depicts the emission-reduction channel's power system, the carbon emissions trading system, and the linkages between them.

The power industry aims to achieve established emission-reduction targets at the lowest possible cost while ensuring power supply for the entire society and stable economic growth (Kumar et al. 2020; Cao et al. 2021; Dong et al. 2021a). Many studies have looked into optimal emission-reduction strategies for power systems at the national (Prebeg et al. 2016; Ioannou et al. 2019; Yu and Fang 2021; Dong et al. 2021b), regional levels (Cheng et al. 2015; Koltsaklis et al. 2014; Nie et al. 2018), and power-sector levels (Chen et al. 2020; Wang and Qie 2020; Liu and Dong 2021). Kang et al. (2020) built a nonlinear technology optimization model to explore the economically optimal carbon capture and storage deployment strategy for China's power sector to achieve 2 °C emission-reduction targets while also satisfying investment demand; the findings provided valuable information for future subsidy settings and carbon-trading markets. Meanwhile, to meet energy demand while mitigating the environmental effects of the power generation system, Ioannou et al. (2019) built a multistage stochastic optimization model consistent with sustainable development goals by incorporating renewable energy, CO_2 emission-reduction targets, and fuel diversity into a series of constraints.

Other studies have investigated CA prices from the standpoint of enterprises' marginal abatement costs. For instance, using the Hamilton–Jacobi–Bellman (HJB) equation to study enterprises' optimal emission-reduction strategies, Xu and Guo (2017) discovered that the CA price and marginal abatement cost are the same, both equal to the discounted value of the excess emission penalty. Accordingly, they obtained the optimal strategy and optimal cost at each decision time point. Given that enterprises' decision-making processes are discrete and CAs cannot be traded at intertemporal periods, Xu and Zhang (2020) developed a discrete emission-reduction decision model for thermal power enterprises and employed dynamic optimization approaches to derive the optimal abatement cost. The marginal abatement cost was used as the guidance price for enterprises to participate in CA trading. Meanwhile, Kollenberg and Taschini (2016) obtained the optimal emission reduction and trading strategies in the market's equilibrium state by solving the dynamic cost minimization problem for enterprises. They provided an analytical foundation for regulators to select appropriate ETS policies.

The following deficiencies exist in existing research on emission-reduction strategies and CA pricing for power-generation enterprises. Most studies use the objective function of minimizing power-generation costs and employ stochastic optimization or objective programming models to determine the optimal emission reduction while ignoring the link between the carbon-trading market and emission-reduction strategies. When building models, due to the influence of fuel price fluctuations and macroeconomic factors, few studies have accounted for stochastic volatility in the energy-switching price, the CA price, and CO_2 emissions. As a result, there are discrepancies between research findings and reality. Moreover, few studies have combined the stochastic volatility model in the financial market with optimal control theory to investigate trends in optimal carbon reduction and the equilibrium price of CAs.

To fill the above-mentioned gaps, this study aims to improve the optimal control model of enterprise costs and introduced stochastic analysis to solve the cost-minimization problem in the thermal power industry. The following are our primary contributions. First, to ensure that the model depicting the energy-switching price is consistent with reality, we used the fractional Brownian motion model to describe its dynamics and concluded that CA prices satisfy the mixed fractional Brownian motion process based on the equilibrium theory of CA trading. Second, unlike previous studies that assigned values to some parameters, this study used historical data from two carbon-trading pilots (Guangdong and Tianjin) as samples to estimate two parameters in the model: the Hurst exponent and the volatility coefficient. By comparing the estimated trajectory to the actual one, we were able to confirm the accuracy of the estimated results and the applicability of the model. Third, we obtained the HJB partial differential equation satisfied by the optimal total compliance cost by combining the dynamic optimization principle with the fractional Itô's formula, and we derived the expressions for optimal emission reduction and equilibrium CA prices. This builds on the work of Xu and Zhang (2020). Furthermore, we simulated the trajectories of optimal emission reduction and equilibrium CA prices over time under different scenarios by varying the peak years. Then, we examined how the main parameters affected optimal emission reduction and equilibrium carbon-emission trading prices.

Model construction

Let $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P\right)$ be the probability space satisfying the general condition filtering, where $\{\mathcal{F}_t\}_{t\geq 0}$ is the right continuous adaptation process. Consider N enterprises participating in the carbon-trading market. Denote I as the set of all of the enterprises; then, $I = \{1, 2, 3, \cdots, N\}$. At any time point t in a compliance period [0, T], enterprises must consider using carbon-abatement technologies or carbon-market trading (buying or selling) to determine the carbon emissions they want to offset. Thus, enterprises face the intertemporal optimization problem of determining the optimal production and trading strategy to achieve the cost-cutting goal. The enterprise i is assumed to receive a certain amount of CAs e_0^i for free at the initial moment of $t = 0$, then

$$x_0^i = y_0^i - e_0^i, \tag{1}$$

where x_0^i is the initial amount of emission reductions required, y_0^i denotes the CO_2 emissions of the enterprise i in the basic as usual (BAU) scenario (i.e., the social development status before China's 11th Five-Year Plan when there was no emission-reduction control policy and trading market) at $t = 0$, and e_0^i denotes the initial CA, which may vary depending on the size of an enterprise. During the compliance period, enterprises must offset their carbon emissions by using clean-energy reduction technologies or purchasing CAs. In the BAU scenario, carbon emissions are assumed to satisfy the following process:

$$dy_t = \mu\left(t, y_t\right)dt + \sigma\left(t, y_t\right)dW_t, \tag{2}$$

where $\mu\left(t, y_t\right)$ and $\sigma\left(t, y_t\right)$ denote the drift and fluctuation terms, respectively, and W_t denotes the $\mathcal{F}_t$-adapted standard Brownian motion (Liang and Huang, 2021). Denote x_t^i as the required carbon-emission reduction of enterprise i observed at the time t within [0, T]:

$$x_t^i = -\int_0^t \left(\alpha_s^i + \beta_s^i\right)dt + \int_0^t y_s ds + E\left(\int_t^T y_s ds|\mathcal{F}_t\right), \tag{3}$$

where α_t^i and β_t^i represent the carbon trading volume and the emission abatement amount of the enterprise i at the time t, respectively; $\alpha_t^i > 0$ denotes that power plant i buys the CAs; otherwise, the allowances are sold. x_T^i is the amount of emission reduction required at the end of the compliance period to meet the requirements. The enterprise intends to complete the compliance emission-reduction target during the compliance period. Thus, the enterprise is compliant at the end of the compliance period; namely, $E_t\left(x_T^i\right) = 0$.

Apply Itô's formula to the Eq. (3):

$$\mathrm{d}x_t^i = -\left(\alpha_t^i + \beta_t^i\right)\mathrm{d}t + G(t)\mathrm{d}W_t, \tag{4}$$

where $G(t)$ is the fluctuation term determined by $\mu\left(t, y_t\right)$ and $\sigma\left(t, y_t\right)$. We assume that $G(t)$ is a t-continuous and bounded function. Changes in demand and allocation prior to emission reductions have different effects on enterprises, whereas all enterprises are subject to systematic shocks. Therefore, for each enterprise $i \in I$, the demand for carbon-emission reduction is assumed to be driven by the same Brownian motion, W_t, whereas the differences in enterprise size and technology, are determined by the distribution of $G(t)$. Referring to Liang and Huang (2020), let $G(t) = \sigma^i$, where σ^i is a constant, denoting the volatility coefficient of emission reduction demand for any enterprise i.

Optimization problem

Let $A(t)$ be the total cost of CA trading for enterprise i. Then, it includes nonnegligible transaction costs $\nu \cdot \left(\alpha_t^i\right)^2$ and CA trading costs $\alpha_t^i \cdot C_t$, where ν is the friction factor of CA trading, and C_t is the trading price.

$$A(t) = \nu \cdot \left(\alpha_t^i\right)^2 + \ C_t \cdot \alpha_t^i. \tag{5}$$

In the absence of carbon-abatement costs, the marginal cost of electricity can be considered the ratio of the energy[2] fuel cost to the efficiency of electricity generation:

$$MC = \frac{FC}{\eta}, \tag{6}$$

where MC is marginal cost, FC is fuel cost, and η is the enterprise's generation efficiency. Considering model complexity, this study excludes costs associated with operations and maintenance, regulatory changes, and manpower (Chevallier et al. 2019). Thus, the fuel cost is also the cost per unit of electricity generated. When we factor in carbon costs like carbon tax and CA trading price, the marginal cost of electricity becomes:

$$MC = \frac{FC}{\eta} + \frac{e}{\eta} \cdot M_t, \tag{7}$$

where e is the carbon emission factor (i.e., carbon emissions per unit of electricity produced), and M_t is the energy-switching price. Energy fuel cost can be expressed as the product of thermal efficiency and energy price, as follows:

$$FC = h \cdot P_t. \tag{8}$$

The switching point can be defined as the emission cost when the marginal costs of burning coal and gas are equal (i.e., $MC_c = MC_g$). Combining the above equations, we determine that M_t depends on each fuel's cost, efficiency, and emissions factor.

$$M_t = \frac{h_g \cdot P_t^g \cdot \eta_c - h_c \cdot P_t^c \cdot \eta_g}{\eta_g \cdot e_c - \eta_c \cdot e_g}. \tag{9}$$

[2] In this study, energy refers to coal and natural gas.

Coal-fired power is more profitable than gas-fired power if the energy-switching price M_t is higher than the CA price; thus, there is no need to replace coal generation with natural gas at this time.

Let $B(t)$ be the total cost of carbon reduction for enterprise i, which consists of two components: the energy-switching cost, $\beta_t^i \cdot M_t$, and the depreciated cost of clean-energy equipment, $\frac{1}{2}\delta \cdot \left(\beta_t^i\right)^2$, used by the enterprises for production:

$$B(t) = \beta_t^i \cdot M_t + \frac{1}{2}\delta \cdot \left(\beta_t^i\right)^2, \tag{10}$$

where δ is the depreciation factor of the abatement equipment. The optimization problem for the compliance cost faced by the enterprise i is as follows:

$$\begin{cases} \omega\left(t, x_t^i, C_t, M_t; \alpha_t^i, \beta_t^i\right) = \min\limits_{\alpha_t^i, \beta_t^i} \ E_t\left[\int_t^T e^{-rs}\left(\nu \cdot \left(\alpha_s^i\right)^2 + C_s \cdot \alpha_s^i + \frac{1}{2}\delta \cdot \left(\beta_s^i\right)^2 + M_s \cdot \beta_s^i\right) ds\right] \\ s.t. E_t\left(x_T^i\right) = 0. \end{cases} \tag{11}$$

where r is the risk-free interest rate. CA trading volume α_t^i and emission-reduction strategy β_t^i, are decision variables. The amount of carbon-emission reduction x_t^i required by the enterprise, CA price C_t, and energy-switching price M_t are all state variables.

Dynamic models for energy-switching and carbon-allowance prices

Market equilibrium consists of abatement and trading strategies for each enterprise, as well as the market clearing price process C_t. Deviations from equilibrium individuals do not result in additional cost savings for any enterprise in equilibrium (Kollenberg and Taschini 2016). The market is assumed to be complete and arbitrage-free. As a result, we can assume that there exists a risk-neutral probability measure Q that is equivalent to the real-world measure P, and that all market participants are risk-neutral. In the risk-neutral world, the expected return on a risky asset is equal to the risk-free rate.

Since energy prices are affected by factors such as seasonality and the supply–demand relationship, energy-switching prices associated with energy prices will vary randomly. It is, therefore, assumed that under risk-neutral probability measure Q, M_t satisfies the following stochastic differential equation model:

$$\frac{\mathrm{d}M_t}{M_t} = r\mathrm{d}t + \sigma \mathrm{d}B_H^Q(t), \tag{12}$$

where $B_H^Q(t)$ is the $\mathcal{F}_t$-adapted fractional Brownian motion process, r is the risk-free rate, and σ is the volatility of the energy-switching price. When $H = 0.5$, the fractional Brownian motion is the standard Brownian motion; when $0 < H < 0.5$, the fractional Brownian motion is anti-persistent; when $0.5 < H < 1$, the fractional Brownian motion is long-run dependent. Carbon emissions in the BAU scenario are independent of the carbon abatement strategy; thus, $corr\left(\mathrm{d}W_t, \mathrm{d}B_H^Q(t)\right)=0$.

Under the condition that the carbon-trading market is cleared, $\int_{i\in I} \alpha_t^i \mathrm{d}I(i) = 0$ (i.e., $\alpha_t^I = 0$). Assume that the state process i of the enterprise is $Z\left(x_t^i, C_t, M_t\right)$; then, according to the derivation process of the abatement and trading strategy for the enterprise in Appendix A, the state variables C_t can be represented as follows.

$$\mathrm{d}C_t = rC_t\mathrm{d}t + \sigma M_t \mathrm{d}B_H^Q(t) + \delta g_t \sigma^I \mathrm{d}W_t, \tag{13}$$

where $g_t = \frac{re^{rt}}{e^{rT} - e^{rt}}$, and σ^I is the distribution of the expected total carbon-emission reductions required in the set of enterprises I. The important information implied by Eq. (13) is that the CA price is driven by the mixed fractional Brownian motion process, which combines mutually independent fractional and standard Brownian motion.

Solution for the optimal model

For ease of expression, denote $x_t^i, C_t, M_t, \alpha_t^i, \beta_t^i$ briefly as x, c, m, α, β. Using the fractional Itô's theorem (Alòs and León 2021) for Eq. (11) and combining Eqs. (4), (12), and (13) with the dynamic optimization principle, the HJB equation for the optimal value function $\omega\left(t, x_t^i, C_t, M_t; \alpha_t^i, \beta_t^i\right)$, of the enterprise i is obtained:

$$\begin{aligned}
&\mathrm{D}_t\omega + rc\mathrm{D}_c\omega + rm\mathrm{D}_m\omega + \frac{1}{2}\left(\sigma^i\right)^2\mathrm{D}_x^2\omega + \left(Ht^{2H-1}\sigma^2 m^2 + \frac{1}{2}\delta^2 g_t^2\left(\sigma^I\right)^2\right)\mathrm{D}_c^2\omega \\
&+Ht^{2H-1}\sigma^2 m^2 \mathrm{D}_m^2\omega + \frac{1}{2}\delta g_t \sigma^i \sigma^I \mathrm{D}_x\mathrm{D}_c\omega + Ht^{2H-1}\sigma^2 m^2 \mathrm{D}_c\mathrm{D}_m\omega \\
&+\inf_{\alpha,\beta}\left\{-(\alpha+\beta)\mathrm{D}_x\omega + \mathrm{e}^{-rt}\left(c\alpha + \nu\alpha^2 + m\beta + \frac{1}{2}\delta\beta^2\right)\right\} = 0,
\end{aligned} \tag{14}$$

Notice that, to minimize the expression in the curly bracket above, for all α and β, it needs to satisfy that

$$\alpha = \frac{1}{2\nu}\left(\mathrm{e}^{rt}\mathrm{D}_x\omega - c\right), \ \beta = \frac{1}{\delta}\left(\mathrm{e}^{rt}\mathrm{D}_x\omega - m\right) \tag{15}$$

resulting in the following lemma.

Lemma 1 *The HJB Eq. (14) can be rewritten in the following form:*

$$\begin{aligned}
&\mathrm{e}^{rt}\left(\mathrm{D}_t\omega + rc\mathrm{D}_c\omega + rm\mathrm{D}_m\omega + \frac{1}{2}\left(\sigma^i\right)^2\mathrm{D}_x^2\omega + \left(Ht^{2H-1}\sigma^2 m^2 + \frac{1}{2}\delta^2 g_t^2\left(\sigma^I\right)^2\right)\mathrm{D}_c^2\omega\right. \\
&\left.+Ht^{2H-1}\sigma^2 m^2 \mathrm{D}_m^2\omega + \frac{1}{2}\delta g_t \sigma^i \sigma^I \mathrm{D}_x\mathrm{D}_c\omega + Ht^{2H-1}\sigma^2 m^2 \mathrm{D}_c\mathrm{D}_m\omega\right) \\
&-\frac{1}{4\nu}\left(\mathrm{e}^{rt}\mathrm{D}_x\omega - c\right)^2 - \frac{1}{2\delta}\left(\mathrm{e}^{rt}\mathrm{D}_x\omega - m\right)^2 = 0,
\end{aligned} \tag{16}$$

Since for any t, $E_t\left(x_T^i\right) = 0$, the terminal condition needs to be added in the process of solving the HJB equation:

$$\lim_{t\to T}\omega\left(t, x_t, C_t, M_t; \alpha_t^i, \beta_t^i\right) = \begin{cases} 0 \ : \ x_t^i = 0 \\ \infty \ : \ x_t^i \neq 0. \end{cases} \tag{17}$$

The solution of the HJB equation can be obtained by differentiation, which leads to the following theorem.

Theorem 1 *The solution of the HJB equation with the terminal condition (17) is.*

$$
\begin{aligned}
\omega(t,x,c,m) &= \frac{r\nu\delta}{(\delta+2\nu)\left(e^{rT}-e^{rt}\right)}x^2 + \frac{e^{-2rt}\left(e^{rT}-e^{rt}\right)\left(2cm-c^2\right)}{2r(\delta+2\nu)} \\
&+\left(m-\frac{c-m}{\delta+2\nu}\right)x + \frac{1}{2\delta(\delta+2\nu)}e^{-2rt-\sigma^2t^{2H}}(\Phi(t)-\Phi(T))m^2 \\
&+\int_t^T \frac{r\delta^2\left(\sigma^I\right)^2-2r\nu\delta\left(\sigma^i\right)^2-r\delta^2\sigma^i\sigma^I}{2(\delta+2\nu)\left(e^{rT}-e^{rs}\right)}\mathrm{d}s,
\end{aligned}
\tag{18}
$$

where $\Phi(t)=\int e^{2rt+\sigma^2t^{2H}}\mathrm{d}t$.

See Appendix B for the proof.

Based on Eq. (15) regarding the relationship between α_t^i, β_t^i *and the minimized cost, the optimal trading and abatement volumes of enterprise i are obtained.*

$$
\alpha_t^i = \frac{re^{rt}\delta}{(\delta+2\nu)\left(e^{rT}-1\right)}\left(y_0^i-e_0^i\right) + \frac{re^{rt}\delta}{\delta+2\nu}\int_0^t \frac{\sigma^i}{e^{rT}-e^{rs}}\mathrm{d}W_s + \frac{M_t-C_t}{\delta+2\nu}, \tag{19}
$$

$$
\beta_t^i = \frac{2re^{rt}\nu}{(\delta+2\nu)\left(e^{rT}-1\right)}\left(y_0^i-e_0^i\right) + \frac{2re^{rt}\nu}{\delta+2\nu}\int_0^t \frac{\sigma^i}{e^{rT}-e^{rs}}\mathrm{d}W_s + \frac{C_t-M_t}{\delta+2\nu}. \tag{20}
$$

Integrating the above equations with respect to i, we obtain the optimal trading and abatement volumes for the carbon market from Eq. (A.2).

$$
\alpha_t^I = 0,\ \beta_t^I = \frac{re^{rt}}{e^{rT}-1}\left(y_0^I-e_0^I\right) + re^{rt}\int_0^t \frac{\sigma^I}{e^{rT}-e^{rs}}\mathrm{d}W_s. \tag{21}
$$

Thus, $C_t = M_t + \delta\beta_t^I$, *implying that the optimal trading and abatement volumes are independent of the Hurst exponent when the carbon market reaches equilibrium and that the equilibrium price of CAs equals the marginal abatement cost.*

From Eq. (12), the energy-switching price at any point in time t can be obtained.

$$
M_t = M_0\exp\left(rt-\frac{1}{2}\sigma^2t^{2H}+\sigma B_H^Q(t)\right), \tag{22}
$$

where M_0 *is the energy-switching price at the initial compliance period, such that the equilibrium price of CAs is*

$$
C_t = M_0\exp\left(rt-\frac{1}{2}\sigma^2t^{2H}+\sigma B_H^Q(t)\right) + \frac{r\delta e^{rt}}{e^{rT}-1}\left(y_0^I-e_0^I\right) + r\delta e^{rt}\int_0^t \frac{\sigma^I}{e^{rT}-e^{rs}}\mathrm{d}W_s. \tag{23}
$$

The desired equilibrium price of CAs agreed upon by all of the enterprises is $E(C_t)$ *denoted by* C_t^d:

$$
C_t^d = M_0e^{rt} + \frac{r\delta e^{rt}}{e^{rT}-1}\left(y_0^I-e_0^I\right). \tag{24}
$$

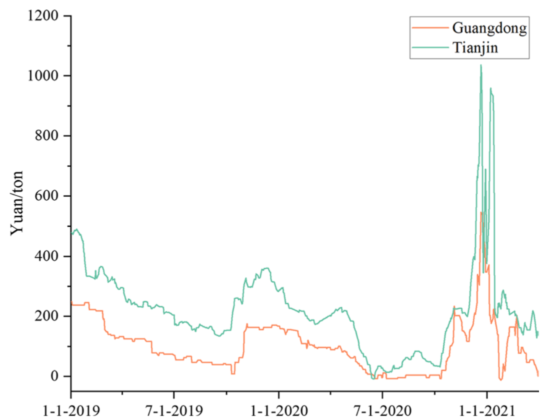

Fig. 2 Energy-switching prices (January 1, 2019–March 31, 2021)

Parameter estimation for the stochastic model

China's national carbon market was officially launched for the trading on July 2021. As a result, there is a scarcity of data on carbon quota trading. Thus, we took the trading pilots Guangdong and Tianjin as samples, and used the energy price data from the pilots to estimate parameters in the model (12).

Logarithmic change in energy-switching and carbon-allowance prices

According to the Intergovernmental Panel on Climate Change, the CO_2 emissions from burnt coal to produce 1 megawatt hour (MWh) of electricity are 0.897 tons and the CO_2 emissions from burnt natural gas to produce 1 MWh of electricity are 0.388 tons; i.e., $e_c = 0.897t_{CO2}/MWh$, $e_g = 0.388t_{CO2}/MWh$. The efficiency values for coal and natural gas are taken as $h_c = 0.378t/MWh$ and $h_g = 101.158\mathrm{m}^3/MWh$ (Xu and Zhang 2020), respectively. In addition, referring to Lu et al. (2021), let $\eta_c = 0.326$ and $\eta_g = 0.311$. Data for coal and natural gas prices were obtained from the WIND database[3]; we selected the market price of power coal (Q5000) in Guangzhou port and the closing price of power coal (Q4500) in Tianjin port, as well as the arrival price of liquefied natural gas (LNG) in Guangzhou and Tianjin.

Equation (9) is used to estimate energy-switching prices for thermal power companies using daily historical data for coal and natural gas prices (January 1, 2019–March 31, 2021). Figure 2 depicts the results. As shown in the figure, energy-switching prices were negative around July 2020, implying that natural gas generation is more advantageous than coal combustion, with energy-switching occurring at a zero-carbon cost. The findings show that for a given coal price, a sufficiently low natural gas price can influence enterprise switching behavior (Chevallier et al. 2019).

Figure 3 depicts the empirical distribution of the logarithmic return, $\log(M_t) - \log(M_{t-1})$ for energy-switching prices, including a fit to the Gaussian distribution of the data. Log returns for both trading pilots have negative skewness and a large positive kurtosis when combined with Table 1. This suggests that the log returns

[3] http://www.wind.com.cn/.

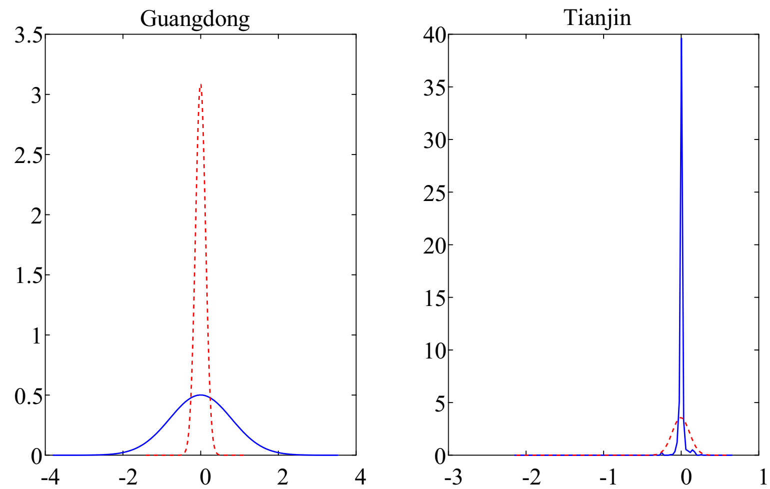

Fig. 3 Empirical distribution of energy-switching prices. Note: The solid line indicates the kernel estimate, and the dashed line indicates the Gaussian fit

Table 1 Overall estimates of logreturns for energy-switching prices

Pilots	Median	Kurtosis	Skewness	Min	Max
Guangdong	0	66.5196	−2.5328	−1.4306	1.1611
Tianjin	0	267.6933	−13.2927	−2.1390	0.6464

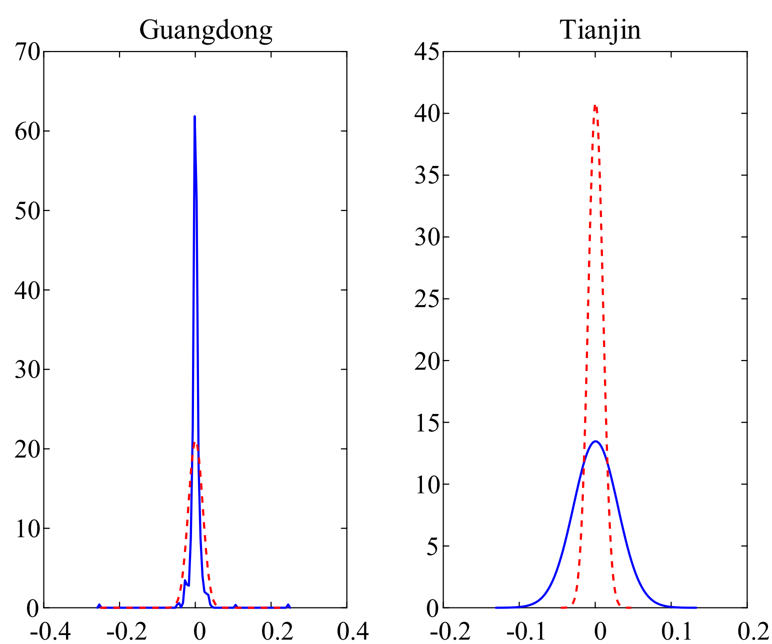

Fig. 4 Empirical distribution of daily trading prices of carbon allowances. Note: The solid line indicates the kernel estimate, and the dashed line indicates the Gaussian fit

Table 2 Overall estimates of the logarithmic returns to carbon-trading prices

Pilots	Median	Kurtosis	Skewness	Min	Max
Guangdong	0.000259	116.4615	− 0.2060	− 0.2532	0.2447
Tianjin	0	11.9994	− 0.1017	− 0.0458	0.0480

are skewed, excessively kurtic, and thick-tailed. Therefore, we depicted the dynamics of energy-switching prices in the fractal market using the fractional Brownian motion model (12), which is characterized by "sharp peaks and thick tails."

Figure 4 shows the empirical distribution in the logarithm changes, $\log(C_t) - \log(C_{t-1})$, in CA prices during the historical period[4] (January 1, 2019–March 31, 2021) for the pilots (Guangdong and Tianjin). Table 2 shows the overall statistics of the log returns to trading prices. Guangdong and Tianjin show negative skewness, and the log returns on trading prices in both pilots are greater than 0. We conclude that the log curves have skewness, excessive kurtosis, and thick tails, which are consistent with the characteristics of fluctuating aggregation and spikes in financial time series (Zhou and Li 2019). As a result of the preceding process, we can deduce a mixed fractional Brownian motion model, combining mutually independent fractional Brownian motion and standard Brownian motion, to depict the logarithmic variation of carbon quota trading prices.

RV method[5] for parameter estimation

The following are the two main steps in the development of parameter estimation. First, R/S analysis (Liu and Huang 2021) was used to estimate the Hurst exponent values. The volatility coefficients were then estimated using the quadratic variance method.

Step 1 R/S method.

Let $X_t = \log(M_t / M_0)$ be the logarithmic price and $Y_l = X_{(l+1)\Delta t} - X_{l\Delta t}, l = 1,2,\ldots,N-1$. Partition a set of time series data $\{Y_l\}$ into K equal-length subintervals of length n. For the kth ($k = 1, 2, \ldots, K$) subinterval, define the extreme difference:

$$R_k = \max_{1 \le j \le n} \sum_{i=1}^{j} \left(Y_{i,k} - \overline{Y}_{i,k}\right) - \min_{1 \le j \le n} \sum_{i=1}^{j} \left(Y_{i,k} - \overline{Y}_{i,k}\right), \tag{25}$$

where $\overline{Y}_{i,k}$ is the arithmetic mean for the data of the kth interval. Define the variance of the kth ($k = 1, 2, \ldots, K$) interval data as

$$S_k^2 = \frac{1}{n} \sum_{i=1}^{n} \left(Y_{i,k} - \overline{Y}_{i,k}\right)^2, \tag{26}$$

The estimated quantity $Q_k^{(HM)} = R_k / S_k$ is the R/S statistic of the Hurst–Mandelbrot's rescaled extreme deviation. Averaging all K such R/S statistics, we obtain

[4] Data were collected from http://www.tanjiaoyi.com/.

[5] We took the initials for the methods of the estimated Hurst exponent and volatility.

Table 3 Estimated parameter values

Pilots	*H*	*σ*
Guangdong	0.5857	0.1700
Tianjin	0.6456	0.1517

$$Q_n = \frac{1}{K}\sum_{k=1}^{K} Q_k^{(HM)}, \tag{27}$$

Since the length n of the subinterval varies, different segmentation cases correspond to different subinterval lengths, and the mean of the corresponding R/S statistic also varies. Long-term empirical practice has shown that

$$Q_n = Cn^H, \tag{28}$$

where C is a constant. Taking the logarithm of both sides of the above equation, we obtain

$$\log(Q_n) = H\log(n) + \log(C). \tag{29}$$

An estimate of the Hurst exponent can be obtained using the least-squares method.

Step 2 Quadratic variance estimation for the volatility coefficient.

Based on the estimated value of the Hurst exponent, an estimate of σ^2 is obtained by the quadratic variance estimation method, as follows:

$$\hat{\sigma}^2 = \frac{\sum_{l=0}^{N-1}\left(X_{(l+1)\Delta t} - X_{l\Delta t}\right)^2}{N(\Delta t)^{2H}}. \tag{30}$$

For fractional Brownian motion process,

$$\left(X_{(l+1)\Delta t} - X_{l\Delta t}\right)^2 = \sigma^2(B_H((l+1)\Delta t) - B_H(l\Delta t)) + o\left((\Delta t)^{2H}\right). \tag{31}$$

Therefore, when $N \to \infty$, $\Delta t \to 0$, we have

$$E\left(\hat{\sigma}^2\right) = E\left(\frac{\sum_{i=0}^{n-1}\left(X_{(i+1)\Delta t} - X_{i\Delta t}\right)^2}{n(\Delta t)^{2H}}\right) \to \sigma^2. \tag{32}$$

We can see from Eq. (32) that the quadratic variance estimate $\hat{\sigma}^2$ is an asymptotic unbiased estimate of σ^2.

Parameter estimation results

Based on the historical data for energy-switching prices of the two carbon-trading pilots, two sets of 510 data each were finally filtered, thus, $N=510$. Let the length n of subintervals be the values {5, 6, 10,…,102}; then, the number of corresponding subintervals K is {102, 85, 51,…,5}. The estimates of the parameters H and σ can be obtained using Eqs.

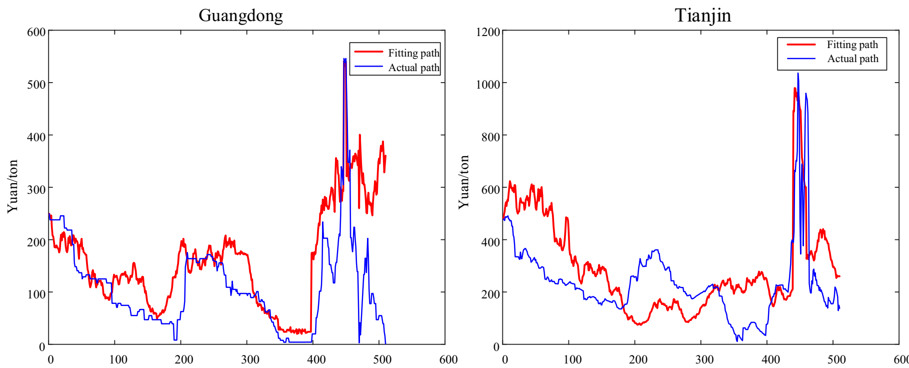

Fig. 5 Fitting for energy-switching prices

(25–32); the results are shown in Table 3. Both carbon-trading pilots have Hurst exponents greater than 0.5, indicating that energy-switching prices are long-term dependent.

Based on Eq. (22) for the energy-switching price at any point in time *t*, the estimated values in Table 3 are regarded as simulation parameters of *H* and *σ*. The risk-free interest rate *r* is calculated as the average of 0.0252 national bond yields over the previous year (October 28, 2020–October 27, 2021). The path in energy-switching prices is fitted using the Monte Carlo method, and compared with the actual trajectory (Fig. 5). The fractional Brownian motion trajectory simulated using the estimated parameter values has a similar trend to the actual path, as shown in Fig. 5. The fitted trajectory is more undulating and fluctuates more frequently than the original trajectory. The bias between the two paths is due to the annual average of national bond yields over the past year being the risk-free rate.

Numerical simulation and analysis of results

In China, the thermal power sector is a major source of carbon emissions. In the short term, the majority of enterprises participating in CA trading will be in the thermal power sector. Using that sector as an example, we ran scenario simulations using the stochastic optimal trading and abatement models developed in the previous section, as well as the stochastic equilibrium model of carbon prices.

Parameter settings

Initial carbon quota setting

To promote high-quality, sustainable socioeconomic development, China must maintain a stable medium- and long-term growth trend in gas power while ensuring that coal power serves as regulatory support and power transfer in the power system. Therefore, we assumed a 10% share for gas-fired power generation in 2040 based on the IEA's (IEA 2017) forecast for China's energy generation share and calculated the average growth rate of the share between 2020 and 2040 from the share for 2020.

In a high-electrification scenario (47–80 billion tons), Zhang et al. (2021b) budgeted CO_2 emissions from China's coal-power sector from 2019 to 2050.We set the CO_2 emissions from coal power in the thermal power industry for 2021–2050 based on the carbon peak time node, with a 2 °C temperature control target. Thus, using MATLAB, we ran a

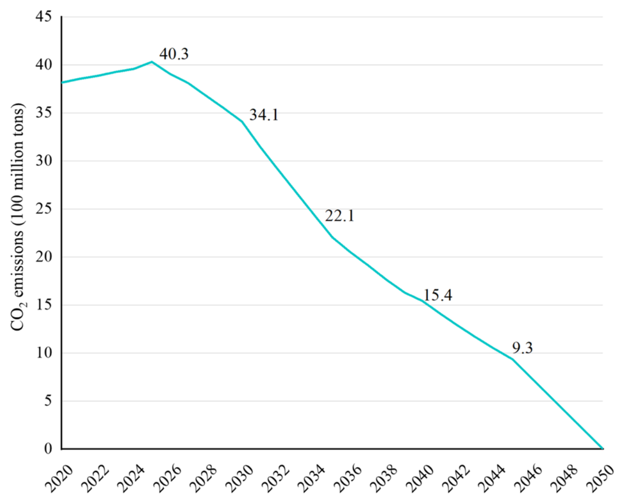

Fig. 6 CO_2 emission budget for the thermal power sector from 2020 to 2050 under a 2 °C target

simulation that inversely performed the proportion of coal generation (which is guaranteed to be less than 40% in 2040 as a function of time t) for the period 2021–2050:

$$\begin{aligned}\text{Annual emissions } &= \text{electricity generation } \times \text{ coal power carbon intensity}\\ &= \text{annual electricity demand } \times \text{ coal power share } \times \text{ carbon intensity.}\end{aligned}$$

Assuming the share of gas power grows from 2040 to 2050 at the average growth rate of 2020 to 2040, CO_2 emissions from energy consumed by the thermal power industry can be projected. An annual smoothing curve of CO_2 emissions from the thermal power industry for the period 2021–2050, as shown in Fig. 6, could be fitted. Total CO_2 emissions peak around 2025 at 4.03 billion tons, followed by a sharp decline to achieve carbon neutrality around 2050. CO_2 emissions in 2040 are used as proxy variables for the initial CAs under the conditions of meeting electricity demand[6], temperature control targets, and energy generation share.

Other parameter settings

(1) To achieve the carbon peak target by 2030 in China, we take 2021–2030 as a compliance period (i.e., $T=10$).
(2) In the BAU scenario, cumulative emissions from the thermal power sector are calculated by multiplying the energy generation share in 2005, the energy carbon intensity, and total electricity demand in the corresponding year.
(3) The depreciation factor of the abatement equipment is taken as $\delta=0.2$ (Xu and Zhang 2020).
(4) The fluctuation factor of the CO_2 emissions reduction required for the thermal power industry is taken as $\sigma^I=0.02$ (Carmona et al. 2009).

[6] According to the "Outlook", for the continuity of the numerical simulation curves, the average annual growth rate of electricity demand for 2021–2030 is taken to be 3.6% on average for 2020–2025 and 2025–2030, and the average annual growth rate of electricity demand for 2030–2050 is taken to be 2%.

Table 4 Values for all parameters

Parameter symbols	Parameters/Units	Parameter values
T	Compliance period (*year*)	10
Pc_0	Initial coal price (*Yuan/ton*)	454.17
Pg_0	Initial natural gas price (*Yuan/m*^{3})	2.41
h_c	Coal thermal efficiency (t_{coal}*/MWh*)	0.378
h_g	Natural gas thermal efficiency (t_{gas}*/MWh*)	101.158
e_c	CO_2 emission intensity for coal power (t_{CO2}*/MWh*)	0.897
e_g	CO_2 emission intensity for gas power (t_{CO2}*/MWh*)	0.388
η_c	Generation efficiency for coal power	0.311
η_g	Generation efficiency for gas power	0.326
r	Risk-free interest rate	0.0252
H	Hurst exponent	0.6456
σ	Volatility coefficient for energy-switching prices	0.1517
δ	Depreciation rate of abatement equipment	0.20
σ^l	Volatility coefficient for emission-reduction demand	0.02

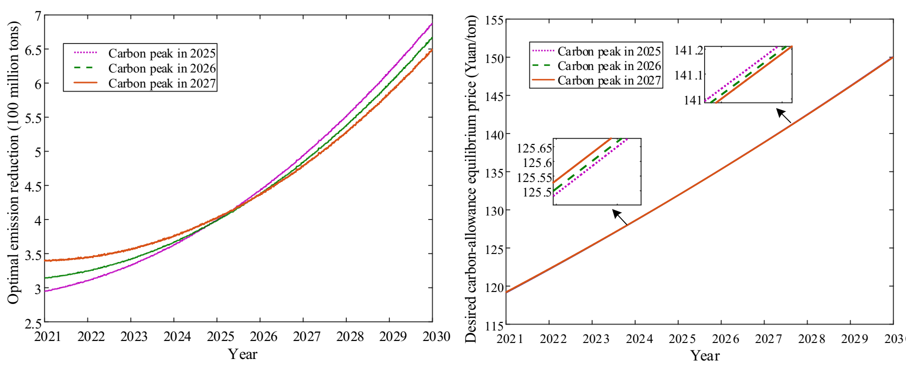

Fig. 7 Optimal emission reductions and desired carbon-allowance equilibrium prices for scenarios of different peak years

(5) The average values of Tianjin's power coal and LNG prices in 2020 are taken as the proxies of initial coal and natural gas prices for China, which are $P_0^c = 454.17\ Yuan/ton$ and $P_t^g = 2.41\ Yuan/\mathrm{m}^3$, respectively.

(6) The volatility coefficient, $\sigma = 0.1517$, and the Hurst exponent, $H = 0.6456$, for energy-switching prices of the thermal power industry in Tianjin are taken as proxy parameters for China.

Energy carbon-emission intensity, energy generation efficiency, and thermal efficiency are all taken the same values as described in "Parameter estimation for the stochastic model" section. The values of all of the parameters are shown in Table 4.

Scenario simulation

We set three different peaking time points for 2025, 2026, and 2027, assuming a constant carbon-peak value for coal power. We obtained the trend in optimal emission reductions and the desired equilibrium prices of CAs in the thermal power industry annually during the compliance period (2021–2030) using scenario simulations under different peak year scenarios (Fig. 7).

In Fig. 7, both optimal emission reductions and desired equilibrium CA prices show an annual increasing trend in different carbon-peak years. This is primarily due to the fact that the abatement strategy causes economic fluctuations. To alleviate the impact of this shock, the government will encourage thermal power companies to make small abatement payments in the beginning and then gradually increase abatement levels. The equilibrium price of CAs is equal to the current cost of the cheapest emission-reduction strategy, and during the abatement process, businesses typically choose the lower-cost reduction technology. Following full implementation of the low-cost emission-reduction strategy, enterprises select the next low-cost emission-reduction technology, resulting in an annual increase in the equilibrium prices of carbon trading (Chevallier et al. 2019). Moreover, in the early stages of the compliance period, the later the peak time, the higher the corresponding optimal emission reductions and CA equilibrium prices. This is primarily due to the fact that, in the early stages, the later the peak time is reached, the greater the pressure on businesses to reduce emissions. Then, they increase emission reductions, which raises abatement costs; thus, the equilibrium prices of CAs also rise. In the latter stages of the compliance period, the opposite is true. Overall, the difference in peak time point has little effect on optimal emission reductions and CA equilibrium prices. This is primarily due to a small change in the peak CO_2 emissions of different years to ensure that the 2 °C temperature control target could be met under high electrification levels.

Parameter sensitivity analysis

Initial CAs influence the change in optimal emission reductions, whereas CA equilibrium prices are affected by factors such as the Hurst exponent and volatility for energy-switching prices, abatement equipment depreciation, and initial carbon quotas. This section focused on the impact of changes in these parameters on optimal emission reductions and CA equilibrium prices.

Effect of the Hurst exponent on the equilibrium price of carbon allowances

In the previous section, we established that energy-switching prices exhibit long-run dependence. The Hurst exponent was set to 0.6456 by default. To investigate the effect of the Hurst exponent on the equilibrium price of CAs, we chose the values 0.5165 and 0.7747 after 20% fluctuations up and down from the default value.

As illustrated in Fig. 8, as the Hurst exponent increases, the curve for the CA equilibrium price oscillates less and the trajectory becomes smoother, while all other parameters remain constant. This is primarily determined by the fractional Brownian motion's long-term dependence. If the equilibrium price of carbon quota trading rises (falls) in the first period, it is more likely to rise (fall) in the second, and the larger the Hurst exponent, the stronger the long-term dependence of fractional Brownian motion. Thus, in the late compliance period, the corresponding equilibrium price with a larger Hurst exponent falls more noticeably. The main factor influencing the Hurst exponent is energy price, which also implies that energy price influences the equilibrium prices of CAs indirectly.

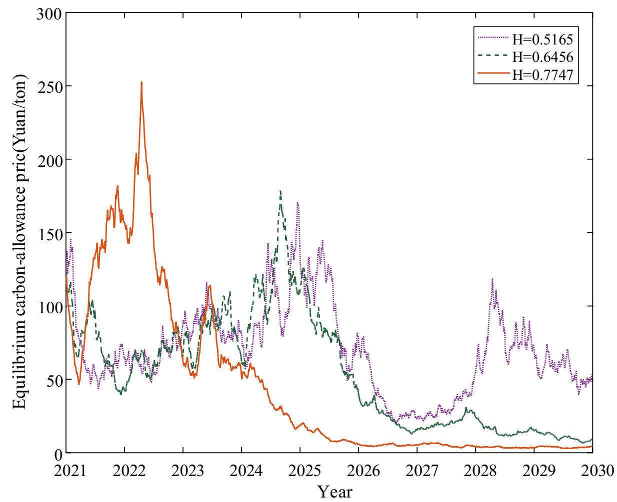

Fig. 8 Equilibrium prices of carbon allowances corresponding to different Hurst exponents

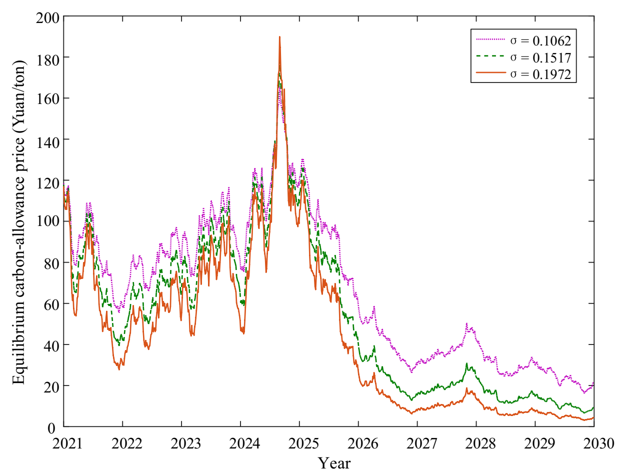

Fig. 9 Equilibrium prices of carbon allowances corresponding to different volatility coefficients

Effect of volatility coefficient for energy-switching prices on the equilibrium price of carbon allowances

Volatility was set to 0.1517 by default. For the sensitivity analysis of the equilibrium price of CAs, the values 0.1062 and 0.1972 after 30% up and down fluctuations from the default value were chosen.

For the majority of the compliance period, the equilibrium price of CAs decreases with increasing volatility, as shown in Fig. 9. The more volatile the market, the more significantly the curve fluctuates. Occasionally, the equilibrium prices of CAs under the three volatilities are approximately equal. This is mainly because when volatility rises, so does the opportunity for energy-switching prices to rise or fall, causing the equilibrium carbon-trading price to change in the same direction; these two outcomes tend to cancel each other out (Hull 2018). As a result, around the peak time point of 2025, the equilibrium price of CAs with high volatility converges to the case with low volatility.

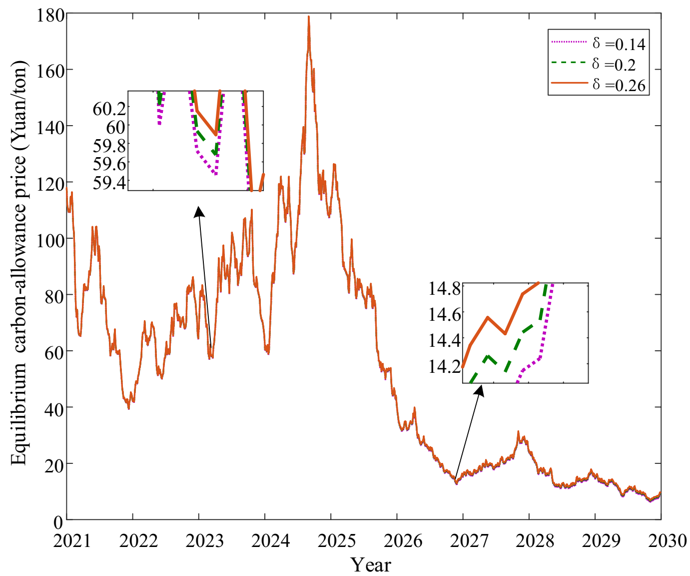

Fig. 10 Equilibrium prices of carbon allowances with different depreciation rates of carbon abatement equipment

Effect of depreciation rate of abatement equipment on the equilibrium price of carbon allowances

The depreciation rate of the enterprises' abatement equipment was set to 0.20 by default. For the sensitivity analysis of the equilibrium price of CAs, values of 0.14 and 0.26 were chosen after 30% up and down fluctuations from the default value.

As illustrated in Fig. 10, the higher the depreciation rate of abatement equipment, the higher the equilibrium price of CAs. This is primarily because a higher depreciation rate raises the variable cost of obtaining one unit of electricity using clean equipment, which raises the variable cost of reducing one unit of emission, causing the marginal abatement cost to rise. However, due to the small depreciation rate value, the overall change is not exceptionally large, and the 30% change in value is also small.

Effect of initial carbon allowances on optimal carbon-emission reductions and the equilibrium price of carbon allowances

We investigated the sensitivity of carbon-emission reductions and equilibrium prices to changes in initial CAs by varying the percentage of energy generation target so that the initial CA fluctuated by 30% up and down.

With all other parameters held constant, Fig. 11*a* shows that the higher the initial CAs in each year, the lower the optimal carbon emission reduction for the thermal power industry. This is primarily because with more initial carbon quotas, more are allocated to each thermal power enterprise, which reduce the pressure on enterprises to lower emissions, and they will not reduce carbon emissions by replacing coal power with gas power. Thus, the thermal power industry's optimal carbon-emission reduction will be reduced accordingly.

As shown in Fig. 11*b*, when the initial carbon quota increases each year, the equilibrium price of carbon quotas for thermal power enterprises decreases slightly. On

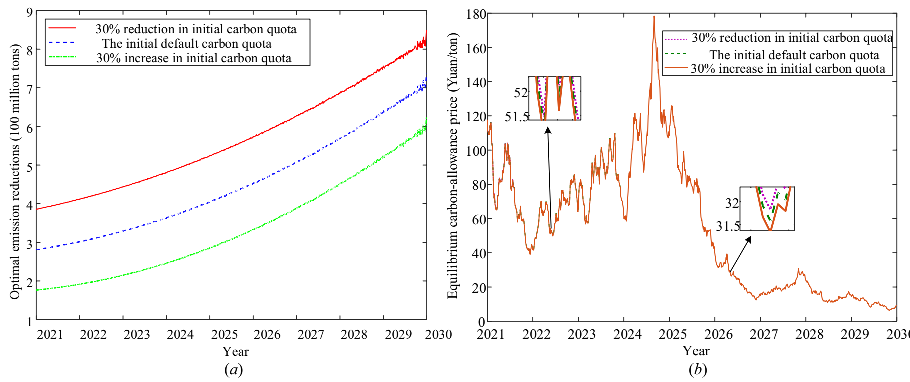

Fig. 11 Optimal carbon-emission reductions and equilibrium prices of carbon allowances for different initial carbon quotas

the one hand, as initial carbon quotas increase, more are granted to individual thermal power enterprises. As the number of tradable quotas in the carbon-trading market grows, the equilibrium price of carbon quota trading falls. On the other hand, with more initial carbon quotas, there is less pressure on each thermal power enterprise to reduce emissions, less initial investment in clean-energy equipment, and more CO_2 released by power generation. Then, enterprises will need to buy more carbon allowances through the carbon market to offset CO_2 emissions. Additionally, investment in clean-energy power generation equipment is relatively large, businesses typically choose to purchase CAs to reduce emissions, eventually raising the equilibrium price of CAs. Under the combined effect of these factors, the equilibrium prices of the carbon market, in which thermal power companies participate, will eventually decrease only slightly.

We draw the following conclusions using numerical simulations.

Despite different carbon peak years for the thermal power industry, the industry's marginal abatement cost will show a low and then high increase trend, as other industries are closely related to the thermal power industry, and the government will inevitably choose to reduce emissions at a low cost first to avoid affecting national economic development. Therefore, both the optimal emission reductions and the desired equilibrium trading prices in the carbon market show an increasing trend over time.

The greater the Hurst exponent, the smoother the equilibrium trading price curve. An increase in the volatility coefficient for energy-switching prices lowers the equilibrium price of CAs, but it also creates a situation similar to low volatility. Increased depreciation of an enterprise's abatement equipment raises the enterprise's marginal abatement cost, raising the equilibrium carbon-trading price. An increase in initial CAs reduces optimal carbon-emission reductions and lowers thermal power companies' equilibrium trading prices. With higher initial CAs, the required emission reductions during the compliance period will be lower, as will the optimal emission reductions. The supply and demand for CAs influence the equilibrium trading price. The simulation results show that supply has a greater impact than demand, causing the equilibrium trading price to fall slightly overall.

Conclusions

Existing studies on the optimal emission reduction and the equilibrium price of CAs for thermal power enterprises are relatively deficient based on a comprehensive consideration of stochastic energy-switching and CA trading prices. Focused on the goal of achieving carbon peak, this study took into account the CA trading cost and abatement cost of energy generation and built a stochastic optimization model with the total enterprise compliance cost minimized. The stochastic optimization model was transformed into a solvable HJB partial differential equation combining the fractional Itô's formula and the dynamic optimization principle. By solving this equation, we obtained the optimal trading and emission-reduction volumes for a single enterprise. Furthermore, the optimal volume of emission reductions and the equilibrium price of CAs for the entire society were determined. Furthermore, using thermal power companies that participate in the national carbon market as an example, we ran scenario simulations and parameter sensitivity analyses. The following are the study's main findings.

First, an empirical distribution of historical data on energy-switching prices in Guangdong and Tianjin revealed "peaks and tails" in the prices. Thus, we chose the fractional Brownian motion model to describe the dynamic changes, and we deduced that CA prices follow the stochastic differential equation model of mixed fractional Brownian motion. The empirical distribution of historical CA price data validated the model's reasonableness. The comparison of the parameter fitting results with real data revealed that the fitted trajectory was similar to the real one, confirming the estimation method's accuracy.

Second, the scenario simulations revealed that the equilibrium price of desired CAs and the optimal emission reductions of thermal power enterprises under different peak years had the same annual trend throughout the compliance period. The later the peak time in the early part of the compliance period, the higher the equilibrium price and optimal emission reduction. The trend reversed in the latter part of the compliance period. This finding indicated that the increasing CA prices would compel thermal power companies to promote emission reductions.

Third, the larger effect of the Hurst exponent determined by energy-switching prices on CA equilibrium prices suggested that the magnitude of energy price changes could significantly affect CA equilibrium prices. An increase in the volatility coefficient could result in a situation where the chances of an increase or decrease in the equilibrium prices of CAs cancel each other out, with the prices eventually convergent to those of low volatility.

Finally, the adjustment to the initial CA clearly had a negative impact on optimal emission reduction. It had a two-way effect on CA equilibrium prices, with the offset having a negative but insignificant effect. The depreciation rate also had a minimal effect on the equilibrium price of CAs.

Policy implications

Our findings make theoretical and practical implications for policy makers. First, the Chinese government should strengthen macrocontrol and appropriately raise the CA price in the carbon market to encourage enterprises to reduce their emissions. In particular, the government may tighten quota supply in order to return the CA price to a reasonable range, increasing the cost of external carbon purchases by thermal power enterprises. As a result, power companies may be forced to take measures such as increasing investment in emission-reduction technologies, improving research and development and application of advanced technologies, and increasing the proportion of clean-energy generation to meet actual electricity demand. This could encourage thermal power enterprises to transition from high-emission coal-fired to green and low-carbon energy sources in order to meet the emission reduction target. Furthermore, the competent authorities could adjust the rules of the carbon offset mechanism flexibly in response to actual market conditions in order to regulate the carbon trading price.

Second, in order to avoid large changes in carbon quota prices and improve the energy market's transmission mechanism to carbon quota prices, the government should macro-regulate energy prices using market-based instruments. This study discovered that fluctuations in energy prices could affect the Hurst exponent, significantly influencing carbon quota prices. Natural gas is priced by the government in China and is therefore insensitive to international market forces; as a result, the government should implement a reasonable pricing method based on market value and gradually link natural prices to more market-oriented alternative energy prices around the world. Furthermore, the government could adjust the natural gas guiding price based on supply and demand in the energy market to deepen the natural gas price reform. Moreover, these measures could prevent gas speculation, avoid market disruption and protect national energy security to stabilize energy-switching costs between coal and natural gas.

Third, the government should develop a reasonable initial CA allocation scheme to help businesses meet their carbon-reduction targets. This study found that initial CAs had a significant impact on thermal power enterprises' carbon-emission reduction. The national carbon emissions trading market is in its early stages, and the allocation system of the carbon emission rights is relatively lax. As a result, the government should establish clear carbon-emission targets for thermal power companies. Initial carbon quotas could be allocated to individual enterprises based on targets in order to mitigate shocks to enterprise production behavior and solve problems of fair competition among enterprises and carbon-market efficiency, motivating enterprises to participate in carbon market trading.

Limitations and suggestions for future research

This study has some limitations. Because the national carbon market was not officially launched for an extended period of time, there is a scarcity of time-series CA price data. This study could only use the Hurst index and the volatility parameter estimated from historical data on natural gas and coal prices in the pilot city of Tianjin as proxy variables for these two parameters in the national energy-switching price model to be consistent with the operating provinces and cities in the carbon market. Notably, future updates to the CA price data on the national carbon trading market will remove this impediment. Furthermore, a jump-diffusion fractional Brownian motion model will be considered in the future to describe the dynamics of the coal-to-gas conversion cost caused by international contingencies, such as epidemics and wars. Furthermore, renewable energy power generation is an efficient way to meet carbon reduction targets and promote sustainable development in the power industry. The conversion of thermal energy to renewable energy sources, such as wind power and photovoltaics, could be a promising research area.

Appendix A

Derivation for carbon-allowance prices to satisfy the dynamic process

For any enterprise i, the emission reduction $\alpha_t^i = \alpha\left(t, x_t^i, C_t, P_t\right)$ and trading strategy $\beta_t^i = \beta\left(t, x_t^i, C_t, P_t\right)$ shall be a function of the state process $Z\left(x_t^i, C_t, M_t\right)$. For convenience, we define $g_t = \frac{re^{rt}}{e^{rT} - e^{rt}}$. For any thermal power company, let its trading and abatement strategies be given by

$$\alpha_t^i = \frac{\delta}{\delta + 2\upsilon} g_t x_t^i + \frac{M_t - C_t}{\delta + 2\upsilon},\ \beta_t^i = \frac{2\upsilon}{\delta + 2\upsilon} g_t x_t^i + \frac{C_t - M_t}{\delta + 2\upsilon}. \tag{A.1}$$

$\alpha_t^I = 0$ yields

$$C_t = M_t + \delta g_t x_t^I. \tag{A.2}$$

Substituting equation (A.1) into Eq. (4), we get the dynamics for the process x_t^i:

$$\mathrm{d}x_t^i = -\frac{re^{rt}}{e^{rT} - e^{rt}} x_t^i \mathrm{d}t + \sigma^i \mathrm{d}W_t. \tag{A.3}$$

Solving for Eq. (A.3), we obtain

$$x_t^i = \frac{e^{rT} - e^{rt}}{e^{rT} - 1} x_0^i + \left(e^{rT} - e^{rt}\right) \int_0^t \frac{\sigma^i}{e^{rT} - e^{rs}} \mathrm{d}W_s. \tag{A.4}$$

Integrating the above equation with respect to i over I yields, together with Eq. (A.2), we have

$$\mathrm{d}C_t = rC_t \mathrm{d}t + \sigma M_t \mathrm{d}B_H^Q(t) + \delta g_t \sigma^I \mathrm{d}W_t. \tag{A.5}$$

Appendix B

Proof for the theorem 1

Proof Theorem 1 can be proven using the method of undetermined coefficients. We observe that the optimal value function can be given by.

$$\omega(t, x, c, m) = A_1(t)x^2 + A_2(t)c^2 + A_3(t)m^2 + B_1(t)cx + B_2(t)mx + B_3(t)cm + D(t).$$

From the boundary condition, we know that.

$$\lim_{t \to T} \frac{1}{A_1(t)} = \lim_{t \to T} \frac{1}{B_1(t)} = \lim_{t \to T} \frac{1}{B_2(t)} = 0, A_2(T) = A_3(T) = B_3(T) = D(T) = 0.$$

Substituting $D_t\omega, D_c\omega, D_m\omega, D_x\omega, D_c^2\omega, D_m^2\omega$ and $D_x^2\omega$ into Eq. (16), we obtain

$$\begin{cases} A_1^{'}(t) = \dfrac{\delta+2\nu}{\nu\delta}e^{rt}A_1^2(t), B_1^{'}(t) + \left(r - \dfrac{\delta+2\nu}{\nu\delta}e^{rt}A_1(t)\right)B_1(t) = -\dfrac{1}{\nu}A_1(t), \\ B_2^{'}(t) + \left(r - \dfrac{\delta+2\nu}{\nu\delta}e^{rt}A_1(t)\right)B_2(t) = -\dfrac{2}{\delta}A_1(t), \\ A_2^{'}(t) + 2rA_2(t) = \dfrac{\delta+2\nu}{4\nu\delta}B_1^2(t)e^{rt} - \dfrac{1}{2\nu}B_1(t) + \dfrac{1}{4\nu}e^{-rt}, \\ B_3^{'}(t) + 2rB_3(t) = \dfrac{\delta+2\nu}{2\nu\delta}e^{rt}B_1(t)B_2(t) - \dfrac{1}{2\nu}B_2(t) - \dfrac{1}{\delta}B_1(t), \\ A_3^{'}(t) + \left(2r + 2Ht^{2H-1}\sigma^2\right)A_3(t) = -2A_2(t)Ht^{2H-1}\sigma^2 - B_3(t)Ht^{2H-1}\sigma^2 \\ +\dfrac{\delta+2\nu}{4\nu\delta}e^{rt}B_2^2(t) - \dfrac{1}{2\delta}e^{-rt} + \dfrac{1}{\delta}B_2(t), \\ \lim\limits_{t\to T}\dfrac{1}{A_1(t)} = \lim\limits_{t\to T}\dfrac{1}{B_1(t)} = \lim\limits_{t\to T}\dfrac{1}{B_2(t)} = 0, A_2(T) = A_3(T) = B_3(T) = D(T) = 0. \end{cases} \tag{B.1}$$

Solving the above expression yields

$$\begin{aligned} &A_1(t) = \frac{r\nu\delta}{(\delta+2\nu)\left(e^{rT}-e^{rt}\right)}, B_1(t) = \frac{\delta e^{-rt}}{\delta+2\nu}, B_2(t) = \frac{2\nu e^{-rt}}{\delta+2\nu}, \\ &A_2(t) = \frac{-e^{-2rt}\left(e^{rT}-e^{rt}\right)}{2r(\delta+2\nu)}, B_3(t) = \frac{e^{-2rt}\left(e^{rT}-e^{rt}\right)}{r(\delta+2\nu)}, \\ &D(t) = \int_t^T \frac{r\delta^2\left(\sigma^I\right)^2 - 2r\nu\delta\left(\sigma^i\right)^2 - r\delta^2\sigma^i\sigma^I}{2(\delta+2\nu)\left(e^{rT}-e^{rs}\right)}\mathrm{d}s, \end{aligned} \tag{B.2}$$

$$A_3(t) = ce^{-2rt-\sigma^2t^{2H}} + \frac{1}{2\delta(\delta+2\nu)}e^{-2rt-\sigma^2t^{2H}}\int e^{2rt+\sigma^2t^{2H}}\mathrm{d}t. \tag{B.3}$$

To simplify the notation, we define $\Phi(t) = \int e^{2rt+\sigma^2t^{2H}}\mathrm{d}t$, then $C = -\frac{1}{2\delta(\delta+2\nu)}\Phi(T)$. Substituting Eqs. (B.2) and (B.3) into the optimal value function deduces the minimization cost for the enterprise i.

Abbreviations

CA	Carbon allowance
GEIDCO	Global Energy Internet Development Cooperation Organization
EU	European Union
LNG	Liquefied Natural Gas
ETS	Emission Trading Scheme
MWh	Megawatt hour
HJB	Hamilton–Jacobi–Bellman
BAU	Basic as usual

Acknowledgements
The authors would like to express their sincere gratitude to the editors and anonymous reviewers for their valuable comments and suggestions.

Author contributions
JS: Conceptualization, Methodology, Software, Data collection and processing, Writing—original draft. FD: Conceptualization, Supervision, Funding acquisition, Validation, Writing—reviewing & editing. Both the authors read and approved the final manuscript.

Funding
We would like to thank Major Program of National Philosophy and Social Science Foundation of China (Grant No. 21ZDA086), National Natural Science Foundation of China (Grant No. 71974188), and Jiangsu Soft Science Fund (Grant No. BR2022007).

Availability of data and materials
The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Competing interests
The authors declare that they have no competing interests.

Received: 30 January 2022 Accepted: 23 October 2022
Published online: 08 January 2023

References

Alòs E, León JA (2021) An intuitive introduction to fractional and rough volatilities. Mathematics 9(9):0994
Arrigoni A, Lu WL, Swishchuk A, Goutte S (2019) Energy-switching using Lévy processes-an application to Canadian and north American data. Available at SSRN: http://dx.doi.org/https://doi.org/10.2139/ssrn.3408174
Batten JA, Maddox GE, Young MR (2021) Does weather, or energy prices, affect carbon prices? Energy Econ 96:105016
Benz E, Trück S (2009) Modeling the price dynamics of CO_2 emission allowances. Energy Econ 31(1):4–15
Brutschin E, Fleig A (2016) Innovation in the energy sector—the role of fossil fuels and developing economies. Energy Policy 97:27–38
Cao QR, Kang W, Sajid MJ, Cao M (2021) Research on the optimization of carbon abatement efficiency in China on the basis of task allocation. J Clean Prod 299:126912
Carmona R, Fehr M, Hinz J (2009) Optimal stochastic control and carbon price formation. SIAM J Control Optim 48:2168–2190
Chen H, Geng HP, Ling HT, Peng S, Li N, Yu SW, Wei YM (2020) Modeling the coal-to-gas switch potentials in the power sector: a case study of China. Energy 192:116629
Cheng R, Xu ZF, Liu P, Wang Z, Li Z, Jones I (2015) A multi-region optimization planning model for China's power sector. Appl Energy 137:413–426
Chevallier J (2009) Carbon futures and macroeconomic risk factors: a view from the EU ETS. Energy Econ 31:614–625
Chevallier J, Goutte S (2017) Estimation of Lévy-driven Ornstein–Uhlenbeck processes: application to modeling of and fuel-switching. Ann Oper Res 255:169–197
Chevallier J, Nguyen DK, Reboredo JC (2019) A conditional dependence approach to CO_2-energy price relationships. Energy Econ 81:812–821
Dong F, Li YF, Zhang XY, Zhu J, Zheng L (2021a) How does industrial convergence affect the energy efficiency of manufacturing in newly industrialized countries. Fresh evidence from China. J Clean Prod 316:128316
Dong F, Pan YL, Li YF, Zhang SQ (2021b) How public and government matter in industrial pollution mitigation performance: evidence from China. J Clean Prod 306:127099
Dong F, Li YF, Gao YJ, Zhu J, Qin C, Zhang XY (2022a) Energy transition and carbon neutrality: exploring the non-linear impact of renewable energy development on carbon emission efficiency in developed countries. Resour Conserv Recycl 177:106002
Dong F, Zhu J, Li YF, Chen YH, Gao YJ, Hu MY, Qin C, Sun JJ (2022b) How green technology innovation affects carbon emission efficiency: evidence from developed countries proposing carbon neutrality targets. Environ Sci Pollut Res 29:35780–35799
Duan K, Ren XH, Shi YK, Mishra T, Yan C (2021) The marginal impacts of energy prices on carbon price variations: evidence from a quantile-on-quantile approach. Energy Econ 95:105131
GEIDCO (2021) Energy and power development plan for 2030 and outlook for 2060. Accessed from https://en.geidco.org.cn/2021/0322/3277.shtml
Guo LY, Feng C (2021) Are there spillovers among China's pilots for carbon emission allowances trading? Energy Econ 103:105574
Hull JC (2018) Properties of stock options. Options, futures, and other derivatives. Pearson, New York, pp 232–233
IEA (2017) World Energy Outlook 2017: China. Accessed from https://www.iea.org/reports/world-energy-outlook-2017-china
IEA (2021) IEA Energy and Carbon Tracker 2021. Accessed from https://www.iea.org/data-and-statistics/data-products
Ioannou A, Fuzuli G, Brennan F, Yudha SW, Angus A (2019) Multi-stage stochastic optimization framework for power generation system planning integrating hybrid uncertainty modelling. Energy Econ 80:760–776
Jin JL, Peng Z, Li CY, Guo XJ, Zhang MM (2019) Low-carbon power dispatch with wind power based on carbon trading mechanism. Energy 170:250–260
Kanen JLM (2006) Carbon trading and pricing. Environmental Finance Publications, London
Kang JN, Wei YM, Liu LC, Han R, Chen H, Li JQ, Wang JW, Yu BY (2020) The prospects of carbon capture and storage in China's power sector under the 2 °C target: a component-based learning curve approach. Int J Greenh Gas Control 101:103149
Kollenberg S, Taschini L (2016) Emissions trading systems with cap adjustments. J Environ Econ Manag 80:20–36

Koltsaklis NE, Dagoumas AS, Kopanos GM, Pistikopoulos EN, Georgiadis MC (2014) A spatial multi-period long-term energy planning model: a case study of the Greek power system. Appl Energy 115:456–482
Kumar P, Banerjee R, Mishra T (2020) A framework for analyzing trade-offs in cost and emissions in power sector. Energy 195:116949
Li TX, Liu P (2021) Li Z (2021) Optimal scale of natural gas reserves in China under increasing and fluctuating demand: a quantitative analysis. Energy Policy 152:112221
Liang J, Huang WL (2020) Optimal control strategy of companies: inheriting period and carbon emission reduction. Math Probl Eng 2020:3461747
Liang J, Huang WL (2021) Optimal control model of an enterprise for single and inheriting periods of carbon emission reduction. Math Financ Econ. https://doi.org/10.1007/s11579-021-00302-4
Liu YJ, Dong F (2021) How technological innovation impacts urban green economy efficiency in emerging economies: a case study of 278 Chinese cities. Resour Conserv Recycl 170:105616
Liu ZB, Huang S (2021) Carbon option price forecasting based on modified fractional Brownian motion optimized by GARCH model in carbon emission trading. N Am Econ Financ 55:101307
Lu WL, Arrigoni A, Swishchuk A, Goutte S (2021) Modelling of fuel- and energy-switching prices by mean-reverting processes and their applications to Alberta energy markets. Mathematics 9(7):709
Nie S, Huang ZC, Huang GH, Yu L, Liu J (2018) Optimization of electric power systems with cost minimization and environmental-impact mitigation under multiple uncertainties. Appl Energ 221:249–267
Pan YL, Dong F (2022a) Dynamic evolution and driving factors of new energy development: fresh evidence from China. Technol Forecast Soc 176:121475
Pan YL, Dong F (2022b) Design of energy use rights trading policy from the perspective of energy vulnerability. Energy Policy 160:112668
Prebeg P, Gasparovic G, Krajacic G, Duic N (2016) Long-term energy planning of Croatian power system using multi-objective optimization with focus on renewable energy and integration of electric vehicles. Appl Energy 184:1493–1507
Rivera NM, Loveridge S (2022) Coal-to-gas fuels witching and its effects on housing prices. Energy Econ 106:105733
Shen MH (2021) On the construction of carbon markets. China Popul Resour Env 31:86–91 **(in Chinese)**
Steinebach Y, Fernández-i-marín X, Aschenbrenner C (2020) Who puts a price on carbon, why and how? A global empirical analysis of carbon pricing policies. Clim Policy 21(3):277–289
Sun JJ, Dong F (2022) Decomposition of carbon emission reduction efficiency and potential for clean energy power: Evidence from 58 countries. J Clean Prod 363:132312
Tan XP, Wang XY (2017a) The market performance of carbon trading in China: a theoretical framework of structure-conduct-performance. J Clean Prod 159:410–424
Tan XP, Wang XY (2017b) Dependence changes between the carbon price and its fundamentals: a quantile regression approach. Appl Energ 190:306–325
Tang L, Wang HH, Li L, Yang KT, Mi ZF (2020) Quantitative models in emission trading system research: a literature review. Renew Sust Energ Rev 132:110052
Wang XP, Qie SY (2020) Research on the timing of CCS investment in supply chain under carbon emission trading. J Ind Eng Eng Manag 34(2):124–130 **(in Chinese)**
Weng QQ, Xu H (2018) A review of China's carbon trading market. Renew Sust Energ Rev 91:613–619
Wilson IAG, Staffell I (2018) Rapid fuel switching from coal to natural gas through effective carbon pricing. Nat Energy 3:365–372
Xu J, Zhang YX (2020) Thermal power enterprise carbon emission reduction strategy based on discrete time and its influence on the price of carbon emission rights. J Ind Eng Eng Manag 34:105–115 **(in Chinese)**
Xu J, Guo SR, Ren QC (2017) Enterprises carbon emission reduction strategy based on mixed energy. J Syst Eng 32:732–737 **(in Chinese)**
Yu BL, Fang DB (2021) Decoupling economic growth from energy-related PM2.5 emissions in China: a GDIM-based indicator decomposition. Ecol Indic 127:107795
Yu BL, Fang DB, Meng JX (2021) Analysis of the generation efficiency of disaggregated renewable energy and its spatial heterogeneity influencing factors: A case study of China. Energy 234:121295
Yu BL, Fang DB, Andrew NK, Xiao K (2022) Exploring the driving mechanism and the evolution of the low-carbon economy transition: Lessons from OECD developed countries. World Econ 45:2766–2795
Zhang XY, Dong F (2023) What affects residents' behavioral intentions to ban gasoline vehicles? Evidence from an emerging economy. Energy 263:125716
Zhang YF, Li S, Luo TY, Gao J (2020) The effect of emission trading policy on carbon emission reduction: evidence from an integrated study of pilot regions in China. J Clean Prod 265:121843
Zhang K, Yao YF, Liang QM, Saren G (2021a) How should China prioritize the deregulation of electricity prices in the context of carbon pricing? A computable general equilibrium analysis. Energy Econ 96:105187
Zhang WR, Zhou YO, Gong Z, Kang JJ, Zhao CH, Meng ZX, Zhang J, Zhang T, Yuan JH (2021b) Quantifying stranded assets of the coal-fired power in China under the Paris agreement target. Clim Policy. https://doi.org/10.1080/14693062.2021.1953433
Zhou KL, Li YW (2019) Influencing factors and fluctuation characteristics of China's carbon emission trading price. Physica A 524:459–474
Zhu BZ, Jiang MX, He KJ, Chevallier JL, Xie R (2018) Allocating CO_2 allowances to emitters in China: a multi-objective decision approach. Energy Policy 121:441–451

Publisher's Note