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Trade credit financing for supply chain coordination under financial challenges: a multi-leader–follower game approach

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Abstract

This study is designed to solve supply chain inefficiencies caused by some members' financial problems, such as capital shortages and financing restrictions in a stochastic environment. To this end, we have established a supply chain finance framework by designing two novel coordinating contracts based on trade credit financing for different problem settings. These contracts are modeled in the form of multi-leader Stackelberg games that address horizontal and vertical competition in a supply chain consisting of multiple suppliers and a financially constrained manufacturer. However, previous studies in the trade credit literature have addressed only simple vertical competition, that is, seller-buyer competition. To solve the proposed models, two algorithms were developed by combining population-based metaheuristics, the Nash-domination concept, and the Nikaido-Isoda function. The results demonstrate that the proposed supply chain finance framework can eliminate supply chain inefficiencies and make a large profit for suppliers, as well as the financially constrained manufacturer. Furthermore, the results of the contracts' analysis showed that if the manufacturer is required to settle its payments to suppliers before the end of the period, the trade credit contract cannot coordinate the supply chain because of a lack of incentive for suppliers. However, if the manufacturer is allowed to extend its payments to the end of the period, the proposed trade credit financing contract can coordinate the supply chain. Finally, the sensitivity analysis results indicate that the worse the financial status of the manufacturer, the more bargaining power suppliers have in determining the contract parameters for more profit.

Keywords: Supply chain coordination, Financial constraint, Multi-leader–follower Stackelberg game, Trade credit financing, Population-based metaheuristics

Introduction

Financial problems such as lack of liquidity, high cost of debt, and limited financing capacity can negatively affect the operational activities of a firm. In the context of a supply chain (SC), these disruptive effects can spread throughout the SC and cause inefficiency for other members as well as the firm involved. Therefore, seeking proper solutions to solve this problem is critical to keeping the business afloat in such situations.

Technically, supply chain coordination is an appropriate solution to the challenges caused by the separate decision-making of SC members. Coordination can be achieved using techniques that persuade the members of a decentralized SC to participate in the SC optimization plan. However, the question remains whether SC coordination can be a solution to these financial difficulties. Moreover, how can SC members be coordinated in such situations?

To answer this question, this study has established a supply chain finance (SCF) decision framework through supply chain coordination, considering the integration of operational and financial issues. To this end, a trade credit financing (TCF) contract was developed for a supply chain consisting of several strategic suppliers and a manufacturer facing financial constraints.

The aforementioned SCF system is designed as a multi-leader–follower Stackelberg game. The Stackelberg game is a category of bi-level optimization problems. These problems have a hierarchical structure in which an optimization problem is a constraint for another problem. Multi-leader–follower games, which are known as equilibrium problems with equilibrium constraints (EPECs), are an extension of Stackelberg's leader–follower duopoly. In such games, there is more than one player at the upper or lower levels. Non-cooperative leaders play a Nash game to maximize their payoff. Thus, each leader's payoff is constrained by both competitors' actions and followers' behavior at the lower level, which forms an equilibrium constraint (Koh 2012). We analyzed the trade credit contract as a Stackelberg game between suppliers as leaders who grant trade credit to the manufacturer as the follower.

As a bi-level problem, the Stackelberg game is strongly Np-hard, even for linear cases (Hansen et al. 1992). Therefore, it is challenging to find a proper solution method for these problems, especially in a multi-leader setting and nonlinear cases. Therefore, to solve the proposed multi-leader–follower Stackelberg game, we developed two effective algorithms by combining population-based metaheuristics: differential evolution (DE) and particle swarm optimization (PSO), the Nash domination (ND) concept, and the Nikaido-Isoda (NI) function.

This study is an extension of the work of Emtehani et al. (2021a), who modeled the integrated financial and operational decisions of a capital-constrained manufacturer under a TCF contract. They addressed the problem from the manufacturer's viewpoint and considered the contract parameters to be constant. In contrast, in this study, the problem is modeled from the suppliers' and the SC's viewpoints to design a coordinating contact to solve SC inefficiencies and improve the suppliers' profits, as well as finance the financially constrained manufacturer.

Few studies have addressed the financial problems in the supply chain coordination field. In addition, most existing studies in this field only consider the vertical competition of two members with a single product. Furthermore, they rarely addressed the limited credit of some enterprises for external financing and their effects on the system's operations. It should be noted that in this study, external financing means financing from any source outside the supply chain, including bank financing, peer-to-peer lending, and so on. (As a new form of loan, online peer-to-peer lending is an electronic marketplace in which individual lenders provide loans to individual borrowers directly through peer-to-peer platforms. For further information about peer-to-peer lending, readers may refer to Wang et al. (2020).)

This study attempts to mitigate the financial challenges of a multi-supplier, multi-product supply chain through supply chain coordination under limited financing credit. To the best of our knowledge, this is the first time that multiple non-cooperative suppliers have been modeled as a supply chain finance system that contains both vertical and horizontal competition. Furthermore, this study examines the conditions under which the TCF contract can coordinate the SC. As a result, a new coordinating contract is designed to achieve channel coordination by combining TCF and revenue-sharing (RS) contracts (referred to as the trade credit and revenue-sharing, TCFRS, contract). Accordingly, the mathematical modeling and solution procedure of the present work are new in the related literature.

The next section provides a review of related studies. The formulation of the problem is explained in detail in Sect. "Model formulation", and the solution methodology is described in Sect. "Solution approach". Section " Results and discussion" presents the results and a discussion. Finally, in Sect. 6, conclusions and some suggestions for problem extensions are presented.

Research background

In this section, we review our research background and contributions. This study relates to two streams of literature. The first deals with the broad area of the operations-finance interface. In particular, we focus on *supply chain coordination under financial considerations* in this area. The second focuses on the solution methods of the *multi-leader–follower Stackelberg competition*.

Supply chain coordination under financial considerations

In general, supply chain coordination mechanisms can be classified according to two SC structures: first, a centralized structure in which the decisions of the whole system are made by a central holding, and second, a decentralized structure in which SC members make their decisions individually to optimize their objectives (Jaber & Osman 2006). Li and Wang (2007) reviewed supply chain coordination mechanisms and classified them based on two factors: supply chain structure and demand status. Arshinder et al. (2008) grouped coordination mechanisms into four categories in their literature review: supply chain contracts, information technology, information sharing, and joint decision making. The coordination between supply chain members has been extensively discussed in the literature. One of the most common mechanisms of supply chain coordination is supply chain contracts such as buyback, revenue-sharing, and quantity flexibility contracts. However, in the presence of financial constraints, these contracts may not be practically applicable. Nonetheless, there are few studies in the literature that investigate supply chain coordination under financial considerations.

Incorporating financial issues into operational decisions has recently received significant interest in the operations research (OR) field. Katehakis et al. (2016) established a joint operational and financial model to analyze the impact of loans and deposits on the inventory decisions of a firm with a single product. Tseng et al. (2019) developed a fuzzy interpretive structural approach to construct a hierarchical model to analyze the attributes that improve the sustainable SCF system in the textile industry. Babich and Kouvelis (2018) reviewed operations, risk management, and financial interactions. They highlighted research gaps and suggested several directions for further studies in this area. Supply chain coordination considering financial problems has recently attracted the attention of researchers. We classified the studies on this subject into three categories, as explained below.

In the first category, researchers established an SCF system by coordinating SC members with the bank to eliminate or reduce the adverse effects of capital shortage on firms and SC performance. For example, Dada and Hu (2008) proposed a nonlinear loan schedule for coordinating newsvendor and bank decisions under capital shortage. Yan and Sun (2013) considered a capital-constrained retailer, a profit-maximizing bank offering finite loans to the retailer, and a supplier. They coordinated this system by designing a wholesale price contract based on limited borrowing credit. Yan et al. (2016) designed a partial credit guarantee contract as a supply chain finance implementation for a system consisting of a manufacturer, a retailer with a lack of liquidity, and a bank. According to this contract, the manufacturer provides a credit guarantee for the loan borrowed by the retailer and bears a part of the retailer's bankruptcy risk. Their analysis indicated that, under an appropriate setting of the model parameters, a partial credit guarantee contract can achieve not only channel coordination but also a super-coordination effect. Shi et al. (2020) addressed an SCF system similar to Yan et al. (2016), except that they used a buyback contract to compensate the bank if the retailer went bankrupt. (Interested readers are referred to Kou et al. (2021) for detailed information regarding bankruptcy prediction and the important features that predict the likelihood of bankruptcy.)

In the second category, the studies examined the effects of financial distress on traditional coordinating contracts and extended these contracts by considering budget constraints to achieve coordination. Moon et al. (2015) found that a revenue-sharing contract in its classical structure cannot coordinate the supply chain under a capital shortage. Therefore, they extended the revenue-sharing contract to apply in the presence of budget constraints. Feng et al. (2015) demonstrated that revenue-sharing and buy-back contracts alone cannot coordinate systems with limited working capital in the absence of a financial market. In response, they designed a combined revenue-sharing and buyback contract for SC coordination. Xiao et al. (2017) developed a generalization of revenue-sharing contracts to coordinate the operations of a supplier and a retailer with financial problems. They also showed that classical revenue-sharing and buy-back contracts cannot coordinate the supply chain without a sufficient initial budget. Yan et al. (2018) investigated the role of the buy-back contract in coordinating a system that includes a supplier, a bank, and a risk-averse retailer. They found that the buy-back contract can even achieve superior coordination when the retailer's risk preference is very high. Peng and Pang (2019) established a buy-back and risk-sharing coordinating contract for a supply chain, including a supplier and distributor with limited capital under yield uncertainty. Yang et al. (2021) examined the coordination effect of wholesale price contracts in the presence of members' risk and capital constraints. Li and Li (2022) developed a BBRS contract to eliminate double marginalization in a new energy vehicle supply chain with a cash-strapped retailer.

The third category addressed the role of internal financing, including TCF and advance payments, in coordinating the SC with financial problems. Lee and Rhee (2011) showed that, if financing costs exist, the BB contract cannot fully coordinate the SC. They applied TCF and buyback contracts to conduct coordination under such circumstances. Kouvelis and Zhao (2012) addressed a supply chain comprising a retailer and a supplier, both of which face budget constraints. Regarding TCF and bank financing, they modeled SC decisions as a Stackelberg game in the presence of default risk. Their results indicated that TCF can improve SC performance but it cannot be considered a coordinating contract. They also concluded that classic coordinating contracts with liquidity constraints continue to be coordinated when using competitively priced bank loans in the absence of credit limits. (See Cachon, (2003) for more information about coordinating contracts in the absence of the capital constraint.) Chen and Wang (2012) used a trade credit contract to enhance the profitability of a capital-constrained supply chain following newsvendor inventory policy. They found that a trade credit contract does not achieve channel coordination unless the capital-constrained retailer makes zero profits. Luo and Zhang (2012) coordinated the decisions of a vendor and a buyer in a deterministic environment using order quantity-dependent TCF under symmetric information. They declared that, under asymmetric information, this coordination scheme failed to coordinate the system. Seifert et al. (2013) reviewed the literature on trade credit and found that it can be used as a coordination mechanism. Devalkar and Krishnan (2019) considered a supply chain under information asymmetry and financial frictions with deterministic demand and analyzed the impact of trade credit on supply chain coordination and moral hazard reduction. Ding and Wan (2020) examined supply chain coordination by considering the supplier's capital shortage and uncertainty in the production yield of the manufacturer. Zhang et al. (2021a, b) examined the coordination effects of RS contracts and advance payments in the form of retail channel price discounts and direct channel price discounts for a dual-channel supply chain with a retailer and a manufacturer with limited capital. They found that the RP contract cannot coordinate the SC, while the retail channel price and direct channel price partially coordinate the SC. Zhang et al. (2021a, b) used trade credit and bank loan financing to eliminate the double marginalization effect in a green SC with a manufacturer and a capital-constrained retailer. Emtehani et al. (2021b) coordinated a capital-constrained three-level SC by deciding upon the operational-financial issues of all members jointly considering TCF and advanced payment in a deterministic environment.

In this study, we considered a multi-product, multi-supplier supply chain under liquidity constraints and limited credit for external financing in a stochastic environment. (Interested readers may refer to Elfarouk et al. (2022) to study the impact of demand uncertainty on SC performance from different aspects.) We adopted the TCF contract proposed by Emtehani et al. (2021a), which contains three options for payment to suppliers. Emtehani et al. (2021a) modeled the operational-financial decisions of a financially constrained manufacturer under a TCF contract, assuming that the contract parameters are constant and predetermined by its suppliers. In other words, they modeled the problem considered in this study from the manufacturer's perspective. In contrast, we modeled the TCF contact from the suppliers' perspective as the contract regulators and examined the conditions under which this contract can coordinate the SC. Moreover, we designed a trade credit financing and revenue-sharing (TCFRS) contract by combining TCF and revenue-sharing contracts to achieve channel coordination where the TCF contract alone cannot coordinate the SC. We modeled both the TCF and TCFRS contracts as multi-leader supplier Stackelberg games and developed two solution procedures based on population-based metaheuristics to solve the proposed models.

All the abovementioned studies considering stochastic environments have addressed the interactions of two vertical competitors in the supply chain under internal financing. They formulated competitor behavior as a classic Stackelberg duopoly. Given that, in practice, SCs are more complex and the interactions of a seller and a buyer are influenced by other members of the SC, it seems essential to study a more complex SC rather than the simplest sample. For this purpose, unlike previous studies, we focused on the interactions of multiple horizontal competitors under the TCF contract and analyzed the downstream member's responses to the decisions of upstream competitors. Accordingly, both horizontal and vertical competition are addressed in this study. Subsequently, a novel problem formulation was applied to address the interactions of players in the supply chain, and new theoretical and practical insights were achieved. Moreover, an effective procedure was developed to solve the proposed complex model.

Multi-leader–follower Stackelberg competition

In the multi-leader–follower Stackelberg game, which is an extension of the well-known Stackelberg duopoly, multiple players compete in a noncooperative Nash game at the upper level (and/or lower level). This problem has many applications in economics, operational research, and other fields. Sherali (1984) was among the first researchers to discuss the existence, uniqueness, and computations of equilibrium solutions for multi-leader–follower games. The methods commonly used in the literature to solve such a problem can be classified into two main categories: mathematical programming approaches and evolutionary algorithms.

In mathematics, the well-known Stackelberg game is a mathematical program with equilibrium constraints (MPEC). Correspondingly, a multi-leader–follower game is formulated as an EPEC. An EPEC is an equilibrium problem that includes a few parametric MPECs involving the strategies of other players as parameters. Equilibrium can be achieved by simultaneously solving all the MPECs. Early work on solving EPECs focused on diagonalization strategies in which a cyclic sequence of MPECs is solved as an equivalent nonlinear program until some equilibrium is found. Some examples are the studies of Hu et al. (2002), Fletcher and Leyffer (2004), and Fletcher et al. (2007). The main weakness of these methods is their inability to converge to the equilibrium solution in some situations, even if one exists. To fill this gap, Su (2004) proposed a sequential nonlinear complementarity algorithm to solve EPECs and proved its convergence. Leyffer and Munson (2010) presented two mathematical solution approaches to solve EPECs using a single optimization problem rather than a sequence of related optimization problems. In the context of SC, Qi et al. (2015) studied the horizontal competition of two suppliers on both wholesale price and reliability and the vertical competition of these two suppliers with their customer. They solved this game using backward induction and analyzed the model in different cases. Hu and Fukushima (2015) provided some applications of mathematical programming methods for solving multi-leader–follower games. However, mathematical programming approaches have major drawbacks. For instance, the solutions are strongly sensitive to the initial point, that is, the algorithm may be trapped in local optima with an improper choice of the initial point. In addition, convergence to the Nash equilibrium may fail if payoff functions are not continuously differentiable. Evolutionary algorithms have been designed to address this problem. Furthermore, they can escape local optimal solutions (Koh 2012).

As in many optimization problems, evolutionary algorithms have attracted researchers in the field of game theory and, in particular, for obtaining the Nash equilibrium in multi-player games. Lung and Dumitrescu (2008) established a concept similar to Pareto domination in evolutionary multi-objective optimization, called Nash domination, to find the Nash equilibrium by applying evolutionary search operators. Sinha et al. (2014) applied a genetic algorithm to solve a multi-leader–follower game in multiple periods with nonlinear and non-smooth functions. He et al. (2016) proposed a DE algorithm to solve nonlinear continuous Nash games. They used a special function called the Nikaido-Isoda function as a fitness function to achieve the Nash equilibrium. Greiner et al. (2017) reviewed the theoretical foundation and applications of meta-heuristics in solving both cooperative and noncooperative games. Zaman et al. (2018) used two evolutionary algorithms, the genetic algorithm (GA) and DE, to find the Nash equilibria in electricity markets. Mahmoodi (2020) modeled the problem of two competitive supply chains as a two-leader–follower game and extended a nested iterative algorithm using DE and threshold accepting algorithms to solve the proposed problem. Mondal and Giri (2021) considered two competing manufacturers and a retailer in a green closed-loop supply chain and formulated their interactions in three scenarios: centralized, Nash game, and manufacturer-led Stackelberg game. They did not establish a solution procedure for the proposed models because of the simplicity of the SC structure, that is, two leaders and a follower, and focused on the behaviors of the players in their study.

In this study, we extended the Nash-domination evolutionary multi-player optimization algorithm proposed by Koh (2012). He used the Nash dominance relation to solve EPECs, using DE as an evolutionary search method. This method was specifically designed for evolutionary algorithms. We extended their method, developed an algorithm to solve the problem proposed in this study, and generalized their method to all population-based metaheuristics. Moreover, for comparison, we applied the NI function instead of the Nash dominance relation in the developed algorithm. (Please refer to Sect. 4.1, for a comprehensive explanation of the Nash dominance relation and NI function.)

Literature gaps and contributions

There is a vast body of literature on supply chain coordination, with a variety of assumptions. However, engaging in financial issues in this area is relatively new, and there are many research gaps. We provide a comparison between the most related previous studies (third category expressed in Sect. 2.1) and the current work in Table 1. Furthermore, some gaps in the literature and our contributions are discussed as follows.

- Previous studies in this field have addressed the interactions between two vertical competitors in the supply chain under internal financing. Unlike previous

Table 1 A comparison between the most related previous studies and the current work

Authors	SC structure	TCF options	Financial decisions	Developing a new Coordinating contract	Vertical competition	Horizontal competition	Financing restriction	External financing
Lee & Rhee, (2011)	Single product Two members	Permissible delay on payments subject to a penalty	✗	✗	✓	✗	✗	✓
Kouvelis & Zhao (2012)	Single product Two members	Discount on early payment, permissible delay on payments subject to a penalty	Bank interest rate	✗	✓	✗	✗	✓
Chen & Wang (2012)	Single product Two members	Permissible delay on payment with no interest	✗	✗	✓	✗	✗	✗
Luo & Zhang (2012)	Single product Two members	Permissible delay on payment with no interest	Trade credit length	✗	✗	✗	✗	✗
Devalkar & Krishnan (2019)	Single product Two members	Permissible delay on payment with no interest	Trade credit length, the portion of early payment receivables	✓	✗	✗	✗	✓
Ding & Wan (2020)	Single product Two members	✗	Advance payment interest rate	✗	✓	✗	✗	✓
C. Zhang et al., (2021a, b)	Single product Two members	✗	✗	✗	✓	✗	✗	✗
Zhang et al., (2021a, b)	Single product Two members	Permissible delay on payments subject to penalty	✗	✗	✓	✗	✗	✓
Emtehani et al., (2021b)	Multiple products Multiple members	Permissible delay on payments subject to penalty	Selection of financing modes, loan amount, payment time, due dates of the interest-free periods, the rate of advanced payment	✗	✗	✗	✓	✓

Table 1 (continued)

Authors	SC structure	TCF options	Financial decisions	Developing a new Coordinating contract	Vertical competition	Horizontal competition	Financing restriction	External financing
Current study	Multiple products Multiple members	Three options; Discount on early payments, permitted delay on payments with no interest, permitted delay on payments subject to penalties	Selection of financing modes, loan amount, selection of trade credit options, payment times to suppliers, due dates of the discounted period, discount rates, due dates of the interest-free periods, penalty rates, revenue sharing coefficient	✓	✓	✓	✓	✓

studies, we focused on the interactions of multiple horizontal competitors under the TCF contract and analyzed downstream members' responses to the decisions of upstream competitors. Accordingly, both horizontal and vertical competitions are addressed in this study and formulated as a multi-leader Stackelberg game.

- We developed two efficient algorithms combining population-based metaheuristics, the Nash domination concept, and the Nikaido-Isoda function to solve the proposed multi-leader Stackelberg game.
- In the supply chain coordination literature on financial considerations, some firms' limited credit for external financing and its effects on operations are mostly ignored. In this study, we consider financing restrictions and capital shortages and discussed their effects on supply chain coordination in the presence of demand uncertainty.
- We examined the conditions under which a trade credit contract can coordinate the supply chain. Consequently, a new coordinating contract is designed for the SC under financial restrictions that fully coordinates the SC.

Model formulation

In this section, the problem is formulated for two scenarios concerning TCF contract terms. In the first scenario, called *Sc-1*, the manufacturer is required to pay each supplier until a predetermined time ($tmax_k$) before the end of period (T). However, in the second scenario, *Sc-2*, the manufacturer is allowed to extend its payment period to the end of the period. We summarize the operational assumptions and financial status of the problem to describe the mathematical model clearly. The

Table 2 list of notations

Indices	
n	Index of the manufacturer's products (original parts) $(n = 1, 2, \ldots, N)$
k	Index of raw materials and related suppliers $(k = 1, 2, \ldots, K)$
Parameters	
y_n	The random variable of demand
p_n	The selling price of product n (per unit)
v_n	The variable production cost for product n (per unit)
h_n	The holding cost for the remaining original parts at the end of the period (per unit)
s_n	The shortage cost for product n at the end of the period (per unit)
w_k	The purchasing cost for material k (per unit)
c_k	The procurement cost of material k by the related supplier (per unit)
m_{nk}	The required amount of material k used for each unit production of original part n
$tmax_k$	The final due date for the payment to supplier k (in days)
r_l	The daily interest rate of external financing
r_m	Rate of return for the manufacturer's investment
I_n	Initial inventory of product n
B_0	Initial budget
ML	Maximum financing capacity
Decision variables of the suppliers	
b_k	The time on or before which the payment of material k will be discounted (in days)
u_k	The discount rate for early payments
d_k	The due date for the interest-free period (in days)
τ_k	The penalty rate per day delay for payment of material k after the interest-free period
Decision variables of the manufacturer	
R_n	Inventory level of product n at the end of the period before shipment to the customers
Q_n	The production quantity of original part n $(Q_n = R_n - I_n)$
Q_k	The order quantity of raw material k $(Q_k = \sum_n m_{nk} Q_n)$
t_k	Payment time for material k to the related supplier (in days)
x_k^i	Binary variable related to pay at each time interval $(i = 1,2,3)$
l	The loan amount borrowed from an external source

model formulation for both scenarios is presented in detail. Table 2 lists the model notations.

Operational assumptions and financial status

The operational assumptions of this study are the same as those of Emtehani et al. (2021a). Suppose a manufacturer produces the original parts of some final products, that is, an original equipment manufacturer (OEM). It orders required components or raw materials from several strategic suppliers (indexed by k) in order quantities of Q_k with a wholesale price w_k, and produces its products (original parts indexed by n) in production quantities of Q_n units with a variable cost of v_n for each unit of product n. Note that $Q_k = \sum_n m_{n,k} Q_n$, where $m_{n,k}$ is the component/raw material k's consumption coefficient for manufacturing one unit of product n. The inventory level of product n before shipment to the customer is R_n, which can be calculated as $R_n = Q_n + I_n$, where I_n is the initial inventory of product n. The manufacturer sells its products to an assembly factory for p_n units of cash per product n. The demand for the original parts is stochastic. Therefore, the manufacturer may face inventory shortage or overage at the end of the period. Unit inventory overage and shortage costs are indicated by h_n and s_n, respectively, in the problem formulation. All members are risk neutral and intend to maximize their profits. We use $\pi_i(.)$ to denote the expected profit of each player (SC member), where the subscript i refers to each supplier (shown by Sup_k), manufacturer (shown by M), and SC. The random demand of each original part (y_n) has a cumulative probability distribution, named $F_n(.)$, and a density probability distribution, named $f_n(.)$, on positive real numbers. It is assumed that $F_n(.)$ is differentiable and increasing. Also, $F_n\left(y_n = 0\right) = 0$.

The manufacturer has an insufficient initial budget (B_0) for performing its operations. Moreover, it has limited credit for financing from financial institutions, that is, the financing capacity of the manufacturer is limited to a maximum value (ML). Thus, this financially constrained manufacturer considers two financing options: borrowing an amount of l from financial institutions with a daily interest rate of r_l, where $l \leq ML$, and using trade credit financing granted by suppliers. It is assumed that the manufacturer's net financial flow (financial inflow minus financial outflow) at the end of the period should be greater than or equal to zero; that is, bankruptcy is not allowed.

The suppliers are independent, but their decisions are affected by each other in terms of financing the manufacturer through the TCF. Thus, the members' integrated operational financial decisions are modeled as a multi-leader–follower Stackelberg game. As a bi-level optimization problem, all suppliers play a non-cooperative Nash game at the upper level as Stackelberg leaders and simultaneously determine the contract parameters by anticipating the manufacturer's response as the Stackelberg follower. The sequence of events in the proposed game is illustrated in Fig. 1.

The manufacturer's problem under the TCF contract

Each supplier offers a TCF contract to the financially constrained manufacturer to increase its purchase amount. The structure of the TCF contract granted to the manufacturer by suppliers is the same as the TCF contract proposed by Emtehani et al. (2021a), except that in *Sc-2,* the payment period is extended to the end of the period. It contains

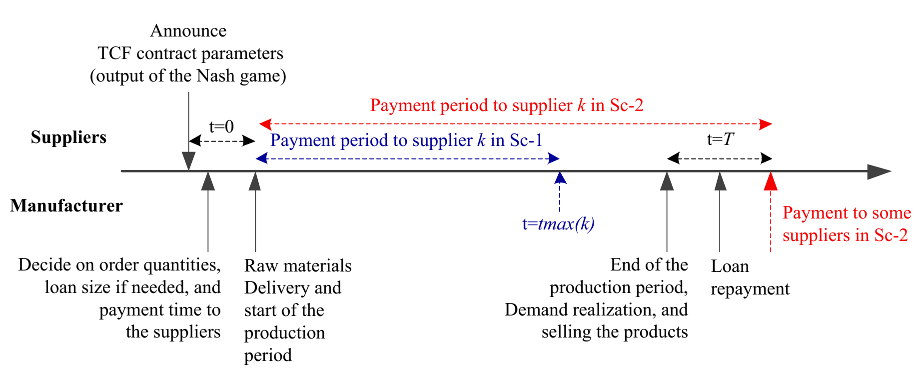

Fig. 1 The sequence of events for the proposed game in *Sc-1* and *Sc-2*

three options for both scenarios. *i*) pay to the suppliers from the time of purchase until b_k is subjected to a discount on raw material k (applied by a discount rate, denoted by u_k, for all units); *ii*) pay in the interest-free period (from $b_k + 1$ until d_k) without any discount or penalty; *and iii*) pay with a penalty from $d_k + 1$ until $tmax_k$. This penalty is applied per day by a daily penalty rate, denoted by τ_k. A binary variable x_k^i is defined to formulate the TCF options. As mentioned before, in *Sc-1*, $tmax_k < T$. Thus, in *Sc-1*, there are three options for payment. However, in *Sc-2*, $tmax_k = T$. Therefore, in *Sc-2*, four options are considered for payment. Note that the fourth option relates to the payment at time T.

The manufacturer's problems according to the TCF contract in *Sc-1* and *Sc-2* are given below as P_M^{Sc-1} and P_M^{Sc-2}. Note that P_M^{Sc-1} is the same as the model proposed by Emtehani et al. (2021a), and P_M^{Sc-2} is an extension of their model concerning the new setting of TCF contract terms.

$\boldsymbol{P_M^{Sc-1}}$:

$$Maximize \pi_M^{Sc-1}\left(R_n x_k^i t_k l\right) = \sum_n p_n E\left[\min\left\{R_n y_n\right\}\right] - \sum_k w_k Q_k \left(2 - (1 + r_m)^{t_k}\right) \left[(1 - u_k)x_k^1 + x_k^2 + (1 + \tau_k)^{t_k - d_k} x_k^3\right] - \sum_n v_n (R_n - I_n) - \sum_n h_n E\left[\max\left\{R_n - y_n 0\right\}\right] - \sum_n s_n E\left[\max\left\{y_n - R_n 0\right\}\right] - l\left[(1 + r_l)^T - 1\right] \tag{1}$$

Subject to:

$$b_k + \left(1 - x_k^1\right)M \geq t_k \forall k = 12 \ldots K \tag{2}$$

$$d_k + \left(1 - x_k^2\right)M \geq t_k \forall k = 12 \ldots K \tag{3}$$

$$b_k + 1 - \left(1 - x_k^2\right)M \leq t_k \forall k = 1, 2, \ldots, K \tag{4}$$

$$d_k + 1 - \left(1 - x_k^3\right)M \leq t_k \forall k = 1, 2, \ldots, K \tag{5}$$

$$\sum_k w_k Q_k (2 - (1 + r_m)^{t_k}) \left[(1 - u_k)x_k^1 + x_k^2 + (1 + \tau_k)^{t_k - d_k} x_k^3\right] + \sum_n v_n (R_n - I_n) \leq B_0 + l \tag{6}$$

$$B_0 + l + \sum_n p_n E\left[min\left\{R_n, y_n\right\}\right] - \sum_k w_k Q_k (2 - (1 + r_m)^{t_k}) \left[(1 - u_k)x_k^1 + x_k^2 + (1 + \tau_k)^{t_k - d_k} x_k^3\right] - \sum_n v_n (R_n - I_n) - \sum_n h_n E\left[max\left\{R_n - y_n, 0\right\}\right] - l(1 + r_l)^T \geq 0 \tag{7}$$

$$l \leq ML \tag{8}$$

$$\sum_i x_k^i = 1 \forall k = 1, 2, \ldots, K \forall i = 1, 2, 3 \tag{9}$$

$t_k \leq t\, \text{max}_k t_k \text{integer} R_n, Q_k, l \geq 0 x_k^i = 0 or 1 (i = 1, 2, 3)$

As stated earlier, in *Sc-2*, because the manufacturer can extend its payments to the end of the period (time T), another option is added to the TCF contact to formulate the problem. This option, denoted by x_k^4, relates to the payments that take place at T. Accordingly, the purchasing cost related to this option is formulated as $\sum_k w_k Q_k (1 + \tau_k)^{T - d_k} x_k^4$, and is added to the costs in the objective function and the cash balance constraint (7). Three new constraints (11, 12, and 13) are added to formulate the fourth option in the model. Note that $t_k = T$ means that the manufacturer pays supplier *k* from sales revenue. Thus, external financing is not used for this payment.

According to the above explanations, the manufacturer's problem in the second scenario is modeled as follows:

$\boldsymbol{P_M^{Sc-2}}$:

$$Maximize \pi_M^{Sc-2}(R_n, x_k^i, t_k, l) = \sum_n p_n E\left[min\left\{R_n, y_n\right\}\right] - \sum_k w_k Q_k (2 - (1 + r_m)^{t_k}) \left[(1 - u_k)x_k^1 + x_k^2 + (1 + \tau_k)^{t_k - d_k} x_k^3\right] - \sum_k w_k Q_k (1 + \tau_k)^{T - d_k} x_k^4 - \sum_n v_n (R_n - I_n) - \sum_n h_n E\left[max\left\{R_n - y_n, 0\right\}\right] - \sum_n s_n E\left[max\left\{y_n - R_n, 0\right\}\right] - l\left[(1 + r_l)^T - 1\right] \tag{10}$$

Subject to:
(2), (3), (4), (5), (6), and (8)

$$T - 1 + \left(1 - x_k^3\right)M \geq t_k \forall k = 1, 2, \ldots, K \tag{11}$$

$$T + \left(1 - x_k^4\right)M \geq t_k \forall k = 1, 2, \ldots, K \tag{12}$$

$$T - \left(1 - x_k^4\right)M \leq t_k \forall k = 1, 2, \ldots, K \tag{13}$$

$$\begin{aligned}
&B_0 + l + \sum_n p_n E\left[min\left\{R_n, y_n\right\}\right] - \sum_k w_k Q_k (2 - (1 + r_m)^{t_k}) \\
&\left[(1 - u_k)x_k^1 + x_k^2 + (1 + \tau_k)^{t_k - d_k} x_k^3\right] - \sum_k w_k Q_k (1 + \tau_k)^{T - d_k} x_k^4 \\
&- \sum_n v_n (R_n - I_n) - \sum_n h_n E\left[max\left\{R_n - y_n, 0\right\}\right] - l(1 + r_l)^T \geq 0
\end{aligned} \tag{14}$$

$t_k \leq t_k$ integer, R_n, Q_k, $l \geq 0$, $x_k^i = 0$ or 1 $(i = 1, 2, 3, 4)$

The suppliers' problem under the TCF contract

Let $\pi_{Sup_k}^{Sc-1}$ and $\pi_{Sup_k}^{Sc-2}$ be supplier k's expected profits in the first and second scenarios, respectively, which can be written as:

$$\begin{aligned}
\pi_{Sup_k}^{Sc-1}(b_k, d_k, u_k, \tau_k, R_n, x_k^i, t_k, l) =& w_k \left[(1 - u_k)x_k^1 + x_k^2 + (1 + \tau_k)^{t_k - d_k} x_k^3\right] \\
&\sum_n m_{nk}(R_n - I_n) - c_k Q_k - w_k (\sum_n m_{nk}(R_n - I_n)) \\
&((1 + r_k)^{t_k} - 1)
\end{aligned} \tag{15}$$

$$\begin{aligned}
&\pi_{Sup_k}^{Sc-2}(b_k, d_k, u_k, \tau_k, R_n, x_k^i, t_k, l) \\
&= w_k \left[(1 - u_k)x_k^1 + x_k^2 + (1 + \tau_k)^{t_k - d_k} x_k^3 + (1 + \tau_k)^{T - d_k} x_k^4\right] \\
&\sum_n m_{nk}(R_n - I_n) - c_k Q_k - w_k (\sum_n m_{nk}(R_n - I_n))((1 + r_k)^{t_k} - 1)
\end{aligned} \tag{16}$$

The last terms in $\pi_{Sup_k}^{Sc-1}$ and $\pi_{Sup_k}^{Sc-2}$ represent the opportunity cost of the tied-up capital by the manufacturer under the TCF contract. Note that r_k is supplier k's minimum attractive rate of return.

As a leader, each supplier plays a Stackelberg game, according to the manufacturer's problem as the follower. Accordingly, each supplier's decision problem for both scenarios can be formulated as a two-level optimization problem as follows:

$$\boldsymbol{P_{Sup_k}^j}:$$

$$Max \pi_{Sup_k}^j (b_k, d_k, u_k, \tau_k, R_n, x_k^i, t_k, l)$$

Subject to:

$$P_M^j = argmax\{\pi_M^j (b_k, d_k, u_k, \tau_k, R_n, x_k^i, t_k, l) s.t: associated constraints\} \tag{17}$$

$$b_k \leq d_k \leq tmax_k \tag{18}$$

We refer to this problem as $P_{Sup_k}^j$, where j denotes *Sc-1* and *Sc-2*. The first constraint (17) is the optimization problem for the manufacturer. This problem is a bi-level,

Table 3 Required notations and basic notions of game theory in the present study

Notation	Description
λ_k	The strategy of each player (supplier) *k*, where $k = 12\ldots K$
$\lambda_{-k} \equiv (\lambda_1 \ldots \lambda_{i-1}\lambda_{i+1} \ldots \lambda_k)$	A combined strategy of all players except the strategy of player *k*
$\lambda \equiv (\lambda_k\lambda_{-k})$	The strategy profile of all players
$^*\pi_k(s)$	Player *k*'s payoff (profit in this study) associated with the strategy profile *s*
Nash equilibrium	A strategy profile in which no player can benefit from deviating its current strategy unilaterally
$G(xy) \equiv cardinalityofset : \{k \in \{1 \ldots K\} \mid \pi_k(y_k x_{-k}) \geq \pi_k(x) y_k \neq x_k\}$	The number of players who can benefit from playing y_k when everyone else plays x_{-k}. Note that *y* and *x* are two strategy profiles
$G(yx) \equiv cardinalityoffset : \{k \in \{1 \ldots K\} \mid \pi_k(x_k y_{-k}) \geq \pi_k(y) y_k \neq x_k\}$	The number of players who can benefit from playing x_k when everyone else plays y_{-k}. Note that *y* and *x* are two strategy profiles

[*] Please note that in this study, the profit function of each player (SC member) is used as its payoff. So, in the rest of the paper, we use profit instead of payoff

nonlinear, mixed-integer optimization problem. In the proposed multi-leader Stackelberg competition, because all the suppliers compete in a non-cooperative Nash game, there are k bi-level problems that should be solved simultaneously. Obviously, by increasing the number of suppliers (as leaders) and products, problem complexity will increase substantially. Therefore, according to the Np-hardness nature of bi-level problems, solving such a game seems to be a complicated task. In Sect. 4, we developed two procedures for solving this problem for both scenarios.

The centralized SC

In this subsection, we consider SC members as centralized entities and jointly model their operational and financial decisions. This model was used as a benchmark to evaluate the efficiency of the TCF contract designed in both scenarios. In the centralized structure, the decisions of all members are in line with the maximization of SC profit. Accordingly, the centralized model is formulated as follows:

$$\begin{aligned} Maximize \pi_{SC}(R_n, Q_k, l) = & \sum_n p_n E\left[\min\{R_n, y_n\}\right] - \sum_n v_n(R_n - I_n) \\ & - \sum_k c_k Q_k - \sum_n h_n E\left[\max\{R_n - y_n, 0\}\right] \\ & - \sum_n s_n E\left[\max\{y_n - R_n, 0\}\right] - l\left[(1 + r_l)^T - 1\right] \end{aligned} \tag{19}$$

$$\sum_n v_n(R_n - I_n) \le B_0 + l \tag{20}$$

$$B_0 + l + \sum_n p_n E\left[\min\{R_n, y_n\}\right] - \sum_n v_n(R_n - I_n) - \sum_n h_n E\left[\max\{R_n - y_n, 0\}\right] - l(1 + r_l)^T \ge 0 \tag{21}$$

$$l \le ML \tag{22}$$

$$R_n, Q_k, l \ge 0$$

Solution approach

To solve a bi-level optimization problem, the lower-level problem should be solved optimally in each iteration of the upper-level problem. In this study, we used the three-phase approach proposed by Emtehani et al. (2021a) to solve the lower-level problem in *Sc-1*

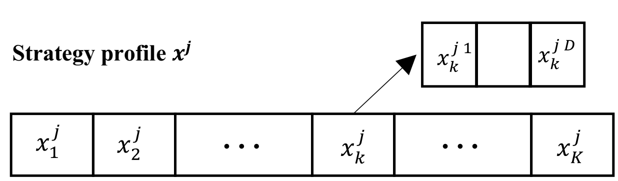

Fig. 2 Representation of the encoding of a strategy profile for all players, each x_k^j, where $k = 1, \ldots, K$, is a D-dimension vector of supplier k's decision variables

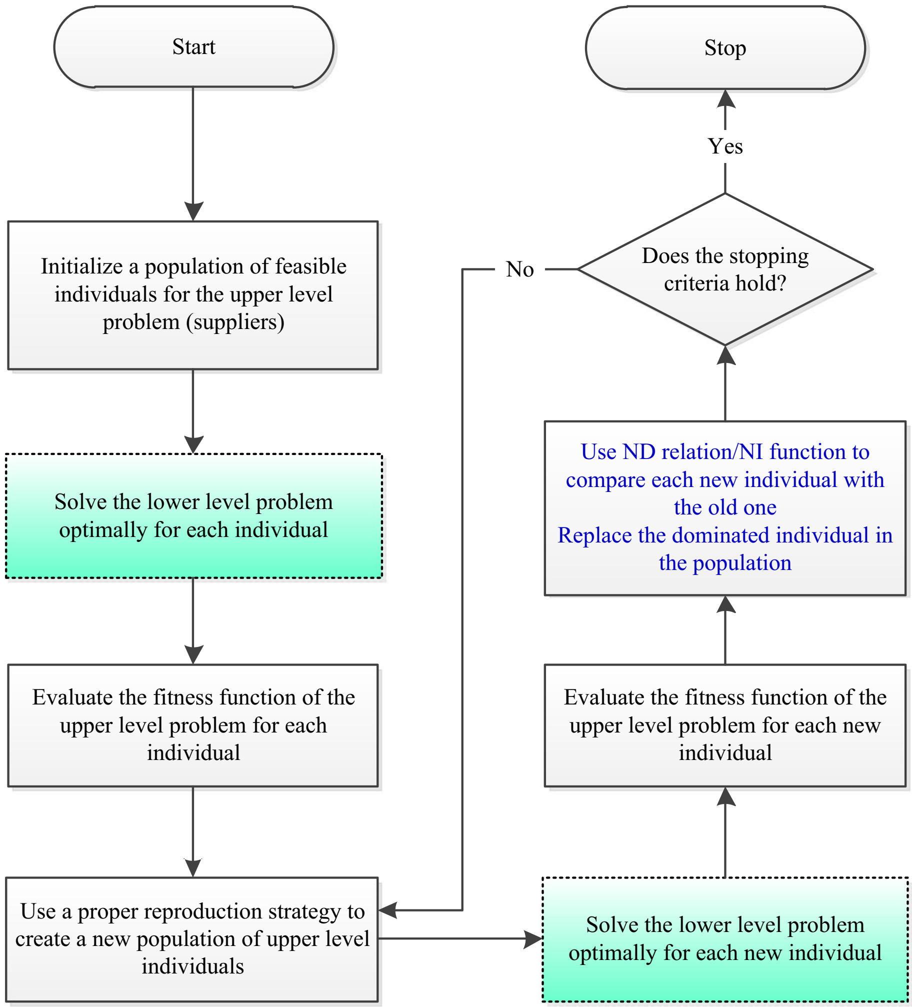

Fig. 3 The flowchart of ND-MLSG-PBM and NI-MLSG-PBM algorithms

and extended it to solve the lower-level problem in *Sc-2* (see Appendix). To solve the main problem (the proposed multi-leader Stackelberg game), we developed two solution procedures based on population-based metaheuristics and used the Nash domination concept and the Nikaido-Isoda function to find the Pareto optimal solutions of the Nash game among leaders. We call these algorithms "ND-MLSG-PBM" and "NI-MLSG-PBM" which refer to "the use of Nash dominance relation/Nikaido-Isoda function for solving Multi-Leader Stachelberg Games based on Population-Based Metaheuristics." In this study, an evolutionary algorithm (DE) and a swarm intelligence algorithm (PSO) were applied in the solution procedures. However, all population-based metaheuristics could be applied to the proposed algorithms.

In this section, the notations and basic notions related to game theory used in this study are presented in Table 3 for more clarity. The Nash dominance relation and Nikaido-Isoda function are described. The mechanism of the proposed algorithms is explained in detail.

Nash dominance relation and Nikaido-Isoda function

The Nash dominance relation was introduced by Lung and Dumitrescu (2008) to find the Nash equilibrium of multi-player Nash games. They declared that this relation makes it possible for evolutionary search operators to converge to multiple solutions in a game.

Nash domination is based on counting the number of players that make a profit by a unilateral deviation from their current strategy in a specific strategy profile. The fewer players that benefit from unilateral deviation in a particular strategy profile, the closer this strategy profile is to the Nash equilibrium of the game.

Suppose that x and y are strategy profiles. According to the above explanations, Nash dominance relations are defined as follows:

a) if $G(x, y) < G(y, x)$, then x dominates y.
b) if $G(x, y) > G(y, x)$, then y dominates x.
c) if $G(x, y) = G(y, x)$, then x and y are Nash non-dominated.

Lung and Dumitrescu (2008) proved that all Nash non-dominated solutions obtained by these pairwise comparisons are in the Nash equilibrium.

To evaluate *G(x,y)* and *G(y,x)*, each player's profit from deviating must be individually computed. For example, if $\pi_k(y_k, x_{-k}) \geq \pi_k(x)$, one is added to *G(x, y)*, and vice versa. (Refer to algorithm 1 in Koh (2012) to see the exact steps of computing *G(x,y)* and *G(y,x)*.)

The Nikaido-Isoda function, developed by Nikaido and Isoda (1955b), is a mathematical tool that is used to transform an equilibrium problem into an optimization problem. The function was first proposed by Nikaido and Isoda (1955a). Suppose that x and y are two strategy profiles for all players. In the Nikaido-Isoda function, given in Eq. (23), each summand (i.e., $\pi_k(y_k, x_{-k}) - \pi_k(x)$) indicates the increase/decrease in a player's profit by unilaterally changing its strategy from x_k to y_k while other players play x_{-k}. So, $\Psi(x, y)$ represents the sum of the changes in all players' profits by deviating unilaterally and playing y while others play x.

$$\Psi(x, y) = \sum_{k=1}^{K} \left[\pi_k(y_k, x_{-k}) - \pi_k(x)\right] \tag{23}$$

According to this function, if $\Psi(x, y) < \Psi(y, x)$, x dominates y and vice versa. It can be inferred from this function that if x is a Nash equilibrium, $\Psi(x, y)$ is non-positive for all feasible y.

ND-MLSG-PBM and NI-MLSG-PBM algorithms

In this subsection, two algorithms developed to solve the proposed problem are described in detail. The ND-MLSG-PBM algorithm is an extension of the Nash domination evolutionary multi-player optimization (NDEMO) algorithm proposed by Koh (2012). He applied the Nash dominance relation to solve EPECs using DE as an evolutionary search method. Their method was specifically designed for evolutionary algorithms. We extended their method and developed the ND-MLSG-PBM algorithm to solve the problem proposed in the previous section and generalized their method for all population-based metaheuristics. Moreover, we developed NI-MLSG-PBM by using the Nikaido-Isoda function instead of the Nash dominance relation to solve the proposed multi-leader Stackelberg game and compared it with ND-MLSG-PBM.

Both the ND-MLSG-PBM and NI-MLSG-PBM algorithms were founded on population-based metaheuristics. The main cycle of population-based metaheuristics includes the reproduction and replacement of the current population. In this study, we used Nash dominance pairwise comparisons and the Nikaido-Isoda function in the replacement phase for the ND-MLSG-PBM and NI-MLSG-PBM algorithms, respectively.

Koh (2012) used evolutionary algorithms to create child vectors using the NDEMO algorithm. In the current study, we used and compared an evolutionary algorithm (DE) and a swarm intelligence algorithm (PSO) for reproduction and generalized both the ND-MLSG-PBM and NI-MLSG-PBM algorithms for every population-based metaheuristic. In both algorithms, each member of the population or swarm, denoted by index j, includes the strategy profile depicted in Fig. 2. Real-value coding is used in the algorithms.

Figure 3 presents an overview of the ND-MLSG-PBM and NI-MLSG-PBM algorithms in the form of a flowchart. The details of the two algorithms are presented in *Algorithm I*. The reproduction strategy applied in *Step 3* is demonstrated through *Algorithms II and III* for PSO and DE, respectively.

Algorithm I, ND-MLSG-PBM, and NI-MLSG-PBM algorithms using population-based metaheuristics.

Step1: a) Random initialization of the whole population; note that each feasible individual (vector x^j, j = 1, ..., Npop, where Npop is the population size) is considered as a strategy profile for all players. b) solve the lower-level problem optimally for each individual.

Step 2: Evaluate the profit of each player (k) for each strategy profile (i.e., $\pi_k(x^j)$).

Step 3: a) Use a reproduction strategy based on the considered population-based metaheuristic algorithm (in this study, Algorithm II for PSO or Algorithm III for DE) and create a feasible child vector in DE/new position in PSO from x^j (the new vector is called strategy profile y^j). Do this for every individual in the current population to reach a new population. b) solve the lower-level problem optimally for each new individual.

Table 4 The ranges of the model parameters for generating test problems in two sizes

Problem size	p_n	μ_n	σ_n	I_n	v_n
1	U[500,3000]	U[2000,10000]	U[50,500]	U[0,3000]	U[40,200]
2	U[5000,20000]	U[300,3000]	U[30,300]	U[0,1000]	U[100,600]
Problem size	h_n	s_n	w_k	c_k	$t\max_k$
1	U[1,20]	U[1,20]	U[20,100]	U[5,80]	U[50,110]
2	U[50,200]	U[50,200]	U[100, 600]	U[30,200]	U[50,110]

Table 5 A summary of the results for the TCFRS contract, WTCF case, and the centralized model for the test problems in size 1

Solution method	ND-MLSG-PBM		NI-MLSG-PBM		WTCF	Centralized SC	Profitability ratio
Metaheuristics	DE	PSO	DE	PSO			
SC profit	13,298,249	13,947,572	13,752,702	14,247,235	10,978,833	14,458,049	30%
Manufacturer's profit	7,246,573	8,129,091	7,868,785	7,904,632	7,011,337	–	13%
Sum of suppliers' profit	5,051,676	5,818,481	5,883,917	6,342,602	3,967,496	–	60%
Sup_1 profit	1,014,368	1,106,621	1,140,625	1,311,047	726,505	–	80%
Sup_2 profit	999,788	1,143,983	1,063,306	1,190,342	803,168	–	48%
Sup_3 profit	816,640	907,396	985,039	1,038,182	627,508	–	65%
Sup_4 profit	888,754	1,064,509	1,072,505	1,155,677	675,808	–	71%
Sup_5 profit	1,332,127	1,595,971	1,622,442	1,647,354	1,134,507	–	45%
φ	52%	48%	46%	47%	–	–	–
Production quantities	5009	5011	5011	5010	2997	5006	67%
	6512	6512	6513	6511	3502	6509	86%
	6811	6813	6813	6810	3930	6805	73%
Loan amount	100,773	131,564	88,859	96,553	4,500,000	28,025	–
Runtime	7085	7465	7213	7901	–	–	–

Table 6 A summary of the results for the TCFRS contract, WTCF case, and the centralized model for the test problems in size 2

Solution method	ND-MLSG-PBM		NI-MLSG-PBM		WTCF	Centralized	Profitability ratio
Metaheuristics	DE	PSO	DE	PSO			
SC profit	74,088,902	75,182,250	74,870,277	77,441,587	41,218,764	78,809,192	88%
Manufacturer's profit	19,579,836	17,821,752	18,077,474	19,011,917	16,056,408	–	18%
Sum of suppliers' profit	54,509,066	57,360,498	56,792,803	58,429,670	25,162,356	–	132%
Sup_1 profit	7,381,778	7,633,354	7,420,006	7,740,857	3,340,146	–	132%
Sup_2 profit	2,476,572	2,781,045	2,527,691	2,616,109	793,104	–	230%
Sup_3 profit	1,989,850	2,097,831	1,959,889	2,183,604	759,144	–	188%
Sup_4 profit	7,357,915	7,389,269	7,589,997	8,116,049	3,484,164	–	133%
Sup_5 profit	8,054,715	8,189,078	7,878,345	8,197,660	3,722,533	-	120%
Sup_6 profit	5,877,603	6,198,213	7,174,773	6,875,245	3,437,680	–	100%
Sup_7 profit	4,947,177	4,978,116	4,770,886	4,803,188	2,080,143	–	131%
Sup_8 profit	3,897,257	4,075,603	3,994,845	4,062,302	1,655,035	–	145%
Sup_9 profit	7,198,385	7,402,571	6,049,393	7,329,803	3,295,371	–	122%
Sup_{10} profit	5,976,806	6,615,418	6,359,986	6,504,853	2,595,035	–	151%
φ	23%	21%	21%	22%	–	–	–
Production quantities	1858	1861	1860	1861	945	1860	97%
	2019	2021	2020	2021	1023	2018	98%
	2430	2431	2432	2430	1316	2425	85%
	1582	1577	1581	1580	877	1576	80%
	1559	1560	1561	1564	958	1555	63%
Loan amount	800,957	744,926	784,026	718,603	20,000,000	396,918	–
Runtime	18,380	16,600	16,890	15,780	–	–	–

Step 4: Evaluate the profit of y_k^j for all $k=1, \ldots,$ K (i.e., $\pi_k(y^j)$) and all $j=1, \ldots,$ Npop.
Step 5 (Specifically for ND-MLSG-PBM): Perform pairwise Nash domination comparison by evaluating G(x^j, y^j) and G(y^j,x^j) between every individual in the current population and its associated child/new position. If $G\left(x^j, y^j\right) \leq G(y^j,x^j)$, keep x^j for the next iteration and discard y^j. Otherwise, replace x^j with y^j.
Step 5 (Specifically for NI-MLSG-PBM): Evaluate $\Psi\left(x^j, y^j\right)$ and $\Psi\left(y^j, x^j\right)$ for every strategy profile in the current population (x^j) and its associated child/new position (y^j). If $\Psi\left(x^j, y^j\right) \leq \Psi\left(y^j, x^j\right)$, keep x^j for the next iteration and discard y^j. Otherwise, replace x^j with y^j.
Step 6: Repeat steps 3—5 until the stopping criteria are reached (e.g., maximum defined iterations or convergence conditions).

Algorithm II: Generating a new strategy profile (y^j) from the current one (x^j) using PSO

Sub-step1: Given a strategy profile (current position) x^j, for the first iteration, initialize the velocity vector for the current strategy profile.
Sub-step 2: Update the velocity vector for each player (k) according to the following equation:

$$v_k^{j,t+1} = \omega v_k^{j,t} + \rho_1 C_1\left(pbest_k^j - x_k^j\right) + \rho_2 C_2\left(gbest_k - x_k^j\right)$$

where t is the current iteration, t + 1 is the next iteration, $pbest_k^j$ is the most profitable strategy of player k in jth individual until t, and $gbest_k$ is the most profitable strategy of player k in the whole swarm until t.
Sub-step 3: Update the strategy of player k (k = 1, …, K) to the new one (y_k^j) by $y_k^j = x_k^j + v_k^{j,t+1}$.
Sub-step 4: Apply repair strategies for those variables that exceed their ranges. Form y^j as the …of y_k^j for k = 1, …, K.
Sub-step 4: Evaluate $\pi_k(y^j)$ and update $pbest_k^j$ and $gbest_k$ if necessary.
Sub-step 5: Continue to step 5 of the ND-MLSG or NI-MLSG algorithm for replacement.

Algorithm III, Generating a new strategy profile (y^j) from the current one (x^j) using DE

Sub-step 1: *Select three strategy profiles in the current population randomly ($x^a x^b and x^c$ such that $x^a \neq x^b \neq x^c \neq x^j$).*
Sub-step 2: Generate a child vector (new strategy profile y^j) from the selected vectors using the following equation:

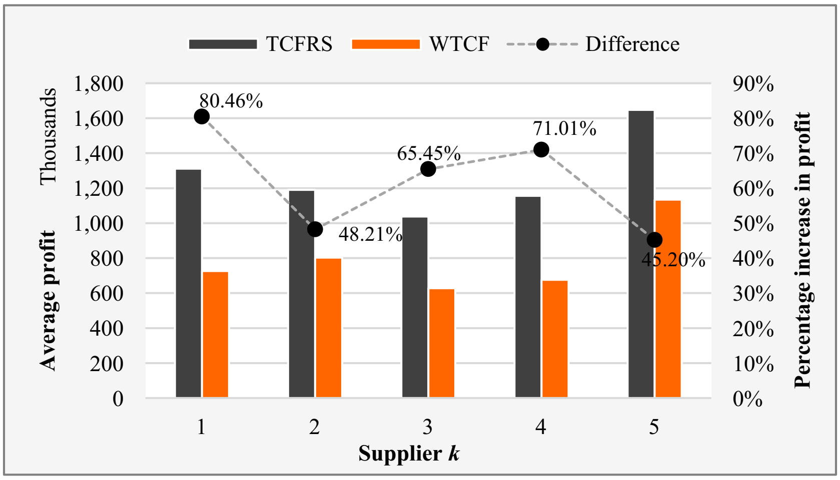

Fig. 4 The effects of the TCFRS contract on the supplier's profit for test problems in size 1

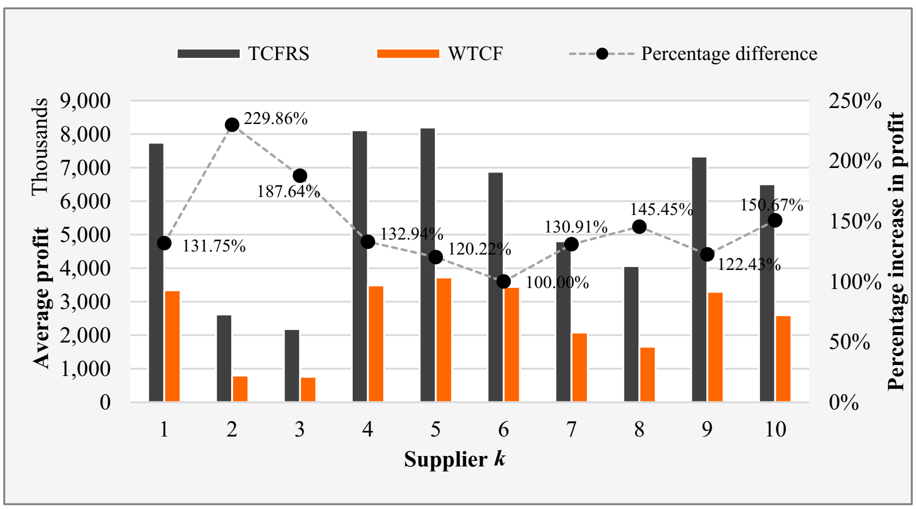

Fig. 5 The effects of the TCFRS contract on the supplier's profit for test problems in size 2

$$y^j = x^a + F\left(x^b - x^c\right)$$

where, parameter $F \in [01]$ represents a scaling factor.

Sub-step 3: Apply repair strategies for those variables that exceed their ranges.

Sub-step 4: Continue to step 4 of the ND-MLSG or NI-MLSG algorithm for replacement.

Interested readers may refer to Talbi (2009) for more details about DE and PSO algorithms.

Results and discussion

In this section, we conducted a numerical analysis to evaluate the efficiency of the proposed TCF contract to improve supply chain performance and achieve supply chain coordination using the solutions of the centralized model as a benchmark. For this

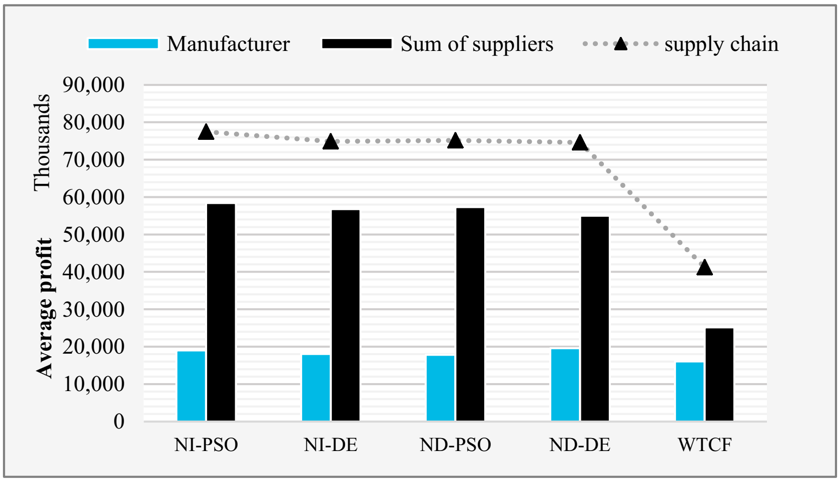

Fig. 6 A comparison between the performance of the proposed solution algorithms in problem size 1

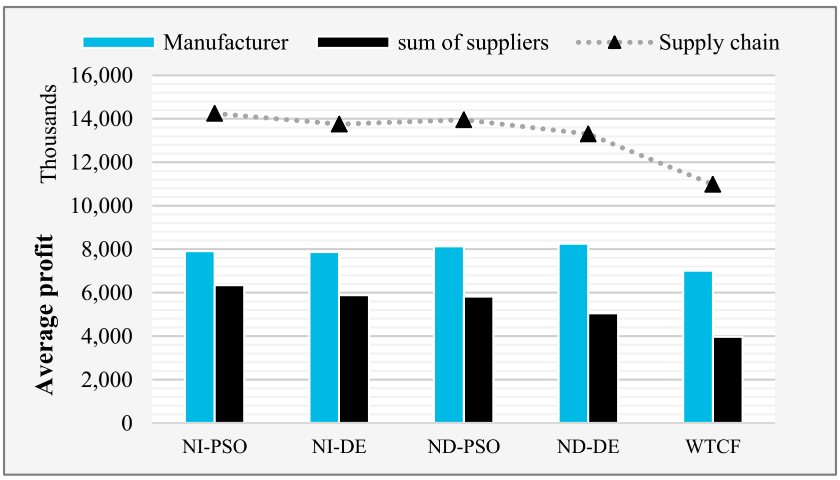

Fig. 7 A comparison between the performance of the proposed solution algorithms in problem size 2

purpose, we generated 60 test problems of two sizes (30 in each) based on the number of original parts and raw material types. All parameters were generated randomly using the uniform distribution shown in Table 4.

In the table, problem size 1 includes three original parts and five raw materials, and problem size 2 includes five original parts and ten raw materials. The random demand variable follows a normal distribution with mean μ_n and standard deviation σ_n. The planning period (T) was 120 days. The initial budget was selected randomly in the ranges [0,3e+6] and [10e+6,15e+6] units of cash for problem sizes 1 and 2, respectively. The maximum allowed external financing (*ML*) was set to 4.5e+6 and 27e+6 units of cash for problem sizes 1 and 2, respectively.

The results of solving the proposed multi-leader Stackelberg game by the proposed algorithms in *Sc-1* and *Sc-2* are presented and discussed in Sects. 5.1 and 5.2, respectively. The model in the centralized structure was also solved by adapting the second and third phases of the solution procedure proposed by Emtehani et al. (2021a).

Table 7 A summary of the results for the TCF contract in Sc-2 and WTCF case for the test problems in size 1

Solution method	ND-MLSG-PBM		NI-MLSG-PBM		WTCF	Profitability ratio
Metaheuristics	DE	PSO	DE	PSO		
SC profit	13,406,571	13,746,626	13,603,767	13,776,851	10,978,833	25%
Manufacturer's profit	7,861,141	7,932,690	7,852,516	7,820,638	7,011,337	12%
Sum of suppliers' profit	5,545,430	5,813,936	5,751,251	5,956,213	3,967,496	50%
Sup_1 profit	1,310,250	1,309,695	1,320,991	1,309,654	726,505	80%
Sup_2 profit	912,030	932,549	947,411	962,822	803,168	20%
Sup_3 profit	954,414	962,855	985,039	1,038,182	627,508	65%
Sup_4 profit	1,045,440	1,085,519	1,045,437	1,110,599	675,808	64%
Sup_5 profit	1,332,296	1,523,318	1,523,292	1,553,509	1,134,507	37%
Production quantities	4997	5001	4998	4997	2997	67%
	6501	6499	6502	6502	3502	86%
	6788	6792	6788	6790	3930	73%
Loan amount	3,881,242	4,482,202	3,881,225	2,197,060	4,500,000	–
Runtime	11,834	11,052	11,286	10,895	–	–

Table 8 A summary of the results for TCF contract in Sc-2 and WTCF case for the test problems in size 2

Solution method	ND-MLSG-PBM		NI-MLSG-PBM		WTCF	Profitability ratio
Metaheuristics	DE	PSO	DE	PSO		
SC profit	71,318,513	72,197,972	72,294,081	72,906,057	41,218,764	77%
Manufacturer's profit	28,134,639	27,441,594	28,199,341	27,570,991	16,056,408	72%
Sum of suppliers' profit	43,183,874	44,756,378	44,094,740	45,335,066	25,162,356	80%
Sup_1 profit	5,616,019	5,572,136	5,572,922	5,837,535	3,340,146	75%
Sup_2 profit	2,715,905	2,807,892	2,811,040	2,681,441	793,104	238%
Sup_3 profit	1,616,847	1,670,772	1,544,008	1,625,720	759,144	114%
Sup_4 profit	4,816,860	4,749,838	4,705,524	4,937,373	3,484,164	42%
Sup_5 profit	6,974,951	7,499,457	7,456,552	7,496,144	3,722,533	101%
Sup_6 profit	4,760,372	5,031,565	4,908,108	5,088,790	3,437,680	48%
Sup_7 profit	3,661,906	3,774,970	3,775,437	3,908,941	2,080,143	88%
Sup_8 profit	3,511,362	3,612,723	3,556,955	3,721,879	1,655,035	125%
Sup_9 profit	4,108,071	4,569,044	4,286,632	4,569,153	3,295,371	39%
Sup_{10} profit	5,401,581	5,467,981	5,477,562	5,468,090	2,595,035	111%
Production quantities	1770	1771	1769	1772	945	88%
	2008	2013	2010	2011	1023	97%
	2403	2408	2411	2409	1316	83%
	1559	1558	1556	1557	877	78%
	1525	1528	1526	1528	958	59%
Loan amount	12,347,904	10,610,823	10,154,530	845,910	20,000,000	–
Runtime	22,570	21,977	22,145	21,460	–	–

Scenario 1

After solving the bi-level model in *Sc-1*, we noticed that the answers are similar to the case without the TCF contract (which we refer to as the WTCF case) for both problem sizes (see the column related to WTCF in Tables 5 and 6). This means that suppliers will not grant TCF to the manufacturer in such a situation. The reason is that, although the

TCF contract in *Sc-1* is beneficial for the manufacturer, it does not make any profit for the suppliers because of the restrictions on the manufacturer's access to external financing. If suppliers grant trade credit to the manufacturer in such a situation, on the one hand, it will increase its order quantities and gain more profit, but still cannot order equivalent to the centralized structure due to financial constraints. On the other hand, the suppliers' profit will decrease because of the discount on sales and the imposed opportunity cost. Since the profit gained from the increase in the manufacturer's order quantities cannot compensate the suppliers, they have no incentive to offer the TCF contract set up in *Sc-1* to the manufacturer. However, if the manufacturer shares a portion of its revenue with suppliers, they may be willing to offer short-term financing through a TCF contract to the manufacturer. Accordingly, we developed a hybrid contract, which we refer to as the TCFRS contract, to test this hypothesis. This contract is a combination of TCF and revenue-sharing contracts. The problem formulation under the TCFRS contract is as follows.

TCFRS contract

The manufacturer's problem under the TCFRS contract is similar to its problem in the TCF contract in *Sc-1*, except that the expected revenue, $\sum_n p_n E\left[\min\left\{R_n y_n\right\}\right]$, in the objective function (Eq. 1) and cash balance constraint (Eq. 7) are multiplied by the revenue-sharing coefficient (φ). Note that $\varphi \in [01]$ is the ratio of the manufacturer's share of its own revenue, which is the decision variable for the TCFRS contract. Each supplier's decision problem under the TCFRS contract, denoted by $P_{sup_k}^{TCFRS}$, is modeled as follows:

$$\boldsymbol{P_{Sup_k}^{TCFRS}}:$$

$$\begin{aligned} Max\pi_{Sup_k}^{TCFRS}\left(b_k d_k u_k \tau_k \varphi R_n Q_k x_k^i t_k l \varphi_k\right) = & w_k Q_k\left[(1-u_k)x_k^1 + x_k^2 + (1+\tau_k)^{t_k - d_k} x_k^3\right] \\ & - w_k Q_k\left((1+r_k)^{t_k} - 1\right) - c_k Q_k + (1-\varphi)\beta_k \\ & \sum_n p_n E\left[\min\left\{R_n y_n\right\}\right] \end{aligned} \tag{24}$$

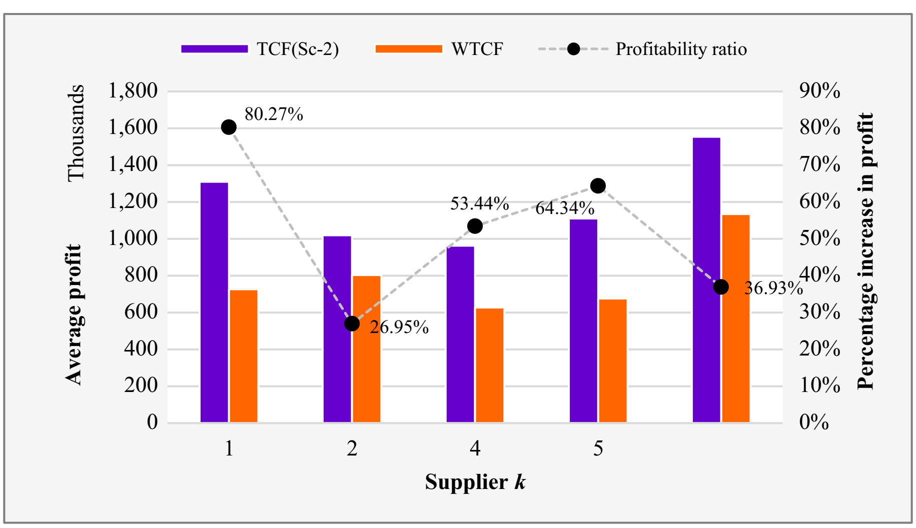

Fig. 8 The effects of TCF contract in *Sc-2* on the supplier's profit for test problems of size 1

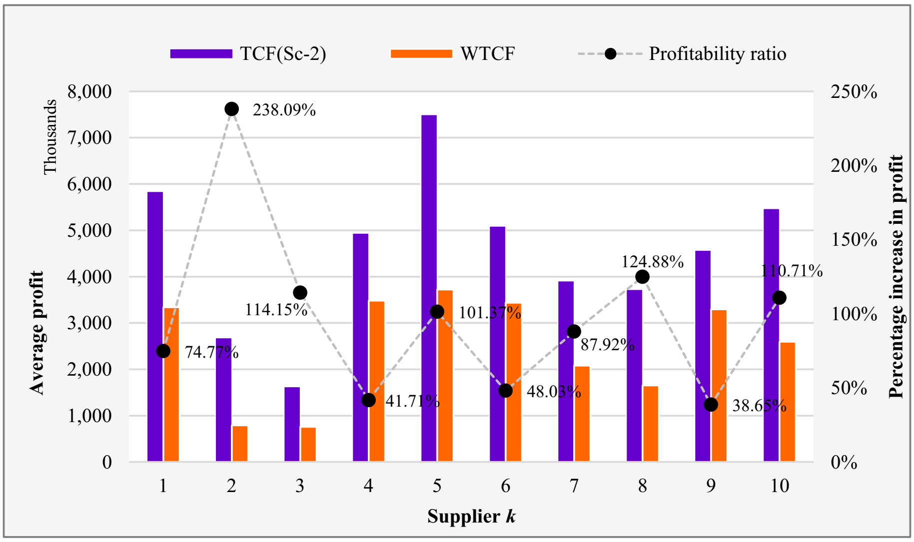

Fig. 9 the effects of TCF contract in *Sc-2* on the supplier's profit for test problems of size 2

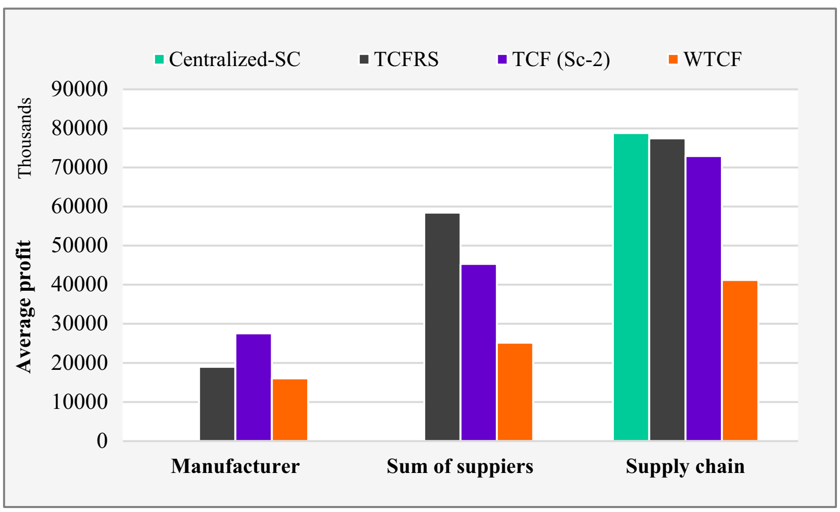

Fig. 10 A comparison between TCFRS contract, TCF contact in *Sc-2*, WTCF case, and centralized structure for test problems of size 1

Subject to:

$$P_M^{TCFRS} = \arg\max\{\pi_M^j\left(b_k d_k u_k \tau_k \varphi R_n x_k^i t_k l\right) \text{st} : associatedconstraints\} \tag{25}$$

$$b_k \leq d_k \leq tmax_k \tag{26}$$

In the objective function, β_k is the ratio of each supplier's share of the manufacturer's expected revenue and is calculated as follows:

$$\beta_k = \frac{\gamma_k}{\sum_k \gamma_k} \tag{27}$$

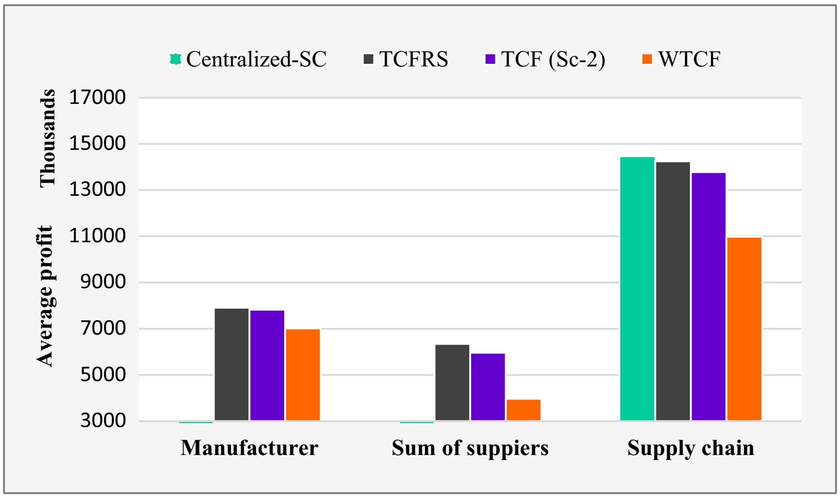

Fig. 11 A comparison between TCFRS contract, TCF contact in *Sc-2*, WTCF case, and centralized structure for test problems of size 2

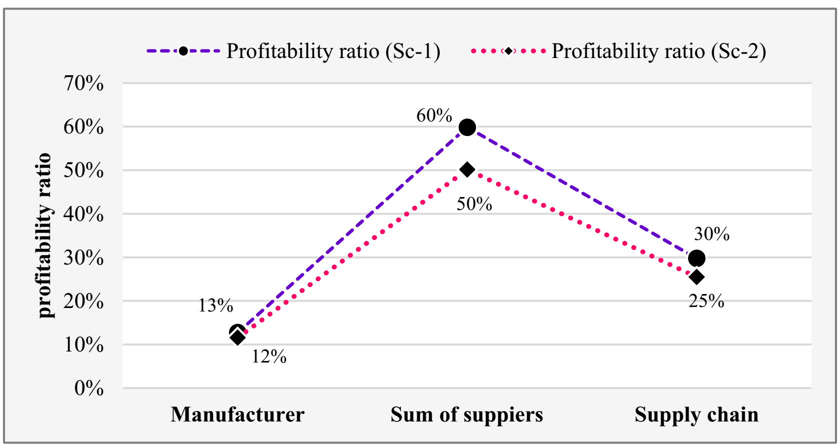

Fig. 12 A comparison of the profitability ratios for SC, suppliers, and the manufacturer in *Sc-1* and *Sc-2* for problem size 1

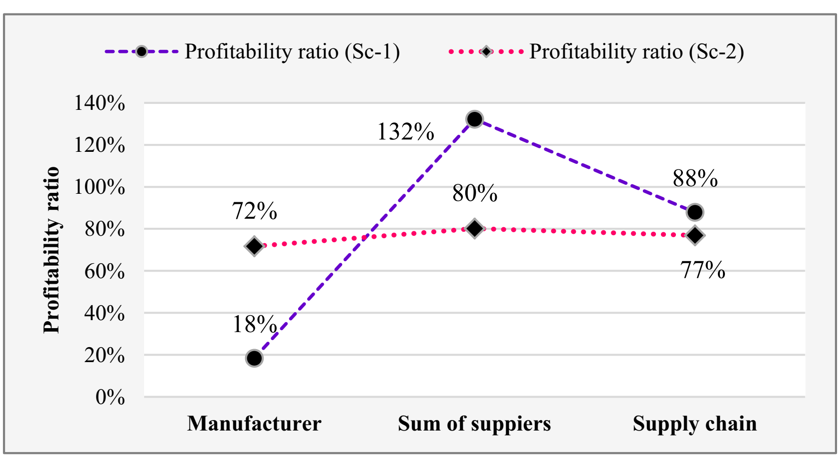

Fig. 13 A comparison of the profitability ratios for SC, suppliers, and the manufacturer in *Sc-1* and *Sc-2* for problem size 2

In the above equation, γ_k is the supplier k's potential loss in the TCF contract before receiving its share of the manufacturer's revenue, which is calculated as:

$$\gamma_k = Z_k^{WTCF} - Z_k^{TCF} \tag{28}$$

where Z_k^{WTCF} is the supplier k's profit without a TCF contract and Z_k^{TCF} is its profit under the TCF contract before receiving its share of the revenue.

Tables 5 and 6 report the results of solving $P_{sup_k}^{TCFRS}$ for both problem sizes using the algorithms proposed in the previous section using the PSO and DE methods. The values reported in each table are the means of the Nash non-dominated solutions of the 30 test problems for the related problem size. The profitability ratio for each member (in the last column) is defined as the percentage increase in its profit in the TCFRS contract compared to the WTCF case. Note that the computations of the profitability ratios are performed using the solutions of the NI-MLSG-PBM algorithm considering PSO for reproduction because they present better solutions than other existing methods.

In Tables 5 and 6, the first four columns represent the data for the TCFRS contract achieved using different solution methods. As observed from the tables, the TCFRS contract significantly increases the suppliers' profit as well as the manufacturer's profit compared with the WTCF case (the fifth column) in both test problem groups.

Figures 4 and 5 illustrate the effects of the TCFRS contract on the supplier's profit for both the problem sizes. The values reported in the charts in the form of percentages represent the profitability ratios of the suppliers in the TCFRS contract. It is observed that by applying the TCFRS contract, the suppliers make a substantial profit and finance the capital-constrained manufacturer. Thus, this contract provides a win–win situation for all members. Moreover, the SC profit under the TCFRS contract almost reaches its profit in the centralized structure, that is, channel coordination is achieved by applying the TCFRS contract.

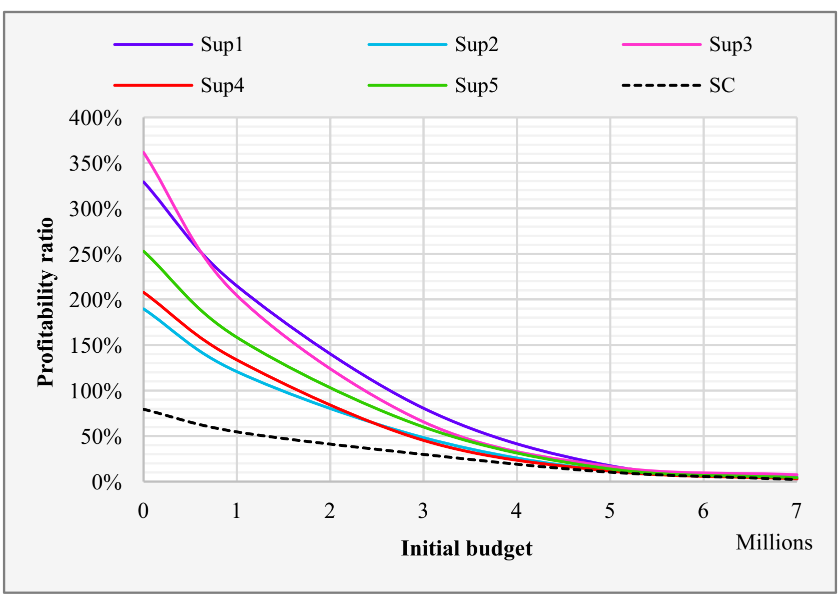

Fig. 14 The result of analyzing the sensitivity of the suppliers' and SC's profitability ratio to the changes in the initial budget

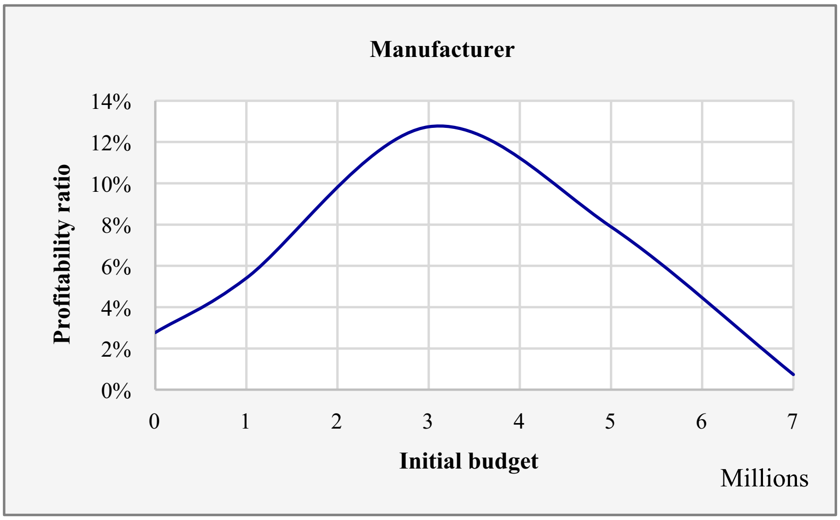

Fig. 15 The result of analyzing the sensitivity of the manufacturer's profitability ratio to the changes in the initial budget

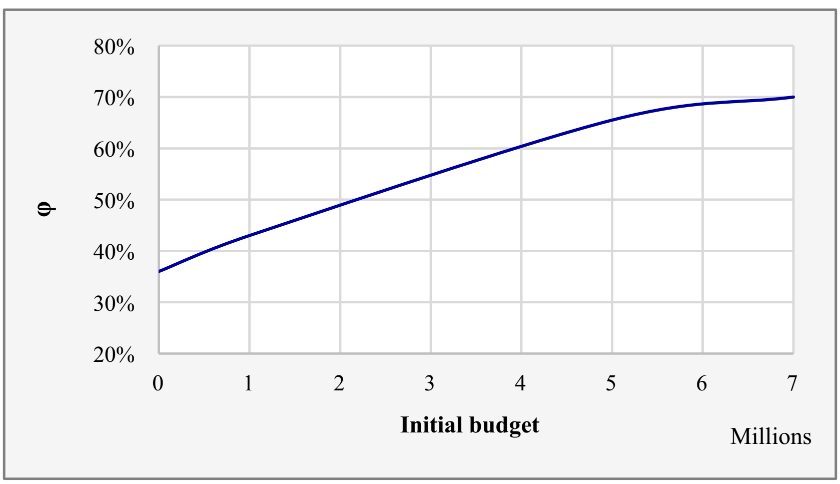

Fig. 16 The result of analyzing the sensitivity of φ to the changes in the initial budget

We used the average total profit of suppliers as a criterion to compare the proposed algorithms. The results are illustrated in Figs. 6 and 7 for both problem sizes. The results indicate that both the NI-MLSG-PBM and ND-MLSG-PBM algorithms provide better solutions with respect to quality and runtime when using PSO for reproduction compared to DE. Furthermore, the Nikaido-Isoda function performs better than the Nash domination relations with respect to both solution quality and runtime while using the same method for reproduction. This result is consistent with He et al. (2016), who compared the Nikaido-Isoda function and Nash domination relations to find the Nash equilibrium of a multi-player game.

Scenario 2

The results of solving the test problems for both problem sizes in Scenario 2 are reported in Tables 7 and 8. Note that the data for the centralized SC in Tables 5 and

Table 9 The results of solving the proposed game for both scenarios with 2 suppliers and 3 products

SC status	Centralized	WTCF	TCFRS contract (*Sc-1*)	Profitability ratio (TCFRS, WTCF)	TCF contract (*Sc-2*)	Profitability ratio (TCF_{Sc-2}, WTCF)
SC profit	15,603,459	11,606,136	15,565,936	34.12%	14,469,205	24.67%
Manufacturer's profit	–	10,149,412	12,528,817	23.44%	11,796,849	16.23%
Sum of suppliers' profit	–	1,456,724	3,037,119	108.49%	2,672,356	83.45%
Sup_1 profit	–	866,892	1,864,633	115.09%	1,658,247	91.29%
Sup_2 profit	-	589,832	1,172,486	98.78%	1,014,109	71.93%
φ	–	–	51%	–	–	–
Production quantities	5009	2605	5010	92.32%	4991	91.59%
	6513	4519	6518	44.25%	6487	43.55%
	6811	3930	6814	73.38%	6789	72.75%
Loan amount	–	1,500,000	301,692	–	410,576	–
Runtime	–	–	4708	–	5165	–

6 are not repeated in Tables 7 and 8. The profitability ratio for each member (in the last column) is defined as the percentage increase in its profit in the TCF contract in *Sc-2* compared to the WTCF case. (Note that the computations of the profitability ratios were performed using the solutions of the NI-MLSG-PBM algorithm considering PSO for reproduction.).

As seen from the tables, the profitability ratios of all members are positive, which indicates an increase in their profit under the TCF contract in *Sc-2* compared to the WTCF case. This means that the TCF contract in *Sc-2* can coordinate the supply chain by considering the aforementioned financial constraints. This is because some of the payments can be settled at the end of the period based on the contract terms. Therefore, the manufacturer can pay sales revenue subject to a penalty. This implies that financing constraints are no longer a bottleneck for the manufacturer's operations. Therefore, the manufacturer will order quantities that are very close to its order quantities in the centralized structure. (Please compare the production quantities for the TCF contract in *Sc-2* in the first four columns of Tables 7 and 8 with the production quantities of the centralized SC in the sixth column of Tables 5 and 6.) According to the model, the manufacturer will choose an optimal combination of financing modes, including external financing and TCF contract options, to maximize its profit, which will lead to more profit for its suppliers.

Similar to *Sc-1*, the best algorithm for solving the problem was the NI-MLSG-PBM algorithm using the PSO method for reproduction, according to the results.

Figures 8 and 9 show the impact of the TCF contract in *Sc-2* on suppliers' profit for both problem sizes. The values reported in percentages represent the profitability ratios of suppliers in the TCF contract (*Sc-2*). Similar to the previous subsection, computations of these values were performed using the solutions of the NI-MLSG-PBM algorithm considering PSO for reproduction.

A comparison between the SC members' profits in the TCFRS contract and the TCF contract in *Sc-2* is conducted for both problem sizes, and the results are illustrated in Figs. 10 and 11. It is observed that the mean of suppliers' profit and SC profit under the TCFRS contract is slightly higher than that under the TCF contract in *Sc-2*. The green column in the figures represents SC profit in the centralized structure.

Figures 12 and 13 show the percentage of increase in the members' profit in the TCFRS and TCF contracts in *Sc-2* compared to the WTCF case. According to the figures, for both problem sizes, the increase in the suppliers' profit is greater than the increase in the manufacturer's profit under both the TCFRS and TCF contracts in *Sc-2*. Moreover, the TCFRS contract is more beneficial for suppliers than the TCF contract in *Sc-2*.

Sensitivity analysis

In this section, the sensitivity of the revenue-sharing coefficient and profitability ratio in the TCFRS contract to the variation in the manufacturer's initial budget is analyzed.

For this purpose, the problem is solved for different values of the initial budget, from zero to seven million units of cash, using the data of the test problems in problem size 1. The results indicate that increasing the manufacturer's initial budget reduces the profitability ratios of the suppliers and the entire SC (see Fig. 14). This implies that the worse the financial status of the manufacturer, the more suppliers benefit from coordination. Thus, suppliers can take advantage of a manufacturer's critical financial situation by offering a TCFRS contract. In the case of the manufacturer, as its initial budget increases, its profit increases until the sum of the initial budget and external financing covers the optimal production quantity. After this point, by increasing the initial budget, its profit in the WTCF case increases and approaches its profit in the TCFRS contract. As can be seen from Fig. 15, the manufacturer's chart first shows an upward trend and then a downward trend.

The results of analyzing the sensitivity of φ (revenue-sharing coefficient) to the initial budget show that the value of φ decreases with decreasing initial budget (see Fig. 16). This implies that the more the manufacturer is financially dependent on suppliers, the more power suppliers must determine φ for higher profit.

The impact of suppliers' numbers on the proposed game

In both scenarios, the proposed game is established for multiple suppliers and one manufacturer. If there was a single supplier, only vertical competition between the supplier and the manufacturer (the traditional Stackelberg leader–follower game) would exist. Therefore, the problem would be different and much easier to solve than with two or more suppliers. In the case of multiple suppliers (the current study), horizontal competition in the form of a Nash game exists between the suppliers in addition to the vertical competition between each supplier and the manufacturer. Also, the greater the number of suppliers, the greater the complexity of the problem. With two suppliers, simpler solution methods may be used, even for complex problems. However, to examine the effects of supplier numbers on the model behavior and results, we also solved the proposed game in both scenarios with two suppliers using the ND-MLSG-PBM algorithm based on PSO. For this purpose, the 30 test problems generated for problem size 1 in Sect. 5 were applied, except that $k=1{,}2$ and the initial budget and *ML* were set to 1e + 6 and 1.5e + 6, respectively. The data reported in Table 9 represent the mean of the Nash nondominated solutions of the 30 test problems. As can be seen in the table, there is no particular change in the nature of the results in comparison with the results of multiple suppliers (Table 5), except for the lower runtime of the algorithm.

Overall, in this study, the number of suppliers (two or more suppliers) did not have any substantial effect on the nature of the results, but it obviously affected the problem complexity and runtime of the proposed algorithms.

Conclusion, implications, and future directions

This study is designed and organized to fill a gap in the previous literature by designing a novel SCF framework based on TCF to eliminate the supply chain inefficiencies raised by members' financial problems, such as capital shortages and financing problems. An exploration of related literature revealed that almost all previous studies focused on the interactions between a seller and a buyer in a simple vertical structure. Given that, in practice, SCs are more complex and the interactions of a seller and a buyer are influenced by other members of the SC, it seems essential to consider the interactions of a bigger piece of the SC rather than the simplest sample. For this purpose, we considered a supply chain consisting of several strategic suppliers of required components and raw materials and a manufacturer whose financial problems negatively affect the profitability of the suppliers as well as its performance. To overcome this challenge, we coordinated SC members by developing two contracts based on trade credit financing. Two scenarios were proposed and tested according to different TCF contract terms. In the first scenario, the payment to the suppliers took place before the end of the period; however, in the second scenario, the manufacturer could delay its payments until the end of the period. In each scenario, the interactions of the SC members were modeled as a multi-leader Stackelberg game that contained both vertical and horizontal competition. The non-cooperative suppliers (as leaders) play a Nash game to determine the contract parameters to maximize their individual profits, considering the manufacturer's response. The manufacturer (as the follower) optimizes its financial and operational decisions as well, in response to suppliers' decisions.

The proposed multi-leader Stackelberg game in both scenarios was Np-hard. To solve this problem, we developed two algorithms by combining population-based metaheuristics, Nash domination relations, and the Nikaido-Isoda functions named NI-MLSG-PBM and ND-MLSG-PBM. In particular, we used DE and PSO for population reproduction in these algorithms. However, these algorithms are generally applicable to population-based metaheuristics. To solve the manufacturer's decision model at the lower level, the three-phase solution approach established by Emtehani et al. (2021a) was used for *sc-1* and extended for *Sc-2*.

The problems in both scenarios were solved using the proposed algorithm. We extracted some theoretical implications and results from solving the proposed models. The results are presented below.

- We examine the conditions under which the TCF contract could coordinate the SC. The results indicate that the TCF contract developed in *Sc-1* cannot coordinate the supply chain because of the lack of incentive for suppliers to grant TCF in such a setting. Therefore, we designed a new coordinating contract called TCFRS, in which the manufacturer shares its revenue with the suppliers at specific rates as the contact parameters. The results of solving the new model indicate that the TCFRS contract significantly increases suppliers' profit compared to the WTCF case and fully coordinates the supply chain. In the second scenario, because the manufacturer pays a portion of its debt to the suppliers from the sales revenue at the end of the period, the existing financial constraints do not disrupt the operations. Therefore, the TCF contract in *Sc-2* coordinates the SC and makes a profit that is very close to the profit in the centralized structure. Moreover, the comparison between the TCFRS contract and the TCF contract *in Sc-2* revealed that the former is more beneficial for the manufacturer and the whole SC than the latter.

- Different problem settings led to different combinations of two financing modes, including internal financing (through TCFRS and TCF contracts) and external financing (bank loan), which are described below.
- In *Sc-1*, the manufacturer borrows up to an allowable limit from external sources to support purchasing and production costs, and there is no internal financing option.
- In *Sc-1*, if the suppliers offer a TCFRS contract, the manufacturer uses joint financing (internal and external financing). In this case, internal financing supports all the purchasing costs of the manufacturer and external financing supports the production costs. Therefore, the loan amount depends on the initial budget of the manufacturer; if the initial budget is sufficient to support production costs, the loan amount is zero. Otherwise, it is not zero, and depends on the initial budget.
- In *Sc-2*, the manufacturer uses joint financing. In this case, since the manufacturer can extend some of (or all of) its payments to the end of the period (subject to a penalty for each day delay), the loan amount (or the combination of financing strategies) depends on the penalty rate of the TCF and the loan interest rate.
- We analyzed the sensitivity of the profitability ratios for all members and the revenue-sharing coefficient to variations in the manufacturer's initial budget in the TCFRS contract. These results suggest that the profitability of suppliers and SC increases by decreasing the initial budget. This implies that the worse the manufacturer's financial situation, the more suppliers benefit from the TCFRS contract. Moreover, we observed that the revenue-sharing coefficient increased by decreasing the manufacturer's initial budget. This also means that the more the manufacturer is financially dependent on suppliers, the more power suppliers have to determine the revenue-sharing coefficient for higher profits.
- A comparison between the solution procedures showed that PSO performs better than DE for reproduction in both algorithms (NI-MLSG-PBM and ND-MLSG-PBM) with respect to solution quality and runtime. Moreover, applying the Nikaido-Isoda function provides better Nash non-dominated solutions than the Nash domination relations; that is, NI-MLSG-PBM provides better solutions with respect to solution quality and runtime compared with ND-MLSG-PBM while using the same method for reproduction.

The main practical implication of the current study is that eliminating the disruptions caused by some financial constraints is possible through supply chain coordination based on trade credit financing with proper setting of the contract parameters.

Moreover, some managerial insights are provided from the suppliers' and the manufacturer's perspectives, revealing that an effective way to overcome financial problems and increase profitability under the pressure of these difficulties is internal financing, that is, financing from supply chain members. In this way, financially constrained companies can convince their suppliers to finance their purchases through large discounts on sales or delayed payments without penalty by offering revenue-sharing. In this case, suppliers generate large profits following this policy. Furthermore, suppliers whose customers are experiencing financial difficulties can make large profits by offering large discounts in return for receiving a portion of their revenue, or by giving them a deadline to pay until the end of the period. Simultaneously, the profitability of their customers will increase and they will have a more efficient supply chain.

We conclude this study by expressing the research limitations and presenting a few directions for the extension of this research. This study has a few limitations in reducing the complexity of the problem. First, it is assumed that the suppliers have a sufficient budget to finance the capital-constrained manufacturer. However, in practice, suppliers may also face financial restrictions. Therefore, considering the budget constraints for suppliers requires a new design for the SCF framework. This could be an interesting subject for future studies. Second, bankruptcy was not addressed in this study. Thus, further studies should include bankruptcy costs and analyze their effects on supply chain coordination. Third, the solution approach proposed to solve the multi-leader Stackelberg game has a high runtime despite its ability to provide good solutions. Hence, it could be a good idea to develop a solution procedure for this problem with less runtime and the same quality. Finally, the SC considered in this study contained multiple suppliers and a single manufacturer. If there is more than one manufacturer, the problem will be more complicated because new horizontal competition (between the manufacturers) will be added to the game. Therefore, this new problem setting requires reformulation and a new solution procedure for a new model. This could be an interesting (but challenging) idea for further studies.

Appendix

Lower level optimization in *Sc-2*

To solve the lower level problem in *Sc-1*, we have used the three-phase approach proposed by Emtehani et al., (Emtehani et al. 2021a) for solving the integrated operational-financial model of a manufacturer with the same assumptions as in *Sc-1* of the current study.

The difference between the manufacturer's model in *Sc-1* and *Sc-2* is that the expression $\sum_k w_k Q_k (1+\tau_k)^{T-d_k} x_k^4$ is added to the purchasing cost in the objective function and the cash flow constraint (constraint 7). So, we have extended the mentioned three-phase approach for solving the lower level problem in *Sc-2*. In the following, the changes made in each phase to adapt the solution procedure to *Sc-2* are described.

Phase 1

In *Sc-1*, to find the optimal values of integer and binary variables (t_k and x_k^i), the best payment time in periods $[0b_k]$, $[b_k+1d_k]$, and $[d_k+1tmax_k]$ were compared. In *Sc-2*, since the manufacturer can extend its payments to the end of the period, the comparison is conducted between periods $[0b_k]$, $[b_k+1d_k]$, $[d_k+1\mathrm{T}-1]$, and time T.

According to the above explanations, all of the lemmas and corollaries inferred from model analysis in their study for phase 1 (in the solution procedure) are also used for *Sc-2*, except that in all of the lemmas and corollaries,$tmax_k$ is substituted by T-1. In addition, some new lemmas are added by comparing the mentioned three time periods with time T. These new lemmas are described below:

Lemma 1 *if* $(1-u_k)\left[2-(1+r_m)^{b_k}\right](1+r_l)^T<(1+\tau_k)^{T-d_k}$, *then paying supplier k at* b_k *is more beneficial than T.*

Proof Emtehani et al., (Emtehani et al. 2021a) defined $W'(t_k)=w_kQ_k\left(2-(1+r_m)^{t_k}\right)\left[(1-u_k)x_k^1+x_k^2+(1+\tau_k)^{t_k-d_k}x_k^3\right](1+r_l)^T$ as the cost which is influenced by the payment time. They used this cost to compare the specific times in the range $[0tmax_k]$ to find the best payment time in this range. In *Sc-2* in this study, the purchasing cost related to the fourth option is added to this cost. So we reach:

$$\begin{aligned}W'_{Sc-2}(t_k)=&w_kQ_k\left(2-(1+r_m)^{t_k}\right)\left[(1-u_k)x_k^1+x_k^2+(1+\tau_k)^{t_k-d_k}x_k^3\right]\\&(1+r_l)^T+w_kQ_k(1+\tau_k)^{T-d_k}x_k^4\end{aligned} \tag{29}$$

It should be noted that if the manufacturer pays to supplier k at T, it will pay from sales revenue and will not pay from external financing.

If $W'_{Sc-2}(b_k)<W'_{Sc-2}(T)$, then b_k is a better option than T. According to Eq.A-1 and after some algebra we reach $(1-u_k)\left[2-(1+r_m)^{b_k}\right](1+r_l)^T<(1+\tau_k)^{T-d_k}$.

Corollary 1 *If* $\left[2-(1+r_m)^{d_k}\right](1+r_l)^T<(1+\tau_k)^{T-d_k}$, *then* d_k *is a better choice for payment time than T, otherwise, T is a better option for payment.*

Corollary 2 *For* $d_k<t_k'<T-1$, *if* $(1+\tau_k)^{t_k'-d_k}\left[2-(1+r_m)^{t_k'}\right](1+r_l)^T<(1+\tau_k)^{T-d_k}$, *then* t_k' *is a better option for payment time than T.*

To find the optimal values of t_k and x_k^i in *Sc-2*, in the continuation of the exact algorithm proposed by Emtehani et al., (Emtehani et al. 2021a) in phase 1, we have found the best payment time between s_k and e_k (the last comparison in the exact algorithm) and name it g_k. Where e_k is the best time for payment in the range $[0d_k]$ and s_k is the best time for payment in the range $[d_k+1tmax_k]$. Then we have conducted a comparison between g_k and T. This comparison is presented in the form of pseudocode to be added to the exact algorithm proposed by Emtehani et al., (Emtehani et al. 2021a) to be used for *Sc-2* in the current study (see Algorithm A-1).

Algorithm A-1, the comparison between g_k and T

// Compare g_k and T.

If $g_k = b_k$

// using lemma A-1

If $(1 - u_k)[2 - (1 + r_m)^{b_k}](1 + r_l)^T < (1 + \tau_k)^{T - d_k}$

$t_k^* = b_k$

$x_k^{1^*} = 1$

Else

$t_k^* = T$

$x_k^{4^*} = 1$

End if

Elseif $g_k = d_k$

// using Corollary A-1

If $[2 - (1 + r_m)^{d_k}](1 + r_l)^T < (1 + \tau_k)^{T - d_k}$

$t_k^* = d_k$

$x_k^{2^*} = 1$

Else

$t_k^* = T$

$x_k^{4^*} = 1$

End if

Elseif $g_k = s_k$

// using Corollary A-2

If $(1 + \tau_k)^{t'_k - d_k}\left[2 - (1 + r_m)^{t'_k}\right](1 + r_l)^T < (1 + \tau_k)^{T - d_k}$

$t_k^* = s_k$

$x_k^{3^*} = 1$

Else

$t_k^* = T$

$x_k^{4^*} = 1$

End if

End if

Phase 2

Following the instructions of phase 2 in Emtehani et al., (Emtehani et al. 2021a) considering the model in *Sc-2*, we reach:

$$R_n^* = F^{-1}\left(\frac{p_n + \pi_n - A'_n - \left(A''_n + v_n\right)(1 + r_l)^T}{p_n + s_n + h_n}\right) \tag{30}$$

and $l* = \sum_n \left(A''_n + v_n(R_n - I_n)\right) - B_0$.

Where, $A'_n = \sum_k m_{nk} w_k (1 + \tau_k)^{T - d_k} x_k^4$, and

$A''_n = \sum_k m_{nk} w_k \left(2 - (1 + r_m)^{t_k}\right)\left[(1 - u_k)x_k^1 + x_k^2 + (1 + \tau_k)^{t_k - d_k} x_k^3\right]$.

Phase 3

All of the steps in phase 3 of the solution procedure of the manufacturer's model in *Sc-2* are the same as phase 3 in the proposed solution approach in Emtehani et al., (Emtehani et al. 2021a) except that $-A_n^{'}$ is added to the numerator of the fraction θ_n.. So, θ_n is rewritten as

$$\theta_n = \frac{p_n - A''_n - A'_n - v_n - h_n \int_0^{R_n} f_n(y_n)\,dy_n - s_n \int_{R_n}^{\infty} f_n(y_n)\,dy_n}{A''_n + v_n} \tag{31}$$

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FE helped to conceive and design the study, conducted the analysis, and drafted the manuscript. NN helped to conceive and design the study and coordinated the research activities. FM helped to conceive and design the study. All authors read and approved the final manuscript.

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Declarations

Competing interests
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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