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Principle of ultrasonic tomography for concrete structures and non-destructive inspection of concrete cover for reinforcement

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Abstract

The authors are trying to develop an ultrasonic CT technique for concrete structures. As the first step of its development, we study how the ultrasonic waves propagate in concrete structures and pose a problem to develop ultrasonic CT, applying which we give a non-destructive inspection technique for concrete cover by application of the idea of the ultrasonic CT. The authors claim that this study gives the foundation of the theory of ultrasonic CT for concrete structures.

Keywords: Ultrasonic tomography, Integral geomerty

1 Introduction

There existing a number of non-destructive inspection techniques for concrete structures which give some rough sketches of the interior structure, few of them provides concrete and complete interior information of concrete structures. A few years ago, the X-ray computerized tomography (CT for short) for concrete structures being practicalized, it costs very expensive and the effect of the X-rays to human bodies and the environment cannot be ignored. Therefore it is not suitable to apply the X-ray CT for the daily maintenance of concrete structures. It is required to develop a safe, cheaply running and concretely interior-describing non-destructive inspection technique for concrete structures, to establish which is our main purpose. In view of the above argument, the authors are trying to develop an ultrasonic CT technique for concrete structures. In this paper, as the first step of its development, we study how the ultrasonic waves propagate in concrete structures and pose a problem to develop ultrasonic CT, applying which we propose a non-destructive inspection technique for concrete cover by application of the ultrasonic CT for concrete structures.

Of course, there existing a number of non-destructive inspection methods for concrete structures applying ultrasonic waves, confer [3], [10] and so on, for example, almost all of them utilize the echo technique. It is very helpful to apply echo technique when we can access the concrete structure from only its one side. Applying this technique, however, we only obtain rough sketches of the inclusion or the cavity which lies very close to the side we can access, which is far from concretely interior-describing non-destructive inspection by which we mean. We can find very few paper like [5] where the study is given in a similar method to ours. Even in the paper [5], its main result is to detect the combination defect in the masonry, which is very rough and strongly depends on a priori information of the structure, which is also far from the non-destructive inspection technique the authors are trying to develop.

This paper consists of the following sections.

§1. Introduction
§2. Motivation of this research
§3. Propagation of the ultrasonic waves in concrete structures
§4. An inverse problem of the acoustic tomography
§5. Application to non-destructive inspection of concrete cover for reinforcement
§6. Conclusion

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In this section, as the introduction of this paper, we introduce the outline of our paper. In the next section, we shall introduce our motivation for this research. We first introduce how we understand the concrete in this paper, based on which there arise many problems to solve. Among them, we shall claim that it is one of the most important problems to establish a concretely interior-describing non-destructive inspection method for concrete structures, not only by practical requirements but also from the viewpoint of our understanding of the concrete. In the third section, we study how the ultrasonic waves propagate in the cement paste and the mortar by the experiments, which gives the fundamentals of our theory. In Section 4, we shall pose an inverse problem for establishment of our ultrasonic CT for concrete structures, for which we shall apply the results of our experiments and their examination discussed in Section 3. The problem posed in this section is also interesting in view of pure mathematics, especially, in view of integral geometry. In the fifth section, we shall develop a non-destructive inspection technique for concrete cover in the reinforced concrete structures by an application of the idea of the ultrasonic tomography discussed in the third and fourth sections. In the final section, we shall summarize our conclusions and mention open problems to be solved for further development.

The authors are grateful to Professor Hisashi Yamasaki for his devoted help for our experiments.

2 Motivation of this research

In this section, we shall introduce our motivation for this research. Before introducing our motivation, the authors claim that

Claim 2.1. *The concrete materials are artificial megaliths.*

Let us first discuss why the authors claim Claim 2.1. Take Valley Temple, Egypt (BC 2500?) and Parthenon, Athens (BC447-432), for example, which are made of megaliths. At that period around those areas, there were plenty of megaliths available, therefore they made Valley Temple and Parthenon of megaliths which are very suitable for edifices. On the other hand, let us turn to Colosseum, Rome (AD 70-80). Its bailey or external wall being made of megaliths, its interior structure is infilled with stones, bricks, sand and Roman cement as bonding material (ash, for example), which we take for the origin of the concrete. It may be because of the shortage of the megaliths in Rome about 2000 years ago. Note that the structure of Colosseum still exists more than 2000 years after its foundation. Hence we claim that the primitive concrete materials applied to the interior infill wall of Colosseum have been playing their important role as the substitutes for the megaliths very well for milleniums, which is one of the reasons why the authors claim Claim 2.1.

We shall introduce our motivation for this research in accordance with Claim 2.1. If we take concrete materials for substitutes for the megaliths, there are superior points to the megaliths' such as

(α) Concrete is easily made and shaped in any form because of its fluidity before it gets hard.
(β) The cost of concrete is very cheap.

However, there are demerits of the concrete such as

(a) Concrete is not tough to tensile strength.
(b) Concrete easily gets cracks in and on itself.

Confer [7] for these merits and demerits of concrete. For the general theory of concrete, confer [2, 9]. The demerit (a) yields necessity to reinforce the concrete. The lifespan of reinforced concrete (RC for short) is said to be about a half century, which is much shorter than that of megaliths'. We have to maintain the concrete very well for a long time for the concrete materials to be substitutes for the megaliths. In view of this point, as well as from the viewpoint of the demerit (b), it is necessary to check the concrete structures by some non-destructive way and repair them effectively for their maintenance. Therefore, the following problem is very important.

Problem 2.1. *In order for concrete materials to be artificial megaliths, it is indispensable to reinforce them and to study how to maintain RC material for a long time.*

There is another approach than Problem 2.1, for concrete materials to be artificial megaliths, that is, we should study a method to make concrete structures, without reinforcement, so solid that they can be artificial megaliths, the study of which is under investigation by the authors. Anyway, in view of Problem 2.1, there arise the following problems.

Problem 2.2. *From the viewpoint of Problem 2.1, we pose the following two problems.*

(1) How to establish an effective, safe and cheaply-running non-destructive inspection technique for RC structures, which detects the interior structure concretely.
(2) How to repair the RC structures in order that they would safely live for a very long time

By Problem 2.2, we mean that it is very important to establish a good method to check RC structures without destructing them and to develop a nice method to maintain the RC structures in order that their lifespan would be very long, for RC materials to play a role as substitutes of megaliths. The problem (1) in Problem 2.2 has a strong connection with our discussion in the fifth section.

3 Propagation of the ultrasonic waves in concrete structures

In this section, as a preparation for our main purpose, we study how the ultrasonic primary waves propagate in the concrete structures. We first introduce our experiments to study how the ultrasonic waves propagate in the cement paste and the mortar. By examining the results of our experiments, we shall study the propagation of the ultrasonic waves in concrete structures of the length about $1m$ or less.

Let us introduce the outline of our experiments.

Outline of our experiments

- Ultrasonic waves;
  - The frequency of the ultrasonic waves is $54kHz$.
  - Velocity of the ultrasonic wave is denoted by $V(m/s)$.
- Length of the test pieces;
  We prepared test pieces of the length $100, 200, 300, 400, 800$ and $1200mm$ in order to check
  - the decay of the acoustic velocity
  - the propagation of the ultrasonic waves
- Inclusions;
  We prepared two types of test pieces of the size $100mm \times 100mm \times 400mm$.
  - Normal test pieces
  - Test pieces with styrofoam $100mm \times 50mm \times (200 or 300mm)$ included in their inside (cf. Fig. 4 below)

  These test pieces are made use of to study the propagation of the ultrasonic waves.
- Number of the test pieces;
  We made three test pieces of each type mentioned above and have taken the average of the observed values of three test pieces in each experiment.

We made the test pieces of cement paste and mortar as shown in Tables 1 and Fig. 1.

Experiment 1.

We first experimented on the normal test pieces, in order to test whether there is decay of the acoustic velocity in accordance with the length of the test pieces. We projected the ultrasonic waves from the inspection points numbered ①, · · · , ⑤ on one end square of the test pieces

Table 1 Mix proportion of the test pieces

Mix proportion of Cement Paste

	Water	Cement	Air	Total
Weight(kg)	553	1382	–	1935
Volume(ℓ)	553	437	10	1000

W/C=40%, Air=1%

Mix proportion of Cement Mortar

	Water	Cement	Sand	Air	Total
Weight(kg)	331	828	1035	–	2195
Volume(ℓ)	331	262	397	10	1000

W/C=40%, S/C=1.25, Air=1%

(see Fig. 1). We name them the source points. We received them at the same-numbered inspection points on the other end square. We name them as the observation points. We have measured the time for the ultrasonic wave to travel between the source and the observation points. The results of these experiments are summed up in Figs. 2 and 3.

Remark that the average of the data on the points ① and ② are treated as the data of the upper points, the average of the data on the points ③ and ④ are treated as the data of the lower points and the point ⑤ is denoted by the center point.

By review-examining the results of Experiment 1, we obtain the following properties.

Property 3.1.

- *By observing the acoustic velocity in the test pieces, we have rediscovered the well known basic property of concrete; the more time goes by, the harder the test pieces are, which is caused by the reaction of hydration of concrete.*
- *We have also rediscovered another well known basic property, the gravity settling of cement and fine aggregate (sand), in terms of the acoustic velocity; the lower the inspection points are, the faster the acoustic*

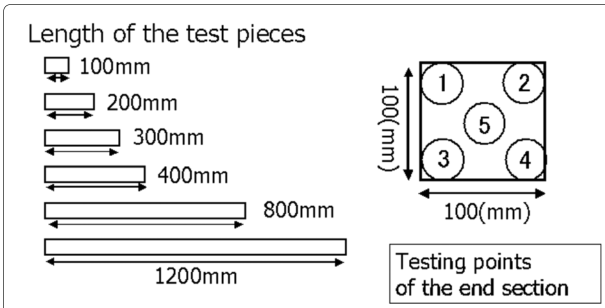

Fig. 1 Length of the test pieces and inspection points

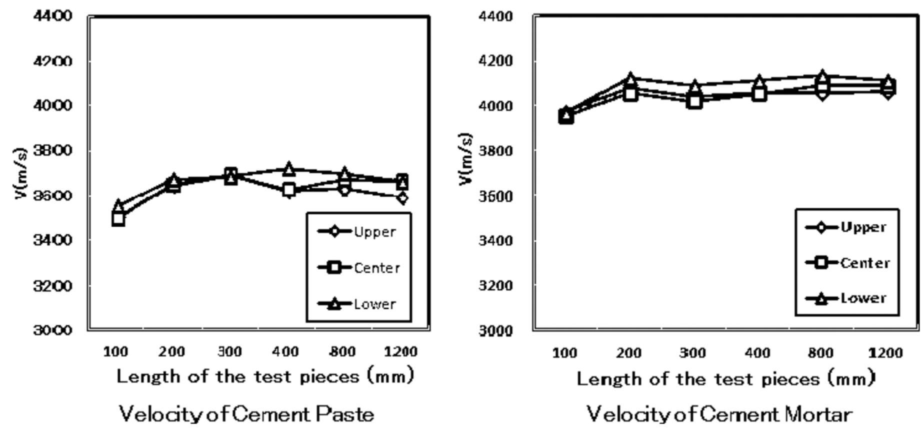

Fig. 2 Normal test pieces (age of a week)

velocity is, which is because of the fact that the lower the points are, the larger their density is, caused by the gravity settling of cement and fine aggregate.

- *We can conclude that for the test pieces of the length less than* 1200*mm, there is no decay of the acoustic velocity from the viewpoint of its first arriving time.*

The last property is essentially important for our study.

Experiment 2.

For this experiment, we utilized the test pieces of the length 400mm(100mm × 100mm × 400mm) with and without the styrofoam of the length 200 or 300mm included in their inside (confer Fig. 4). We conducted the same experiments as Experiment 1, in order to study how the ultrasonic waves propagate in the concrete structures, whose results are review-examined in the following.

In Experiment 2, the (formal) velocity V, which is calculated by

$$V = \frac{\text{length of the test piece}(meters)}{\text{arriving time}(seconds)}, \tag{1}$$

in the lower points is smaller than that of the upper points (confer Table 2 below), applying which we studied the propagation of the ultrasonic waves in the test pieces. We hypothesized that the propagation of the ultrasonic waves in the test pieces is as the following hypothesis which is also shown in Fig. 5.

Hypothesis 3.1. Let $\Omega \subset \mathbb{R}^3$ be a domain where the test pieces of the cement paste or the mortar locates, and $f(x)$ be the propagation speed of the ultrasonic wave at the point $x \in \Omega$. For $\alpha, \beta \in \partial\Omega$, we denote by $\gamma_{\alpha,\beta}$ a route from α to β contained in Ω. The primary wave of the ultrasonic one which travels from the point $\alpha \in \partial\Omega$ to the point $\beta \in \partial\Omega$ takes the route where the travel time is given by

$$\min_{\gamma_{\alpha,\beta}} \int_{\gamma_{\alpha,\beta}} 1/f(x)d\gamma, \tag{2}$$

This route is called 'the fastest route'.

A similar claim to Hypothesis 3.1 being made by H. Yamamoto et al. [12], it is not exactly the same as Hypoth-

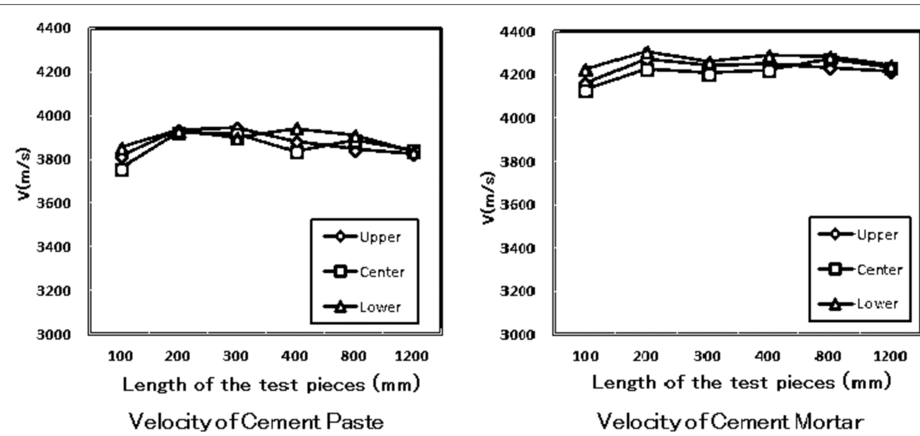

Fig. 3 Normal test pieces (age of 4 weeks)

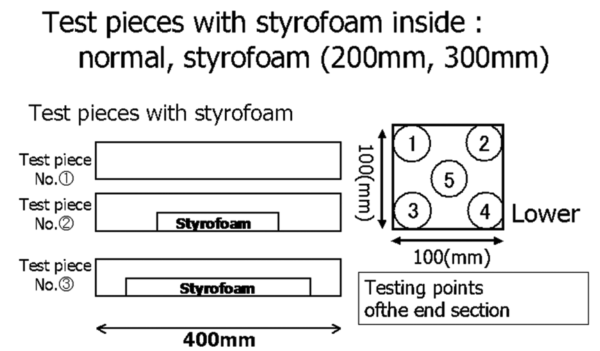

Fig. 4 Test pieces with styrofoam

esis 3.1. In [12], they tried to establish an ultrasonic CT, however, there has not been a satisfactorily practical ultrasonic CT technique for concrete structures yet. The authors claim that the verification of Hypothesis 3.1 is very important to establish a satisfactorily practical ultrasonic CT technique. In the next section we pose an inverse problem (Problem 4.1 below) by virtue of Hypothesis 3.1, which is an essential problem to establish a satisfactorily practical ultrasonic CT technique.

In view of Hypothesis 3.1, we have modified the length of the orbit along which the ultrasonic wave propagates, that is, the modified velocity V' is given by

$$V' = \frac{0.406(meters)}{\text{arriving time}(seconds)} \quad (3)$$

for the lower points in the test pieces with styrofoam of the length 200mm and by

$$V' = \frac{0.412(meters)}{\text{arriving time}(seconds)} \quad (4)$$

for the lower points in the test pieces with styrofoam of the length 300mm. Confer Fig. 5 for the image of these modifications. The results of Experiment 2 with the modification of the velocities are summarized in Table 2, Figs. 6 and 7. Remark that in Table 2, the fields of the modified velocity are left blank where the modification is not necessary.

Let us review-examine the results of Experiment 2, that is, we shall study the modified acoustic velocities shown in Table 2, Figs. 6 and 7. The modified velocities at the lower points being not exactly the same as the test pieces without styrofoam, it is because, in the test pieces with styrofoam, gravity settling of cement and fine aggregate (sand) would not happen by the existence of the styrofoam. As another reason of this phenomenon, it is known that the air concentrate near the formwork in manufacturing process of concrete structures [2, 9]. In this experiment, the boundary surface between the cement paste (or the mortar) and the styrofoam plays the role of the formwork and the air concentrates there. Therefore, the primary ultrasonic wave takes a little longer way around than the route shown in Fig. 5, which results in that the modified velocities of the middle and the lower points are a little slower than the upper points. The exact evaluation how longer the fastest route is, is a very difficult problem.

In view of this review-examination, we conclude that Hypothesis 3.1 is right. Let us summarize the conclusions of Experiments 1 and 2.

Conclusion 3.1 (Conclusion of Experiments 1 and 2)**.**

- *The primary wave of the ultrasonic one takes the fastest route in the cement paste, the mortar and the concrete.*
- *In the concrete structures of the length less than 1200mm, there is no decay of the speed of the ultrasonic waves with respect to the length.*

Remarks 3.1. It is our newer idea than the existing ones [6, 8] to focus on the first arrival time of the ultrasonic primary waves and to pose a problem for the development of the acoustic CT, which may yield a concretely interior-describing non-destructive inspection method. We shall discuss this problem in the next section. We also note that the ultrasonic CT is a part of the acoustic CT. In this

Table 2 Tables of modification of the velocity (age of 4 weeks)

Cement Paste							
V	No-Styrofoam	Styrofoam 200mm	Styrofoam 300mm	V′	No-Styrofoam	Styrofoam 200mm	Styrofoam 300mm
Upper	3777	3800	3868	Upper			
Center	3788	3824	3874	Center			
Lower	3809	3700	3731	Lower		3756	3843
Cement Mortar							
Upper	4223	4208	4223	Upper			
Center	4204	4160	4192	Center			
Lower	4229	4080	4034	Lower		4141	4155

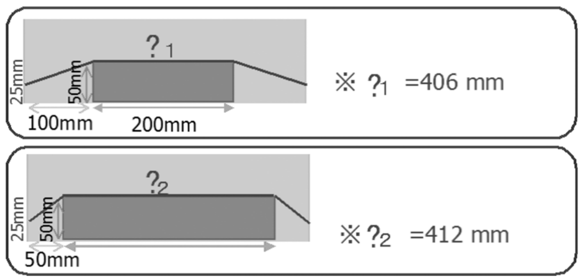

Fig. 5 Propagation of the ultrasonic waves

paper, we are trying to establish a CT technique applicable to wider problems with other acoustic techniques than the ultrasonic one. As an example of such another acoustic CT technique, the authors are studying to develop a CT technique with electromagnetic acoustic pulses (confer Remark 3.2 below). We also note that the propagation of the ultrasonic primary wave mentioned in the first conclusion in Conclusion 3.1 is the same as ultrasound CT for medical application (cf. [1] for example).

Remarks 3.2. Having introduced the results of our experiments on the data by application of ultrasonic primary waves, we have almost the same results on electromagnetic acoustic pulse primary waves, which enables us to treat electromagnetic acoustic pulses in the same way as ultrasonic waves for our study. We shall study how to treat electromagnetic acoustic pulses in our forthcoming paper.

4 An inverse problem of the acoustic tomography

In this section, we pose an inverse problem for the establishment of an ultrasonic tomographic technique for concrete structures. The research to develop an ultrasonic tomography was begun by J. F. Greenleaf at al. [4] and many researches have been published since then. Confer [12] for the references on such researches. For the time being, however, there has not been developed a satisfactorily practical ultrasonic CT technique yet. The authors claim, as in the previous section, that the verification of Hypothesis 3.1 is the first step to establish a satisfactorily practical ultrasonic CT technique, since it yields a key problem, Problem 4.1 below, for establishment of a satisfactorily practical ultrasonic CT technique.

As was studied in the previous section, we know that the the primary wave of the ultrasonic one takes the fastest route in the concrete structures of the length less than $1.2m$ and there is no decay in the velocity according to the length of the test pieces, which is what Conclusion 3.1 claims. In view these properties, we pose the following problem, which is the main problem in this paper.

Problem 4.1 (Problem to develop an acoustic CT for concrete structure). *Let $\Omega \subset \mathbb{R}^3$ be a domain and $f(x)$ be the propagation speed of the ultrasonic wave at the point $x \in \Omega$. For $\alpha, \beta \in \partial\Omega$, we denote by $\gamma_{\alpha,\beta}$ a route from α to β contained in Ω. In this case, reconstruct $f(x)(x \in \Omega)$ out of the data*

$$\min_{\gamma_{\alpha,\beta}} \int_{\gamma_{\alpha,\beta}} 1/f(x)d\gamma, \tag{5}$$

for $\forall\alpha, \beta \in \partial\Omega$.

Let us explain what is meant by Problem 4.1. We first note that we take Ω for some concrete structure. By $\partial\Omega$, we denote the boundary of Ω, that is, $\partial\Omega$ is the surface of the concrete structure Ω. For a fixed route $\gamma_{\alpha,\beta}$, from α to β through Ω with $\alpha, \beta \in \partial\Omega$, the integral $\int_{\gamma_{\alpha,\beta}} 1/f(x)d\gamma$ represents the travel time of the ultrasonic wave when it propagates along $\gamma_{\alpha,\beta}$. It is because that the travel time of the sound (of the ultrasonic wave) is proportional to the reciprocal of the propagation speed of the sound.

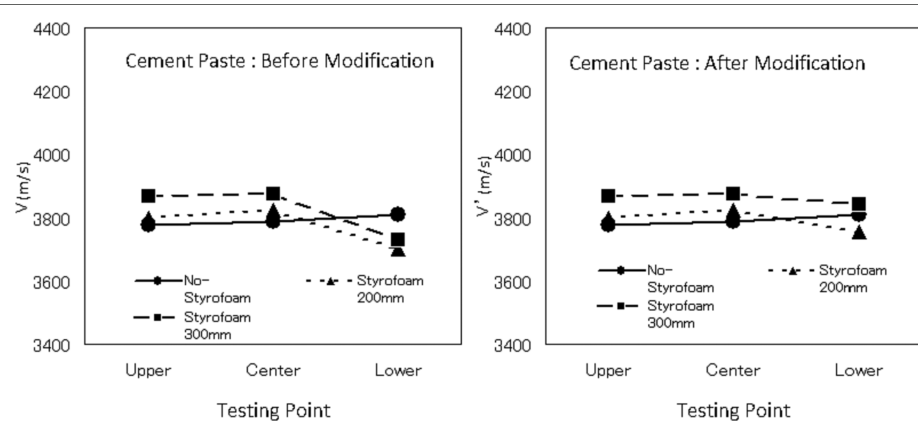

Fig. 6 Test pieces of cement paste with styrofoam (age of 4 weeks)

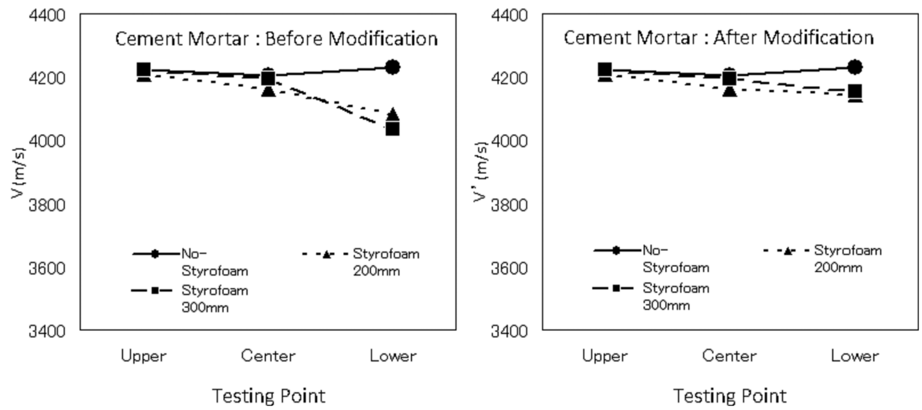

Fig. 7 Test pieces of mortar with styrofoam (age of 4 weeks)

By Conclusion 3.1, what we can obtain by observation is the travel time of the ultrasonic wave which propagates along the route where the travel time would be the shortest in Ω, which is what is meant in (5). If we can reconstruct $f(x)$, $(x \in \Omega)$ out of the data (5), then we can concretely reconstruct the interior of the concrete structures in a non-destructive way. Let us summarize this argument as follows.

Claim 4.1. *By Problem 4.1 we mean the problem "Reconstruct the acoustic velocity" $f(x)$ at the all points $x \in \Omega$ out of the data of the acoustic arrival time between the all pairs of the points on the boundary. Solution to this problem enables us to reconstruct the interior structure concretely, that is, at each point of the interior of the concrete structure, we detect what is at the point, the cement paste, the fine aggregate, the coarse aggregate or the air, completely except the non-essential points mentioned in Remark 4.1 below. As mentioned above, it is very important and useful to develop such safe and cheaply-running non-destructive inspection method for concrete structures which enables us to detect their interior structures concretely, the authors claim that study of Problem 4.1 would bring a breakthrough for non-destructive inspection of concrete structures.*

Study of Problem 4.1 is very important not only for solution of Problem 2.1 and problems to be introduced in the next section, but to establish an ultrasonic CT for general concrete structures including RC ones.

Let us give some remarks on Problem 4.1.

Remarks 4.1 (Remarks on Problem 4.1)**.**

- In Problem 4.1, we give $\min_{\gamma_{\alpha,\beta}} \int_{\gamma_{\alpha,\beta}} 1/f(x)d\gamma$ as the observed travel time of the primary wave between the two points $\alpha, \beta \in \partial\Omega$, based on Conclusion 3.1. It being obvious that $\inf_{\gamma_{\alpha,\beta}} \int_{\gamma_{\alpha,\beta}} 1/f(x)d\gamma$ exists, this infimum is attained as real observation of the travel time of the primary wave. Therefore the minimum, $\min_{\gamma_{\alpha,\beta}} \int_{\gamma_{\alpha,\beta}} 1/f(x)d\gamma$, exists.
- It is impossible to reconstruct the information of some points x's where $f(x)$'s are very small. For example, we cannot reconstruct the acoustic velocity of the styrofoam if it is included near the center of the test piece since no acoustic wave would go through it because of Conclusion 3.1. However, it does not matter very much. Since it is very important to determine the part where the acoustic velocity is so small that the ultrasonic wave would not go through it, where the exact velocity in such area would not matter. Remark that, in practice, it being very important to determine that the place where the acoustic velocity is relatively small, for example, to determine the place where the steel is corroded or there exists a cavity in the RC structures, it is not so important to determine the exact acoustic velocity itself in the place where it is relatively small.
- It is an interesting problem to determine the optimal subset reconstructible by the acoustic CT established by the study of Problem 4.1.

Study of Problem 4.1 is also important in view of both pure and applied mathematics, especially from the viewpoint of integral geometry. Let us mention how important Problem 4.1 is in view of the research in pure and applied mathematics.

Remarks 4.2 (Importance of Problem 4.1 in mathematics)**.**

- It is a very interesting problem to establish an reconstruction formula for Problem 4.1 in view of pure mathematics, especially, in integral geometry.
- It is another interesting problem in Problem 4.1 to determine the subset of Ω where the reconstruction

is impossible because it has no intersection with any γ giving (5). This problem is also interesting in view of integral geometry.

- In practice, we have to study various incomplete data problems of Problem 4.1 by the restriction arisen from various reasons in practical application, which is interesting in view of pure mathematics, especially in view of integral geometry with incomplete data, which is also very important in applied mathematics.

Let us study a simple problem related to Problem 4.1. For simplicity, let us assume that there is a cavity of a disc whose center and radius are unknown in the two dimensional homogeneous object. The motivation to study this problem is as follows. Almost all standards of concrete materials are determined by the mortar part structure since it is the most important how to make the mortar part in the construction of the concrete materials. The authors claim that we can take the mortar as a homogeneous object since the particles of the cement and the fine gravel are so small that together with the water and the air they make a homogeneous material from macro viewpoint. It can be very basic and important problem to study a cavity of a disc in the mortar, which yields the following problem.

Theorem 4.1. *Let us study the 2-dimensional case. Assume that a homogeneous rectangle contains a cavity of a disk in its interior. In this case, the cavity is reconstructed by appropriate three data of the ultrasonic tomography.*

In this theorem, by the term appropriate three data we mean the data which would determine the disc. For simplicity, we assume the acoustic velocity in the homogeneous rectangle is 1 and the cavity is of radius r and centered at (x_0, y_0). In Fig. 8 below, the length of the detour of the acoustic wave is given by

$$L_i = \left(\left(X_i^1 - x_i^1\right)^2 + \left(Y_i^1 - y_i^1\right)^2 + \right)^{\frac{1}{2}} + \left(\left(X_i^2 - x_i^2\right)^2 + \left(Y_i^2 - y_i^2\right)^2 \right)^{\frac{1}{2}} + r\theta_i, \tag{6}$$

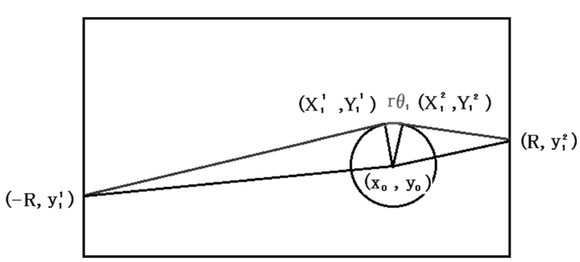

Fig. 8 Propagation of ultrasonic waves

where $i = 1, 2, 3$, $j = 1, 2$, $\left(x_i^1, y_i^1\right)$ are the points where we project the acoustic wave and (x_i^2, y_i^2) are the points where receive the acoustic wave ($x_i^1 = -R$, $x_i^2 = R$).

We can solve the system (6) of three equations to determine x_0, y_0 and r, since we can determine X_i^j, Y_i^j by

$$\begin{cases} \left(X_i^j - x_0\right)^2 + \left(Y_i^j - y_0\right)^2 = \left(X_i^j - x_i^j\right)^2 + \left(Y_i^j - y_i^j\right)^2 + r^2 \\ \left(X_i^j - x_0\right)\left(X_i^j - x_i^j\right) + \left(Y_i^j - y_0\right)\left(Y_i^j - y_i^j\right) = 0. \end{cases} \tag{7}$$

and θ_i by

$$\theta_i = \cos^{-1} \frac{\left(X_i^1 - x_0\right)\left(X_i^2 - x_0\right) + \left(Y_i^1 - y_0\right)\left(Y_i^2 - y_0\right)}{\sqrt{\left(X_i^1 - x_0\right)^2 + \left(Y_i^1 - y_0\right)^2}\sqrt{\left(X_i^2 - x_0\right)^2 + \left(Y_i^2 - y_0\right)^2}}. \tag{8}$$

Note that there being quadratic equations in the systems (6) and (7), they have two solutions, however, by virtue of the condition that the detour is the shortest way we can determine the unique solution. We also note that *theta*$_i$ in (8) must be acute.

It is very easy to extent Theorem 4.1 to the 3-dimensional case.

Theorem 4.2. *In the three dimensional case, a cavity of a ball in its interior of a homogeneous object is reconstructed by appropriate four data of the ultrasonic tomography.*

In this case, what we have to do is determine the four unknowns, the coordinate (x_0, y_0, z_0) of the center and the radius r. It is sufficient to have appropriate four data. The reconstruction algorithm is easily obtained by extending the algorithm for the 2-dimensional case.

For practical application, we shall extend Theorem 4.2 for the case where the cavity consists of several balls and develop a method to approximate a general cavity by a sum of several balls in the future.

5 Application to non-destructive inspection of concrete cover for reinforcement

In this section, we shall introduce some examples where the study of Problem 4.1 is very important. The main application in this section, we propose a non-destructive inspection method for concrete cover in the RC structures. In the interior of RC structures, there are a number of steel rods inbedded for reinforcement. It is very important to prevent the steel from getting corroded, for which it is very helpful if we can develop a good, cheaply running non-destructive testing method for concrete cover for reinforcing steel. For simplicity, let us assume a simple structure where only one steel rod is inbedded as designed in Fig. 9, which is a section of the three dimensional structure. Based on the above assumption, we shall establish

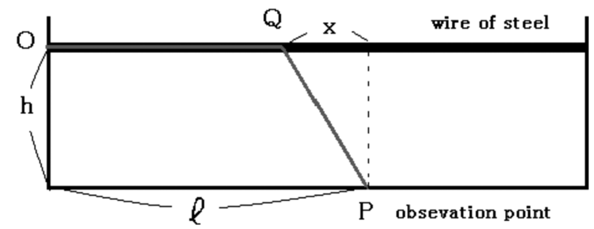

Fig. 9 A cavity of a circle in the 2-dimensional molar structure

the basic theory for our non-destructive inspection technique for concrete cover. By modification, our method can be applied for real RC structures.

Concrete structure being inhomogeneous, we homogenize the acoustic velocity in the concrete by application of the idea of the least square solution (confer the discussion below), by which we denote the velocity in the steel by V and that in the concrete by v. In general, it is known that $4000m/s < v < 5200m/s$ and $5500m/s < V < 6500m/s$. Therefore we can assume that $v < V$.

We first determine the acoustic velocity in the steel V by application of Conclusion 3.1. By homogenization of the acoustic velocity in the concrete mentioned above, the primary ultrasonic wave projected from the point $O(0, h)$ propagates along the spline $\overline{OQ} \cup \overline{QP}$ and is received at the point $P(l, 0)$, whose travel time we denote by t_0. If we project the ultrasonic wave from the point O and receive it at the point $P_i(l + l_i, 0)$ with travel time t_i, then we obtain approximate acoustic velocity in the steel by $l_i/(t_i - t_0)$. Take observation points $P_1, \cdots, P_n$ as many as possible and take the least square solution

$$V = \frac{l_1(t_1 - t_0) + \cdots + l_n(t_n - t_0)}{(t_1 - t_0)^2 + \cdots + (t_n - t_0)^2} \tag{9}$$

to the system of linear equation which trivially has no solution

$$V(t_1 - t_0) = l_1, \cdots, V(t_n - t_0) = l_n. \tag{10}$$

We also comment that the least square solution V is the minimizer for the function

$$(V(t_1 - t_0) - l_1)^2 + \cdots + (V(t_n - t_0) - l_n)^2. \tag{11}$$

We then determine the homogenized acoustic velocity in the concrete. For given V and v, x and the length L of the segment QP in Fig. 8 are calculated as

$$x = \frac{vh}{\sqrt{V^2 - v^2}}, L = \frac{Vh}{\sqrt{V^2 - v^2}}. \tag{12}$$

We first let $v_0 = 3500m/s$ then the ultrasonic wave projected from the point $O_i(0, h_i), \bar{h}_i < h$ linearly propagates in the concrete to the observation point $R_j(r_j, 0), r_j < \frac{vh}{\sqrt{V^2 - v_0^2}}$. Take as many pairs of source and observation points $\{O_i, R_j\}$ which gives the length L_{ij} of the segment where the ultrasonic wave propagates, and the travel time t_{ij}. Therefore we obtain the first step value v_1 of the homogenized acoustic velocity in the concrete by

$$v_1 = \frac{\sum t_{ij} L_{ij}}{\sum t_{ij}^2}. \tag{13}$$

Replacing v_0 by v_1 and repeating the same procedure, we obtain the second step values $v_2 = \bar{v}$, which we determine as the homogenized acoustic velocity in the concrete. If necessary, we make the sequence $\{v_k\}$ until it approximately converges, for example, we stop the procedure when $v_{k+1} - v_k < 0.01(m/s)$.

By the knowledge of the acoustic velocities, V in the steel and $\bar{v}$ in the concrete, and the route where the ultrasonic wave propagates, we can propose a non-destructive inspection technique for concrete cover by application of Conclusion 3.1.

In Fig. 8, when we observe the travel time the primary ultrasonic wave between the points O and P, the calculated travel time is

$$\frac{l - \frac{\bar{v}h}{\sqrt{V^2 - \bar{v}^2}}}{V} + \frac{Vh}{\bar{v}\sqrt{V^2 - \bar{v}^2}} \tag{14}$$

and the one in the segment QP is

$$\tilde{t} = \frac{Vh}{\bar{v}\sqrt{V^2 - \bar{v}^2}}. \tag{15}$$

If the velocity $\tilde{v}$ in the segment QP is much smaller than the homogenized velocity $\bar{v}$, for example, $\bar{v} - \tilde{v} > 200m/s$, then one or more of the following could happen.

- There is a cavity in the segment QP.
- There is a water route around the steel.
- Some part of the steel may get corroded.

In any case, we have to shave the concrete cover and have to repair the defect. Shaved concrete cover can be easily re-fixed by filling the cavity by shaving with better concrete. Therefore, what is important is to establish how to find where to shave in a non-destructive way, an answer of which is given as the above way.

6 Conclusion

In this section, we summarize our conclusions in the present paper.

Conclusion 6.1 (Conclusion of this paper)**.**

- *For development of the ultrasonic CT, we studied how the primary ultrasonic wave propagates in the cement paste and the mortar (Conclusion 3.1).*
- *Applying the property of the primary ultrasonic wave, we have posed a problem for the development of the ultrasonic CT (Problem 4.1).*
- *Our ultrasonic CT for concrete structures is safe, cheaply-running and concrete non-destructive*

inspection method for concrete structures, which enables us to reconstruct the interior of the concrete structures concretely (Claim 4.1).

- *The problems posed in this paper are also interesting and important in view of the study of mathematics (Remarks 4.1 and 4.2).*
- *As an application of our ultrasonic CT, we developed a non-destructive inspection method for concrete cover in RC structures (Section 5), where homogenization of acoustic velocity in the concrete by the idea of the least square solutions played an important role.*

Theoretical approach, as well as computational one, to Problem 4.1 is under investigation. As an approach to Problem 4.1, the main problem in this paper, we are trying a similar method that G. N. Hounsfield applied for practicalization of the computerized tomography. Confer [11] for the idea by G. N. Hounsfield to practicalize CT.

For the verification of our non-destructive inspection testing technique introduced in the fifth section, we shall make another test pieces of reinforced concrete with some pieces of styrofoam in its interior and perform experiments to detect them by our technique, we shall also apply our technique to the real expressways which shall be introduced in our forthcoming paper.

It is also required, in view of practical application, to generalize our non-destructive inspection techniques for concrete cover, in more complicated RC structures, which in under investigation with real expressways by the authors and West Nippon Expressway Shikoku Company Limited.

Abbreviations
CT: computerized tomography; RC: reinforced concrete

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Authors' contributions
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Competing interests
The authors declare that they have no competing interests.

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References
1. Arakawa, M., Kanai, H., Ishikawa, K., Nagaoka, R., Kobayashi, K., Saijo, Y.: A method for the design of ultrasonic devices for scanning acoustic microscopy using impulsive signals. Ultrasonics. **84**, 172–179 (2018)
2. Dunham, C. W.: The theory and practice of reinforced concrete (third edition). McGrow-Hill, New York (1953)
3. Gorzelańczyk, T., Hola, J., Sadowski, L., Schabowicz, K.: Methodology of nondestructive identification of defective concrete zones in unilaterally accessible massive members. J. Civ. Eng. Manag. **19**, 775–786 (2013)
4. Greenleaf, J. F., Johnson, A. A., Lee, S. L., Herman, G. T., Wood, E. H.: Algebraic reconstruction of spatial distribution of acoustic absorption within tissue from their two-dimensional acoustic projections. Acoust. Hologr. **5**, 591–603 (1974)
5. Hobbs, B.: Ultrasonic NDE for assessing the quality of structural brickwork. J. Nondestruct. Test. Eval. **12**, 75–85 (1995)
6. Jones, R., Facaoaru, I.: Materials and Structures. Res. Test. **2**, 275 (1969)
7. Li, Z.: Advanced Concrete Technology. Wiley, Hoboken (2011)
8. Motooka, S., Okujima, M.: Study on Detection of Buried Steel Bar in Concrete with Electromagnetic Impact Driving Method. Proc. 2nd Symp. Ultrason. Electron., 144–146 (1981)
9. Neville, A. M.: Properties of concrete (fifth edition). Pearson Education Limited, Edinburgh (2011)
10. Schabowicz, K.: Ultrasonic tomography – The latest nondestructive technique for testing concrete members – Description, test methodology, application example. Arch. Civ. Mech. Eng. **14**, 295–303 (2014)
11. Takiguchi, T.: How the computerized tomography was practicalized. Bull. JSSAC. **21**, 50–57 (2015)
12. Yamamoto, H., Saito, H., Tabei, M., Ueda, M.: Calculation of projection data for the ray ultrasonic computed tomography by the ray tracing method. J. Acoust. Soc. Jpn. (E). **12**, 157–167 (1991)