ORIGINAL RESEARCH

Back analysis of cohesion and friction angle of failed slopes using probabilistic approach: two case studies

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Abstract

In general, geotechnical investigations deal with the back analysis of the geotechnical parameters on the basis of the field observation. According to the back analysis method introduced in the present study, geometry of a number of failed slopes have been carefully mapped and then statistical features of the groundwater level, the bulk unit weight and other measurable parameters have been determined. After that, a series of two-dimensional models for back analysis of the failures have been established. Moreover, statistical analyses based on probabilistic approaches have been utilized to estimate the variation ranges of the shear strength variables. The study provided the probabilistic back analysis of the slope failure with the two case studies in the copper mines of Anjerd and Daraloo. Results indicated that the approach to the probabilistic back analysis has been effective in the analysis of the slope failures wherein considerable uncertainty has been observed in the shear strength and other influential variables. Furthermore, the probabilistic back analysis presented a lot of information in comparison to the approach to the deterministic back analysis and thus had more reasonable matching with the practice of geotechnical engineering in the real world. Therefore, this method would be beneficial and practical for stability analysis, redesigning the slopes, designing the new slopes under the same geotechnical conditions and promote the construction safety. The probabilistic back analysis could be also used to estimate the shear parameters and analyze the stability of slopes as a cost-effective and high reliability method.

Keywords: Failed slopes, Probabilistic back analysis, Shear strength variables, Anjerd and Daraloo copper mines

Introduction

One of the common problems in the geotechnical engineering is the slope stability in the open pit mines and waste dumps [1]. Therefore, authors presented diverse techniques for solving the issues related to the slope stability. They commonly selected methods like the Limit Equilibrium Method (LEM) and Strength Reduction Method (SRM). Though these methods have been originally deterministic, they could be readily adjusted for suiting the probabilistic models. According to the deterministic analysis, stability has been evaluated with regard to the factor of safety based on the fixed parameter values. However, according

to the probabilistic analysis, all variables have been assigned to a statistical distribution, and stability has been assessed with regard to a reliability index or a probability of failure [2]. There is enough information that the probabilistic analysis has been considered one of the rational approaches for engineering design because it could present more information in comparison to the single deterministic analysis [3].

Some studies demonstrated that the soil and rock masses had been a complex engineering material formed by a combination of various mechanical and physical properties. Hence, it would be of high importance to determine suitable values with a specific level of confidence for shear strength parameters of the rock mass in the design and sustainability of structures fabricated in the rock and soil. Thus, determination methods of the shear strength parameters could be categorized into engineering experience analogy method, back analysis method, and test method [4, 5]. However, the shear strength parameters reported in the back analysis ensured higher reliability than the parameters achieved by laboratory or in-situ testing while using for designing the remedial measures. Back analysis in many cases has been considered as one of the efficient tools and occasionally the only instrument for examining the strength characteristics of a soil deposit [6, 7]. According to the geotechnical engineering, inverse analysis that is on the basis of the field observations has been a robust device to identify the soil and rock physical and mechanical parameters [8–10].

A major reliable approach for determining shear strength of the slope material at the time of failure has been called the limit equilibrium back analysis of a failed slope [11]. In addition, the methodologies applied for the back analysis could be categorized into two groups; that is, probabilistic [12, 13] and deterministic methods [14]. In a deterministic technique, the slope stability model has been commonly supposed to be accurate, and therefore back analysis aimed at finding some parameters causing the slope failure. Nonetheless, according to a probabilistic back analysis, results indicated that the slope stability model could not be fully precise and multiple combinations of the slope stability parameters could lead to the slope failure [15]. Based on the conventional back analysis, internal friction angle or cohesion has been supposed for calculating the other parameter with regard to a factor of safety equal to 1.0. Hence, it is necessary for the shear strength features achieved from back analysis of the field situations have greater reliability in comparison to the properties observed from the laboratory experiments and thus experts in the field paid considerable attention to all techniques contributing to the back analysis procedure [16]. As a result, the shear strength parameters shown in the back analysis of the slopes have been admitted as the ones with higher consistency than the parameters achieved by the laboratory or in-situ testing in the course of the remedial measure design [7]. Julian Osorio et al. (2021) performed the influence of the variability of the drained friction angle and the cohesion on the active earth pressure coefficient through probabilistic analysis concerning shear strength parameters. The probability of failure obtained through the c-φ variability can be considered inadmissible even if the required FS are met [17].

The proposed method for the probabilistic back analysis

Back analysis is one of the integral parts of the remedial work for redesigning the failed slopes and for new projects in similar types of material. Moreover, in the back analysis of failure, the factor of safety of a slope can be defined in Limit Equilibrium Method (LEM) as the ratio between resistance and disturbance along a potential slip surface. By fixing

the factor of safety at 1.0, a condition would be provided, in which the conservative design assumptions have been un-conservative in back analysis. In addition, successful back calculation requires accurate information regarding geometry, material properties, and pore pressure distribution.

Therefore, understanding the processes and mechanisms driving instability would be crucial for conducting a successful back analysis approach. According to Sonmez et al. (1998), the rock slope stability analysis methods, which depended on the jointing conditions, could be usually categorized into two key groups of the methods appropriate for the slopes in the hard jointed rocks and the methods appropriate for the slopes in the strongly jointed and weak rock masses [5]. Therefore, the limit equilibrium technique has been considered to be applied for the two above methods. Moreover, the back analysis in the first group aimed at determining the shear strength parameters along the sliding discontinuity. Nonetheless, for applying the limit equilibrium discipline for the rock slopes in the second group, the jointed rock mass has been supposed as one of the continuums and its shear strength parameters have been back analyzed by the techniques devised for the soil slope stability.

Experts in the field proposed a variety of approaches for back analyzing the soil and rock slopes [18]. For example, Sancio (1981) and Sonmez et al. (1998) intruded the shear strength parameters of a failed slope, which have been back calculated by the geotechnical engineers and engineering geologists in below processes [5, 19]:

- To assume the value of the cohesion or friction angle (Fig. 1a).

- To utilize a major cross section of a failed slope and the other cross-section in proximity of the major cross section in a similar failed slope or utilize two cross sections in two failed slopes with the same geological and hydro-geological conditions for establishing two equations and assess the values of cohesion and friction angle (Fig. 1b).
- Due to the changes in the mechanical features of a similar material in various places, to use more than two slope cross sections for obtaining $(n-1)/2$ points of intersection (solution) for n curves $c(\phi)$ (Fig. 1c).
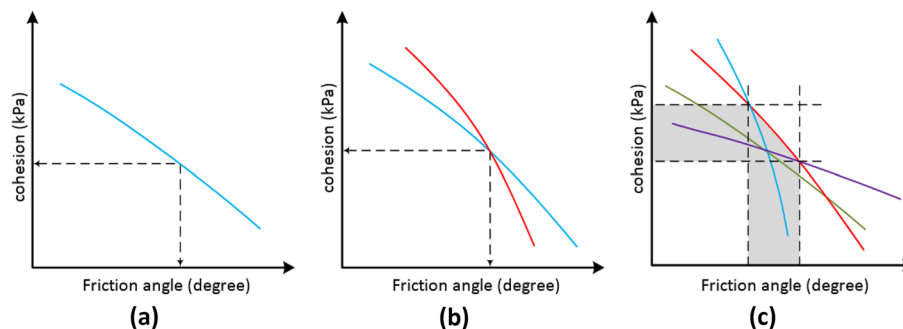

Fig. 1 Major back analysis approaches utilized for the slope forming substances following the linear failure envelopes. **a** The ranged derived from the cohesion and friction angle and determination of the cohesion from the supposed friction angle; **b** A single solution for two slides with distinct geometry; **c** Several solutions for four slides with distinct geometry [5, 19]

Researchers introduced two methods for statistical back analysis of the slope failure. One of the methods on the basis of the probabilistic analysis and a simplified method on the basis of the sensitivity analysis. Although the simplified method would be more convenient to be practically applied, it could be just applied as the slope stability model has been nearly linear [13]. Therefore, for back analyzing the multiple sets of the slope stability parameters concurrently under uncertainty, it is possible to implement the back analysis in a probabilistic way wherein uncertain parameters have been modelled as the random variables and thus their distribution has been enhanced with regard to the information of the observed slope failure. Now, the process of the statistical back analysis based on the probabilistic method of failed slope is presented (Fig. 2):

- Mapping: The primary geometry of slope and failed slope geometry coordinates has been accurately mapped.
- Determining the values and statistical distribution of effective parameters: At this stage, measurable properties of the soil or rock mass, including bulk unit weight, groundwater conditions, and other variables have been evaluated. For shear parameters, an initial mean value would be assumed based on the engineering experience and judgment. Moreover, a uniform distribution of shear strength parameters would be assumed with the equal generation of the samples in the entire ranges of the values for both parameters. It is notable that the probabilistic analysis could be utilized
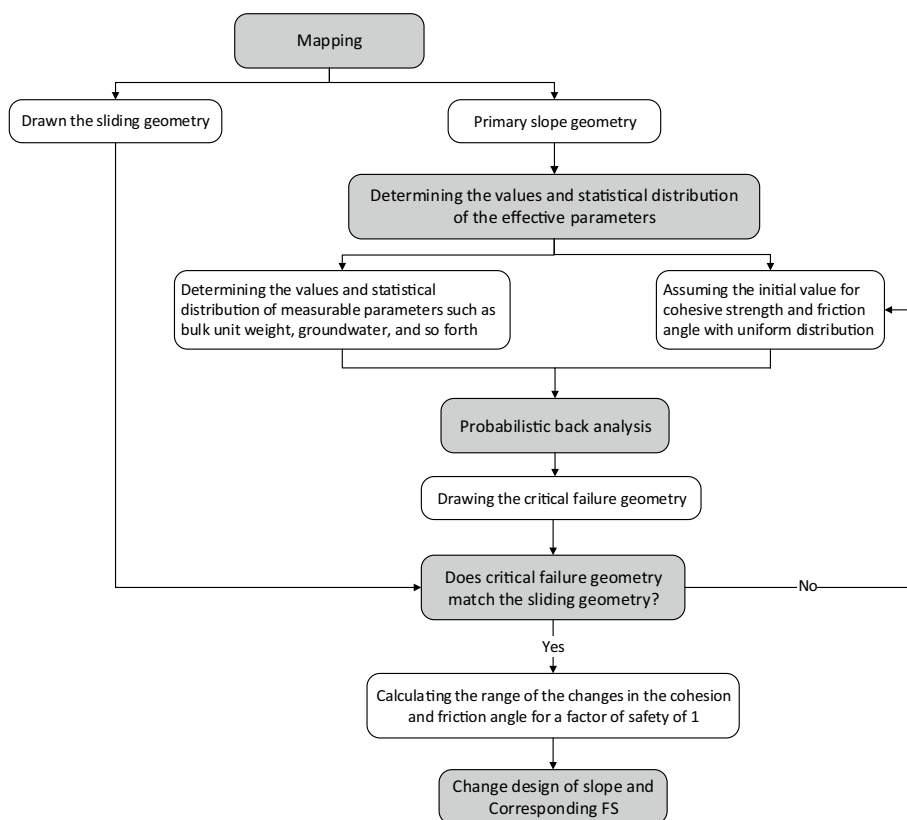

Fig. 2 The flow chart of the probabilistic back analysis of the shear strength parameters

for back analysis of both shear strength parameters. The approach determined an association between the cohesion and friction angle, causing a safety factor equal to 1 for a certain failure surface; that is, instead of a single correct response, an infinite number of solutions has been found; that is, pairs of value for cohesion and friction to the problems.

- Probabilistic back analysis: In contrast with deterministic analyses, where the soil properties are characterized as fixed values, probabilistic analyses consider the uncertainty of soils within a slope. In a probabilistic analysis, slope stability is evaluated through considering the variation of soil properties, allowing the user to predict the probability of failure. The probability of failure in rock slope is often estimated by a flexible and versatile method called "Monte Carlo" simulation. By entering all the required parameters, probabilistic back analysis would be run on the model, and the critical failure geometry would be drawn.

- Comparison of the critical failure geometry with the failed slope geometry: In this stage, the pairs of value have been selected to represent a range in the dimensionless parameter friction angle and cohesion; however, the values did not inevitably generate a factor of safety equal to 1. If the critical failure geometry is the same as the slope failure geometry, the range of the changes in the shear strength parameters would be determined for a factor of safety of 1 and selected for final stability analysis of the modified slope. However, if the critical failure is not the same as the slope failure, the values of the shear parameters would be corrected and Step 3 would be reiterated.

- Final slope stability analysis: Therefore, this method outputs could be utilized with confidence when the failure surface is so large and passes across each material. Stability would be analyzed on the final slope and its factor of safety and failure probability would be determined.

The present study used the probabilistic back analysis method to estimate the shear parameters of the eastern failed slope in Anjerd copper mine and the waste dumps in Daraloo copper mine (Fig. 3). Laboratory or in-situ measurements could be utilized to estimate the bulk unit weight; however, estimation of the shear parameters of the materials would be very complex and difficult so that using in-situ tests would be uneconomical. Based on the mapping of the exact location of the existing failures in the two studied cases, this report determined the shear parameters of the materials in these slopes. Here, the calculation procedure would be presented.

Case study 1: Anjerd copper mine

Overview

According to the studies, Anjerd copper mine is located in Ahar county, Azarbaijan province, Iran. The mine area has been estimated ~ 23 km northwest of Ahar city and 150 km northeast of Tabriz city (Fig. 3). Outcrops in this mine include andesite, tuff, lapilli tuff, monzodiorite, granodiorite, quartz, monzonite, basaltic-andesitic dikes, and quaternary sediments. The mineralized zone with the engineering controlled had a width of ~ 700 m and length of ~ 1000 m. Anjerd copper mine area has a rough morphology so that the height difference is about 300 m in this region. It is cold and is located under a semi-arid area. The region has cold, frosty winters and mild summers. The temperature

Anjerd Copper mine

Daraloo Copper mine

Fig. 3 A view of Anjerd and Daraloo copper mines with their location

in the region is higher than that of other areas, so that it reaches $-30°$ in winter and $+35°$ in summer. The average annual rainfall in this region is 381 mm and the average of frost days is 131 days. Moreover, the average annual relative humidity of Anjerd mine is between 60 and 70%.

Failed slope in the eastern wall

Figure 4 depicts the plan of Anjerd copper mine, hydrogeological borehole (BH1-Piez.) and the failure and sliding locations. Due to the soil-like material of the eastern wall and the presence of water in the slope face, the circular failures took place in the wall. Because of the access road and the main ramp of the mine at the top and bottom of the wall, it is important to analyze its stability and stabilization. Hence, back analysis has been used to analyze the slope stability. Now, the analysis process is explained.

Data collection

Hydrology

As mentioned earlier, a piezometer was installed to monitor the level of groundwater during a day. Monitoring groundwater and regional observations indicated the high water level on the eastern wall and flow from the slope face (Sliding III in Fig. 4). Considering the site investigation, piezometers monitoring, weather condition, and the mountainous terrain of Anjerd region, a truncated normal distribution was used to model the groundwater as shown in Fig. 5a. In these graphs, horizontal axis represents the normalized underground water level which is the ratio of measured water
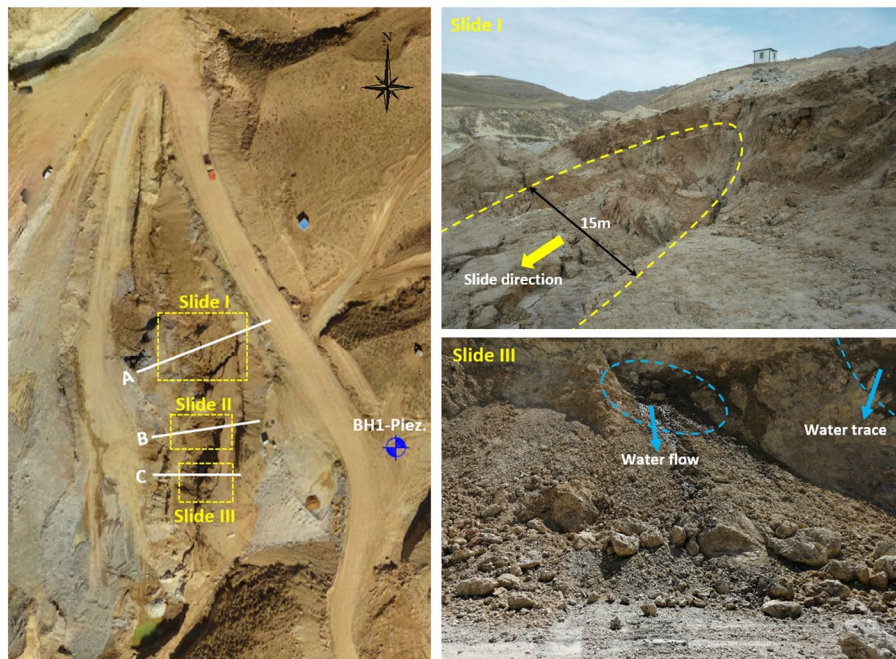
Fig. 4 A view of circular failures and water flow in the eastern wall of Anjerd copper mine. Three cross sections have been selected to estimate the shear parameters of the eastern wall of Anjerd copper mine: **A**, **B** and **C**
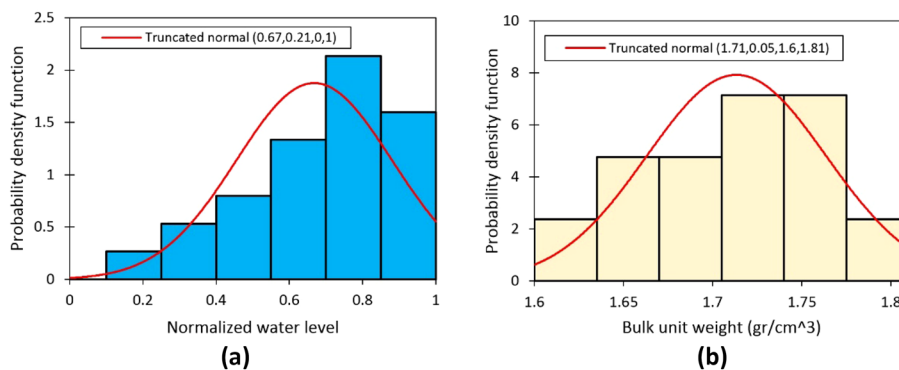

Fig. 5 Probability distribution of **a** Normalized groundwater level; **b** Bulk unit weight of soil mass

level to maximum water level. The mean values and coefficients of variations in the mentioned distribution equaled 0.67% and 31%, respectively.

Geotechnical features

One of the important parameters in the stability analysis is the bulk unit weight of materials so that its minimum, average, and maximum values for the eastern wall equaled 1.6, 1.71, and 1.81 g/cm^3, respectively. Statistical analysis on the samples showed that the truncated normal distribution with the coefficient of variation of 3% yielded the best result (Fig. 5b). Moreover, the analysis of the soil grading of this area
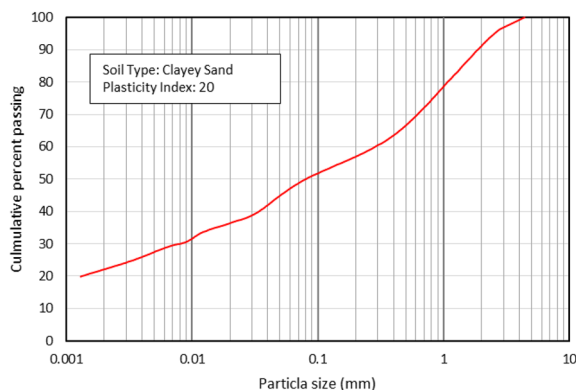
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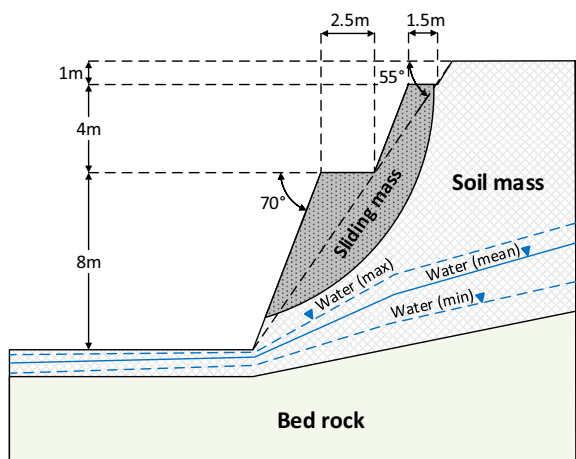**Fig. 6** Grain size distribution of the eastern wall soil

Fig. 7 Modelling section "A" for back analysis of the eastern wall of Anjerd copper mine

indicated that the soil type is the clayey sand and its plasticity index is 20%, reflecting the high plasticity content of the soil above this wall (Fig. 6).

Probabilistic back analysis of the strength parameter

In this stage, three cross sections have been selected to estimate the shear parameters of the eastern wall of Anjerd copper mine (Fig. 4). Figure 7 indicates the failure geometry of the section "A".As seen in the figure, the bench face angle is 70 degree, the overall slope angle is 55 degree, and the overall height of the slope is 13 m. Notably groundwater and bulk unit weight according to "Data collection" section and Mohr-Coulomb criterion for the failure criterion have been utilized in these analyses. For back analysis, initial values for the angle of internal friction and cohesion have been selected based on the engineering experience and judgment so that the failure corresponded to the sliding location. For probabilistic back analysis, the truncated normal distribution has been chosen for the bulk unit weight and groundwater parameters, but for cohesion and internal friction angle, uniform distribution has been selected. Therefore, the initial values of the shear data have been chosen in a way that the critical failure corresponded to the sliding location on the wall and a factor of safety equaled 1.

Figure 8 shows the results of the probabilistic back analysis of these cross sections for a factor of safety of 1 by LEM method. As shown by the figure, the values of the internal friction angle varied between 27.4 and 32.2 degree and the values of the corresponding cohesion varied between 6.3 and 10.2 kPa. Therefore, the average values of the internal friction angle and cohesion equaled 30.2 degree and 8.3 kPa, respectively.

Stability analysis of the final eastern wall

With regard to the eastern wall back analysis, soil shear parameters in this wall have been considered for analyzing the stability of the final eastern wall. Hence, the probabilistic analysis has been performed by Monte Carlo simulation.

Regarding the frequency distribution, to which datasets for specific geotechnical parameters have been approximated, a general symmetry could be frequently found that completely agreed with the normal distribution [20]. However, the assumed (not averaged) values of the variation coefficients on the shear parameters by Cherubini (1997) have been 10–20% for the internal friction angle and 20–30% for the cohesion [21]. In addition, Zhang et al. (2013) estimated the coefficient of variation of internal friction angle and cohesion of soil of 10% and 20%, respectively [22]. Table 1 reports the statistical parameters and geotechnical properties of this soil mass.

Figure 9a depicts the specifications of the 2D cross section at site "A" after stabilization. This wall have been established in two different slopes. The bench faces angle equaled 39 and 60 degree in the soil and rock masses, respectively. Moreover, the overall slope height equaled 34 m and its overall slope angle has been 29 degree. The wall has been analyzed as shown in Fig. 9a and the factor of safety and failure probability equal to 1.49 and 0.002, respectively. Therefore, the method of reduction of slope angle has been utilized to stabilize this slope, and this method has been run to stabilize the slope without any particular concerns (Fig. 9b).
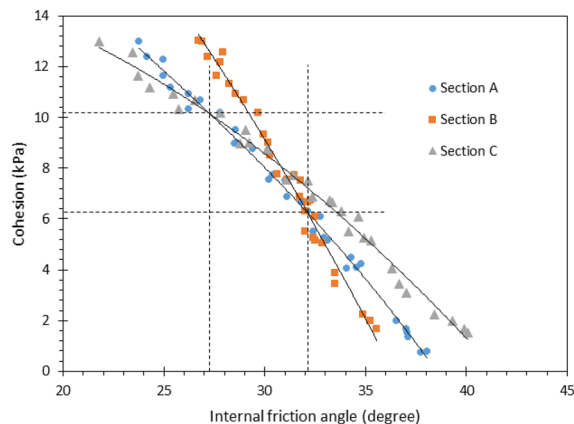

Fig. 8 Shear strength parameters of cross sections of the eastern wall of Anjerd copper mine for a factor of safety of 1 by LEM method

Table 1 Summary of the statistics of the eastern wall data of Anjerd copper mine

Variable	Probability distribution	Mean	Coefficient of variation (%)	Standard division	Minimum	Maximum
Internal friction angle (degree)	Truncated normal	30.2	10	3.02	27.4	32.2
Cohesion (kPa)	Truncated normal	8.3	20	1.66	6.3	10.2
Bulk unit weight (g/cm^3)	Truncated normal	1.71	3	0.05	1.6	1.81
Normalized groundwater level	Truncated normal	0.67	31	0.21	0	1
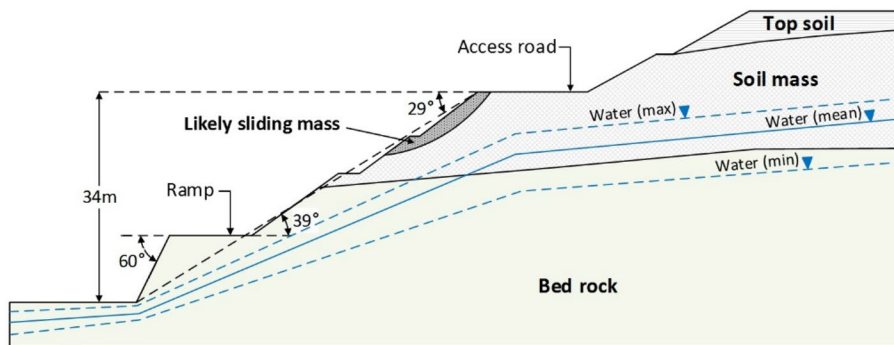
(a)

(b)

Fig. 9 a Stability analysis of the final eastern wall; **b** A view of the final eastern stable wall

Case study 2: Daraloo copper mine

Overview

Daraloo copper open pit mine is located at 45 km northeast of Rabor city and 120 km southern of Kerman city in Iran (Fig. 3). Daraloo copper deposits lie in a northwest-trending fault zone, 10 km long and 0.5-1 km wide in the southern section of the Kerman copper belt. Explored deposit in this mine is ovral shape with almost 1100 m major diameter and 400 m minor diameter in NW- SE direction. The porphyritic intrusion is dacitic which is hosted by andesites. It is hydrothermally altered with a distinctive

alteration zonation ranging including potassic, phyllic and propylitic. The mine reservoir is estimated 133 million tones with an average grade 0.4% that approximately 95 million tons of the extractable materials by open pit method. In order to access the deposit, 10 Mm^3 pre-stripping overburden are required to be excavated. Currently 91% of the overburden is excavated and dumped so far. Moreover, during the mining operation, 6 Mt waste must be dumped around the mine annually. Large volumes of dumped waste materials around the mine, necessitates the stability evaluation for the dumps. Stability analysis and construction of these dumps, requires the determination of the mechanical and physical properties of the waste materials.

Materials properties of waste dumps

In open pit mines, mine waste management and optimal use of the space around open pit mine for waste dump are one of the most important tasks during the open pit mine operation. The waste materials are hauled directly to the dumps after blasting without any screening, therefore particle size distribution (PSD) of the waste materials is usually diverse. Particles as fine as clays and as large as the massive blocks are observed within these materials. The particle shapes of the waste materials are also diverse. Bulk unit weight and strength parameters of the soils are highly affected by PSD and particle shapes. Consequently for waste dumps, with such a diverse PSD and particle shape, estimation of the bulk unit weight and shear strength parameters is extremely challenging. Moreover, bulk unit weight and strength parameters and stability of open pit excavations are interconnected. This magnifies the importance of the estimation of the bulk unit weight and strength parameters. In-situ measurements of bulk unit weight and back analysis of slides within dumps to determine shear strength parameters of the materials could produce more realistic results for such materials with diverse PSDs. Estimation of these properties using laboratory tests results have some significant uncertainties, due to the varying particle size distribution of waste materials.

Average bulk unit weight and shear strength parameters of the waste materials are determined using in-situ measurement and probabilistic back analysis of certain slides in waste dump in Daraloo copper open pit mine.

In-situ large scale tests for estimation of the bulk unit weight of waste dumps

Bulk unit weight is one of the main required parameters for stability analysis of waste dump in open pit mines. In some investigations, an approximate value is considered for bulk unit weight in stability analysis based on the existing guidelines or previous studies. Although, the range of bulk unit weight for waste materials is not too wide but this value from one mine to others has certain differences. For comprehensive studies of waste dumps in Daraloo open pit copper mine and in order to increase the back analysis precision of shear strength parameters, 12 large scale in-situ tests were performed to measure the bulk unit weight.

Figure 10a illustrates a rectangular trench with 4 m×2 m×2 m dimensions which was excavated in wastes using a mechanical shovel and loaded carefully in trucks. Later, total weight of the excavated soil was measured. Then, walls and floor of trench were treated and length, width and height of the excavated area were measured
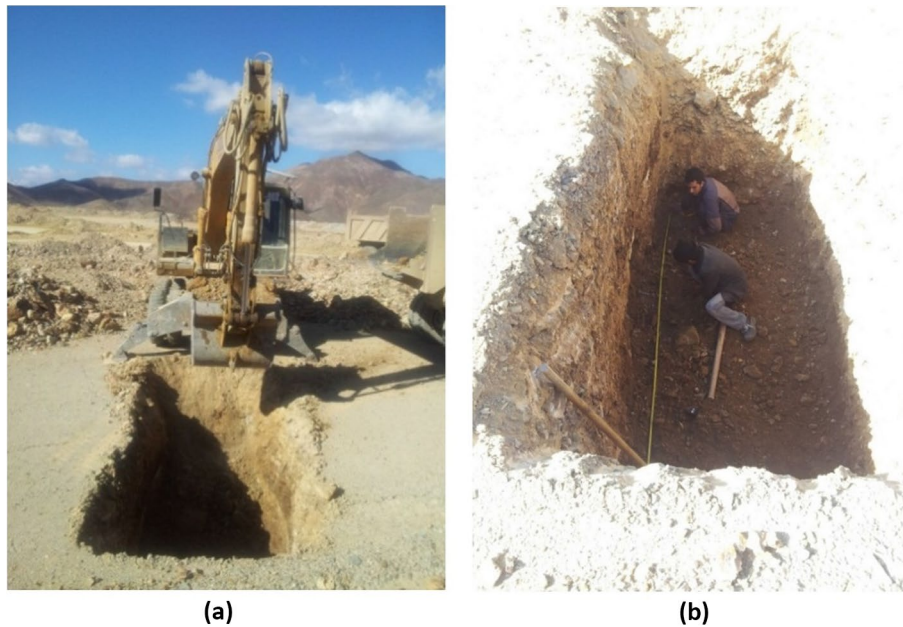
Fig. 10 **a** Excavation of the trench within mining waste to estimate in-situ bulk unit weight of waste dump; **b** Treatment and measurement of hole dimensions to calculate volume of the trench
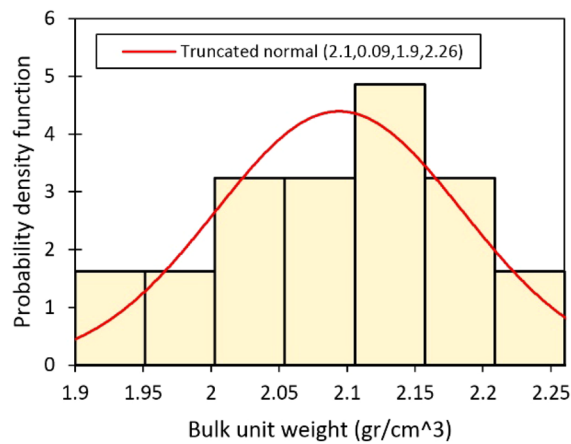

Fig. 11 Probability distribution of in-situ bulk unit weight of waste material

(Fig. 10b). Finally, by calculating the trench's total volume and the soil weight, bulk unit weight of the waste materials was estimated. Probability distribution of the data are shown in Fig. 11.

As depicted in Fig. 11, the range of in-situ bulk unit weight in waste materials of Daraloo mine is between 1.9 and 2.26 g per cubic centimeter and its average value is 2.1 g per cubic centimeter. The changes in bulk unit weight are related to the soil compaction in the site. Some factors such as the location of the test, PSD, moisture content and rainfall can affect compaction of waste materials in different places. Moreover, histogram of the result data shows that a truncated normal distribution with approximately 4.3% coefficient of variation can be fit to the data (Fig. 11). Results

of the in-situ tests and resulted graphs are used to determine the average value and the range of the waste soil bulk unit weight which was used later for back analysis of the shear strength parameters.

Estimation of shear parameters with probabilistic back analysis

Daraloo copper mine waste materials were dumped in two sites (Fig. 12) in recent years. Local observation and field investigations show that certain circular failure have occurred in certain locations in these dumps. For estimating shear strength parameters of the dumps, first, tension cracks (Fig. 13) and their locations were surveyed. According to the hydrogeological evaluations in the waste dumps, we used the dry soil.

   Initial values of the shear parameters have been chosen in a way that the critical failure at each cross section corresponded to the failed slope. For the probabilistic back analysis, for the bulk unit weight and groundwater parameters has been chosen the truncated normal distribution and for cohesion and internal friction angle has been selected uniform distribution.

   In next step, a suitable section of the dump is chosen and 2D circular slide models are designed using former topographic maps. Then, all slides were analyzed in order to determine the shear strength parameters in the certain regions in the dumps. Figure 14 shows section $B^{'}$ of the designed in studied sections presented in Fig. 12.

   Shear strength parameters which have been calculated based on the intersection of the SF = 1 line and monograms in Fig. 15 by LEM method. This figure shows that shear strength parameters of the waste materials considering the SF = 1 for different slides have a vast range. However, we expect a low range of shear strength parameters for the sections in the dumps; because waste materials are crushed by the same explosive pattern, hauled system, loading and deposited approach. Accordingly, as depicted in Fig. 15, range of internal friction angle of the materials is between 23.4
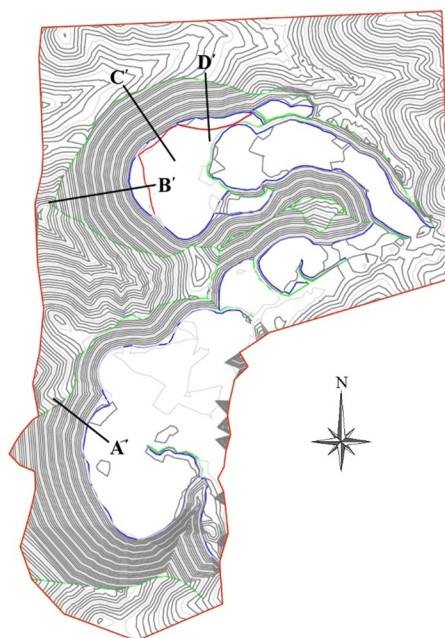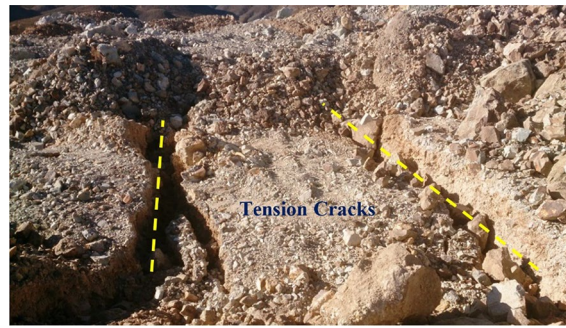

Fig. 12 Dumps plan and chosen sections for back analysis

Fig. 13 A typical view of tension cracks within one of waste dumps of Daraloo copper mine
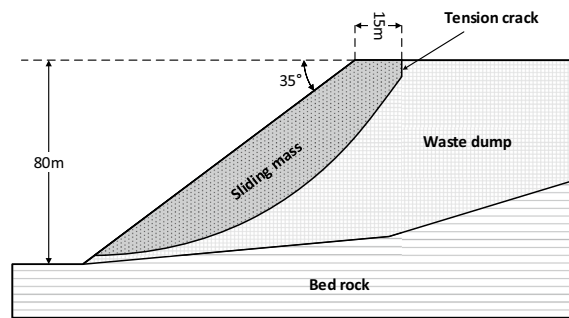

Fig. 14 Section"B" designed model for back analysis of the waste dump
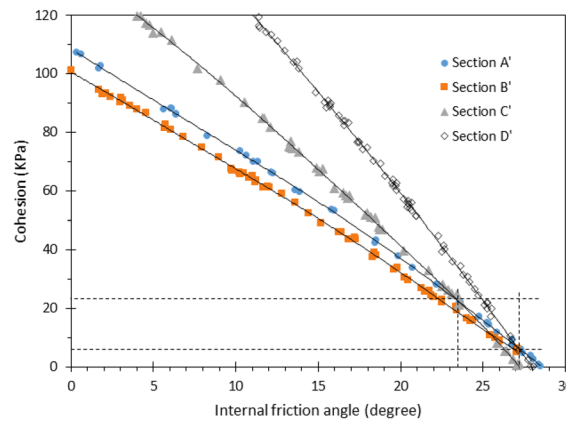

Fig. 15 Values of possible shear strength parameters for the circular failure in the analyzed sections of the waste materials considering the SF = 1 for different slides have a vast range

and 27.2 degree and their cohesion ranges between 6.5 and 22.5 kPa. This graph shows the medium value for internal friction angle of the dumps is 25.2 degree in Daraloo copper mine and the medium value of cohesion of the materials is about 14.9 kPa. Using the obtained results, the waste dumps was stability analyzed and redesigned.

Conclusion

According to the probabilistic back analysis of a mapped failure surface, shear strength parameters of the slopes and waste dumps in Anjerd and Daraloo copper mine have been determined and outputs have been utilized to redesign the failed slope and design the new slopes under the same geotechnical conditions. Therefore, it could be concluded that:

- The slope failure has been frequently back analyzed for improving our knowledge of the parameters of a slope stability model. Therefore, for back analyzing several sets of the slope stability parameters concurrently under uncertainty, it could be run based on a probabilistic way so that uncertain parameters would be modeled as the random variables and their distribution has been uniform; therefore, distributing other effective parameters would be enhanced with the observed slope failure information.
- Practically, using the simplistic technique on the basis of the sensitivity analysis would be more convenient; therefore, it could be applied as the slope stability model has been nearly linear. It is notable that the probabilistic back analysis would have higher efficiency in the case of the analysis of the slope failures wherein considerable uncertainty has been found in the shear strength and other influential parameters.
- The probabilistic back analysis offered a lot of information in comparison to the ones presented by the deterministic back analysis. It also relatively matched the geotechnical engineering in the real world. Hence, this method has been considered to be beneficial and functional to analyze stability and re-design the slopes; thus, it could increase the construction safety.
- Probabilistic back analysis method could be used to estimate the shear parameters and stability analysis of mineral and construction slopes as a cost-effective and high speed method with high reliability.

Data availability
The authors confirm that the data supporting the findings of this study are available within the article.

Declarations

Competing interests
The authors RN, AD, VB certify that they have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript. The authors RN, AD, VB report the following details of affiliation or involvement in an organization or entity with a financial or non-financial interest in the subject matter or materials discussed in this manuscript. Please specify the nature of the conflict on a separate sheet of paper if the space below is inadequate.

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