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Analytical technique for stability analyses of the rock slope subjected to slide head toppling failure mechanisms considering groundwater and stabilization effects

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Abstract

The contributions of the current analytical models on the prediction of the stability of the slope subjected to slide head toppling failure mechanisms, have always focused on the idealized geometry comprising regular blocks dipping into the slope face. Besides, the influence of groundwater and stabilizations from the lowermost block of the slope have been overlooked in the available literature. In this article, the analytical solutions that incorporates the kinematic mechanisms of the jointed rock slope under the influence of groundwater and stabilizing the lowermost block subjected to slide head toppling are derived based on the limit equilibrium. Furthermore, a real slide head toppling failure case history was studied to illustrate the effectiveness of the presented analytical solutions. The investigation results indicate that the presence of groundwater in the jointed rock slope, lowers the distributions of the normal and shear forces thereby inducing slide head toppling. Reinforcing the lowermost block of the slope, enhances the distributions of the normal and shear forces thus improving the stability of the jointed rock slope. The study results depict that the presented analytical solutions can provide an accurate and efficient stability analyses of the jointed rock slope subjected to slide head toppling failure mechanisms considering the presence of groundwater and stabilization effects.

Keywords: Jointed rock slopes, Stability technique, Analytical solution, Groundwater, Stabilizations

Introduction

In slope engineering field, understanding of rock slope failure mechanisms has increased considerably during the last four decades in response to the continued development of the commercial dams, highways, steep surface mine slopes, bridges and urban populations in mountainous areas. It is a well-known fact that analytical methods for analyzing sliding failure mechanisms of the rock slope are vigorous and adequate in the existing geotechnical literature [1–3]. Compared to sliding failure mechanisms, the analytical methods for analysing failure mechanisms by slide head toppling failure mechanisms that exists are not robust [4–9].

Although much efforts have been made in the current analytical models for the prediction of the stability of the jointed rock slope subjected to slide head toppling failure mechanisms, their contributions are based on the idealized geometry comprising regular blocks dipping into the slope face. Besides, the discontinuity bounding the potential toppling blocks is assumed to be regular running from the topmost surface of the slope model, and then daylighting near the toe of the slope. However, in some physical situations [10–12], the discontinuity bounding the potential slide head toppling blocks maybe irregular due to the variation of geological conditions. In that case, the potential discontinuity bounding the potential toppling blocks may be counter-tilted within the rock mass due to the changes in the geological characteristics and daylights on an unanticipated point on the face of the slope as shown in Figs. 1 and 2. This may lead to mixed failure mechanisms involving toppling and other failure mechanisms i.e. block-flexure toppling, toppling-circular and slide head toppling [13–17].

This article places emphasis on the mixed failure mechanisms involving slide head toppling failure mechanisms. The source of errors in the current analytical models could be on the assumptions of the weak plane bounding the potential failure blocks. Yet, slide head toppling failure is still commonly experienced in sedimentary rock formation in many commercial dams, highways and surfaces mine slopes around the world [18–21]. Besides, very few scholars have conducted research on mixed failure modes involving sliding of the toe region and toppling of the blocks on the upper section of the slope.

It is flawed to evaluate the stability of the jointed rock slope susceptible to slide head toppling using the current analytical model. Bowa and Xia 2019 [12], proposed

an analytical models based on a 2D Model that apply basic trigonometry principles to resolve the geometry of the jointed rock slope with counter-tilted discontinuity subjected to slide head block toppling failure mechanisms. Then, an iterative process followed in which forces acting on each block are determined. Thereafter, the stability of each block dipping into the face of the slope are resolved using limit equilibrium approach. The computation process starts from the set of blocks on the upper reaches, followed by the set of blocks on the middle section and ending with the set of blocks on the lower reaches of the slope. Comparisons between stability against toppling and sliding of each block are made, with the maximum forces of these two values dictating the failure mechanisms of the next block. The overall slope is considered unstable if the lowermost block is either sliding or toppling. There are several factors that have been overlooked in the current analytical models that includes joint infill materials, joint spacing, persistency, size of the block, geometry of the block, groundwater, stabilizations from the lowermost block. In this article, the groundwater and stabilization effects on the potential toppling and sliding blocks were overlooked in their proposed analytical model [22–24] have been discussed. An improved analytical model is hence necessary that incorporates the kinematic mechanisms of the jointed rock slope with counter-tilted discontinuity subject to mixed failure mechanisms involving slide head toppling under the effect of groundwater and stabilizing influence on the lower block based on limit equilibrium methods.

The rest of this paper is organized as follows. The analytical solutions that incorporates the kinematic mechanisms of the jointed rock slope subjected to slide head toppling are derived based on the limit equilibrium. This accounts for the influence of groundwater and stabilizing the lowermost block. Furthermore, a real slide head toppling failure, a case history of Nchanga open pit was studied to validate the

Derivations of the analytical models

Detail A: Detail of Water force for the n^{th} block only

Stable

Toppling

Sliding

See detail A

Fig. 3 Groundwater forces, reinforcement forces and moments acting on the jointed rock slope subjected to slide-head-toppling mechanism

Rock slope geometry

In order to examine the stability of the slope subjected to slide-head-toppling failure mechanisms, the geometry of the slope ought to be resolved using the basic trigonometry principles. The geometry of the jointed slope is resolved by first approximating the angle of the bases of the rock blocks subjected to the failure modes. Scholars [25–27] indicated that the toppling blocks tend to be stepped on the base planes and the approximation of the base plane angle is usually taken as the sum of the initial weak plane angle and the stepped angle which varies between 10° and 30° . Considering the weak plane angle during toppling failure, variations of stepped angles and the counter-tilted angle, the angle of the base planes (ψb) is estimated using Eq. 1.

$$\psi b \approx \left(\frac{\psi c + \psi p}{2} + 10^\circ \right) \text{ to } \psi b \approx \left(\frac{\psi c + \psi p}{2} + 30^\circ \right). \quad (1)$$

The approximated angle of the base planes (ψb), is then used for estimating the number of rectangular blocks, dipping into the slope using Eq. 2.

$$n = \frac{H}{\Delta x} \left[\operatorname{cosec}(\psi b) + \left(\frac{\cot(\psi b) - \cot(\psi f)}{\sin(\psi b - \psi f)} \right) \sin \psi s \right]. \quad (2)$$

The height of the n th block, the y_n below the crest of the slope is resolved using Eq. 3 while the height of the n th block, the y_n above the crest of the slope is resolved using Eq. 4.

$$y_n = n(a_1 - b) \quad (3)$$

$$y_n = y_{n-1} - a_2 - b. \quad (4)$$

The constants a_1 , a_2 , b are resolved using by Eqs. (5–7) respectively;

$$a_1 = \Delta x \tan(\psi f - \psi c) \quad (5)$$

$$a_2 = \Delta x \tan(\psi p - \psi s) \quad (6)$$

$$b = \Delta x \tan(\psi b - \psi p). \quad (7)$$

Other typical dimensions of the rock blocks needed to be resolved prior to conducting block stability analysis are the application points denoted M_n and L_n shown in Fig. 4. Goodman and Shi [28] presented a simplistic set of analytical equations to resolve the application points M_n and L_n . The analytical equations are reproduced in Eqs. (8–12) to compute the application points. When the n th blocks are below the crest, then M_n and L_n are resolved using Eqs. (8, 9) respectively.

$$M_n = y_n \quad (8)$$

$$L_n = y_n - a_1. \quad (9)$$

Fig. 4 Limiting equilibrium conditions for failure modes of the n th block: **a** Forces acting on the potential stable block; **b** Forces acting on the potential toppling block; **c** Forces acting on the potential sliding block

When the n th block is the crest, then Mn and Ln are resolved using Eqs. (10 - 11) respectively.

$$Mn = yn - a2 \quad (10)$$

$$Ln = yn - a1 \quad (11)$$

When the n th block is above the crest, then Mn and Ln are resolved by Eqs. (12-13) respectively.

$$Mn = yn - a2 \quad (12)$$

$$Ln = yn. \quad (13)$$

Stability of the rock blocks

In Fig. 4a, the rock columns on the upper section of the slope are classified as stable considering that the center of gravity lies within the base of the rock columns. On the intermediate section of the jointed rock slope shown in Fig. 4b, the center of gravity lies outside the base plane of the rock columns and the possibility of toppling failure mechanisms is high. On the lower section of the jointed rock slope in Fig. 4c, there are high possibilities of rock columns undergoing failure mechanisms by sliding. This is because the height to width ratio of each rock column is presumably greater than unitary and

each block is receiving the toppling force from the intermediate section. The internal friction of the rock mass and friction angles between rock columns vary depending on the characteristics of the local lithology. In this article, it is assumed that the friction angle between the rock columns (ϕd) and the friction angles on the weak planes bounding the potential failure blocks are equal ($\phi p = \phi c$). By applying the limit equilibrium analysis method, the shear forces on each rock block are computed using the derived Eqs. 14-15. Considering the perpendicular and side friction angles to the two bases (ψp , ψc) of a rock column with weight, Wn , the resulting normal and shear forces are computed using Eqs. 16-17 respectively.

$$Qn = Pn \tan \phi d \quad (14)$$

$$Qn - 1 = Pn - 1 \tan \phi d \quad (15)$$

$$\begin{aligned} Rn &= Wn \cos \psi p + (Pn - Pn - 1) \tan \phi d \\ Rn &= Wn \cos \psi c + (Pn - Pn - 1) \tan \phi d \end{aligned} \quad (16)$$

$$\begin{aligned} Sn &= Wn \sin \psi p + (Pn - Pn - 1) \\ Sn &= Wn \sin \psi c + (Pn - Pn - 1) \end{aligned} \quad (17)$$

The forces Pn, t required to prevent toppling of block n considering the initial and counter-tilted angle of the weak plane in a situation where the rock column is under rotational equilibrium state, are computed using Eqs. (18-19) respectively. For Equations derivations refer to Appendix 1 Eqs A1.1- A1.10.

$$Pn, t = \frac{\frac{Wn}{2}(yn \sin \psi p - \Delta x \cos \psi p) + Pn + 1(Mn - \Delta x \tan \phi)}{Ln} \quad (18)$$

$$Pn, t = \frac{\frac{Wn}{2}(yn \sin \psi c - \Delta x \cos \psi c) + Pn + 1(Mn - \Delta x \tan \phi)}{Ln} \quad (19)$$

In a case where the rock column on the lower section of the slope is determined to be undergoing sliding failure mechanisms, the magnitude forces, Pn, s , necessary to prevent sliding of rock column n taking into account initial and counter-tilted angle of the weak plane is computed using Eqs. (20-21) respectively. For Equations derivations refer to Appendix 2 Eqs A2.1- A2.16.

$$Pn, s = Pn + 1 + \frac{Wn \cos \psi p (\tan \psi p - \tan \phi)}{1 - \tan^2 \phi} \quad (20)$$

$$Pn, s = Pn + 1 + \frac{Wn \cos \psi c (\tan \psi c - \tan \phi)}{1 - \tan^2 \phi} \quad (21)$$

Block stability considering the influence of the groundwater

In some physical situations, groundwater exists as an external force that may be acting on the slope and may influence the stability of the jointed rock slope subjected to slide head toppling failure mechanisms. In this article, it is assumed that the joint persistency is low and does not allow groundwater to drain off at the face of the slope. It is envisaged that there high transient water pressures will develop. It was further assumed that the rock mass in the model has low conductivity and high water pressures was possible. It is therefore necessary to incorporate its effect on the stability of the slope in the existing analytical model. Figure 5a, b show three groundwater forces (V_1 , V_2 , V_3) and as well as the forces P_n and P_{n-1} produced by the blocks above and below the crest of the jointed rock slope. The current Equations are then modified by incorporating the groundwater forces and resolving all the forces normal and parallels to the base of the blocks based on limit equilibrium principles. Considering the weight of the n th block and the perpendicular and sides friction angles on the weak plane bounding the n th block, and the presence of groundwater on both sides.

Then the total normal forces before counter-tilting of the weak plane is resolved using Eq. 22 and after counter-tilting of the weak plane is resolved using Eq. 23 respectively.

$$R_n = W_n \cos \psi_p + (P_n - P_{n-1}) \tan \phi_d + (V_1 - V_2) \tan \phi_d - V_3 \cos \psi_p. \quad (22)$$

$$R_n = W_n \cos \psi_c + (P_n - P_{n-1}) \tan \phi_d + (V_1 - V_2) \tan \phi_d - V_3 \cos \psi_c \quad (23)$$

Considering the weight of the n th block and the perpendicular and sides friction angles on the weak plane bounding the n^{th} block, then the total shear forces before counter-tilting of the weak plane is resolved using Eq. 24 and after counter-tilting of the weak plane is resolved using Eq. 25 respectively.

$$S_n = W_n \sin \psi_p + (P_n - P_{n-1}) - (V_1 - V_2) - V_3 \sin \psi_p \quad (24)$$

$$S_n = W_n \sin \psi_c + (P_n - P_{n-1}) - (V_1 - V_2) - V_3 \sin \psi_c. \quad (25)$$

When the rock block has potential of undergoing toppling failure mechanisms under the influence of groundwater forces, then the magnitude $P_{n,t}$ to cause toppling of block n under limit equilibrium condition before counter-tilting of the weak plane is resolved using Eq 26. When the weak plane is counter-tilted, then the magnitude $P_{n,t}$ to cause toppling of block n under limit equilibrium condition is resolved using Eq. 27.

$$P_{n,t} = \frac{\frac{W_n}{2}(y_n \sin \psi_p - \Delta x \cos \psi_p) + \frac{V_1 y_1}{3} + \frac{\gamma_w \Delta x^2}{6} * \cos \psi_p (y_2 + 2y_1) - \frac{V_2 y_2}{3} + P_n + 1(M_n - \Delta x \tan \phi)}{L_n} \quad (26)$$

$$P_{n,t} = \frac{\frac{W_n}{2}(y_n \sin \psi_c - \Delta x \cos \psi_c) + \frac{V_1 y_1}{3} + \frac{\gamma_w \Delta x^2}{6} * \cos \psi_c (y_2 + 2y_1) - \frac{V_2 y_2}{3} + P_n + 1(M_n - \Delta x \tan \phi)}{L_n}. \quad (27)$$

When the rock block has potential of undergoing sliding failure mechanisms under the influence of groundwater forces (while ignoring the influence of intact rock/mass parameters, like joint spacing) then the magnitude $P_{n,s}$ to cause sliding of block n under limit equilibrium condition before counter-tilting of the weak plane is resolved using Eq. 28. When the weak plane is counter-tilted, then the magnitude $P_{n,s}$ to cause sliding of block n under limit equilibrium condition is resolved using Eq. 29.

$$P_{n,s} = P_n + 1 + \frac{W_n \cos \psi_p (\tan \psi_p - \tan \phi) + V_1 - V_3 \tan \phi - V_2}{1 - \tan^2 \phi} \quad (28)$$

$$P_{n,s} = P_n + 1 + \frac{W_n \cos \psi_c (\tan \psi_c - \tan \phi) + V_1 - V_3 \tan \phi - V_2}{1 - \tan^2 \phi}. \quad (29)$$

where, $V_1 = \frac{1}{2} \gamma W \cos \psi_p y_1$; $V_2 = \frac{1}{2} \gamma W \cos \psi_p (y_1 + y_2) \Delta x$ and $V_3 = \frac{1}{2} \gamma W \cos \psi_p y_2^2$.

The stability analyses of the blocks under the influence of groundwater based on the limit equilibrium stability analysis then proceeds as before using the modified versions for forces Pn, t and Pn, s as demonstrated in Eqs. 28 and 29 respectively.

Anchor force required to stabilize a slope with groundwater

For the stability of the toppling slope to be improved, reinforcing the lowermost unstable block using anchors and bolts is a common remedial measure (Wyllie [22]). The optimum anchor force is determined using analytical methods based on limit equilibrium principles for comparative cost valuations. Considering the plunge angle ψT at which the anchor cable is reinforced through the lowermost block at a distance $L1$ above the toe of the slope. The stability analyses of the jointed rock slope subjected to sliding/toppling of blocks has been conducted considering the influence of groundwater and stabilizing effect on the lowest block. The optimum anchor force required to stabilize the toppling blocks considering the influence of groundwater effect before counter-tilting of the weak plane is resolved using Eq. 30 and after counter-tilting of the weak plane the optimum anchor force required to stabilize the toppling blocks considering the influence of groundwater effect is resolved using Eq. 31 respectively.

$$T, t = \frac{\frac{Wn}{2} (yn \sin \psi p - \Delta x \cos \psi p) + \frac{V1y1}{3} + \frac{\gamma w \Delta x^2}{6} * \cos \psi p (y2 + 2y1) - \frac{V2y2}{3} + Pn + 1(Mn - \Delta x \tan \phi)}{Ln \cos(\psi p + \psi T)} \quad (30)$$

$$T, t = \frac{\frac{Wn}{2} (yn \sin \psi c - \Delta x \cos \psi c) + \frac{V1y1}{3} + \frac{\gamma w \Delta x^2}{6} * \cos \psi c (y2 + 2y1) - \frac{V2y2}{3} + Pn + 1(Mn - \Delta x \tan \phi)}{Ln \cos(\psi c + \psi T)} \quad (31)$$

The optimum anchor force required to stabilize the sliding blocks considering the influence of groundwater effect before counter-tilting of the weak plane is resolved using Eq. 32 and after counter-tilting of the weak plane is resolved using Eq. 33 respectively.

$$Ts = \frac{P1(1 - \tan^2 \phi) - W1(\tan \phi \cos \psi p - \sin \psi p) + V1 - V3 \tan \phi - V2}{(\tan \phi \sin(\psi p + \psi T) + \cos(\psi p + \psi T))} \quad (32)$$

$$Ts = \frac{P1(1 - \tan^2 \phi) - W1(\tan \phi \cos \psi c - \sin \psi c) + V1 - V3 \tan \phi - V2}{(\tan \phi \sin(\psi c + \psi T) + \cos(\psi c + \psi T))} \quad (33)$$

Once the force T is applied to the lowermost block under the influence of groundwater based on limit equilibrium method, the normal forces on the sides of the blocks and the base bounding the potential failure blocks before and after counter-tilting of the weak plane are resolved using Eqs. 34 and 35 respectively,

$$Rn = Wn \cos \psi p + (Pn - Pn - 1) \tan \phi d + (V1 - V2) \tan \phi d - V3 \cos \psi p + T \sin(\psi T + \psi p) \quad (34)$$

$$Rn = Wn \cos \psi c + (Pn - Pn - 1) \tan \phi d + (V1 - V2) \tan \phi d - V3 \cos \psi c + T \sin(\psi T + \psi c). \quad (35)$$

When the force T is applied to the lowermost block under the influence of groundwater based on limit equilibrium method, the shear forces on the sides of the blocks and base bounding the potential failure blocks before and after counter-tilting of the weak plane are resolved using Eqs. 36 and 37 respectively,

$$Sn = Wn \sin \psi p + (Pn - Pn - 1) - (V1 - V2) - V3 \sin \psi p - T \cos(\psi P + \psi T) \quad (36)$$

$$Sn = Wn \sin \psi c + (Pn - Pn - 1) - (V1 - V2) - V3 \sin \psi c - T \cos(\psi c + \psi T). \quad (37)$$

Engineering applications: North wall of Nchanga Open Pit Mine

A typical example of slide head toppling failure of the jointed rock slope is a surface mine slope at Nchanga Open Pit in Zambia shown in Fig. 6. Figure 6, indicates a set of stable blocks in the Dolomite rock formations on the topmost surface, a set of toppling blocks in the Shale with Grit formations on the intermediate section and set of sliding blocks in the variations of Shale with Grit types 1,2 and 3 rock formation. The geological settling of the Nchanga open pit is shown in Fig. 7. It is presumed that the toppling blocks are bounded by the weak plane on the intermediate sections of the slope. The sliding blocks in the variations of Shale with Grit types 1,2 and 3 rock formation zone are bounded by the potential counter-tilted weak plane shown in Fig. 7.

Fig. 7 Post-failure photograph of slide head toppling of Northwall of the Nchanga Open Pits

Structurally, the Nchanga Syncline regional structure trending east–west and plunging at 5° – 7° to the north controls the Nchanga Stratigraphy. The Nchanga Open Pit's South limb has shallow dipping of between 25° and 35° due north with local variation of shallow to steep dips while north limb dips at 60° . The sub-surface geology and characteristics of Nchanga Open pits from the topmost surface to the bottom is provided in Table 1. However the porosity, average permeability of each rock unit and joint characteristics were not available and not included in this table. Slide head toppling failure occurred in shale with grit formation. At Nchanga open pit, Cobalt and Copper ore is hosted in Feldspathic Quartzite formation.

Resolving the slope geometry

Considering the initial weak plane angle that is counter-tilted in the intermediate section of the slope, the angle of the base planes is estimated using Eq. 1 as 55° . The number of blocks dipping into the face of the slope is estimated using Eq. 2 as 14. The height of the n th block below and above the crest are resolved using Eqs. 3 and 4 respectively. The constants a_1 , a_2 and b are computed using Eqs. 5, 6 and 7 as 5, 5 and 1 respectively. The geometries of the failed slope are shown in the conceptual model in Fig. 8. Stabilization of the toe of the slope prior to failure is considered in the conceptual model.

Resolving of application points

However, the analytical models by Goodman and Shi [28] presented in Eqs. 8–13 for resolving the positions of the application points M_n and L_n are considered to be very conservative.

Table 1 The general stratigraphy and rock characteristics

Rock Unit	Rock layer (m)	Density Kg/m ³	Internal friction Angle (°)	Cohesion (MPa)	Rock mass rating (RMR)	Hydro-geological unit	Description
Shale with Grit	50	2500	35	400	68	Aquiclude	Fair to good quality, bedded rock mass
Shale with Grit type 1	40	2300	30	360	49	Aquiclude	Weak and bedded rock mass
Shale with Grit types 2 and 3	20	2200	30	360	49	Aquiclude	Weak and bedded rock mass
Chingola Dolomite	15	2400	33	380	64	Aquifer	Fair to good quality, bedded rock mass
Dolomitic Schists	20	2800	42	540	75	Minor Aquifer	Good quality, bedded rock mass
Banded Shale	18	2600	37	490	68	Aquiclude	Fair to good quality rock mass, seam at the base, bedded
Feldspathic Quartzite	18	2900	45	800	82	Aquiclude	Very good rock mass, massive
Banded sandstone	15	1900	28	150	47	Aquifer	Weak rock mass with calcite infill material, bedded
Pink quartzite	5	3200	59	1300	84	Aquiclude	Very good quality with random joints
Arkose	15	3300	67	4000	96	Minor Aquifer	Very strong and competent rock mass
Basements (Gneiss and red granite)	> 400	3000	39	1150	90	Impermeable	Strong and Competent with some schistosity

With some laboratory tests conducted by Aydan and Kawamoto [30]; their results showed that the position of the application point of force on the $(n + 1)$ th block is given in the general form of $Mn = \chi \cdot hn$, while Ln is a function of failure mode. In the case where only block toppling failure mechanisms of the slope occurs, Goodman and Bray [29] assumed that $\chi = 1.0$. While in the case where shear-sliding is determined to be the only failure mechanisms, Aydan et al. ([30]; Aydan et al. [31] and Amini et al., [32] assumed that $\chi = 0.5$. Zheng et al. [33] recommends the use of back analysis to determine χ where side forces could be acting on the bordering blocks to improve the accuracy of (χ) on the physical model. Once the χ is determined through back analysis, the application point resolved using Goodman and Shi [29] is recalculated. Determinations of the applications point of the n th and $(n + 1)$ th blocks is an iterative process and highly

Table 2 Parameters of rock mass and joints

Material type	Material properties				
	Density, kg/m ³	Bulk modulus, (GPa)	Shear modulus, (GPa)	Cohesion, (MPa)	Friction Angle, (°)
Shale with Grit	2500	11.25	8.4	400	35
Shale with Grit type 1	2300	10.85	7.65	360	30
Shale with Grit type 2&3	2200	9.88	8.5	360	30
Chingola Dolomite	2400	11.02	7.2	380	33
Joint material 1 (joint material on the interface between 2 units)	1500	0.84	0.93	10	35
Joint material 2(joint material along weak plane bounding the rock units)	1300	0.59	0.76	8	35

dependent on the position on the slope model on which the force is being applied. The application point L_n is a function of the failure mode and could be resolved using analytical equations presented by Goodman and Shi [29].

In this article, the use of intelligent algorithm was applied by using the user-defined FISH function in 3DEC program, ITASCA [34] (could not share the FISH function in 3DEC program due to limitations in rights to share) to determine χ where side forces could be acting on the bordering blocks to improve the accuracy of (χ) on the physical

Table 3 Magnitude and location of normal force of contacts between adjacent rock columns (blocks) above the weak plane

χ	Rock Column (5 and 6)		Rock Column (7and 8)		Rock Column (11 and 12)	
	Y-c(m)	N-f(kN)	Y-c(m)	N-f(kN)	Y-c(m)	N-f(kN)
	73	105	80.6	70	90.6	96
	45.6	102.6	78.3	65	95	85
	42.8	98.5	75.9	62	98.5	83
	41.6	98	74.5	58		
	41.2	97.5	74.1	57.4		
			74	56		
χ	0.92		0.94		0.96	

*Y-c and N-f denote Y-coordinate and normal force of contacts between adjacent rock columns, respectively, which were listed from the slope surface to the failure surface. χ denotes the point where total force is assumed to be acting

model. To determine the value of this parameter for the example, back-analysis was performed using the discrete element code -3DEC (Itasca [34] based on the parameters given in Table 2. A user-defined FISH function (i.e. c-result) was written to obtain the magnitude and location of the normal force of contacts between adjacent blocks, based on the point where the side total forces acting could be determined. Table 3 provides the obtained magnitude and location of normal forces acting on the contacts between adjacent rock columns (blocks) above the weak plane. The magnitude and location of the normal force of contacts between adjacent block columns considered were considered as boundaries defining the failure mechanisms. These boundary blocks are between rock columns 5 and 6; Rock columns 7 and 8; and rock columns 11 and 12.

Based on the user-defined FISH function, the point where total side force acts between rock columns 5 and 6, 7 and 8, as well 11 and 12 were determined respectively, as shown in Table 4. Taking the total side force between columns 5 and 6, as an example, the calculation process is illustrated below. Location of total side force of rock columns 5 and 6 can be resolved using Eq. 38.

$$Y_{5,6} = \frac{\sum (y - c)_i (N - f)_i}{\sum (N - f)_i} \quad (38)$$

$$Y_{5,6} = 72.8.$$

Then, χ can be determined

$$\chi = \frac{72.8 - 48}{73 - 48}$$

$$\chi = 0.92$$

.

The results of the numerical simulation indicated that this value (χ) was approximately equal to 0.92. Thus, χ was assumed to be equal to 0.92. Therefore the position of the application of force on the 5th block can be given in the general form of $Mn = \chi \cdot hn$

Table 4 Limited equilibrium analysis of a block toppling slope- listing block dimensions, forces and stability failure modes without the influence of groundwater and stabilizing from the toe of the slope

Block, n	yn(m)	yn/ Δ x	Mn(m)	Ln(m)	Pn – t	Pn – s	Pn	Rn	Sn	Sn/Rn	Failure mode
14	– 2	–0.2			0	0	0	353.6	353.6	1.00	Stable
13	4	0.4			0	0	0	707.1	707.1	1.00	
12	10	1.0	2	10	0	0	0	1767.8	1767.8	1.00	
11	16	1.6	8	16	0	0	0	2456.9	2298.1	0.97	Toppling
10	22	2.2	14	22	530.5	– 1664.7	530.5	3399.6	3190	0.94	
9	26	2.6	18	26	1229.4	– 1758.7	1229.4	4102.6	3891.3	0.95	
8	32	3.2	24	27	1934.3	– 1475.8	1934.3	4544.8	4068.6	0.90	
7	28	2.8	28	23	3522.6	– 1393.1	3522.6	3808	3319.2	0.87	
6	24	2.4	24	19	5153.1	609.3	5153.1	2605.8	1904.9	0.73	
5	20	2	20	15	5490.8	6534.3	6534.3	4225.6	3053.3	0.72	Sliding
4	16	1.6	16	11	5304.4	6140.2	6140.2	3826.5	3079.7	0.80	
3	12	1.2	12	7	4520	5146.1	5146.1	3558.1	3292.6	0.93	
2	8	0.8	8	3	2948.2	3365.6	3365.6	3855	4313.6	1.12	
1	4	0.4	4		– 218.3	– 96	– 96	666.3	355.3	0.53	

while Ln is a function of the failure mode in this case sliding. The position of application of the force on the 6th block can be resolved using Eq. 39.

$$Mn = \chi .hn. \quad (39)$$

where, hn is dependent on the point of applications on the slope model and could be obtained using ordinary methods presented in Eqs. (8, 10 and 12) respectively. Now that the value of $\chi = 0.92$ and $hn = 26$ therefore $Mn = 0.92 * 26 \approx 24$ for the 6th column as shown in Table 4. Determinations of the applications point Mn of the n th and $(n + 1)$ th blocks is an iterative process and highly dependent on the position on the slope model on which the force is being applied. The application point Ln is a function of the failure mode and could be resolved using the ordinary equations Eqs. (9; 11 and 13). The jointed rock slope subjected to slide head toppling is assumed to be acting under no noticeable external forces (earthquake, groundwater) acting on the slope and that the friction angles are $\phi p = \phi d = \phi c = 30^\circ$. Considering that the blocks (14, 13 and 12) are short and that the height to width ratio is below 1.0 and their center of gravity lies within their bases, the three blocks are regarded as being stable. For blocks between 6 and 11, the height to width ratio is above 1.0 and the blocks topples. In that case the force required for toppling of block 11, is assumed to be $P11 = 0$ while $P10$ to $P6$ are computed using Eqs. 18 respectively. For blocks between 5 and 1, the weak plane bounding the potential failure blocks is counter-tilted, therefore the height to width ratio is reduced and the blocks are receiving force from toppling blocks from the upper and intermediate sections. These blocks will undergo sliding failure mechanisms. The normal and shear forces are computed using Eqs. 16 and 17 respectively and results are presented in Table 4.

The forces Pn, t required to prevent toppling of block n are computed using Eqs. 18 while forces Pn, s to require to prevent sliding of block n under limit equilibrium condition is computed using Eq. 28.

There were three scenarios that were considered for the computations of the distributions of the normal and shear forces presented. These include; a) The geometric model parameters and the slope was assumed to be dry and unsupported; b) The geomaterials parameters and the slope is assumed to be wet and unsupported and; c) The geomaterials parameters and the slope is wet and supported (Table 5).

Parametric analyses

Once the distribution of forces has been well define in the potential failure regions, the distributions of the normal and shear forces on the bases of the blocks are computed using Eqs. 16 and 17 respectively. The influence of external forces such as groundwater and stabilization effects on the lowermost block on the distributions of forces were considered in the parametric analyses. The distributions of the normal and shear forces on the bases of the blocks considering a) no any other external forces; b) Influence of groundwater and c; stabilization from the lowermost blocks are shown in Figs. 9, 10 and 11.

Table 5 Slope with varying parameters

Type	Variables, Fixed and varying nth block	parameters	Mn (m)	Ln(m)	Rn(kN)	Sn(kN)
Geometric model parameters	n = 14	a1 = 5 m; a2 = 5 m; b = 1	–	–	353.6	353.6
	n = 13	m; $\Delta x = 10$ m; H = 90	–	–	707.1	707.1
	n = 12	m; $\psi f = 65^\circ$; $\psi d = 60^\circ$;	2	10	1767.8	1767.8
	n = 11	$\phi p = \phi c = \phi d = 30^\circ$;	8	16	2456.9	2298.1
	n = 10	$\psi p = 45^\circ$; $\psi p = 35^\circ$;	14	22	3399.6	3190
	n = 9	$\gamma_{roc} = 26 \text{ kN/m}^3$	18	26	4102.6	3891.3
	n = 8	Wn = 500 kN/m Dry Slope, unsupported	24	27	4544.8	4068.6
	n = 7		28	23	3808	3319.2
	n = 6		24	19	2605.8	1904.9
	n = 5		20	15	4225.6	3053.3
	n = 4		16	11	3826.5	3079.7
	n = 3		12	7	3558.1	3292.6
	n = 2		8	3	3855	4313.6
	n = 1		4	–	666.3	355.3
Geomaterials parameters	n = 14;	a1 = 5 m; a2 = 5 m; b = 1	–	–	5.31	– 203.88
	n = 13	m; $\Delta x = 10$ m; H = 90	–	–	5.31	– 203.88
	n = 12	m; $\psi f = 65^\circ$; $\psi d = 60^\circ$;	2	10	5.31	– 203.88
	n = 11	$\phi p = \phi c = \phi d = 30^\circ$;	8	16	– 300.987	– 734.38
	n = 10	$\psi p = 45^\circ$; $\psi p = 35^\circ$	14	22	$\gamma_{roc} = 26 \text{ kN/m}^3$	– 902.89
	n = 9	$\gamma_{water} = 10 \text{ kN/m}^3$	18	26	– 398.27	– 912.55
	n = 8		24	27	– 403.37	– 912.55
	n = 7	Wn = 500 kN/m; 50%	28	23	– 911.2	– 1792.19
	n = 6	Wet Slope: variation of	24	19	– 935.89	– 1834.25
	n = 5	y1 = 10 m, y2 = 5 m, unsupported	20	15	– 976.53	– 1585.1
	n = 4		16	11	233	190.35
	n = 3		12	7	576.41	790.35
	n = 2		8	3	1033.44	790.35
	n = 1		4	–	1951.14	3171.45
Geomaterials parameters	n = 14	a1 = 5 m; a2 = 5 m; b = 1	–	–	0	0
	n = 13	m; H = 90 m; $\Delta x = 10$	–	–	91.91	91.50
	n = 12	m; $\psi f = 65^\circ$; $\psi d = 60^\circ$;	–	–	91.91	91.50
	n = 11	$\phi p = \phi c = \phi d = 30^\circ$;	2	10	91.91	91.50
	n = 10	$\psi p = 45^\circ$; $\psi p = 35^\circ$;	8	16	– 213.4	– 212.4
	n = 9	$\gamma_{roc} = 26 \text{ kN/m}^3$	14	22	$\gamma_{water} = 10 \text{ kN/m}^3$	– 310.50
	n = 8		18	26	– 311.67	– 315.40
	n = 7		24	27	– 316.77	– 315.40
	n = 6	Wn = 500 kN/m, 50%	28	23	– 824.6	– 823.5
	n = 5	Wet Slope: y1 = 10 m;	24	19	– 849.20	– 848.00
	n = 4	y2 = 5 m; Supported, T	20	15	– 889.93	– 889.30
	n = 3	$\psi T = 30^\circ$, T = 100 kN;	16	11	319.60	319.50
	n = 2		12	7	663.01	662.00
	n = 1		8	3	1120.04	1120.00
	n = 1		4	–	2037.74	2037.50
	n = 1		4	–	86.60	86.60

Results and discussions

In a case where it is assumed that there are no external forces acting on the jointed rock slope subjected slide head toppling, the resulting distributions of normal and shear forces are high shown in Fig. 9 compared to the cases (Figs. 10 and 11) where it assumed that there are some external forces are acting on the jointed rock slope subjected slide head toppling. The highest Distribution of normal (R) and shear (S) forces on the bases of blocks without any external influence under limit equilibrium methods are 4600 kN on rock column 8 and 4200 kN on rock column 2 respectively. The Normal forces are above the shear forces throughout the slope. Since the normal forces are not equal to the shear forces on the blocks near the toe of the slope, entails unstable slope.

There may be environmental settings where there are external forces acting on the slope and there is need to examine their effect on slope stability. In this article we examined external forces that include groundwater forces acting on the sides and bases of the

blocks. Another external force that has been considered in this article are rock anchors that are secured in stable ground beneath the toppling mass and then tensioned against the face.

A feature of the proposed limit equilibrium analysis is that any number of external forces can be added to the analysis, provided that their magnitude, direction and point of application are known. Considering Fig. 5a-c which shows a portion of a toppling slope in which there is a sloping water table. The forces acting on block n include the force Q inclined at an angle below the horizontal, and three water forces V_1 , V_2 and V_3 , as well as the forces P_n and P_{n-1} produced by the blocks above and below. Assuming that the blocks are in a state of limiting equilibrium, the force just sufficient to prevent sliding/toppling of block n have resolved. The detrimental effects of the presence of groundwater in the jointed rock on the distributions of forces were investigated. It is evident (see Fig. 10) that the presence of groundwater diminishes the distributions of the normal and shear forces on the bases of the blocks. The highest Distribution of normal (R) and shear (S) forces on the bases of blocks with groundwater as external influence under limit equilibrium methods are 2000 kN on rock column 2 and 21000 kN on rock column 2 respectively. Figure 10 indicates that the presence of groundwater along joints widens the distribution gap between the normal and shear forces thereby increasing the forces that increase the potential failure of the block. It is highly thought that the presence of Water pressure from existence of groundwater pressure in tension cracks or similar near vertical fissures reduces stability of the slope by diminishing the shear strength of potential failure surfaces as shown in Fig. 10 above. Water pressure in tension cracks or similar near vertical fissures reduces stability by increasing the forces that induce sliding.

In the case where there are changes in groundwater levels of some rock formation, particularly shales, can cause accelerated weathering and a decrease in shear strength would lead in slope instability.

In examining rock or soil slopes, it may be a mistake to assume that ground water is not present if no seepage appears on the slope face. The seepage rate may be lower

than the evaporation rate, and hence the slope surface may appear dry and yet there may be water at significant pressure within the rock mass. It is water pressure, and not rate of flow, which is responsible for instability in slopes and it is essential that measurement or calculation of this water pressure forms part of site investigations for stability studies.

Another parametric study involved studying the influence of stabilizing the lowermost block of the jointed rock slope subjected to slide head toppling. Stabilization of the lowermost blocks provide a resisting forces between joints. It lowers the distribution gap between the normal and shear forces thereby increasing the cohesion forces that reduces the potential failure of the block. The distribution of normal and shear forces on the bases of blocks are nearly the same as shown in Fig. 11. This would provide the forces necessary to overcome the frictional resistance at the block bases thereby stabilizing the slope.

Concluding remarks

The analytical solutions that incorporates the kinematic mechanisms of the jointed rock slope with counter-tilted discontinuity subjected to slide head toppling are derived based on the limit equilibrium. Then, the parametric studies were conducted to investigate the potential effects of the groundwater and stabilizing influence on the distribution of the normal and shear forces of the jointed rock slopes subjected to slide head toppling failure mode. The influence of groundwater and stabilizations from the lowermost blocks of the slope which have been overlooked in the available literature. These have since been studied and incorporated in this article. The study results indicates that groundwater lowers distributions of the normal and shear forces thereby inducing slope failure of the jointed rock slope subjected to slide- head toppling. Stabilization from the lowermost block of the slope, reduces the gap between the normal and shear forces thereby enhancing the stability of the slope.

Lastly, a slide head toppling case at Nchanga Zambia was investigated in order to illustrate the validity and effectiveness of these presented analytical solutions. The study results depict that the presented analytical solutions can provide an accurate and efficient way for stability analyses of the jointed rock slope subjected to slide head toppling failure mechanisms considering groundwater and stabilization effects.

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Authors' contributions

VMB collected literature data and drafted the manuscript. WG developed the analytical model. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

Appendix 1- Limiting Equilibrium for Toppling of the nth Block

To find the magnitude of P_n , we set the moment about the pivot point 0 to zero (refer Fig. 4.) Generally, we assume that the interfaces' friction angle of the rock columns denoted (ϕ_d) is equal to the friction angle on the bases (ϕ_p, ϕ_c), thus $\phi_d = \phi_p = \phi_c = \phi$. Therefore, before counter-tilting of the weak plane along base plane ψ_p with respect to the horizontal, we have

$$\sum M_{0\oplus} = 0 \quad (40)$$

$$= \frac{y_n}{2} W_n \sin \psi_p - \frac{\Delta x}{2} W_n \cos \psi_p + M_n P_n + 1 - \Delta x (P_n + 1 \tan \phi) - L_n P_n \quad (41)$$

$$= \frac{W_n}{2} (y_n \sin \psi_p - \Delta x \cos \psi_p) + P_n + 1 (M_n - \Delta x \tan \phi) - L_n P_n. \quad (42)$$

This can be rewritten as;

$$L_n P_n = \frac{W_n}{2} (y_n \sin \psi_p - \Delta x \cos \psi_p) + P_n + 1 (M_n - \Delta x \tan \phi). \quad (43)$$

Therefore, we get the following equation for P_n

$$P_{n,t} = \frac{\frac{W_n}{2} (y_n \sin \psi_p - \Delta x \cos \psi_p) + P_n + 1 (M_n - \Delta x \tan \phi)}{L_n}. \quad (44)$$

Appendix 1- Limiting Equilibrium for Sliding of the nth Block

To determine the magnitude of P_n due to counter-tilting of the weak plane along base plane ψ_c with respect to the horizontal, we set the moment about the pivot point 0 to zero. We have;

$$\sum M_{0\oplus} = 0 \quad (45)$$

$$= \frac{y_n}{2} W_n \sin \psi_c - \frac{\Delta x}{2} W_n \cos \psi_c + M_n P_n + 1 - \Delta x (P_n + 1 \tan \phi) - L_n P_n \quad (46)$$

$$= \frac{W_n}{2} (y_n \sin \psi_c - \Delta x \cos \psi_c) + P_n + 1 (M_n - \Delta x \tan \phi) - L_n P_n \quad (47)$$

This can be rewritten as;

$$L_n P_n = \frac{W_n}{2} (y_n \sin \psi_c - \Delta x \cos \psi_c) + P_n + 1 (M_n - \Delta x \tan \phi).$$

Therefore, we get the following equation for $P_{n,t}$ due to counter-tilting of the weak plane

$$P_{n,t} = \frac{\frac{W_n}{2} (y_n \sin \psi_c - \Delta x \cos \psi_c) + P_n + 1 (M_n - \Delta x \tan \phi)}{L_n}. \quad (48)$$

Appendix 2- Limiting Equilibrium for Sliding of the nth Block

To determine the magnitude of P_n , we set the sum of the forces in both the horizontal and vertical directions to zero (taking the horizontal axis to be along the base plane, at the angles ψ_p or ψ_c with respect to horizontal refer Fig. 4 c in the manuscript).

Generally, we assume that the interfaces' friction angle of the rock columns denoted (ϕ_d) is equal to the friction angle on the bases (ϕ_p, ϕ_c), thus $\phi_d = \phi_p = \phi_c = \phi$. Therefore, before counter-tilting of the weak plane along base plane ψ_p with respect to the horizontal, we have.

$$\sum F_x = 0 = P_n - P_{n+1} - W_n \sin \psi_p + S_n \quad (49)$$

$$\sum F_y = 0 = R_n + P_n \tan \phi - W_n \cos \psi_p - P_{n+1} \tan \phi. \quad (50)$$

where S_n is the shear force along the Column-base contact. From the Mohr–Coulomb criterion, we have

$$\tau_n = c + \sigma_n \tan \phi \quad (51)$$

where; τ_n Shear stress along the column-base contact σ_n Normal stress at the column-base contact.

Thereafter we assumed cohesion (c) is equal to zero and divide both sides of the equation by the column-base contact area denoted by A . Thus, we get

$$S_n = R_n \tan \phi \quad (52)$$

where; R_n normal force across the column-base contacts ($W_n \cos \psi_p$).

By solving the F_y equation for R_n and substituting into the F_x equation before counter-tilting of the weak plane along base plane ψ_p with respect to the horizontal, we have,

$$P_n - P_{n+1} - W_n \sin \psi_p + W_n \cos \psi_p \tan \phi - (P_n - P_{n+1}) \tan^2 \phi = 0 \quad (53)$$

$$(P_n - P_{n+1})(1 - \tan^2 \phi) - W_n \sin \psi_p + W_n \cos \psi_p \tan \phi = 0. \quad (54)$$

This leads;

$$(P_n - P_{n+1})(1 - \tan^2 \phi) = W_n \cos \psi_p (\tan \psi_p - \tan \phi) \quad (55)$$

$$(P_n - P_{n+1}) = \frac{W_n \cos \psi_p (\tan \psi_p - \tan \phi)}{1 - \tan^2 \phi}. \quad (56)$$

Therefore, we get the following equation for P_n ,

$$P_n = P_{n+1} + \frac{W_n \cos \psi_p (\tan \psi_p - \tan \phi)}{1 - \tan^2 \phi}. \quad (57)$$

However, after counter-tilting of the weak plane along base plane ψ_c with respect to the horizontal, we have.

$$\sum F_x = 0 = P_n - P_{n+1} - W_n \sin \psi_c + S_n \quad (58)$$

$$\sum F_y = 0 = R_n + P_n \tan \phi - W_n \cos \psi_c - P_{n+1} \tan \phi. \quad (59)$$

By solving the F_y equation for R_n and substituting into the F_x equation after counter-tilting of the weak plane along base plane ψ_c with respect to the horizontal, we have,

$$P_n - P_{n+1} - W_n \sin \psi_c + W_n \cos \psi_c \tan \phi - (P_n - P_{n+1}) \tan^2 \phi = 0. \quad (60)$$

$$(P_n - P_{n+1})(1 - \tan^2 \phi) - W_n \sin \psi_c + W_n \cos \psi_c \tan \phi = 0. \quad (61)$$

This leads;

$$(P_n - P_{n+1})(1 - \tan^2 \phi) = W_n \cos \psi_c (\tan \psi_c - \tan \phi) \quad (62)$$

$$(P_n - P_{n+1}) = \frac{W_n \cos \psi_c (\tan \psi_c - \tan \phi)}{1 - \tan^2 \phi} \quad (63)$$

Therefore, we get the following equation for P_{ns} .

$$P_{n,s} = P_{n+1} + \frac{W_n \cos \psi_c (\tan \psi_c - \tan \phi)}{1 - \tan^2 \phi}. \quad (64)$$

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