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d-Neighborhood system and generalized F-contraction in dislocated metric space

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Abstract
This paper, gives an answer for the Question 1.1 posed by Hitzler (Generalized metrics and topology in logic programming semantics, 2001) by means of "Topological aspects of d-metric space with d-neighborhood system". We have investigated the topological aspects of a d-neighborhood system obtained from dislocated metric space (simply *d*-metric space) which has got useful applications in the semantic analysis of logic programming. Further more we have generalized the notion of F-contraction in the view of d-metric spaces and investigated the uniqueness of fixed point and coincidence point of such mappings.

Keywords: *d*-Metric space, *d*-Neighborhood system, $\mathcal{GF}$-Contraction.

Mathematics Subject Classification: 47H10, 54H25

Background

Metrics appear everywhere in Mathematics: Geometry, Probability, statistics, coding theory, graph theory, pattern recognition, networks, computer graphics, molecular biology, theory of information and computer semantics are some of the fields in which metrics and/or their cousins play a significant role. The notion of metric spaces introduced by Frechet (1906), is one of the helpful topic in Analysis. Banach (1922) proved a fixed point theorem for contraction mapping in a complete metric space. The Banach contraction theorem is one of the primary result of functional analysis. After Banach contraction theorem, huge number of fixed point theorems have been established by various authors and they made different generalizations of this theorem.

Matthews (1985) generalized Banach contraction mapping theorem in dislocated metric space. Hitzler (2001) introduce the notion of dislocated metric (d-metric) space and presented variants of Banach contraction principle for various modified forms of a metric space including dislocated metric space and applied them to semantic analysis of logic programs. Hitzler (2001) has applied fixed point theorems for self maps on dislocated metric spaces, quasi dislocated metric spaces, generalized ultra metric spaces in his thesis “Generalized Metrics and Topology in Logic Programming Semantics”. In this context, Hitzler raised some related questions on the topological aspects of dislocated metrics.

Recently, Sarma and Kumari (2012) initiated the concept of d-balls and established topological properties on d-metric space. In the context of d-metric space, many papers have been published concerning fixed point, coincidence point and common fixed point theorems satisfying certain contractive conditions in dislocated metric space (see Karapinar and Salimi 2013; Kumari et al. 2012a, b; Zoto et al. 2014; Ahamad et al. 2013; Ren et al. 2013) which become an interesting topic in nowadays.

Of late several weaker forms of metric are extensively used in various fields such as programming languages, qualitative domain theory and so on.

Motivated by above, we give an answer for the Question 1.1 posed by Hitzler, further more we discuss some topological properties in d-neighborhood system obtained from dislocated metric space. Moreover, we generalize the notion of F-contraction initiated by Wardowski (2012) and we prove fixed point theorem. Our established results generalize similar results in the framework of dislocated metric space. Further more, we provide coincidence theorem in the setting of d-neighborhood systems.

Preliminaries and notations

First, we collect some fundamental definitions, notions and basic results which are used throughout this section. For more details, the reader can refer to Hitzler (2001).

Definition 2.1 Let X be a set. A relation $\lhd \subseteq X \times \mathcal{P}(X)$ is called a *d-membership relation* (on X) if it satisfies the following property for all $x \in X$ and $A, B \subseteq X : x \lhd A$ and $A \subseteq B$ implies $x \lhd B$.

Definition 2.2 Let X be a set, let $\lhd$ be a d-membership relation on X and let $\mathcal{U}_x \neq \phi$ be a collection of subsets of X for each $x \in X$. We call $(\mathcal{U}_x, \lhd)$ a *d-neighborhood system* for x if it satisfies the following conditions.

(i) If $U \in \mathcal{U}_x$, then $x \lhd U$
(ii) If $U, V \in \mathcal{U}_x$, then $U \cap V \in \mathcal{U}_x$
(iii) If $U \in \mathcal{U}_x$, then there is a $V \subseteq U$ with $V \in \mathcal{U}_x$ such that for all $y \lhd V$ we have $U \in \mathcal{U}_y$
(iv) If $U \in \mathcal{U}_x$, and $U \subseteq V$, then $V \in \mathcal{U}_x$.

Each $U \in \mathcal{U}_x$ is called a d-neighborhood of x. Finally, let X be a set and $\lhd$ be a d-membership relation on X and, for each $x \in X$, let $(\mathcal{U}_x, \lhd)$ be a d-neighborhood system for x. Then $(X, \mathcal{U}, \lhd)$ (*or simply* X) is called a d-topological space, where $\mathcal{U} = \{\mathcal{U}_x / x \in X\}$.

Proposition 2.3 *Let X be a nonempty set. A distance on X is a map* $d : X \times X \to [0, \infty)$. *A pair* (X, d) *is known as dislocated metric space* (Simply d-metric space) *if d satisfies the following conditions*

(d_1) $d(x, y) = 0 \Longrightarrow x = y$
(d_2) $d(x, y) = d(y, x)$
(d_3) $d(x, z) \leq d(x, y) + d(y, z)$ *for all* x, y, z *in* X

If $x \in X$ and $\epsilon > 0$, the set $\mathcal{B}_\epsilon(x) = \{y / y \in X$ and $d(x, y) < \epsilon\}$ is called the ball with center at x and radius ϵ.

Proposition 2.4 *Let (X, ϱ) be a d-metric space. Define the d-membership relation $\lhd$ as the relation $\{(x, A)/$ there exists $\epsilon > 0$ for which $\mathcal{B}_\epsilon(x) \subseteq A\}$. For each $x \in X$, let $\mathcal{U}_x$ be the collection of all subsets A* of *X such that $x \lhd A$. Then $(\mathcal{U}_x, \lhd)$ is a d-neighborhood system for x; for each $x \in X$.*

Definition 2.5 Let $(X, \mathcal{U}, \lhd)$ be a d-topological spaces and let $x \in X$. A net (x_γ) d-converges to $x \in X$ if for each d-neighborhood U of x we have that x_γ is eventually in U, that is, there exists some γ_0 such that $x_\gamma \in U$ for each $\gamma > \gamma_0$.

Definition 2.6 Let (X, ϱ) be a d-metric space and let $(X, \mathcal{U}, \lhd)$ be a d-topological spaces as in Proposition 2.4. Let (x_n) be a sequence in X. Then (x_n) converges in (X, ϱ) if and only if (x_n) d-converges in $(X, \mathcal{U}, \lhd)$.

Definition 2.7 Let X and Y be d-topological spaces and let $f : X \to Y$ be a function. Then f is d-continuous at $x_0 \in X$ if for each d-neighborhood V of $f(x_0)$ in Y there is a d-neighborhood U of x_0 in X such that $f(U) \subseteq V$. We Say f is *d-continuous* on X if f is *d-continuous* at each $x_0 \in X$.

Theorem 2.8 *Let X and Y be d-topological spaces and let $f : X \to Y$ be a function. Then f is a d-continuous if and only if for each net (x_γ) in X which d-converges to some $x_0 \in X$,$(f(x_\gamma))$ is a net in Y which d-converges to $f(x_0) \in Y$.*

Proposition 2.9 *Let (X, ϱ) and (Y, ϱ') be d-metric spaces, let $f : X \to Y$ be a function and let $(X, \mathcal{U}, \lhd)$ and $(Y, \mathcal{V}, \lhd')$ be the d-topological spaces obtained from (X, ϱ), respectively (Y, ϱ') as in Proposition 2.4. Then f* is *d-continuous at $x_0 \in X$ if and only if for each $\epsilon > 0$ there exists a $\delta > 0$ such that $f(\mathcal{B}_\delta(x_0)) \subseteq \mathcal{B}_\epsilon(f(x_0))$.*

Definition 2.10 Let (X, ϱ) be a d-metric space, let $f : X \to X$ be a contraction with contractivity factor γ and let $(X, \mathcal{U}, \lhd)$ be the d- topological space obtained from d-metric (X, ϱ) as in Proposition 2.4. Then f is d-continuous.

Topological aspects of *d*-metric space with *d*-neighborhood system

The following question was put forth in Hitzler Thesis.

(Question 1.1). Question: Is there a reasonable notion of d-open set corresponding to the notions of d-neighborhood, d-convergence and d-continuity.

We provided an answer for the above open question by constructing below theorems.

Theorem 3.1 *Let $(X, \mathcal{U}, \lhd)$ be a d-topological space. Define $\mathfrak{J} = \{V/$ for each $x \in V$ there exists $A \in \mathcal{U}_x$ such that $A \subset V\}$. Then $\mathfrak{J}$ is a topology on X.*

Proof Clearly $\mathfrak{J}$ contains X and $\emptyset$.

Let $\{V_\alpha\}$ be an indexed family of non-empty elements of $\mathfrak{J}$.

Let $x \in \cup V_\alpha$ which implies that $x \in V_\alpha$ for some α.

Thus there exists $A \in \mathcal{U}_x$ such that $A \subset V_\alpha \subset \cup V_\alpha$. Which implies that $\cup V_\alpha \in \mathfrak{J}$.

Let $\{V_{\alpha_i}\}_{i=1}^n$ be any finite intersection of elements of $\mathfrak{J}$.

We have to prove that $\cap_{i=1}^{n} V_{\alpha_i} \in \mathfrak{J}$. To obtain this, first we prove that if $G_1, G_2 \in \mathfrak{J}$ then $G_1 \cap G_2 \in \mathfrak{J}$. Let $x \in G_1 \cap G_2$.

Which implies that $x \in G_1$ and $x \in G_2$ then there exists $A_1 \in \mathcal{U}_x$ such that $A_1 \subset G_1$ and there exists $A_2 \in \mathcal{U}_x$ such that $A_2 \subset G_2$.

Which implies $A_1 \cap A_2 \in \mathcal{U}_x$ and $A_1 \cap A_2 \subset G_1 \cap G_2$.

Thus $G_1 \cap G_2 \in \mathfrak{J}$. Hence by induction, we get $\cap_{i=1}^{n} V_{\alpha_i} \in \mathfrak{J}$. □

Definition 3.2 Let $(X, \mathcal{U}, \lhd)$ be a d-topological space and $A \subseteq X$ be a *d-open* if for every $x \in A$ there exists $\mathcal{U} \in \mathcal{U}_x \ni \mathcal{U} \subset A$.

Definition 3.3 Let $(X, \mathcal{U}, \lhd)$ be a d-topological space and $A \subseteq X$ is *d-open* then A^c is *d-closed*.

Definition 3.4 Let $(X, \mathcal{U}, \lhd)$ be a d-topological space and $A \subseteq X$. A point x in A is called an *interior point* of A if $x \lhd A$.

Remark Interior point of A is an *open set*.

Definition 3.5 Let $(X, \mathcal{U}, \lhd)$ be a d-topological space and $A \subseteq X$. A point x in X is said to be *limit point* of A if for every $U \in \mathcal{U}_x$ there exist $y \neq x$ in A such that $y \lhd U$.

Definition 3.6 Let (X, d) be a d-metric space and $f : X \to X$. If there is a number $0 < \alpha < 1$ such that $d(f(x), f(y)) \leq \alpha d(x, y) \forall x, y \in X$ then f is called a *contraction*.

Definition 3.7 (Sarma and Kumari 2012) Let (X, d) be a d-metric space and $f : X \to X$ be a mapping. Write $V(x) = d(x, f(x))$ and $Z(f) = \{x / V(x) = 0\}$. We call points of $Z(f)$ as *coincidence point* of f. Clearly every point of $Z(f)$ is a fixed point of f but the converse is not necessarily true.

Theorem 3.8 *A subset $F \subseteq X$ is said to be d-closed iff a net (x_γ) in F d-converges to x then $x \in F$.*

Proof Suppose $F \subseteq X$ is d-closed.

Let (x_γ) be a net in F such that $\lim d(x_\gamma, x) = 0$.

We shall prove that $x \in F$.

Let us suppose $x \notin F$ which implies that $x \in X - F$, which is open.

Thus there exists $A \in \mathcal{U}_x$ such that $A \subset X - F$.

As $A \in \mathcal{U}_x$ there exists $\epsilon > 0$ such that $\mathcal{B}_\epsilon(x) \subset A$.

Since $\lim d(x_\gamma, x) = 0$ there exists γ_0 such that $d(x_\gamma, x) < \epsilon$ for $\gamma \geq \gamma_0$.

Hence $x_\gamma \in \mathcal{B}_\epsilon(x) \subset A \subset X - F$. A contradiction.

It follows that $x \in F$.

Conversely, assume that if a net (x_γ) in F *d-converges* to x then $x \in F$.

We shall prove that $F \subseteq X$ is d-closed.

i.e $X - F$ is d-open.

For this we have to prove that for every $x \in X - F$ there exists $A \in \mathcal{U}_x$ such that $A \subseteq X - F$.

Suppose for some $x \in X - F$ there exists $A \in \mathcal{U}_x$ such that $A \nsubseteq X - F$.
Let $x_A \in A - (X - F)$.
As $\mathcal{U}_x$ is a direct set under set inclusion $A \leq B$ *if* $B \subseteq A$.
Thus $\{x_A/A \in \mathcal{U}_x\}$ is a net.
Let $\epsilon > 0,\ A_0 = \mathcal{B}_\epsilon(x) \in \mathcal{U}_x$.
If $A \geq A_0$, $A \subseteq A_0$,
Thus $x_A \in A_0$ implies that $d(x_A, x) < \epsilon$.
It follows that $\lim d(x_A, x) = 0$.
Which implies that $x \in F$. A Contradiction.
So for all $x \in X - F$ there exists $A \in \mathcal{U}_x$ such that $A \subseteq X - F$.
Which completes the proof. □

Remark For each $\delta > 0, \mathcal{B}_\delta(x)$ is a d-neighborhood of x.

Theorem 3.9 *Let $(X, \mathcal{U}, \lhd)$ be a d-topological space and let $\mathcal{U}_x$ be the collection of all subsets U of X such that $x \lhd U$. Then $\mathcal{U}_x$ is said to be a basis for a topology on X if*

(i) *For each $x \in X$, there exists $U \in \mathcal{U}_x$ such that $x \lhd U$.*
(ii) *If $x \lhd U_1 \cap U_2$ there exists $U_3 \in \mathcal{U}_x$ such that $x \lhd U_3$ and $U_3 \subseteq U_1 \cap U_2$.*

Proof (i) is clear.
Since $x \lhd U_1 \cap U_2$ implies $U_1 \cap U_2 \in \mathcal{U}_x$.
So there exists $\epsilon > 0$ such that $\mathcal{B}_\epsilon(x) \subset U_1 \cap U_2$.
Since balls are d-neighborhood, choose $U_3 = \mathcal{B}_{\frac{\epsilon}{2}}(x) \in \mathcal{U}_x$.
Then $x \lhd U_3$ and $\mathcal{B}_{\frac{\epsilon}{2}}(x) \subset U_1 \cap U_2$. □

Lemma 3.10 *Let X be any set and $\mathcal{B}, \mathcal{B}'$ be basis for the topologies $\mathfrak{J}$ and $\mathfrak{J}'$ respectively. Then the following are equivalent.*

(i) *$\mathfrak{J}'$ finer than $\mathfrak{J}$ $(\mathfrak{J} \subseteq \mathfrak{J}')$*
(ii) *For each $x \in X$ and each basis element $B \in \mathcal{B}$ with $x \in B$ there exists a basis element $B' \in \mathcal{B}'$ such that $x \in B'$ and $B' \subseteq B$.*

Theorem 3.11 *Let $(X, d, \mathfrak{J})$ be the topology induced from the d-topological space $(X, \mathcal{U}, \lhd)$ obtained from d-metric as in Proposition 2.4, $\mathfrak{J}_d$ be the topology induced by the d-metric then $\mathfrak{J} = \mathfrak{J}_d$.*

Proof Let $\mathcal{V}_\epsilon(x) = \mathcal{B}_\epsilon(x) \cup \{x\}$. Then the collection $\mathfrak{B} = \{\mathcal{V}_\epsilon(x)/x \in X\}$ is a basis for $\mathfrak{J}_d$, and $\mathcal{U}_x = \{U \subset X/x \lhd U\}$ is a basis for $\mathfrak{J}$. Clearly $\mathfrak{J}_d \subset \mathfrak{J}$, since $\mathcal{V}_\epsilon(x)$ is a d-neighborhood.
Let $x \in X$ and $U \in \mathcal{U}_x$ such that $x \in U$.
Since $x \lhd U$ there exists $\epsilon > 0$ such that $\mathcal{B}_\epsilon(x) \subseteq U$.
Which implies $\{x\} \cup \mathcal{B}_\epsilon(x) \subseteq U$.
So $\mathcal{V}_\epsilon(x) \subseteq U$. Hence $\mathfrak{J} \subset \mathfrak{J}_d$. □

Theorem 3.12 *Let $(X, \mathcal{U}, \lhd)$ be a d-topological space and $A \subseteq X$ and $x \in X$ the following are equivalent, assume $\mathcal{B}_\epsilon(x) \neq \phi$ for every $\epsilon > 0$.*

(1) *There exists $(x_n) \in A$ such that $\lim d(x_n, x) = 0$*
(2) *For every $U \in \mathcal{U}_x$ there exists $y \neq x$ in A such that $y \lhd U$.*

Proof Let $U \in \mathcal{U}_x$ there exists $\epsilon > 0$ such that $\mathcal{B}_\epsilon(x) \subseteq U$.
Since (1) holds, $\lim d(x_n, x) = 0$.
Which implies that, there exists N such that $d(x_n, x) < \epsilon \forall n \geq N$.
Let $y = x_N$ and $r = \epsilon - d(x_N, x)$ then $\mathcal{B}_r(y) \subset \mathcal{B}_\epsilon(x) \subset U$.
It follows that $\mathcal{B}_r(y) \subset U$.
So $y \lhd U$. Hence (2) holds.
Assume that (2) holds. Let $U = \mathcal{B}_{\frac{1}{n}}(x)$, there exists $x_n \neq x$ in A such that $x_n \lhd U = \mathcal{B}_{\frac{1}{n}}(x)$.
i.e there exists $\epsilon_n < \frac{1}{n}$ such that $\mathcal{B}_{\epsilon_n}(x_n) \subset \mathcal{B}_{\frac{1}{n}}(x)$.
Let $y_n \in \mathcal{B}_{\epsilon_n}(x_n)$.
Which implies that $d(x_n, y_n) < \frac{1}{n}$ and $d(x, y_n) < \frac{1}{n}$.
Hence $d(x_n, x) < d(x_n, y_n) + d(y_n, x) < \frac{2}{n}$.
Which yields $\lim d(x_n, x) = 0$. Hence (1) holds. □

Theorem 3.13 *Let $(X, \mathcal{U}, \lhd)$ be the d-topological space obtained from d-metric (X, ϱ) as in Proposition 2.4 .Then balls are d-open.*

Proof Let $\mathcal{B}_\epsilon(x)$ be a ball with center at x and radius ϵ.
It sufficies to prove that $\mathcal{B}_\epsilon(x)$ is d-open.
i.e we shall prove for every $y \in \mathcal{B}_\epsilon(x)$ there exists $U \in \mathcal{U}_y$ such that $U \subset \mathcal{B}_\epsilon(x)$.
Since $y \in \mathcal{B}_\epsilon(x)$ implies $d(x, y) < \epsilon$.
Choose $\delta = \epsilon - d(x, y)$.
As $\mathcal{B}_\delta(y)$ is a d-neighborhood, now let $U = \mathcal{B}_\delta(y)$.
So it is sufficient to prove that $U \subset \mathcal{B}_\epsilon(x)$.
Let $z \in U$.
This implies that $d(y, z) < \delta < \epsilon - d(x, y)$.
Then $d(x, z) < \epsilon$.
It follows that $z \in \mathcal{B}_\epsilon(x)$.
Hence $U \subset \mathcal{B}_\epsilon(x)$. □

Theorem 3.14 *Let $(X, \mathcal{U}, \lhd)$ be a d-topological space obtained from d-metric (X, ϱ) as in Proposition 2.4. Then $(X, d, \mathfrak{J})$ is a Haussdorff space.*

Proof Suppose $x \neq y \Rightarrow d(x, y) > 0$.
Let us choose $\delta = d(x, y)$.
Let $\mathcal{B}_{\frac{\delta}{2}}(x), \mathcal{B}_{\frac{\delta}{2}}(y)$ be the d-neighborhoods of x and y respectively.
It sufficies to prove $\mathcal{B}_{\frac{\delta}{2}}(x) \cap \mathcal{B}_{\frac{\delta}{2}}(y) = \phi$.
Let $z \in \mathcal{B}_{\frac{\delta}{2}}(x) \cap \mathcal{B}_{\frac{\delta}{2}}(y)$.
Which implies that $z \in \mathcal{B}_{\frac{\delta}{2}}(x)$ and $z \in \mathcal{B}_{\frac{\delta}{2}}(y)$.
So $d(x, z) < \frac{\delta}{2}$ and $d(y, z) < \frac{\delta}{2}$.

It follows $d(x,y) < \delta = d(x,y)$.
Which is a contradiction. □

Theorem 3.15 *Let $(X,\mathcal{U},\lessdot)$ be a d-topological space obtained form d-metric (X,ϱ) as in Proposition 2.4. Then singleton sets are d-closed in $(X,d,\mathfrak{J})$.*

Proof Let $x \in X$, we have to prove that $\{x\}$ is d-closed or it is sufficies to prove $X - \{x\}$ is d-open.
i.e for each $y \in X - \{x\}$ there exists $U \in \mathcal{U}_y$ such that $U \subseteq X - \{x\}$.
Since $y \neq x$, implies $d(x,y) > 0$.
Which yields $x \notin \mathcal{B}_\epsilon(y)$.
Thus, there is a d-neighborhood,$\mathcal{B}_\epsilon(y) \in \mathcal{U}_y$ such that $\mathcal{B}_\epsilon(y) \subseteq X - \{x\}$.
Hence $\{x\}$ is d-closed. □

Corollary 3.16 *Let $(X,\mathcal{U},\lessdot)$ be a d-topological space obtained form d-metric (X,ϱ). Then $(X,d,\mathfrak{J})$ is a T_1-space.*

Corollary 3.17 *Let $(X,d,\mathfrak{J})$ be a d-topological space. Then the collection $\{\mathcal{B}_\epsilon(x)/x \in X\}$ is an open base at x for X.*

Main theorems

Wardowski (2012) introduced a new type of contraction called F-contraction and proved a new fixed point theorem concerning F-contraction and supported by computational data illustrate the nature of F-contractions. In this section, we present a theorem which generalizes the Wardowski's theorem.

Definition 4.1 (Wardowski 2012) Let $F : R^+ \to R$ be a mapping satisfying,

(i) F is strictly increasing, i.e for all $\alpha, \beta \in R^+$ such that $\alpha < \beta, F(\alpha) < F(\beta)$
(ii) For each sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive numbers $lim_{n\to\infty}\alpha_n = 0$ iff $lim_{n\to\infty}F(\alpha_n) = -\infty$
(iii) There exists $k \in (0,1)$ such that $lim_{\alpha\to 0^+}\alpha^k F(\alpha) = 0$

A mapping $T : X \to X$ is said to be an F-contraction if there exists $\tau > 0$ such that for all $x, y \in X, d(Tx, Ty) > 0 \Rightarrow \tau + F(d(Tx, Ty)) \leq F(d(x,y))$.

Theorem 4.2 (Sgroi and Vetro 2013) *Let (X, d) be a complete metric space and let $T : X \to X$ be an F-contraction then T has a unique fixed point $x^* \in X$ and for every $x_0 \in X$ a sequence $\{T^n x_0\}_{n\in\mathbb{N}}$ is convergent to x^*.*

In the literature one can find some interesting papers concerning F-contractions; (see for example Cosentino and Vetro 2014; Sgroi and Vetro 2013; Secelean 2013; Paesano and Vetro 2014; Hussain and Salimi 2014).

Definition 4.3 By $\mathcal{G}$ we denote the set of all monotone decreasing real functions $g : [0,\infty) \to [0,\infty)$, such that $g(x) = 0$ iff $x = 0$ and $lim_{t\to 0^+} g(t) = 0$.

Lemma 4.4 *Let $g \in \mathcal{G}$ and $\{\epsilon_n\} \subseteq [0, \infty)$, then from $g(\epsilon_n) \to 0$ it follows that $\epsilon_n \to 0$.*

Proof Routine. □

Theorem 4.5 *Let (X, d) be a d-metric space, $x \in X$, $\{x_n\} \subseteq X$ and $g \in \mathcal{G}$ satisfying subadditive property. Define $d^* : X^2 \to [0, \infty)$ by $d^*(x, y) = g(d(x, y))$ for any $x, y \in X$. Then*

(1) *(X, d^*) is a d-metric space.*
(2) $\lim d(x_n, x) = 0$ *iff* $\lim d^*(x_n, x) = 0$.
(3) *(X, d) is complete iff (X, d^*) is complete.*

Proof Let $d^*(x, y) = 0$.
Which yields $g(d(x, y)) = 0$ implies $d(x, y) = 0$.
So $x = y$. $d^*(x, y) = d^*(y, x)$ follows from $g(d(x, y)) = g(d(y, x))$.
Now consider $d^*(x, z) = g(d(x, z))$
$\leq g(d(x, y) + d(y, z))$
$\leq g(d(x, y) + g(d(y, z))$ since g is subadditive.
$= d^*(x, y) + d^*(y, z)$.
It follows that $d^*(x, z) \leq d^*(x, y) + d^*(y, z)$.
Hence d^* is a d-metric. This completes the proof of (1).
Let $\lim d(x_n, x) = 0$. It follows that $\lim g d(x_n, x) = g(0) = 0$.
Which implies $\lim d^*(x_n, x) = 0$.
Suppose $\lim d^*(x_n, x) = 0$. By above lemma, it follows that $\lim d(x_n, x) = 0$. Which completes the proof of (2).
Let us suppose that (X, d) is complete. Thus for every $\epsilon > 0$ there exist $n_1 \in N$ such that $d(x_n, x_m) < \epsilon$ for all $m, n \geq n_1$.
Which yields $\lim d(x_n, x_m) = 0$.
Which implies $\lim d^*(x_n, x_m) = \lim g d(x_n, x_m) = g(0) = 0$; because g is continuous at 0. So $\{x_n\}$ is a Cauchy sequence in (X, d^*). By using (2), we get (X, d^*) is complete.
Conversely suppose that (X, d^*) is complete.
Let $\{x_n\}$ is a Cauchy sequence in (X, d^*).
Then for every $\epsilon > 0$ there exist $n_1 \in N$ such that $d^*(x_n, x_m) < \epsilon$ for all $m, n \geq n_1$.
Thus $\lim d^*(x_n, x_m) = 0$.
It follows that $\lim g d(x_n, x_m) = 0$.
By above Lemma, $\lim d(x_n, x_m) = 0$.
Which implies that $\{x_n\}$ is a Cauchy sequence in (X, d). By using (2) we conclude that (X, d) is complete. □

Definition 4.6 Let $F : R^+ \to R$ be a mapping satisfying,

(i) F is strictly increasing, i.e for all $\alpha, \beta \in R^+$ such that $\alpha < \beta$, $F(\alpha) < F(\beta)$.
(ii) For each sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive numbers $lim_{n\to\infty}\alpha_n = 0$ iff $lim_{n\to\infty}F(\alpha_n) = -\infty$
(iii) There exists $k \in (0, 1)$ such that $lim_{\alpha\to 0^+}\alpha^k F(\alpha) = 0$. A mapping $T : X \to X$ is said to be an $\mathcal{G}F$-contraction if there exists $\tau > 0$ such that for all $x, y \in X$, $g(d(Tx, Ty)) > 0 \Rightarrow \tau + F(g(d(Tx, Ty))) \leq F(g(d(x, y)))$.

Theorem 4.7 *Let (X, d) be a complete d-metric space and let $T : X \to X$ be an $\mathcal{G}F$-contraction. Then T has a unique fixed point.*

Proof Define $d^* : X^2 \to [0, \infty]$ by $d^*(x, y) = g(d(x, y))$ for any $x, y \in X$ and $g \in \mathcal{G}$. By lemma 4.4 and theorem 4.5 it follows that (X, d^*) is a d-metric space.

We have, when $d^*(Tx, Ty)) > 0$ implies $\tau + F(d^*(Tx, Ty)) \leq F(d^*(x, y))$.

Then by using same proof as in Theorem 4.2, we can conclude that T has a unique fixed point. □

Theorem 4.8 *Let (X, ρ) be a complete d-metric space and let $f : X \to X$ be a contraction and $(X, \mathcal{U}, \lhd)$ be the d-topological space obtained from (X, ϱ). Then f has a unique coincidence point for f.*

Proof Let $x_0 \in X$. Choose $x_{n+1} = f(x_n) = f^n(x_0)$.

Then $f^n(x_0)$ is a cauchy sequence and converges in (X, ϱ) to some point u.

i.e $u = \lim f^n(x_0)$. Since f is a contraction it is also d-continuous, by Proposition 2.10, $f(u) = lim f^{n+1}(x_0)$.

Hence $d(u,f(u)) = lim_{n\to\infty} d(f^n(x_0), f^{n+1}(x_0)) < lim_{n\to\infty} \alpha^n d(x_0, f(x_0)) = 0$. Since $0 < \alpha < 1$, $d(u,f(u)) = 0$.

Thus u is a coincidence point of f. □

Uniqueness

Let us suppose that v be the another coincidence point such that $d(v,f(v)) = 0$.

Thus $f(v) = v$ and $f(u) = u$. By using triangle inequality, $d(v, u) \leq d(v,f(v)) + d(f(v),f(u)) + d(f(u), u) \leq \alpha d(v, u)$ which implies $d(v, u) = 0$.

Hence $u = v$.

Authors' contributions
All authors participated in the design of this work and performed equally. All authors read and approved the final manuscript.

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Competing interests
The authors declare that they have no competing interests.

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