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Financial structure, cycle, and instability

Kenshiro Ninomiya*

*Correspondence:
kninomiya@rikkyo.ac.jp

Faculty of Economics, Rikkyo University, 3-34-1 Nishi-Ikebukuro, Toshima, Tokyo 171-8511, Japan

Abstract

The subprime loan mortgage crisis has revived scholarly interest in Minsky's financial instability hypothesis. The related mathematical models present two types of Minskian financial structures. We construct macrodynamic models that consider both structures and discuss financial instability and cycles. We also demonstrate that one of the financial cycles occurs when a real factor stabilizes the economy. The burden of interest-bearing debt is an important determinant of the cycle. We posit that the escalating financial fragility in this cycle is a more appropriate interpretation of the Minskian financial structure that refers to hedging, speculative and Ponzi behaviors. We further demonstrate that another financial structure destabilizes the economy. If the instability occurs at the point of fragility, then the economy may deteriorate into financial crisis. Fragility then becomes instability.

Keywords: Minskian financial structure, Financial fragility, Business cycle, Financial instability

JEL classification: E12, E32, E33, E43

1 Introduction

The financial instability hypothesis proposed by Hyman P. Minsky (1975, 1982a, 1986) has attracted renewed attention following the subprime loan mortgage crisis. Many authors, mainly post-Keynesian economists, employ two types of financial structures in their mathematical models.

Taylor and O'Connell (1985) formulated that lenders' liquidity preferences intensify with a decrease in the expected profit rate (ρ). They hypothesized that an increase in the expected profit rate (ρ) reduces the interest rate (i). They also asserted that a true Minsky crisis occurs when the value of derivatives (i_ρ) turns significantly negative. Kregel (1997) emphasized that the margins of safety proposed by Minsky (1982a) are significant for financial instability.[1] When an economic boom reduces lenders' risks, banks, including commercial ones, promote lending despite erosion in the margin of safety.

Ninomiya (2007, 2018) considered these factors in a Kaldorian business cycle model and discusses financial instability as a cycle. Ninomiya and Tokuda (2017) demonstrated that Japan's financial structure has been fragile since the mid-1990s by expanding upon the work of Taylor and O'Connell (1985) and applying VAR analysis. Ninomiya and

[1] Minsky (1982a), pp.65–66. See Nabeshima (2017) for more details.

Tokuda (2012) demonstrated that Korea's financial structure stabilized after the Asian monetary crisis.

On the contrary, Kregel (2008) does not regard the subprime mortgage crisis as traditional Minskian financial instability. Ninomiya and Tokuda (2021) demonstrated that the US financial structure stabilized before the crisis.[2] We identify financial structures similar to the aforementioned structures (Japan and Korea) as the lenders' risk type (LR).

Minsky emphasizes increasing financial fragility, which refers to hedging, speculation, and Ponzi finance. His financial instability hypothesis is an endogenous financial business cycle theory. Therefore, related mathematical models interpret enlargement in firms' debt burdens as the source of increasing financial fragility and introduce a dynamic equation for debt burden into Kaldorian business cycle, Goodwin, and Kaleckian models.[3]

Asada (2001) and Ninomiya (2015, 2018) developed a macrodynamic model that takes interest-bearing debt into account as a source of financial instability and cycles. However, Asada (2001) did not consider the LR financial structure. Furthermore, Asada (2001), Ninomiya (2015, 2018), and many other studies did not explicitly define the Minskian financial structure.

Nishi (2012b) and Ninomiya and Tokuda (2017) examined debt burden and financial instability by applying a VAR analysis.[4] Nishi (2021) focused on debt dynamics at the industrial level and discussed financial fragility, thereby separating the complete period (1960–2018) into two at 1998 to demonstrate fluctuations in the debt ratio over the business cycle for 1960–2018 in Japan.

Some studies explicitly consider the latter type of Minskian financial structure,[5] which we identify as the hedge, speculative and Ponzi type (HSP). Nishi (2012a) proposed a revised Minskian financial structure and introduces the burden of interest-bearing debt into a Kaleckian model. Although his definition of hedge finance differs from Foley's (2003), it promises to be widely accepted. Nonetheless, he focused on the long run without discussing financial cycles and assumes a constant interest rate. He did not consider an LR financial structure.[6]

Sasaki and Fujita (2014) consider dividends in a Kaleckian model and suggest that cyclical fluctuations can occur such that the financial structure of firms changes periodically between speculative finance and Ponzi finance. Since we do not consider dividends, we adopt Nishi's definition. Note also that Sasaki and Fujita (2014) also assume

[2] Kregel (2008) asserted that safety cushions had been insufficient from the beginning of the subprime loan crisis, and the crisis did not represent a conventional Minsky process. Ninomiya and Tokuda (2017) contended that their investigation may support this argument.

[3] See, for example, Keen (1995), Asada (2001, 2006), Ninomiya (2006), Hein (2007), Charles (2008a), Sasaki and Fujita (2014), Ninomiya (2018), and Asada et al. (2019).

[4] Nishi (2012a) investigated the dynamic relationships among income distribution, debt ratio, and capital accumulation in the Japanese economy. Ninomiya and Tokuda (2017) considered the instability of confidence and structural change.

[5] See Foley (2003), Lima and Meirelles (2007), Charles (2008b), and Sasaki and Fujita (2014).

[6] Some models depend on the endogenous money theory (horizontalist) of the post-Keynesian school, thereby assuming that the interest rate is constant. Controversy within the post-Keynesian school remains regarding the endogenous money supply theory. The horizontalists believe that the central bank supplies high-powered money completely passively, whereas the structuralists believe that there is a limited supply. Naito (2011) asserted that Minsky was primarily a structuralist.
Minsky (1982b) contended that an increase in the interest rate triggers recession from a euphoric boom. See Nabeshima (2017) for more details. Accordingly, we emphasize the LR financial structure formulated by Taylor and O'Connell (1985).

a constant interest rate and do not consider the LR financial structure. Although Ninomiya and Tokuda (2017) considered both the LR and HSP financial structures, they did not explicitly examine the HSP financial structure.

This paper constructs simple macrodynamic models, introduces two types of Minskian financial structures (LR and HPS) and discusses financial instability and cycles. Multiple studies include the LR or HSP financial structure in their models, whereas this study explicitly considers the HSP financial structure and focuses on the business cycle because the financial instability hypothesis is an endogenous theory of the business cycle. We present a numerical simulation of financial cycles and describe an HSP financial structure. One of the cycles indicates that the financial factor has a stabilizing role in the economy, although the financial regime becomes more fragile from hedge finance to speculative and Ponzi finance. By contrast, another cycle indicates that interest-bearing debt burden has a destabilizing role in the economy. Further, examining monetary and fiscal policy interventions is worthwhile for coping with financial instability. We emphasize the importance of considering both LR and HSP structures in dynamic systems.

The remainder of this paper is organized as follows. Section 2 introduces the two types of Minskian financial structures, thereby presenting a basic macrodynamic model in which the interest rate is constant, finally exploring financial instability and cycles, and considering only the HSP structure in the model. Section 3 presents extended models featuring an endogenous interest rate, which include both LR and HSP financial structures. Section 4 concludes.

2 Financial structures and basic dynamics

We first clarify LR and HSP Minskian financial structure. Real gross profit Π is defined as follows:

$$\Pi = Y^d - \frac{W}{p}N, \tag{1}$$

where Y^d is the demand side of goods, W is the nominal wage, p is the price level and N represents the level of employment. Following Asada (1995), we assume that disequilibrium in the goods market is compensated by inventory fluctuation and the demand side is always realized ($Y^d = Y$).

We also assume that the economy is oligopolistic and the price level p is decided by the mark-up principle as follows:

$$p = (1+\tau)\frac{WN}{Y}, \tag{2}$$

where τ is the mark-up rate. Therefore, real gross profit Π is:

$$\Pi = Y - \frac{W}{p}N = \frac{\tau}{1+\tau}Y = \theta Y, \tag{3}$$

where θ is the rate of profit sharing.

We assume that the real gross profit Π is distributed to firms, and an interest payment iD is distributed to rentiers. Firms retain their remaining profit as internal reserves V, obtained by

$$V = \Pi - iD = \theta Y - iD, \tag{4}$$

where i is the interest rate and D denotes firms' debt burdens.[7] We assume that all interest payments are saved.

Following Nishi (2012a), who formulated the HSP-type Minskian financial structure, we formalize the financial regimes as follows:

$$\Pi \geqq \dot{D} + iD \qquad (\text{hedge finance}), \tag{5}$$

$$\Pi \geqq iD \qquad (\text{speculative finance}), \tag{6}$$

$$\Pi < iD \qquad (\text{Ponzi finance}), \tag{7}$$

where $\dot{D}$ denotes the change in debt burden D. For example, hedge finance means that internal reserves $V(= \Pi - iD)$ exceed the increase in debt burden D. Ponzi finance means that a firm's gross profit (net operating revenue Π) cannot cover its interest payment iD.

Suppose that investment demand must be financed by adding debt if it is not financed via internal reserves. The dynamic equation expressing debt burden D becomes

$$\dot{D} = I - V = I - (\theta Y - iD). \tag{8}$$

The investment function I is defined as

$$I = g_1 Y - g_2 iD - g_0, \quad g_i > 0, \tag{9}$$

where g_1 represents animal spirits or appropriate investment opportunities. For example, a paucity of appropriate opportunities reduces g_1 even though income Y rises. $-g_2$ implies that a firm curtails investment demand because its debt burden rises.[8] $-g_0$ is a depreciation that indicates that I falls when Y is sufficiently small.

We, first, begin to examine a basic dynamic system and assume that the interest rate is constant in the system as follows:

[7] Minksy (1986) explained the difference between financial instability hypothesis and Kalecki (1971) with respect to profits as follow: "The financial instability hypothesis identifies profits, determined as Kelecki shows, as a cash flow that does or does not validate past financial commitments; it integrates Kalecki's vision of the dynamic determination of profits with the capitalist institutional fact of a liability structure inherited from the past that commits current and future profits" (p.118).

[8] Investment function (9) is based on Kaldorian business cycle models. Using the S-sharped investment function for income, Kaldor (1940) presented an endogenous business cycle. Meanwhile, Chang and Smyth (1971) used the Poincaré–Bendixson theorem to investigate the work of Kaldor (1940). Asada (1995) did not assume the S-shaped investment function and developed a Kaldorian business cycle model in an open economy.
However, investment function (9) depends on debt burden. This formulation is based on Adachi (1994) and Asada (1997, 2001). Adachi (1994) formulated the discounted present value of expected returns PV from investment as follows:
$PV = \frac{Q}{i+\sigma}$,
where Q is the average expected returns, i is the interest rate, and σ is the risk premium. Adachi (1994) assumed that the risk premium increases with an increase in debt burden and presented a simple optimization model that posits the dependence of investment on debt burden. Asada (1997, 2001) used an optimization model to extend the 'Penrose effect' (Uzawa 1969) to debt burden and show how investment depends on debt burden. Adachi (1994) and Asada (1997, 2001) based their study on the principle of increasing risk (Kalecki, 1937). Furthermore, Asada and Semmler (1995) and Nakamura (2002) discussed the investment based on Kalecki (1937).
The Cabinet Office (2006) empirically discussed the relationship between the investment and the interest-bearing debt burden during 1990–2004 in Japan. After the collapse of the bubble economy in 1991, the problem of non-performing loans had become more severe in Japan.

$$i = i_0. \tag{10}$$

By ordering (3)–(10), we obtain the following financial regimes:

$$D \geqq \frac{(g_1 - 2\theta)}{(g_2 - 2)i_0} Y - \frac{g_0}{(g_2 - 2)i_0} \qquad \text{(hedge finance)}, \tag{11}$$

$$D \leqq \frac{\theta}{i_0} Y \qquad \text{(speculative finance)}, \tag{12}$$

$$D > \frac{\theta}{i_0} Y \qquad \text{(Ponzi finance)}. \tag{13}$$

Figure 1 presents one of the regions in (D, Y) space to the different regimes. The boundary of hedge finance (11) depends on the signs of $g_1 - 2\theta$ and $g_2 - 2$. For example, the coefficient of Y is positive and the intercept on the Y axis is negative when $g_2 - 2 > 0$ and $g_1 - 2\theta > 0$. We assume that an economy is expanding, and both investment demand I and internal reserves V increase. $g_1 - 2\theta > 0$ indicates that growth in investment demand I exceeds that in internal reserves V; therefore, debt burden D increases. By contrast, $g_2 - 2 > 0$ indicates that the increasing burden of interest-bearing debt iD significantly induces the decline in investment demand I. These two effects lead to a decline in debt burden (Fig. 1-1).

Note that the region of speculative finance (2) in Fig. 1-1 satisfies the hedge finance condition. For example, the reduction in investment demand I prevents an increase in debt burden D. Meanwhile, a decrease in investment demand I induces the decrease in income Y, what we refer to as a recession. Therefore, the economy of the speculative finance (2) region may be as fragile as that of the speculative finance (1) region.

Conversely, the coefficient of Y is negative and the intercept on the Y axis is positive when $g_2 - 2 < 0$ and $g_1 - 2\theta > 0$. We again assume that the economy is expanding. $g_2 - 2 < 0$ indicates that the increase in the burden of interest-bearing debt iD induces the decline in investment demand I but not significantly. This effect leads to an increase in debt burden (Fig. 1-2).

The boundary between speculative and Ponzi finance depends on the parameter θ and the interest rate i_0. The region of Ponzi finance expands when θ falls or i_0 rises. The fall in θ reduces internal reserves, and the rise in i_0 enlarges the burden of interest-bearing debt. Therefore, firms' financial conditions deteriorate.

Note that the region of Ponzi (2) in Fig. 1-2 satisfies the condition of hedge finance through the reduction in debt burden D. Accordingly, the decline in investment demand I is highly significant in covering payment obligations iD. The economy of the Ponzi (2) region is as serious as that of the Ponzi (1) region.

We, next, formulate the basic dynamic system assuming that interest rate is constant. This means that we cannot consider the LR structure in the basic dynamic system.

Real wage income H_w is obtained from Eq. (3) as follows:

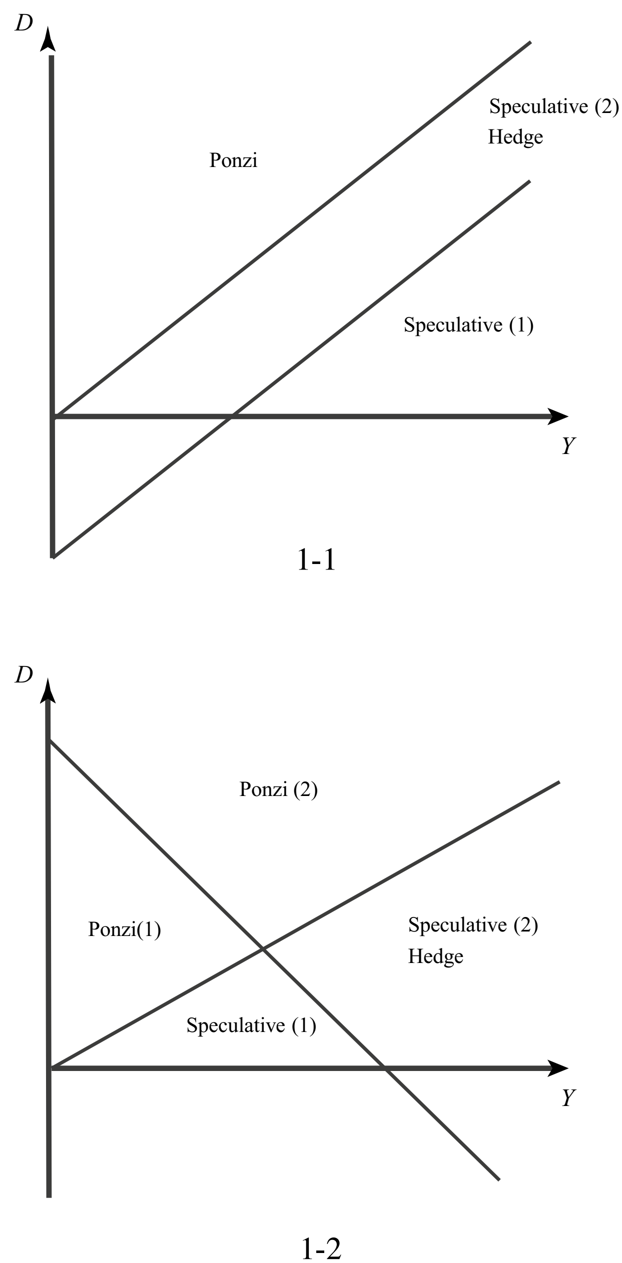

Fig. 1 Financial regimes

$$H_w = \frac{W}{p}N = \frac{1}{1+\tau}Y = (1-\theta)Y, \quad 0 < \theta < 1. \tag{14}$$

The income-tax function T is assumed as follows:

$$T = tY, \quad 0 < t < 1, \tag{15}$$

where t represents tax rate.[9]

The consumption function C is assumed to be a linear function of $H_w - T$:

$$C = c(H_w - T) + C_0 = c(1-\theta-t)Y + C_0, \quad 0 < c < 1, \quad C_0 > 0, \tag{16}$$

where c is the marginal propensity to consume and C_0 is basic consumption.

The dynamic equation for income Y is formulated as

$$\dot{Y} = \alpha(C + I + G - Y), \quad \alpha > 0. \tag{17}$$

Equation (17) describes the quantity adjustment in the goods market, and α is the speed of adjustment.

Ordering (8), (9), (10), (15), (16) and (17) obtains the following dynamic system (S_a.1):

$$\dot{Y} = \alpha[c(1-\theta-t)Y + C_0 + g_1 Y - g_2 i_0 D - g_0 + G - Y] \tag*{(S_a.1.1),}$$

$$\dot{D} = g_1 Y - g_2 i_0 D - g_0 - \theta Y + i_0 D \tag*{(S_a.1.2).}$$

We adopt the following assumption:

$$g_1 - s > 0, \tag*{(A.1)}$$

where $s = 1 - c(1-\theta-t)$. Assumption A.1 indicates that the real factor destabilizes the economy.[10] Kaldorian business cycle models employ a similar assumption.[11]

The loci of $\dot{Y} = 0$ and $\dot{D} = 0$ are as follows:

$$D_{(\dot{Y}=0)} = \frac{g_1 - s}{g_2 i_0} Y + \frac{C_0 + G - g_0}{g_2 i_0}, \tag{18}$$

$$D_{(\dot{D}=0)} = \frac{g_1 - \theta}{(g_2 - 1) i_0} Y - \frac{g_0}{(g_2 - 1) i_0}. \tag{19}$$

The locus of $\dot{Y} = 0$ is positive by assumption A.1, but the locus of $\dot{D} = 0$ depends on the sign of $g_2 - 1$. The slope is negative when $g_2 - 1 < 0$ (Fig. 2-1) and positive when $g_2 - 1 > 0$ (Fig. 2-2).[12]

The Jacobian matrix of the dynamic system (S_a.1) at equilibrium can be expressed as:

$$J_a = \begin{pmatrix} \alpha(g_1 - s) & -\alpha g_2 i_0 \\ g_1 - \theta & (1 - g_2) i_0 \end{pmatrix}. \tag{20}$$

Therefore, we obtain:

[9] For simplicity, we ignore corporate income tax in this paper; however, considering corporate income tax is crucial for future studies expanding on this paper. A reduction in corporate income tax improves firms' financial condition, which may have the effect of deterring financial instability.

[10] If $g_1 - s > 0$, we obtain $\frac{d\dot{Y}}{dY} > 0$. By contrast, If $g_1 - s < 0$, we obtain $\frac{d\dot{Y}}{dY} < 0$.

[11] See, for example, Asada (1995) and, Ninomiya (2007, 2018).

[12] Equilibrium income Y_a^* is $Y_a^* = \frac{g_2(C_0+G) - (C_0 - g_0 + G)}{(s-\theta)g_2 + g_1 - s}$.

Equilibrium debt burden D_a^* is $D_a^* = \frac{g_0(g_1 - s) + (C_0 + G - g_0)(g_1 - \theta)}{i_0\{g_2(g_1 - \theta) - (g_1 - s)(g_2 - 1)\}}$.

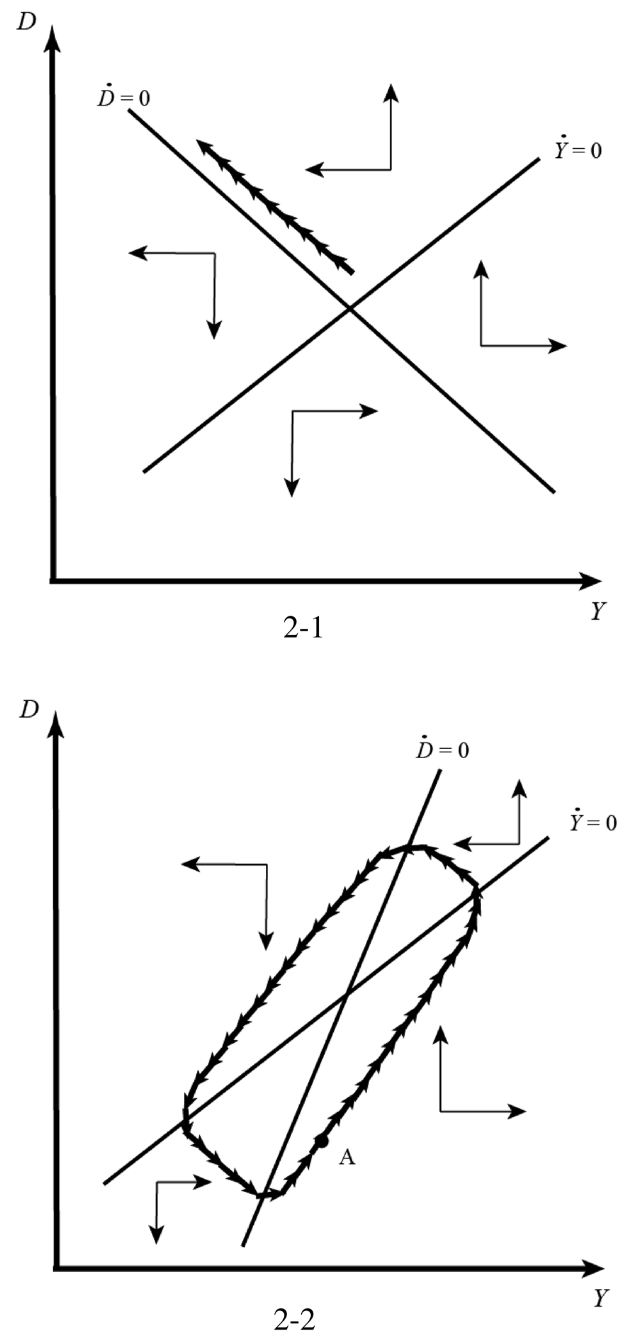

Fig. 2 Dynamic system S_a

$$\mathrm{tr}J_a = \alpha(g_1 - s) + (1 - g_2)i_0, \tag{21}$$

$$\det J_a = \alpha i_0[g_1 - s + (s - \theta)g_2] > 0. \tag{22}$$

We obtain $\det J_a > 0$ by adopting assumption A.1.[13] Therefore, the stability of the system (S_a.1) depends on only the tr J_a.

[13] $s - \theta = \{1 - c(1 - \theta - t)\} - \theta = (1 - \theta)(1 - c) + ct > 0$.

The dynamic system (S_a.1) becomes unstable when $1 - g_2 > 0$ in Fig. 2-1 through the following mechanism. Suppose income Y descends below equilibrium during an economic downturn. The decrease leads to a decline in profit Π and an expansion in debt burden D. Expansion in D restrains investment demand I. However, debt burdens rise because the upsurge in interest payments iD exceeds the decline in investment demand I. Therefore, D rises with the decline in Y. This mechanism indicates that financial factors destabilize the economy alongside real factors. Note that the burden of interest-bearing debt iD is a crucial contributor to financial instability.

$$Y \downarrow \Rightarrow \Pi \downarrow \Rightarrow D \uparrow \Rightarrow I \downarrow < iD \uparrow \Rightarrow D \uparrow .$$

In addition, there is one parameter value α_a at which Hopf bifurcation occurs when $1 - g_2 < 0$.[14] Figure 2-2 shows at least one closed orbit around the equilibrium in the system (S_a.1) in this case, when α is close to α_a (Appendix 1).[15] This is a financial cycle with income Y and debt burden D. This cycle occurs via the following mechanism. Suppose the economy occupies Point A in Fig. 2-2. Income Y and debt burden D increase at Point A. Rising D restrains investment demand I, and the economy enters recession. In this instance, however, erosion in investment demand I exceeds the greater burden of interest-bearing debt iD. Therefore, debt burden D shrinks. The financial factor stabilizes the economy.[16]

$$D \uparrow \Rightarrow I \downarrow (Y \downarrow) > iD \uparrow \Rightarrow D \downarrow .$$

Figure 3 presents the relation between an HSP structure (Fig. 1) and the dynamic in Fig. 2. That is, Fig. 3 shows escalating financial fragility during a business cycle. Suppose the economy operates under a hedge finance regime at Point A in Fig. 3-1. Income Y increases, and the financial regime shifts from hedge to speculative (Point B). The debt burden D also expands,[17] and the financial regime shifts to Ponzi finance (Point C).[18] Consequently, the economy enters depression (Point D).

Figure 3-2 shows the other process of escalating financial fragility during the business cycle. We emphasize that the financial factor has a stabilizing role in the economy, although the financial regime becomes more fragile from the hedge finance to the speculative finance and the Ponzi finance.[19]

Next, we simulate the financial cycle in the basic dynamic system numerically. By enumerating parameters as $c = 0.6$, $\theta = 0.5$, $t = 0.2$, $C_0 = 20$, $g_1 = 1.5$, $g_2 = 3$, $i = 1(\%)$, $g_0 = 35$ and $G = 25$, we rewrite dynamic system (S_a.2) as follows (see Appendix 2):

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[14] See Gandolfo (1997) for more information on the Hopf bifurcation theorem.

[15] Two types in Hopf bifurcations exist. If the closed orbit exists in the region of $\alpha < \alpha_a$, it is an unstable periodic orbit. By contrast, if the closed orbit exists in the region of $\alpha > \alpha_a$, it is a stable periodic orbit. The unstable periodic orbit is called a subcritical Hopf bifurcation, whereas the stable periodic orbit is called a supercritical Hopf bifurcation. We are unable to confirm the type of cycles in this paper.

[16] Only this cycle occurs when the interest rate is constant.

[17] As previously stated, the reduction in investment demand I refrains from increasing in debt burden D. Therefore, the financial regime shifts to the speculative finance (2) region, which satisfies the hedge finance condition. However, a decrease in investment demand I induces the recession.

[18] If g_2 becomes small at Point C, the economy might fall into a financial crisis and the debt burdens D would continue to increase.

[19] The cycle in the dynamic system (S_a.1) is similar to Asada (2001), although the interest rate is constant in the cycle.

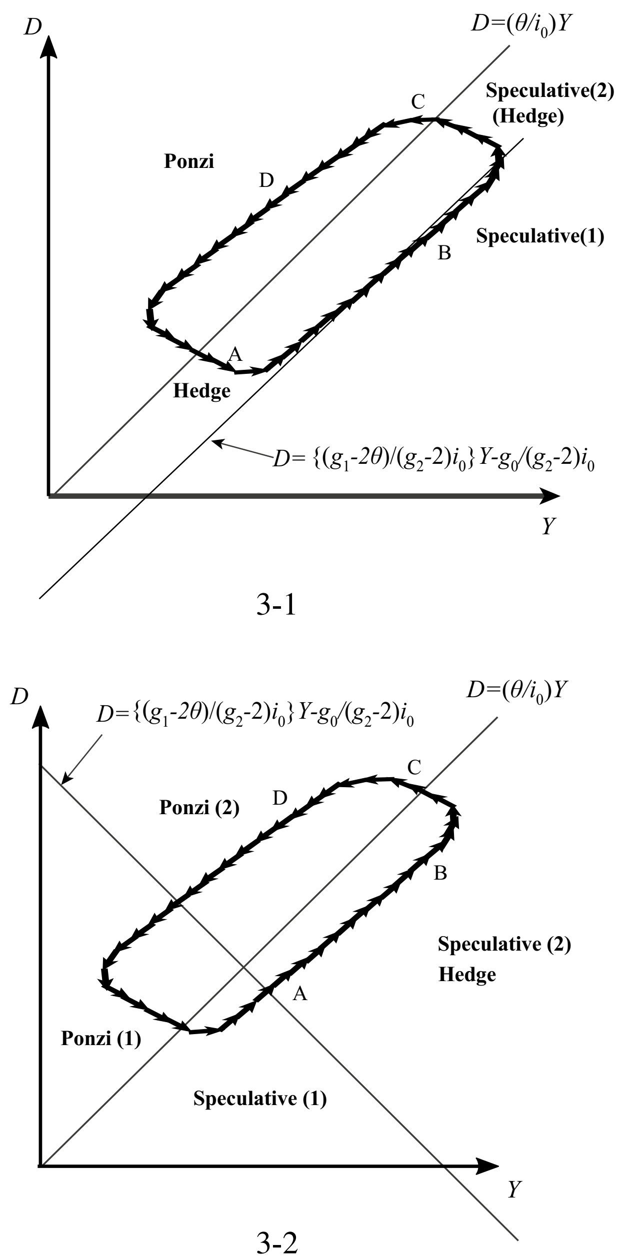

Fig. 3 Financial regimes and cycles

$$\dot{Y} = \alpha[0.68Y - 3D + 10] \qquad (S_a.2.1),$$

$$\dot{D} = Y - 2D - 35 \qquad (S_a.2.2).$$

By considering (11), (12) and (13), the financial regimes are

$$D \geqq 0.5Y - 35 \qquad \left(\text{hedge finance}\right), \tag{23}$$

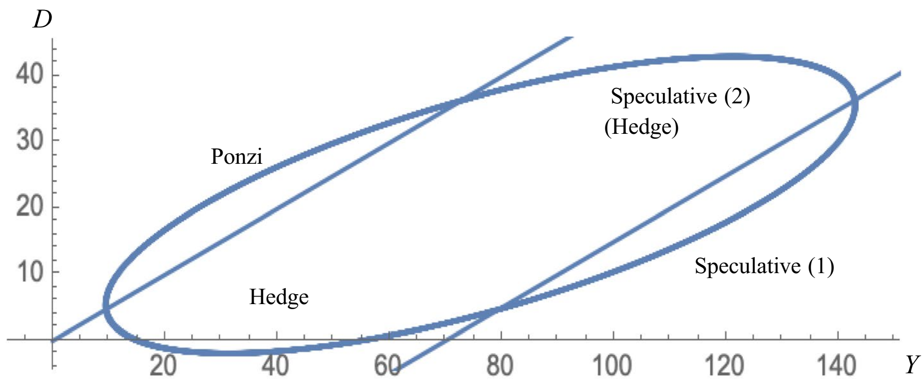

Fig. 4 Numerical simulation system S_a

$$D \leqq 0.5Y \qquad \text{(speculative finance)}, \tag{24}$$

$$D > 0.5Y \qquad \text{(Ponzi finance)}. \tag{25}$$

Figure 4 illustrates there is a closed orbit in the dynamic system (S_a.2) when $\alpha = 2.941$. [20] The equilibrium value of Y is $Y^* \fallingdotseq 82.31$. Figure 4 also illustrates the relation between the financial cycle and the financial structure. The hedge finance regime of (23) satisfies $g_1 - 2\theta > 0$ and $g_2 - 2 > 0$. Figure 4 also illustrates the escalating fragility of an HSP financial structure in the business cycle.[21]

3 Lenders' risks and instability

Section 2 assumed a constant interest rate, which prevented us from investigating the LR financial structure in the dynamic system (S_a.1). We now consider a dynamic system in which the interest rate is an endogenous variable. That is, in addition to the HSP structure, the LR financial structure is considered.

The money demand function M^d is

$$M^d = L(Y, i), \quad L_Y \equiv \frac{\partial L}{\partial Y} \gtreqless 0, \quad L_i \equiv \frac{\partial L}{\partial i} < 0, \tag{26}$$

where $L_Y < 0$ implies that lenders' liquidity preference intensifies with the decrease in income Y. This effect expresses an aspect of LR. We call this LR the "Taylor and O'Connell type (T-O type) LR".[22]

Following Rose (1969) and Ninomiya (2007, 2015, 2016, 2018), we define the money supply function M^s as:

[20] We cannot determine whether a closed orbit exists in the regions of $\alpha < \alpha_a$ or $\alpha > \alpha_a$.

[21] The cycle in Fig. 4 contains the period $D < 0$. This indicates that firms lend or firms' bank balances exceed their outstanding loans because investment demand is extremely low.

[22] $L_Y < 0$ is based on Taylor and O'Connell (1985). As we mentioned, Taylor and O'Connell (1985) formulated that lenders' liquidity preferences (the money demand function) intensify with a decrease in the expected profit rate (ρ). They hypothesized that raising the expected profit rate (ρ) would lower the interest rate (i). They claimed that a true Minsky crisis occurs when the value of derivatives (i_ρ) falls dramatically. Ninomiya (2007, 2018) and Ninomiya and Tokuda (2017) investigated T-O type LRs using a simple microeconomic framework.

$$M^s = \mu(Y, i)H, \quad \mu_Y \equiv \frac{\partial \mu}{\partial Y} > 0, \quad \mu_i \equiv \frac{\partial \mu}{\partial i} > 0, \tag{27}$$

where μ is a monetary multiplier. $\mu_Y > 0$ implies that the money supply increases when a bank lends to an expanding economy. This effect is also an expression of the T-O type LR.[23] The monetary multiplier μ includes commercial bank behavior.[24] We assume that high-powered money H is constant ($H = \bar{H}$).

Ordering (26) and (27), the interest rate i is determined by equilibrium in the money market as follows:[25]

$$L(Y, i) = \mu(Y, i)\bar{H}. \tag{28}$$

Totally differentiating Eq. (28) with respect to interest rate i and income Y, we obtain

$$i = i(Y), \quad i_Y \left(\equiv \frac{di}{dY}\right) = -\frac{L_Y - \mu_Y \bar{H}}{L_i - \mu_i \bar{H}} \gtreqless 0. \tag{29}$$

Equation (29) also shows that interest rate i is reflected by LRs. This is a financial structure of the LR type.

As mentioned, LRs are expressed by L_Y and μ_Y. The sign of i_Y depends on the sign of $L_Y - \mu_Y \bar{H}$. We obtain $i_Y < 0$ when $L_Y - \mu_Y \bar{H} < 0$. For example, we obtain $i_Y < 0$ when μ_Y is significant. The monetary multiplier μ includes the behavior of commercial banks. Kregel (1997) emphasized that the margins of safety proposed by Minsky are significant for financial instability. When an economic boom reduces LRs, lenders, including commercial banks, promote lending despite erosion in margins of safety.

We also obtain $i_Y < 0$ when $L_Y < 0$. This is similar to Taylor and O'Connell's (1985) study. They presented that an economy would fall into a financial crisis when a decline in expected profit rates aggravated the financial condition of firms and increased household preference for liquidity.

Ninomiya (2007, 2016) introduced the factors $L_Y < 0$ and $\mu_Y > 0$, and discusses financial instability when $i_Y < 0$. He indicated that the economy becomes unstable even when the real factor $(g_1 - s)$ stabilizes the economy when $i_Y < 0$. We call this instability the "Taylor–O'Connell type (T-O type) financial instability". The mechanism of this instability is as follows. We suppose that an economy is in recession. A decline in income Y raises the interest rate i. An increase in interest rate i restrains investment demand I, and a financial crisis ensues.

$$Y \downarrow \Rightarrow i \uparrow \Rightarrow I \downarrow \Rightarrow Y \downarrow.$$

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[23] See Ninomiya (2007, 2018) and Ninomiya and Tokuda (2017) for more details.

[24] Lima and Meirelles (2007) and Ryoo (2013) introduce the effect of bank profitability on credit supply.

[25] Although this paper only focuses on lenders' risks, the borrowers' risks proposed by Minsky are also crucial. Okishio (1986), Ninomiya (2006), and Ninomiya and Tokuda (2017) demonstrated that the interest rate is determined by the bond market as follows:
$-\left[(I - S) + \left(M^d - M^s\right)\right] = 0, \quad \text{(F.22)}$
where S is savings, investment I represents the borrowers' behavior, and investment depends on the income and interest-bearing debt burden. Ninomiya (2006) considered the borrowers' risks. We adopt Eqs. (28) and (42) to simplify the analysis in this paper and would like to consider borrowers' risks in more detail in future research.

By ordering (8), (9), (15), (16), (17), and (29), we obtain the following dynamic system (S_b.1):

$$\dot{Y} = \alpha[c(1-\theta-t)Y + C_0 + g_1 Y - g_2 i(Y)D - g_0 + G - Y] \qquad (S_b.1.1),$$

$$\dot{D} = g_1 Y - g_2 i(Y)D - g_0 - \theta Y + i(Y)D \qquad (S_b.1.2).$$

The Jacobian matrix of the dynamic system (S_b.1) at equilibrium can be expressed as:

$$J_b = \begin{pmatrix} \alpha[(g_1 - s) - g_2 i_Y D] & -\alpha g_2 i \\ g_1 - \theta + (1-g_2) i_Y D & (1-g_2)i \end{pmatrix}. \tag{30}$$

Therefore, we obtain

$$\mathrm{tr}J_b = \alpha\{(g_1 - s) - g_2 i_Y D\} + (1-g_2)i, \tag{31}$$

$$\det J_b = \alpha i[(g_1 - s) + (s-\theta)g_2] > 0. \tag{32}$$

We obtain $\det J_b > 0$. Therefore, the stability depends on the sign of tr J_b as indicated:

$$\begin{aligned} &(1)\ \ g_1 - s - g_2 i_Y D > 0, \quad 1 - g_2 > 0 \Rightarrow \mathrm{tr}J_b > 0 : \text{Unstable}, \\ &(2)\ \ g_1 - s - g_2 i_Y D < 0, \quad 1 - g_2 < 0 \Rightarrow \mathrm{tr}J_b < 0 : \text{Stable}, \end{aligned}$$

Stability depends on the signs of $1-g_2$ and $g_1 - s - g_2 i_Y D$. The inequity $g_1 - s - g_2 i_Y D > 0$ indicates that the goods market destabilizes the economy. This is usually assumed in closed Kaldorian models. We should note that the condition is satisfied even when $i_Y < 0$ and the absolute value is significant. This means that the LR financial structure is unstable and the financial factor may stabilize the economy when $g_1 - s - g_2 i_Y D > 0$.

There is one parameter value α_b at which Hopf bifurcation occurs when $1 - g_2 < 0$. There is at least one closed orbit around equilibrium in the system (S_b.1), when α is close to α_b (Appendix 3). Cycle 1 is similar to the cycles in the basic dynamic system (S_a).[26] We should note that the HSP financial structure stabilizes the system (S_b.1). The system (S_b.1) is unstable when $g_1 - s - g_2 i_Y D > 0$ and $1 - g_2 > 0$. We emphasize that the fragile HSP financial structure also destabilizes the economy when $1 - g_2 > 0$.

In contrast, $g_1 - s - g_2 i_Y D < 0$ indicates that marginal propensity to invest ($g_1 - g_2 i_Y D$) is smaller than marginal propensity to save (s). In other words, the indirect effect ($g_2 i_Y D$) is significant. Therefore, the goods market stabilizes the economy despite the destabilizing real factor ($g_1 - s > 0$). This means that the LR financial structure makes the economy stable. Therefore, the dynamic system (S_b) is stable when $g_1 - s - g_2 i_Y D < 0$ and $1 - g_2 < 0$. The HSP financial structure also stabilizes the economy when $1 - g_2 < 0$.

There is one parameter value α_b at which Hopf bifurcation occurs when $g_1 - s - g_2 i_Y D < 0$ and $1 - g_2 > 0$, which means that the HSP financial structure is fragile. There is at least one closed orbit around equilibrium in System (S_b.1), when α

[26] Cycle 1 is also similar to Asada (2001). However, he did not consider the T-O type financial instability.

is close to α_b (Appendix 3). In other words, Cycle 2 is quite different from Cycle 1 and Asada (2001).

Although economic boom reduces safety margins, lenders continue to lend due to the decrease in the T-O type LRs. The economy might fall into "euphoria". However, recession exacerbates the T-O type LRs, and they may curtail lending rapidly and drastically. A financial crisis could occur if T-O type financial instability occurs as fragility progresses from hedge finance to speculative finance and Ponzi finance. Fragility then becomes instability.[27] As mentioned, the system $(S_b.1)$ is unstable when $g_1 - s - g_2 i_Y D > 0$ and $1 - g_2 > 0$. Again, we emphasize it is essential to consider both types of financial structures in dynamic systems.

It is worthwhile to describe monetary policy interventions for coping with financial instability. The dynamic system (S_a) shows stability conditions under a constant interest rate. That is, the economy mirrors the system (S_a) if central bank policy targets the interest rate. We regard interest rate targeting useful in avoiding T-O type financial instability.

Next, we present a numerical simulation in the case of $i_Y > 0$ by giving an example and specify Eq. (29) as:

$$i = i_1 Y, \quad i_1 > 0. \tag{33}$$

By considering Eq. (33), we obtain the following dynamic system $(S_b.2)$[28]:

$$\dot{Y} = \alpha[c(1-\theta-t)Y + C_0 + g_1 Y - g_2 i_1 YD - g_0 + G - Y] \qquad (S_b.2.1),$$

$$\dot{D} = g_1 Y - g_2 i_1 YD - g_0 - \theta Y + i_1 YD \qquad (S_b.2.2).$$

By ordering (3)–(9), and (33), we obtain these financial regimes in the dynamic system $(S_b.2)$:

$$Y \geqq -\frac{g_0}{(2\theta - g_1) + (g_2 - 2)i_1 D} \qquad \text{(hedge finance)}, \tag{34}$$

$$D \leqq \frac{\theta}{i_1} \qquad \text{(speculative finance)}, \tag{35}$$

$$D > \frac{\theta}{i_1} \qquad \text{(Ponzi finance)}. \tag{36}$$

The boundary of hedge finance (34) depends on the signs of $2\theta - g_1$, $2 - g_2$ and i_1. We offer the following numerical simulation as an example because there are many patterns.

[27] According to Minsky (1975), "lender's risk does appear on signed contracts." For any set of market conditions, as applied to a particular firm, lender's risk takes the form of increased cash flow requirements in debt contracts as the debt-to-total-asset ratio increases. Lender's risk manifests itself in financial contracts in various forms: higher interest rates, shorter terms to maturity, the requirement to pledge specific assets as collateral, and restrictions on dividend payments and further borrowing. The lender's risk rises with an increase in the ratio of debt to equity financing or the ratio of commitment cash flows to total prospective cash flow" (p. 110).
A recession increases the ratio of commitment cash flows to total prospective cash flow. Therefore, T-O type LRs will also rise.

[28] Equilibrium income Y_b^* is $Y_b^* = \frac{g_0 - (1-g_2)(C_0+G)}{(g_1-\theta)+(1-g_2)(1-s)}$.
Equilibrium debt burden D_b^* is $D_b^* = \frac{(C_0+G)(g_1-\theta)+(1-s)g_0}{\{g_0-(C_0+G)(1-g_2)\}i_1}$.

The boundary between speculative and Ponzi finance depends on the parameter θ and the parameter of interest rate i_1. The region of Ponzi finance expands when θ decreases or i_1 rises. The decrease in θ reduces internal reserves. The rise in i_1 enhances the burden of interest-bearing debt via the increase in the T-O type LRs.

We present a numerical simulation of the financial cycle. We enumerate parameters as $c = 0.8, \theta = 0.6, t = 0.2, C_0 = 15, g_1 = 2, g_2 = 1.1, i_1 = 0.1, g_0 = 35$, and $G = 10$. Therefore, we rewrite the dynamic system (S_b.2) as follows (see Appendix 4):

$$\dot{Y} = \alpha[1.16Y - 0.11YD - 10] \qquad (S_b.3.1),$$

$$\dot{D} = 1.4Y - 0.01YD - 35 \qquad (S_b.3.2).$$

By considering (34), (35) and (36), the financial regimes are:

$$Y \geqq -\frac{35}{-0.8 - 0.09D} \qquad (\text{hedge finance}), \tag{37}$$

$$D \leqq 6 \qquad (\text{speculative finance}), \tag{38}$$

$$D > 6 \qquad (\text{Ponzi finance}). \tag{39}$$

Figure 5 shows that there is a closed orbit in dynamic system (S_b.3) when $\alpha = 0.672$ and the financial structure is HSP.[29] The equilibrium value of Y is $Y^* \fallingdotseq 26.33$. This simulation is an example of Cycle 2. Figure 5 also shows the escalation of financial fragility in the business cycle. In addition, the financial factor destabilizes the economy in Cycle 2. Therefore, escalating financial fragility depicted in Cycle 2 is a more appropriate interpretation of an HSP Minskian structure. As previously mentioned, the financial factor may stabilize the economy in Cycle 1.

In the dynamic system (S_b), we suppose that the interest rate i depends on the income Y. However, some studies supposed that the interest rate i depends on the debt burden D.[30] We also construct the following dynamic system (S_c) in which the interest rate i depends on the debt burden D.

We define the money supply function M^s as:

$$M^s = \mu(i, D)H, \quad \mu_i \equiv \frac{\partial \mu}{\partial i} > 0, \quad \mu_D \equiv \frac{\partial \mu}{\partial D} < 0. \tag{40}$$

$\mu_D < 0$ implies that the money supply shrinks as banks, concerned about firms' increased debt burden D, curtail lending. This effect also expresses LRs.

The money demand function M^d is:

$$M^d = L(i, D), \quad L_i \equiv \frac{\partial L}{\partial i} < 0, \quad L_D \equiv \frac{\partial L}{\partial D} > 0, \tag{41}$$

[29] The closed orbit exists in the region of $\alpha < \alpha_b$; therefore, this seems to be a subcritical Hopf bifurcation.

[30] See, for example, Asada (2006) and Adada et al. (2019). Asada (2006) said "the private debt and public bond are imperfect substitutes, and the difference of the rates of interest of these assets reflects the difference of the 'degrees of the risk' of these assets (p. 469).

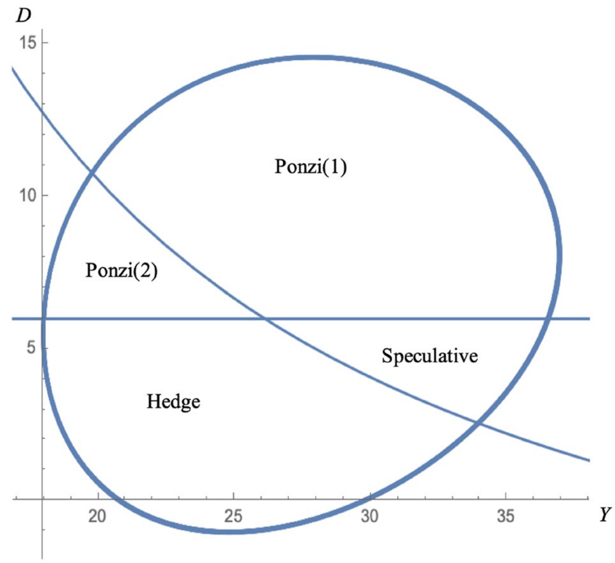

Fig. 5 Numerical simulation system S_b

where $L_D > 0$ also implies that lenders' liquidity preferences intensify with the decrease in income Y and the increase in firms' debt burden D. This effect also expresses LRs.

Ordering (40) and (41), we determined the interest rate i by equilibrium in the money market as follows:

$$L(i, D) = \mu(i, D)\bar{H}. \tag{42}$$

Totally differentiating Eq. (42) with respect to the interest rate i and debt burden D, we obtain

$$i = i(D), \quad i_D\left(\equiv \frac{\partial i}{\partial D}\right) = -\frac{L_D - \mu_D \bar{H}}{L_i - \mu_i \bar{H}} > 0. \tag{43}$$

Since $i_D > 0$, we specify Eq. (43) as follows[31]:

$$i = i_2 D, \quad i_2 > 0. \tag{44}$$

By ordering (8), (9), (15), (16), (17) and (44), we obtain dynamic system (S_c):

$$\dot{Y} = \alpha[c(1 - \theta - t)Y + C_0 + g_1 Y - g_2(i_2 D)D - g_0 + G - Y] \qquad (S_c.1),$$

[31] For example, Asada (2001) assumed that.
$i^* = i + \varphi(d), \varphi(d) \geq 0, \varphi'(d) \geq 0,$ (F.33)
where i^* is the interest rate that applied initial debt. i is the market rate of interest. d is the debt–capital ratio. As previously stated, Asada (1997, 2001) discussed investment based on the increasing risk principles (Kalecki, 1937). It seems that the principle of increasing risk captured mainly borrower's risk. Equation 43 indicates only the lenders' risks. That is, the interest rate affects the investment decision. Asada (1997) captured (F.33) as a lender's risk based on Kalecki (1937) and Minsky (1986).
Ninomiya (2015, 2016) and Ninomiya and Tokuda (2017) investigated the case of $i_D < 0$ by using (F.22) in footnote 22 and an investment function that is dependent on debt burden, such as Eq. (9). It seems that $i_D < 0$ is an extremely rare case. However, Ninomiya and Tokuda (2017) implied $i_D < 0$ during a serious recession after the collapse of the bubble economy in Japan.

$$\dot{D} = g_1 Y - g_2(i_1 D)D - g_0 - \theta Y + (i_2 D)D \qquad (S_c.2).$$

The Jacobian matrix of the system (S_c) at equilibrium can be expressed as:

$$J_c = \begin{pmatrix} \alpha(g_1 - s) & -2\alpha g_2 i_2 D \\ g_1 - \theta & 2(1 - g_2) i_2 D \end{pmatrix}. \tag{45}$$

Therefore, we obtain

$$\mathrm{tr} J_c = \alpha(g_1 - s) + (1 - g_2) 2 i_2 D, \tag{46}$$

$$\det J_c = \alpha 2 i_2 D[g_1 - s + (s - \theta) g_2] > 0. \tag{47}$$

We also obtain $\det J_c > 0$ by adopting assumption A.1 in this case. Therefore, stability of the system depends solely on the sign of tr J_c. The dynamic system (S_c) is unstable when $1 - g_2 > 0$ (Fig. 6-1). However, there is one parameter value α_c at which Hopf bifurcation occurs when $1 - g_2 < 0$. In this case, at least one closed orbit around equilibrium in the system (S_c) occurs when α is close to α_c (Fig. 6-2) (Appendix 5). These properties are identical to those in the dynamic system (S_a) because $i_D (= i_2) > 0$ also stabilizes the dynamic system (S_c).[32]

By ordering (3)–(9), (44), we obtain the following financial regimes in the dynamic system (S_c):

$$Y \geqq \frac{(2 - g_2) i_2}{(2\theta - g_1)} D^2 - \frac{g_0}{(2\theta - g_1)} \qquad (\text{hedge finance}), \tag{48}$$

$$Y \geqq \frac{i_2}{\theta} D^2 \qquad (\text{speculative finance}), \tag{49}$$

$$Y < \frac{i_2}{\theta} D^2 \qquad (\text{Ponzi finance}). \tag{50}$$

The boundary of hedge finance also depends on the sign of $2\theta - g_1$ and $2 - g_2$. For example, the coefficient of D^2 and the intercept are positive when $2 - g_2 < 0$ and $2\theta - g_1 < 0$. The former indicates that the decline in investment demand I via the increase in debt burden D exceeds the rising burden of interest-bearing debt iD. This effect leads to the decrease in debt burden. In this case, therefore, the region of hedge finance also expands with the economy.

The boundary between speculative and Ponzi finance depends on the parameter θ and the parameter of interest rate i_2. The region of Ponzi finance expands when θ falls or i_2 rises. A decrease in θ reduces internal reserves. Parameter i_2 captures LRs. For example, an increase in i_2 enhances the burden of interest-bearing debt via the increase in LRs. Therefore, the increase in i_2 reduces the region of hedge finance and enlarges the region of Ponzi finance.

32 The cycle in the dynamic system (S_c) is very similar to Asada (2001).

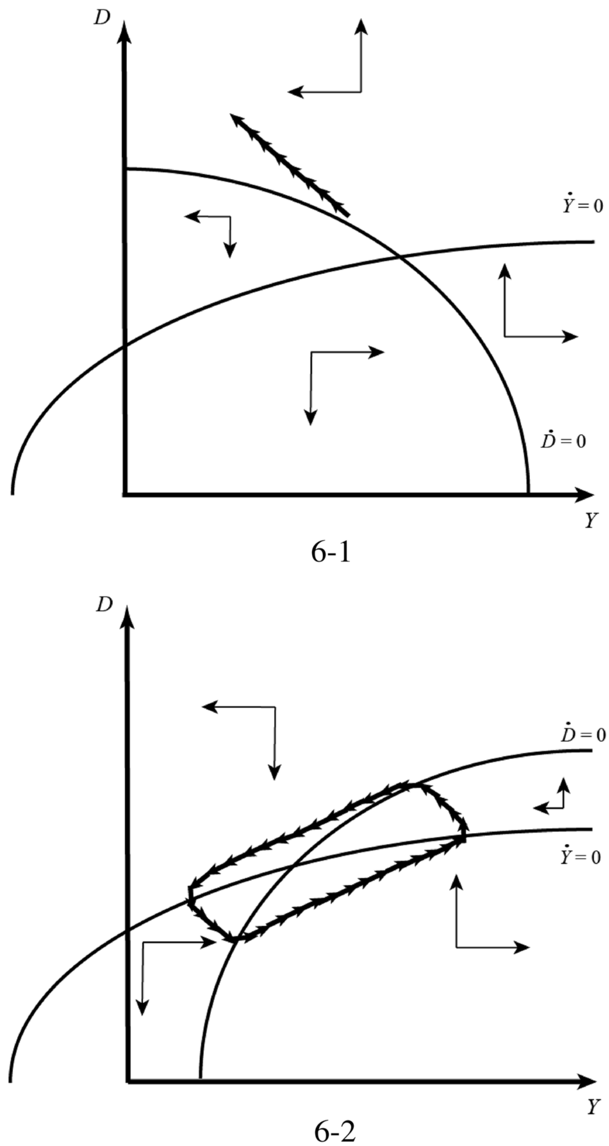

Fig. 6 Dynamic system S_c

Figure 7 presents one relationship between an HSP structure and the cycle in Fig. 6-2. Figure 7 also shows escalating financial fragility in the business cycle. The progression of fragility in the dynamic system (S_c) resembles that in the dynamic system (S_a), although parameter i_2 contains LRs. We should note that $i_D(=i_2) > 0$ stabilizes the dynamic system (S_c).[33]

[33] If we follow the definition of lender's risk by Minsky (1975), it might be more appropriate to use $\mu_D < 0$ in Eq. (40) and $L_D > 0$ in Eq. (41) as the formulation of lenders' risks. Although $i_D(=i_2) > 0$ stabilizes the dynamic system (S_c), the decrease in the absolute value of μ_D, for example, makes the stabilizing effect small.

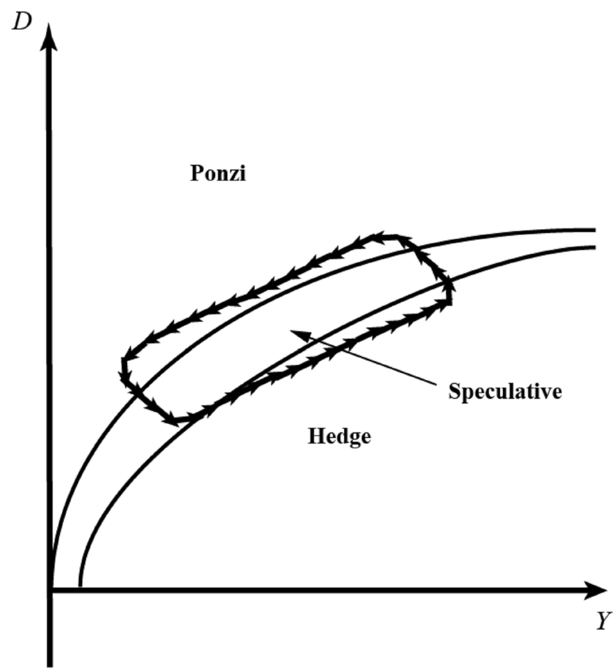

Fig. 7 Financial regimes and cycle: system S_c

4 Conclusion

This study considered two types of Minskian financial structures—LR finance and HSP finance—and discussed financial instability and cycles. Kregel (1997) emphasized the significance of margins of safety for financial instability. LRs affect margins of safety and interest rates. We also simulated financial cycles numerically.

We examined three instances of dynamic systems: (1) when the interest rate is constant (system (S_a)); (2) when it depends on income (system (S_b)); and (3) when it depends on debt burdens (system (S_c)). The system (S_a) can display only the process of escalating financial fragility, which refers to HSP finance during a business cycle. The systems (S_b) and (S_c) can examine LR type financial structures. However, the progression of financial fragility in the system (S_c) resembles that in the system (S_a), although the system (S_c) considers the effects of lenders' risks. We noted that the financial factor has a sabilizing effect in the business cycles of the system (S_a) and (S_c).

In contrast, one cycle in the system (S_b) occurs when the financial factor causes economic instability. Therefore, we posit that one of the process of increasing financial fragility in the system (S_b) is a more appropriate interpretation of an HSP Minskian financial structure. Furthermore, we presented Taylor–O'Connell type (T-O type) financial instability occurring in the system (S_b). If instability occurs during the progression of increasing financial fragility of the HSP type, then the economy may deteriorate into financial crisis. Fragility becomes instability. Targeting the interest rate helps to avoid the T-O type financial instability. We emphasized the significance of considering both financial structures in dynamic systems.

However, the models in this paper are only two-dimensional systems in debt burden D and income Y. We need to consider the dynamics of price and income share, and examine monetary policy to avoid this instability.

We assume that the rate of profit sharing θ is constant. Sasaki and Fujita (2014) show that the range of fluctuations in business cycles depends on the retention ratio. We also should develop our model with a formulation that considers the borrower's risk and differs from the money market equilibrium condition.[34] Furthermore, our study is a theoretical analysis following Ninomiya and Tokuda (2012, 2021), who examined T-O type financial instability using VAR analysis. In a future study, we will examine HSP-type instability empirically based on this research.

Appendix 1[35]

Suppose $1 - g_2 < 0$. The characteristic equation of system (S_a) is

$$\lambda^2 + (-\mathrm{tr}J)\lambda + (\mathrm{det}J) = 0.$$

A necessary condition of the Hopf bifurcation for complex roots is det $J_a > 0$, which is satisfied from (22). Regarding tr J_a, we find that

$$\mathrm{tr}J_a \gtreqless 0 \Leftrightarrow \alpha_a \gtreqless 0, \qquad \alpha_a = \frac{-(1-g_2)i_0}{g_1 - s}.$$

Roots of the characteristic equation are

$$\lambda_{1,2} = -\frac{1}{2}(-\mathrm{tr}J) \pm \sqrt{(-\mathrm{tr}J)^2 - 4(\mathrm{det}J)}.$$

Because tr $J_a = 0$ for critical value α_a of the parameter, the characteristic equation has a pair of pure imaginary roots, $\lambda_{1,2} = \pm i\sqrt{(\mathrm{det}J_a)}$ (where $i = \sqrt{-1}$). Roots of the above equation remain a complex conjugate for $(-$ tr $J_a)$ sufficiently small, namely for α sufficiently near α_a.

We obtain

$$\left.\frac{d(trJ_a/2)}{d\alpha}\right|_{\alpha=\alpha_a} = \frac{g_1 - s}{2} \neq 0.$$

From the preceding discussion, all conditions for Hopf bifurcation are satisfied at Point $\alpha = \alpha_a$.

Appendix 2

We specify the consumption function (16) and investment function (9) as follows:

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[34] See Fazzari et al. (2008). Godley and Lavoie (2007) and Dos Santos and Zezza (2008) develop a stock-flow-consistent model. Okishio (1986) examines stock-flow relations among the central bank, commercial banks, firms and households, and, presents it as an *IS-BB* analysis.

[35] The method of the proof in Appendices 1, 2 and 3 is based on Gandolfo (1997).

$$C = c(1-\theta-t)Y + C_0 = 0.6(1-0.5-0.2)Y + 15, \tag{51}$$

$$I = g_1 Y - g_2 iD - g_0 = 2Y - 3D - 35, \tag{52}$$

where $c = 0.6, \theta = 0.5, t{=}0.2, C_0 = 15, g_1 = 2, g_2 = 3, i = 1(\%)$, and $g_0 = 35$.

Ordering (8), (17), (51), (52) and $G = 30$, we obtain

$$\dot{Y} = \alpha[0.6(1-0.5-0.2)Y + 15 + 2Y - 3D - 35 + 30 - Y],$$

$$\dot{D} = 2Y - 3D - 35 - 0.5Y + 1D.$$

Therefore, we obtain the dynamic system (S_a.2).

Appendix 3

det $J_b > 0$ is satisfied from (32) when $i_1 > 0$. Regarding tr J_b, we find that

$$\mathrm{tr}J_b \lesseqgtr 0 \Leftrightarrow \alpha_b \lesseqgtr 0, \text{when } (g_1 - s) - g_2 i_2 D > 0 \text{ and } 1 - g_2 < 0,$$

$$\mathrm{tr}J_b \lesseqgtr 0 \Leftrightarrow \alpha_b \gtreqless 0, \text{when } (g_1 - s) - g_2 i_2 D < 0 \text{ and } 1 - g_2 > 0,$$

$$\alpha_b = \frac{-(1-g_2)i_2 Y}{(g_1 - s) - g_2 i_2 D} > 0.$$

Because tr $J_b = 0$ for the critical value α_b of the parameter. We obtain

$$\left.\frac{d(trJ_b/2)}{d\alpha}\right|_{\alpha=\alpha_b} = \frac{(g_1 - s) - g_2 i_1 D}{2} \neq 0.$$

From the proceeding discussion, all conditions in which Hopf bifurcation occurs are satisfied at the point $\alpha = \alpha_b$.

Appendix 4

We specify the consumption function (16) and investment function (9) as follows:

$$C = c(1-\theta-t)Y + C_0 = 0.8(1-0.6-0.2)Y + 15, \tag{53}$$

$$I = g_1 Y - g_2 iD - g_0 = 2Y - 1.1iD - 35, \tag{54}$$

where $c = 0.8, \theta = 0.6, t = 0.2, C_0 = 15, g_1 = 1.5, g_2 = 1.1$, and $g_0 = 35$. We also specify Eq. (33) as follows:

$$i = i_1 Y = 0.1Y. \tag{55}$$

Ordering (8), (17), (53), (54), (55) and $G = 10$, we obtain

$$\dot{Y} = \alpha[0.8(1 - 0.6 - 0.2)Y + 15 + 2Y - 1.1 * 0.1YD - 35 + 10 - Y],$$

$$\dot{D} = 2Y - 1.1 * 0.1YD - 35 - 0.6Y + 0.1YD.$$

Therefore, we obtain the dynamic system (S_b.3).

Appendix 5

Suppose $1 - g_2 < 0$. det $J_c > 0$ is satisfied from (47). Regarding tr J_c, we find that

$$\mathrm{tr}J_c \lesseqgtr 0 \Leftrightarrow \alpha_c \lesseqgtr 0, \qquad \alpha_c = \frac{-(1-g_2)2i_1D}{g_1 - s} > 0.$$

Because tr $J_c = 0$ for the critical value α_c of the parameter. We obtain

$$\frac{d(trJ_c/2)}{d\alpha}_{\alpha=\alpha_c} = \frac{g_1 - s}{2} \neq 0.$$

From the preceding discussion, all conditions in which Hopf bifurcation occurs are satisfied at Point $\alpha = \alpha_c$.

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Declarations

Competing interests
The author declares that I have no competing interests.

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