Advances in Continuous and Discrete Models

RESEARCH Open Access

Fractional discrete Temimi–Ansari method with singular and nonsingular operators: applications to electrical circuits

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Abstract

The goal of this article is to present a recently developed numerical approach for solving fractional stochastic differential equations with a singular Caputo kernel and a nonsingular Caputo–Fabrizio and Atangana–Baleanu (ABC) kernel. The proposed method is based on the discrete Temimi–Ansari method, which is combined with three different numerical schemes that are appropriate for the new fractional derivative operators. The proposed technique is used to investigate the effects of Gaussian white-noise and Gaussian colored-noise perturbations on the potential source and resistance in fractional stochastic electrical circuits. The proposed method's robustness and efficiency were demonstrated by comparing its results to those of the stochastic Runge–Kutta method (SRK). The valuable point in this article is that the resulting numerical scheme is able to combine two powerful methods that can be extended into more complex stochastic models. The comparison of different fractional derivatives using Mathematica 12 software has been obtained and the simulation results demonstrate the merit of the contributed method.

MSC: 65C30; 65L12; 26A33; 43A08; 35R11

Keywords: Temimi–Ansari method (TAM); Brownian motion process; Stochastic differential equations; RC electrical circuit; Caputo–Fabrizio derivative; Atangana–Baleanu derivative; Caputo derivative

1 Introduction

Stochastic differential equations SDEs are a combination of deterministic differential equations and a noise term. Researchers' attention has been attracted to SDEs for being useful to model phenomena in various fields, such as physics, engineering, economics and finance, population, and biology.

In an electrical circuit, there are two types of noise: external noise and internal noise. External noise denotes oscillations outside the deterministic system subjected to external factors. Adding a noise term to the right side of a deterministic equation is a well-known example with numerous engineering applications. Internal noise, such as burst noise, low-frequency noise, shot noise, and thermal noise, is created by the discontinuous nature of electrical signals [1]. In the internal noise, the magnitude of the random field at a given point and time is not depending on its magnitude at other points or times. Internal noise is referred to as white noise. Clearly, there is no electrical circuit without thermal noise at a certain temperature. Random impacts from both external and internal noise can be obtained by replacing the deterministic model's input and internal parameters with random processes [2]. Many researchers have investigated the effects of this type of noise in electrical circuits, see [3] for more information. Colored noise is used as an external noise in which the interaction of the random field between differential points and times can be nonzero. Many researchers have studied replacing white noise with colored noise in a stochastic differential equation [4, 5]. Following Ito's work [6] this type of random differential equation can be referred to as stochastic differential equations. The solution of stochastic differential equations explains Markov diffusion processes [7].

Fractional calculus has recently been applied in a variety of fields, including thermoelectricity, fluid dynamics, economics, control system design, reaction–diffusion equations, signal processing, and many others [8, 9]. The main idea behind using fractional differentiation is to describe memory effects using the property of fractional-order derivatives. Fractional-order derivatives and integrals consider system memory, patrimonial properties, and nonlocal divided effects; these effects are critical for depicting real-world problems. Furthermore, fractional-order derivatives are eventually nonlocal operators; the next state of a model depends not only on its current state but also on all of its previous states. Variables are rated using fractional derivatives as a result of the states. That is, the integral represented the store memory as an inverse operator.

The Riemann–Liouville and Liouville–Caputo fractional-order derivatives and integrals are not the only ones known. These definitions include a single function that represents the kernel and describes the system's memory effects. As none of them can accurately describe the full effects of memory effects, Caputo–Fabrizio (CF) and Atangana–Baleanu (ABC) proposed new operators with local and nonsingular kernels that use the exponential decay law rather than the power law. These nonsingular fractional derivatives can efficiently model the memory effect and systems with partial waste or squandering [10]. In recent years, fractional calculus has been applied to the development of the fractional-derivatives model, which represents the behavior of various types of fractional-derivative electrical circuit models [11–15].

In the following section, we will present three popular definitions for fractional operators, which will be used in this article to model the RC model.

Definition 1 ([16]) For $\nu : [a,b] \to R^m, m-1 < \alpha \le m$ and $m \in N$, the Caputo αth-order fractional derivative and the fractional integral are, respectively, defined by

$$
\begin{aligned}
{}_a^C D_t^{\alpha} \nu(t) &= \frac{1}{\Gamma(m-\alpha)} \int_a^t \nu^{(m)}(s)(t-s)^{m-\alpha-1}\, ds, \\
{}_a I_t^{\alpha} \nu(t) &= \frac{1}{\Gamma(\alpha)} \int_a^t \nu(s)(t-s)^{\alpha-1}\, ds.
\end{aligned}
$$

However, the main disadvantage of the Caputo derivative is that it provides a unique kernel in the maintenance of domestic dynamics. To overcome this stiffness, the following definitions introduced the Caputo–Fabrizio (CF) operator with the exponential kernel [12] and the ABC derivative with the Mittag–Leffler (ML) function [13]. In [11–14, 16–19] some unique applications of these nonsingular fractional derivatives are discussed.

Definition 2 ([17]) For $v \in H^1(a,b)$, $0 < \alpha \leq 1$, the CF αth αth-order fractional derivative and the fractional integral are, respectively, defined by

$$
\begin{aligned}
{}_{a}^{CF}D_t^{\alpha} v(t) &= \frac{1}{(1-\alpha)} \int_a^t \frac{dv(s)}{ds} \operatorname{Exp}\left[-\frac{\alpha}{1-\alpha}(t-s)\right] ds, \\
{}_{a}^{CF}I_t^{\alpha} v(t) &= (1-\alpha)v((t) + \alpha \int_a^t v(s)\, ds.
\end{aligned}
$$

Definition 3 ([16, 17]) For $v \in H^1(a,b)$, $0 < \alpha \leq 1$, the ABC αth-order fractional derivative and the fractional integral are, respectively, defined by

$$
\begin{aligned}
{}_{a}^{ABC}D_t^{\alpha} v(t) &= \frac{\beta(\alpha)}{(1-\alpha)} \int_a^t \frac{dv(s)}{ds} E_{\alpha}\left[-\frac{\alpha}{1-\alpha}(t-s)^{\alpha}\right] ds, \\
{}_{a}^{ABC}I_t^{\alpha} v(t) &= \frac{(1-\alpha)}{\beta(\alpha)} v((t) + \frac{\alpha}{\beta(\alpha)\Gamma(\alpha)} \int_a^t v(s)(t-s)^{\alpha-1}\, ds,
\end{aligned}
$$

where E_{α} is the Mittag–Leffler (ML) function and $\beta(\alpha)$ is the normalization function with $\beta(0) = \beta(1) = 1$. for facilitation we take $\beta(\alpha) = 1$.

The Temimi–Ansari approach has been used to investigate several forms of differential equations [20, 21]. The discrete Temimi–Ansari method (DTAM) was developed by the authors in [22, 25] to solve a wide class of stochastic nonlinear differential equations where the classic TAM is coupled with the finite-difference numerical scheme.

In this article, a variant of the Discrete Temimi–Ansari method, known as the fractional Discrete Temimi–Ansari method (FDTAM), is presented. The Discrete Temimi–Ansari method's remarkable technique is combined with three numerical schemes of the fractional operator with a singular kernel of Caputo type and nonsingular kernels of Caputo–Fabrizio and Atangana–Baleanu ABC types. This method is characterized by combining two powerful methods for obtaining approximate numerical solutions for fractional stochastic models. It is important to note that the proposed method is efficient in reducing the computational work compared to traditional numerical methods, owing to the high precision of the numerical result.

The proposed technique was successfully used to investigate the effects of Gaussian white-noise and Gaussian colored-noise perturbations on the potential source and resistance of fractional linear electrical circuits with fractional order as follows:

$$
{}_{0}D_t^{\alpha} v(t) = F(t,v) + G(t) + f(t)n(t), \qquad v(0) = a, \tag{1}
$$

where the fractional time derivative ${}_{0}D_t^{\alpha} v(t)$ can be of the type ${}_{0}^{C}D_t^{\alpha}$ or ${}_{0}^{CF}D_t^{\alpha}$ or ${}_{0}^{ABC}D_t^{\alpha}$, the fractional-order of a derivative is $\alpha \in [0,1]$ and $t > 0$. $v(t)$ the unknown function, t represents the independent variable, $F(t,v)$ and $G(t)$ are linear or nonlinear functions, and $n(t)$ is Gaussian white noise that can be derived from the Wiener process $\omega(t)$ by:

$$
n(t) = \varrho \frac{d\omega(t)}{dt}. \tag{2}
$$

The expectation $E[n(t)] = 0$, and finite variance $\operatorname{Var}[n(t)] = \varrho^2$, assume $\varrho = 1$.

This article is organized as follows: The numerical schemes for the fractional discrete Temimi–Ansari Method (FDTAM) are presented in Sect. 2. The proposed method was applied to the fractional stochastic linear electric circuit in Sect. 3 under the effects of Gaussian white and colored noise. Finally, Sect. 4 contains the conclusions.

2 Fractional discrete Temimi–Ansari method (FDTAM) and convergence analysis

In this section, we present a new variation of the Discrete Temimi–Ansari method DTAM for handling stochastic linear electrical circuits with fractional order, which includes resistances, inductances, capacitances, and voltage sources for a new fractional operator with a singular kernel of Caputo type and nonsingular kernels of Caputo–Fabrizio and Atangana–Baleanu ABC types.

Consider the following differential equation in the form

$$L\big[v(t)\big] + N\big[v(t)\big] + g(t) = 0, \tag{3a}$$

$$\text{with initial conditions} \quad I\big(v, \big(d^j v\big)/\big(dt^j\big)\big) = 0, \tag{3b}$$

where L and N exemplify the linear and the nonlinear operators, respectively, and $g(t)$ exemplifies the nonhomogeneous term. The Temimi–Ansari method was used to solve the differential Eq. (1) as follows:

To obtain the initial approximate function $v_0(t)$, this is the solution to the following initial-value problem

$$L\big[v_0(t)\big] + g(t) = 0, \qquad I\big(v_0, \big(d^j v_0\big)/\big(dt^j\big)\big) = 0. \tag{4}$$

To acquire the next sacrificial function $v_1(t)$, the following problem must be solved

$$L\big[v_1(t)\big] + N\big[v_0(t)\big] + g(t) = 0, \qquad I\big(v_1, \big(d^j v_1\big)/\big(dt^j\big)\big) = 0. \tag{5}$$

Also, the nth approximate functions $v_n(t)$ can be evaluated in the same way. Then,

$$L\big[v_n(t)\big] + N\big[v_{n-1}(t)\big] + g(t) = 0, \quad n = 2, 3, \ldots, \qquad I\big(v_n, \big(d^j v_n\big)/\big(dt^j\big)\big) = 0. \tag{6}$$

The produced iterative solution becomes close to the exact solution as the number of iterations increases

$$v(t) = \lim_{n \to \infty} v_n(t). \tag{7}$$

The authors of [20, 22, 23] present an expanded investigation of error analysis and convergence standards for the TAM approach applied to an ordinary differential equation and its extension to systems of differential equations.

To present a study of convergence, we will start by:

$$\begin{cases} \xi_0 = v_0(t), \\ \xi_1 = \Psi[\xi_0], \\ \xi_2 = \Psi[\xi_0 + \xi_1], \\ \vdots \\ \xi_n = \Psi[\xi_0 + \xi_1 + \cdots + \xi_{n-1}]. \end{cases} \tag{8}$$

By defining the factor $\Psi[v(t)]$ as

$$\Psi\big[\xi_n(t)\big] = v_n(t) - \sum_{i=0}^{n-1} v_i(t), \quad i = 1, 2, 3, \dots, \tag{9}$$

given that $v_n(t)$ is the solution for TAM.

Using these standards, convenient provisions for the convergence of TAM are discussed by the following theorems.

Theorem 1 *The chain solution $v(t) = \lim_{n\to\infty} v_n(t)$ will appear as the exact solution to the given problem if this chain solution is convergent.*

Proof See [18, 22]. □

Theorem 2 *Assume that Ψ stated in Eq.* (9), *is a factor from H to H, where H is a Hilbert space. The chain solution $v(t) = \lim_{n\to\infty} v_n(t)$ converges if $\exists 0 < \eta < 1$ such that*

$$H\big\|\Psi[\xi_0 + \xi_1 + \cdots + \xi_n]\big\| \le \eta\big\|\Psi[\xi_0 + \xi_1 + \cdots + \xi_{n-1}]\big\| \quad \forall \eta \in \mathbb{N} \cup \{0\}.$$

This notion is a specific state of the fixed-point notion and it is enough to prove the convergence of TAM.

Proof See [18, 22]. □

Theorem 3 *Whether the chain solution $\sum_{i=0}^{\infty} v_i(t)$ is convergent to $v(t)$, then the maximum error $E_n(t)$ will be*

$$E_n(t) \le \frac{1}{1-\rho}\rho^n \|v_0\|, \tag{10}$$

where the chain $\sum_{i=0}^{n-1} v_i(t)$ is employed to solve a wide class of nonlinear problems.

Proof See [18, 22]. □

The acquired solution by the TAM converges to the exact solution as: $\exists\ 0 < \eta < 1$ such that

$$D_n = \begin{cases} \frac{\|\xi_n\|}{\|\xi_{n-1}\|} & \|\xi_n\| \neq 0, \\ 0, & \|\xi_n\| = 0. \end{cases} \tag{11}$$

The power-chain solution $\sum_{n=0}^{\infty} v_n(t)$ converges to the exact solution $v(t)$ when $0 \leq D_n < 1, \forall n = 0, 1, 2, \ldots$

Only for $\alpha = 1$, can the solution to the stochastic Eq. (1) be achieved using the TAM technique, however, due to the complexity of integrating random functions, only a few iterations are possible. As the fractional time derivative ${}_0D_t^\alpha$ t can be singular or nonsingular of type ${}_0^CD_t^\alpha$ or ${}_0^{CF}D_t^\alpha$ or ${}_0^{ABC}D_t^\alpha$, we propose the fractional discrete Temimi–Ansari (FDTAM) method to solve stochastic nonlinear differential Eq. (1) for a fractional operator with a local singular kernel type Caputo and a nonsingular kernel of Caputo–Fabrizio and Atangana–Baleanu ABC types as follows:

2.1 Liouville–Caputo sense

The FDTAM scheme for a Caputo fractional operator to approximate the solution for problem (1) will be as follows:

Assume: an n-point uniform mesh on $[0, T]$ as $\{i : i = 1, \ldots, n\}$, $0 < t_1 < t_2 < \cdots < t_n = T$ with $t_i - t_{i-1} = q$. Let $h \in (0, q]$ be a fixed constant for a fixed h.

The finite-difference form to approximate $\frac{d\omega(t_{j+1})}{dt}$ is given by

$$\frac{d\omega(t_{i+1})}{dt} = \frac{\omega_{i+1} - \omega_i}{h}. \tag{12}$$

The generalized Euler's scheme to approximate ${}_0^CD_t^\alpha v_0(t_{j+1})$ is given by

$$v_0(t_{i+1}) = v_0(t_i) + \frac{h^\alpha}{\Gamma(\alpha+1)}\left[G(t_i) + f(t_i)\frac{\omega(t_{i+1}) - \omega(t_i)}{h}\right], \tag{13}$$

where $v_0(t_{i+1}) = v_0^{i+1}, v_0(t_i) = v_0^i, \omega(t_{i+1}) = \omega_{i+1}$ and $\omega(t_i) = \omega_i$. Therefore, the first refined equation to approximate the initial approximate function v_0 (t_{i+1}) is

$$v_0^{i+1} = v_0^i + \frac{h^\alpha}{\Gamma(\alpha+1)}\left[G(t_i) + f(t_i)\frac{\omega_{i+1} - \omega_i}{h}\right]. \tag{14a}$$

The next discrete approximate function v_1 (t_{i+1}) and the nth discrete approximate functions v_n (t_{i+1}) can be computed as follows

$$v_1^{i+1} = v_1^i + \frac{h^\alpha}{\Gamma(\alpha+1)}\left[F\left(t_i, v_0^i\right) + \left(G(t_i) + f(t_i)\frac{\omega_{i+1} - \omega_i}{h}\right)\right], \tag{14b}$$

$$v_n^{i+1} = v_n^i + \frac{h^\alpha}{\Gamma(\alpha+1)}\left[F\left(t_i, v_{n-1}^i\right) + \left(G(t_i) + f(t_i)\frac{\omega_{i+1} - \omega_i}{h}\right)\right]. \tag{14c}$$

The solution will be calculated through k runs of various patterns of the Wiener process ω(t), and then the refined scheme (14a)–(14c) can be written in the following form:

$$v_{0,k}^{i+1} = v_{0,k}^i + \frac{h^\alpha}{\Gamma(\alpha+1)}\left[G(t_i) + f(t_i)\frac{\omega_{i+1} - \omega_i}{h}\right], \tag{15a}$$

$$v_{1,k}^{i+1} = v_{1,k}^i + \frac{h^\alpha}{\Gamma(\alpha+1)}\left[F\left(t_i, v_{0,k}^i\right) + \left(G(t_i) + f(t_i)\frac{\omega_{i+1} - \omega_i}{h}\right)\right], \tag{15b}$$

$$v_{n,k}^{i+1} = v_{n,k}^i + \frac{h^\alpha}{\Gamma(\alpha+1)}\left[F\left(t_i, v_{n-1,k}^i\right) + \left(G(t_i) + f(t_i)\frac{\omega_{i+1} - \omega_i}{h}\right)\right]. \tag{15c}$$

We must select a time step h to guarantee system convergence of (15a)–(15c). Setting the convergence gauge of the fixed-point iteration by dividing the right-hand side of Eq. (15b) we obtain:

$$\left(\frac{h^{\alpha}}{\Gamma(\alpha+1)}\right)\frac{\partial F(t_i, v_{0,k}^i)}{\partial v_{0,k}^i} < 1, \tag{16}$$

$$h < \left(\frac{\Gamma(\alpha+1)}{\frac{\partial F(t_i, v_{0,k}^i)}{\partial v_{0,k}^i}}\right)^{\frac{1}{\alpha}}. \tag{17}$$

Let $g_1 = \frac{\partial F(t_i, v_{0,k}^i)}{\partial v_{0,k}^i}$, then we have:

$$h < \left(\frac{\Gamma(\alpha+1)}{g_1}\right)^{\frac{1}{\alpha}}. \tag{18}$$

The condition $h < (\frac{\Gamma(\alpha+1)}{g_1})^{\frac{1}{\alpha}}$ is an appropriate condition for the time step used in the Liouville–Caputo sense for convergence. The other Eqs. (15a) and (15c) can use the same condition because of symmetry.

2.2 Caputo–Fabrizio sense

The FDTAM scheme for a Caputo–Fabrizio fractional operator to approximate the solution for the problem (1) will be as follows:

The Caputo–Fabrizio scheme to approximate ${}_0^{CF}D_t^{\alpha} v_0(t_{j+1})$ is given by

$$\begin{aligned} v_0(t_{i+1}) = v_0(t_i) &+ \left(\frac{1-\alpha}{\beta(\alpha)} + \frac{3\alpha h}{2\beta(\alpha)}\right)\left[G(t_i) + f(t_i)\frac{\omega(t_{i+1}) - \omega(t_i)}{h}\right] \\ &+ \left(\frac{1-\alpha}{\beta(\alpha)} + \frac{\alpha h}{2\beta(\alpha)}\right)\left[G(t_{i-1}) + f(t_{i-1})\frac{\omega(t_i) - \omega(t_{i-1})}{h}\right], \end{aligned} \tag{19}$$

where $v_0(t_{i+1}) = v_0^{i+1}, v_0(t_i) = v_0^i, \omega(t_{i+1}) = \omega_{i+1}$, and $\omega(t_i) = \omega_i$. Therefore, the first iterative equation to approximate the initial approximate function $v_0\ (t_{i+1})$ is

$$\begin{aligned} v_0^{i+1} = v_0^i &+ \left(\frac{1-\alpha}{\beta(\alpha)} + \frac{3\alpha h}{2\beta(\alpha)}\right)\left[G(t_i) + f(t_i)\frac{\omega_{i+1} - \omega_i}{h}\right] \\ &+ \left(\frac{1-\alpha}{\beta(\alpha)} + \frac{\alpha h}{2\beta(\alpha)}\right)\left[G(t_{i-1}) + f(t_{i-1})\frac{\omega_i - \omega_{i-1}}{h}\right]. \end{aligned} \tag{20a}$$

The next Caputo–Fabrizio approximate function $v_1\ (t_{i+1})$ and the nth Caputo–Fabrizio approximate functions $v_n\ (t_{i+1})$ can be computed as follows

$$\begin{aligned} v_1^{i+1} = v_1^i &+ \left(\frac{1-\alpha}{\beta(\alpha)} + \frac{3\alpha h}{2\beta(\alpha)}\right)\left[F\left(t_i, v_0^i\right) + \left(G(t_i) + f(t_i)\frac{\omega_{i+1} - \omega_i}{h}\right)\right] \\ &+ \left(\frac{1-\alpha}{\beta(\alpha)} + \frac{\alpha h}{2\beta(\alpha)}\right)\left[F\left(t_{i-1}, v_0^{i-1}\right) + \left(G(t_{i-1}) + f(t_{i-1})\frac{\omega_i - \omega_{i-1}}{h}\right)\right], \end{aligned} \tag{20b}$$

$$v_n^{i+1} = v_n^i + \left(\frac{1-\alpha}{\beta(\alpha)} + \frac{3\alpha h}{2\beta(\alpha)}\right)\left[F\left(t_i, v_{n-1}^i\right) + \left(G(t_i) + f(t_i)\frac{\omega_{i+1} - \omega_i}{h}\right)\right]$$

$$+\left(\frac{1-\alpha}{\beta(\alpha)}+\frac{\alpha h}{2\beta(\alpha)}\right)\left[F\left(t_{i-1},v_{n-1}^{i-1}\right)+\left(G(t_{i-1})+f(t_{i-1})\frac{\omega_i-\omega_{i-1}}{h}\right)\right]. \tag{20c}$$

The solution will be calculated using k runs of various Wiener process ω(t) patterns, and the refined scheme (20a)–(20c) can be written as follows:

$$v_{0,k}^{i+1}=v_{0,k}^{i}+\left(\frac{1-\alpha}{\beta(\alpha)}+\frac{3\alpha h}{2\beta(\alpha)}\right)\left[G(t_i)+f(t_i)\frac{\omega_{i+1}-\omega_i}{h}\right]$$
$$+\left(\frac{1-\alpha}{\beta(\alpha)}+\frac{\alpha h}{2\beta(\alpha)}\right)\left[G(t_{i-1})+f(t_{i-1})\frac{\omega(t_i)-\omega(t_{i-1})}{h}\right], \tag{21a}$$

$$v_{1,k}^{i+1}=v_{1,k}^{i}+\left(\frac{1-\alpha}{\beta(\alpha)}+\frac{3\alpha h}{2\beta(\alpha)}\right)\left[F\left(t_i,v_{0,k}^{i}\right)+\left(G(t_i)+f(t_i)\frac{\omega_{i+1}-\omega_i}{h}\right)\right]$$
$$+\left(\frac{1-\alpha}{\beta(\alpha)}+\frac{\alpha h}{2\beta(\alpha)}\right)\left[F\left(t_{i-1},v_{0,k}^{i-1}\right)+\left(G(t_{i-1})+f(t_{i-1})\frac{\omega_i-\omega_{i-1}}{h}\right)\right], \tag{21b}$$

$$v_{n,k}^{i+1}=v_{n,k}^{i}+\left(\frac{1-\alpha}{\beta(\alpha)}+\frac{3\alpha h}{2\beta(\alpha)}\right)\left[F\left(t_i,v_{n-1,k}^{i}\right)+\left(G(t_i)+f(t_i)\frac{\omega_{i+1}-\omega_i}{h}\right)\right]$$
$$+\left(\frac{1-\alpha}{\beta(\alpha)}+\frac{\alpha h}{2\beta(\alpha)}\right)\left[F\left(t_{i-1},v_{n-1,k}^{i-1}\right)+\left(G(t_{i-1})+f(t_{i-1})\frac{\omega_i-\omega_{i-1}}{h}\right)\right]. \tag{21c}$$

For facilitation we take $\beta(\alpha)=1$.

2.3 Atangana–Baleanu sense

The FDTAM scheme for the Atangana–Baleanu fractional operator will be as follows to approximate the solution to problem (1):

The Atangana–Baleanu scheme to approximate ${}_{0}^{ABC}D_t^{\alpha}v_0(t_{j+1})$ is given by

$$v_0(t_{i+1})=v_0(t_i)+\left(\frac{h^\alpha}{\beta(\alpha)\Gamma(\alpha)}\right)\left(1+\frac{(1-\alpha)\Gamma(\alpha)}{h^\alpha}\right)\left[G(t_i)+f(t_i)\frac{\omega(t_{i+1})-\omega(t_i)}{h}\right]$$
$$+\left(\frac{h^\alpha}{\beta(\alpha)\Gamma(\alpha)}\right)\left[G(t_{i-1})+f(t_{i-1})\frac{\omega(t_i)-\omega(t_{i-1})}{h}\right], \tag{22}$$

where $v_0(t_{i+1})=v_0^{i+1}, v_0(t_i)=v_0^i, \omega(t_{i+1})=\omega_{i+1}$, and $\omega(t_i)=\omega_i$. Therefore, the first iterative equation to approximate the initial approximate function v_0 (t_{i+1}) is

$$v_0^{i+1}=v_0^{i}+\left(\frac{h^\alpha}{\beta(\alpha)\Gamma(\alpha)}\right)\left(1+\frac{(1-\alpha)\Gamma(\alpha)}{h^\alpha}\right)\left[G(t_i)+f(t_i)\frac{\omega_{i+1}-\omega_i}{h}\right]$$
$$+\left(\frac{h^\alpha}{\beta(\alpha)\Gamma(\alpha)}\right)\left[G(t_{i-1})+f(t_{i-1})\frac{\omega(t_i)-\omega(t_{i-1})}{h}\right]. \tag{23a}$$

The next Atangana–Baleanu approximate function v_1 (t_{i+1}) and the nth Atangana–Baleanu approximate functions v_n (t_{i+1}) can be computed as follows:

$$v_1^{i+1}=v_1^{i}+\left(\frac{h^\alpha}{\beta(\alpha)\Gamma(\alpha)}\right)\left(1+\frac{(1-\alpha)\Gamma(\alpha)}{h^\alpha}\right)\left[F\left(t_i,v_0^{i}\right)+\left(G(t_i)+f(t_i)\frac{\omega_{i+1}-\omega_i}{h}\right)\right]$$
$$+\left(\frac{h^\alpha}{\beta(\alpha)\Gamma(\alpha)}\right)\left[F\left(t_{i-1},v_0^{i-1}\right)+\left(G(t_{i-1})+f(t_{i-1})\frac{\omega_i-\omega_{i-1}}{h}\right)\right], \tag{23b}$$

$$v_n^{i+1}=v_n^{i}+\left(\frac{h^\alpha}{\beta(\alpha)\Gamma(\alpha)}\right)\left(1+\frac{(1-\alpha)\Gamma(\alpha)}{h^\alpha}\right)\left[F\left(t_i,v_{n-1}^{i}\right)+\left(G(t_i)+f(t_i)\frac{\omega_{i+1}-\omega_i}{h}\right)\right]$$
$$+\left(\frac{h^\alpha}{\beta(\alpha)\Gamma(\alpha)}\right)\left[F\left(t_{i-1},v_{n-1}^{i-1}\right)+\left(G(t_{i-1})+f(t_{i-1})\frac{\omega_i-\omega_{i-1}}{h}\right)\right]. \tag{23c}$$

The solution will be calculated using k runs of various Wiener process ω(t) patterns, and the refined scheme (23a)–(23c) can be written as follows:

$$v_{0,k}^{i+1} = v_{0,k}^{i} + \left(\frac{h^{\alpha}}{\beta(\alpha)\Gamma(\alpha)}\right)\left(1 + \frac{(1-\alpha)\Gamma(\alpha)}{h^{\alpha}}\right)\left[G(t_i) + f(t_i)\frac{\omega_{i+1}-\omega_i}{h}\right] + \left(\frac{h^{\alpha}}{\beta(\alpha)\Gamma(\alpha)}\right)\left[G(t_{i-1}) + f(t_{i-1})\frac{\omega(t_i)-\omega(t_{i-1})}{h}\right], \tag{24a}$$

$$v_{1,k}^{i+1} = v_{1,k}^{i} + \left(\frac{h^{\alpha}}{\beta(\alpha)\Gamma(\alpha)}\right)\left(1 + \frac{(1-\alpha)\Gamma(\alpha)}{h^{\alpha}}\right)\left[F\left(t_i, v_{0,k}^{i}\right) + \left(G(t_i) + f(t_i)\frac{\omega_{i+1}-\omega_i}{h}\right)\right] + \left(\frac{h^{\alpha}}{\beta(\alpha)\Gamma(\alpha)}\right)\left[F\left(t_{i-1}, v_{0,k}^{i-1}\right) + \left(G(t_{i-1}) + f(t_{i-1})\frac{\omega_i-\omega_{i-1}}{h}\right)\right], \tag{24b}$$

$$v_{n,k}^{i+1} = v_{n,k}^{i} + \left(\frac{h^{\alpha}}{\beta(\alpha)\Gamma(\alpha)}\right)\left(1 + \frac{(1-\alpha)\Gamma(\alpha)}{h^{\alpha}}\right)\left[F\left(t_i, v_{n-1,k}^{i}\right) + \left(G(t_i) + f(t_i)\frac{\omega_{i+1}-\omega_i}{h}\right)\right] + \left(\frac{h^{\alpha}}{\beta(\alpha)\Gamma(\alpha)}\right)\left[F\left(t_{i-1}, v_{n-1,k}^{i-1}\right) + \left(G(t_{i-1}) + f(t_{i-1})\frac{\omega_i-\omega_{i-1}}{h}\right)\right]. \tag{24c}$$

Computing the mean and variance of the resulting sequences $\{v_{n,1}, v_{n,2}, v_{n,3}, \dots, v_{n,k}\}$ will yield the solution's mean and variance.

Finally, we obtain the mean and variance of the solution by taking the mean and variance of the solution sequences $\{v_{n,1}, v_{n,2}, v_{n,3}, \dots, v_{n,k}\}$. These smart proposed numerical schemes are more effective than conventional TAM in solving fractional stochastic nonlinear differential equations with different fractional operators.

3 Applications

3.1 The mathematical model of an RLC circuit

An RLC circuit is an electrical circuit that uses a voltage or current source to drive a resistor, a capacitor, and an inductor. According to Kirchhoff's law, the charge $v(t)$ at time t at a fixed location in an electrical circuit fulfills the differential equation [15, 24].

$$L\frac{d^2v(t)}{dt^2} + R\frac{dv(t)}{dt} + \frac{1}{C}v(t) = q(t), \qquad v(0) = v_0, \qquad \frac{dv(0)}{dt} = I_0, \tag{25}$$

where L is inductance, R is resistance, C is capacitance, and $q(t)$ is the potential source at time t. Now, we may have a state where some of the coefficients are not in the deterministic form. The circuit is under an external voltage $q(t)$, which is a superposition of a periodic signal and white noise

$$q^*(t) = q(t) + \beta n(t). \tag{26}$$

Using the stochastic version of the voltage source, the model of RLC can be written as:

$$L\frac{d^2v(t)}{dt^2} + R\frac{dv(t)}{dt} + \frac{1}{C}v(t) = q(t) + \beta n(t), \qquad v(0) = v_0, \qquad \frac{dv(0)}{dt} = I_0, \tag{27}$$

where β is the intensity of the noise.

The solution of the stochastic RLC circuit model (27) with white Gaussian noise $n(t)$ produced using a strong Kloeden–Platen–Schurz scheme of order 1.5, which is included in the Mathematica-12 software, is shown in Fig. 1.

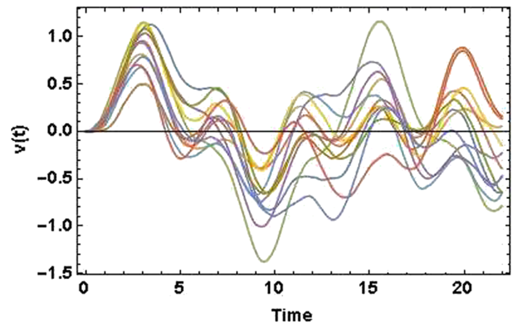

Figure 1 The solution of a stochastic RLC circuit with white noise

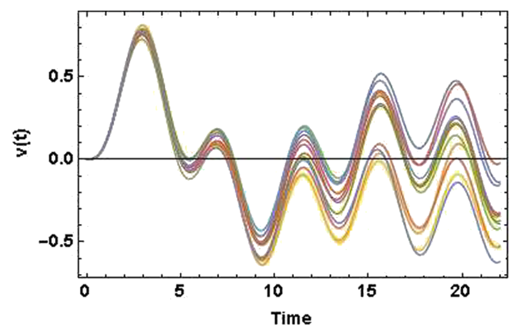

Figure 2 The solution of a stochastic RLC circuit with colored noise

White noise cannot be considered a stochastic process where the Wiener-process route is not differentiable everywhere, but it can be approximated by conventional stochastic processes with wide spectral bands, which are generally known as colored-noise processes. The Ornstein–Uhlenbeck process $n_1(t)$ with zero mean and variance $\sigma = 0.7$ is the most well-known example of this type of noise. It is generated using a linear stochastic differential equation driven by white noise as follows:

$$\frac{dn_1(t)}{dt} = -\frac{1}{\tau_s} n_1(t) + \sigma \sqrt{\frac{2}{\tau_s}} n(t), \tag{28}$$

where $\tau_s = 100$ ms and $n(t)$ is a white Gaussian noise with zero mean and variance one.

Equation (28) can be written in the Ito integral form as

$$dn_1(t) = -\frac{1}{\tau_s} n_1(t)\, dt + \sigma \sqrt{\frac{2}{\tau_s}}\, d\omega(t), \tag{29}$$

where $\omega(t)$ is the Wiener process.

Replacing the white noise $n(t)$ in Eq. (27) with the colored noise model (29) we obtain the solution presented in Fig. 2.

3.2 The mathematical model of RC circuit

The condensation of charge in the RC circuit's capacitor, which is modeled by an ordinary differential equation and its stochastic equivalents, is solved for the stochastic case in this section. Each electrical circuit made up of an inductor, a resistor, and a capacitor can be framed by an ordinary differential equation in which the differential operators' parameters are functions of the circuit elements. By adding white noise to the potential input source, the deterministic ordinary differential equation can be converted to a stochastic one.

Assuming that $v(t)$ is the charge on the capacitor and $q(t)$ is the potential source applied to the input of the RC circuit. Using Kirchoff's second law,

$$q(t) = I(t)R + \frac{v(t)}{C}. \tag{30}$$

Since $I(t) = \frac{dv(t)}{dt}$, we obtain the following equation:

$$\frac{dv(t)}{dt} + \left(RC^{-1}\right)v(t) = R^{-1}q(t), \tag{31a}$$

with initial condition

$$v(0) = v_0, \tag{31b}$$

where $v(0)$ is the initial charge stored in the capacitor. The potential source and resistance may not be deterministic but of the form:

$$q^*(t) = q(t) + \beta n(t), \tag{32}$$

and

$$R^* = R + \text{“noise”} = R + \zeta n_1(t), \tag{33}$$

where $n(t)$ is the Gaussian white noise with mean zero and variance one, and $n_1(t)$, is a zero mean, exponentially correlated stationary process, ζ, β is the noise intensity, and they are positive constants that signal the stochastic case's deviation from the deterministic one, and the correlated process $n_1(t)$, is the colored noise driven by Eq. (29).

Substituting (32) and (33) into (31a)–(31b) yields:

$$\frac{dv(t)}{dt} + \left(\frac{1}{C(R + \zeta n_1(t))}\right)v(t) = \left(\frac{1}{(R + \zeta n_1(t))}\right)\left(q(t) + \beta n(t)\right), \tag{34a}$$

with initial condition

$$v(0) = v_0. \tag{34b}$$

The general model of the stochastic RC circuit can be written in the general form

$$\frac{dv}{dt} = F(t,v) + G(t) + f(t)n(t), \quad v(0) = a. \tag{35}$$

Since the fractional derivatives ultimately include memory, then it is controversial to apply a fractional extension on Eq. (35). If we exchange the time derivative of Eq. (35) by the

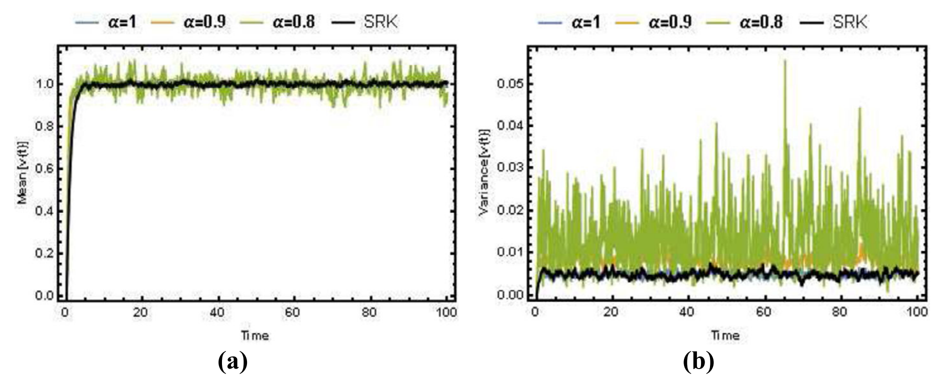

Figure 3 The mean and variance of a fractional stochastic RC Eq. (36) for several values of α by 100 iterations of CDTAM

Caputo (CD) or Caputo–Fabrizio (CF) or Atangana–Baleanu ABC fractional derivatives, then we obtain Eq. (1).

Consider the fractional linear electrical circuits based on the new fractional operator with a Caputo local singular kernel and nonsingular kernels of the Caputo–Fabrizio and Atangana–Baleanu ABC types. The time-domain fractional equations considered derivatives in the range $\alpha \in (0,1]$. The numerical solutions were obtained using the FDTAM approach, and some examples are provided, which involve electrical circuits with resistances, inductances, capacitances, and voltage sources.

Case 1. External noise

Consider the Fractional Stochastic RC Equation with only external noise in the potential source of the form

$$ {}_0D_t^{\alpha} v(t) + Av(t) = B + Dn(t), \qquad v(0) = v_0, t \geq 0, \tag{36} $$

where $A = (RC)^{-1}$, $B = R^{-1}q(t)$, and $D = \beta R^{-1}$.

The parameters used in the stochastic model of the RC circuit are as follows [2]; $R = 10\Omega; C = 1F; q(t) = q = 20V; \beta = 1$.

Application is made of the constructed fractional discrete Temimi–Ansari method (FDTAM) schemes (15a)–(15c), (21a)–(21c), and (24a)–(24c) to the fractional stochastic RC equation for Caputo, Caputo–Fabrizio, and Atangana–Baleanu ABC types (36). Using $h = 0.01$, Figs. 3(a), 4(a), 5(a) show the expectation of sequences $v_{200,1}, v_{200,2}, v_{200,3}, \dots, v_{200,1000}$ for different orders of the fractional-derivative operator $\alpha = 1, 0.9, 0.8$, and the solution by the stochastic Runge–Kutta method (SRK) at $\alpha = 1$. Figs. 3(b) 4(b), and 5(b) depict variances by proposed schemes for different orders of the fractional derivative operator $\alpha = 1, 0.9, 0.8$, and the solution by stochastic Runge–Kutta method (SRK) at $\alpha = 1$. These figures confirmed that the results of (FDTAM) schemes are excellently compatible with the stochastic Runge–Kutta method (SRK) at $\alpha = 1$. Table 1 compares the approximate solutions of the fractional stochastic RC model with external noise for the fractional derivatives CD, CF, and ABC at $\alpha = 1$, $\alpha = 0.9$, and $\alpha = 0.8$ to the solution of stochastic Runge–Kutta (SRK) method.

Case 2. Internal and external noise

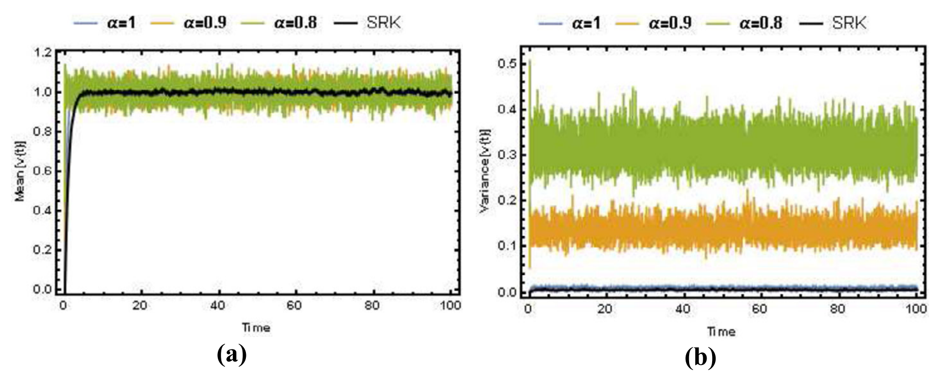

Figure 4 The mean and variance of a fractional stochastic RC Eq. (36) for several values of α by 100 iterations of CFTAM

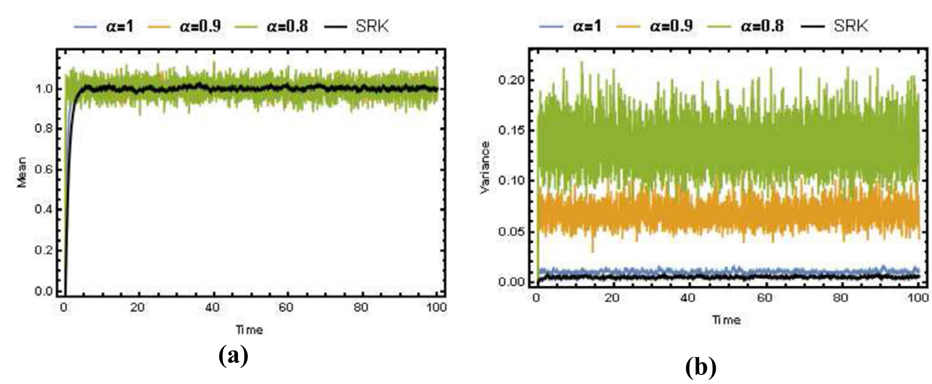

Figure 5 The mean and variance of a fractional stochastic RC Eq. (36) for several values of α by 100 iterations of CFTAM

Consider the Fractional Stochastic RC model with both internal and external noise in potential source and resistance, respectively, of the form

$$ {}_0D_t^{\alpha} v(t) + \left(\frac{1}{C(R+\zeta n_1(t))}\right) v(t) = \left(\frac{1}{(R+\zeta n_1(t))}\right)\bigl(q(t_i) + \beta n(t)\bigr). \tag{37} $$

Applying the constructed Fractional Discrete Temimi–Ansari method (FDTAM) schemes (15a)–(15c), (21a)–(21c) and (24a)–(24c), respectively, for Caputo, Caputo–Fabrizio, and Atangana–Baleanu ABC types on the fractional stochastic RC Eq. (37). By selecting $h = 0.01$, Figs. 6(a), 7(a), 8(a) show the expectation of sequences $\{v_{100,1}, v_{100,2}, v_{100,3}, \dots, v_{100,1000}\}$ for different orders of the fractional derivative operator: $\alpha = 1, 0.9, 0.8$. Figs. 6(b), 7(b), 8(b) show the variances by the proposed schemes for different orders of the fractional derivative operator: $\alpha = 1, 0.9, 0.8$. These figures confirmed that the results of (CDTAM), (CFTAM), and (ABTAM) are compatible with each other in an excellent modality. Table 2 shows a time comparison of the approximate solutions of the fractional stochastic RC model with external and internal noise for CD, CF, and ABC fractional derivatives at $\alpha = 1$, $\alpha = 0.9$, and $\alpha = 0.8$.

A new fractional stochastic method for approximating a fractional stochastic RC circuit under the effects of Gaussian white noise and Gaussian colored noise perturbations on the

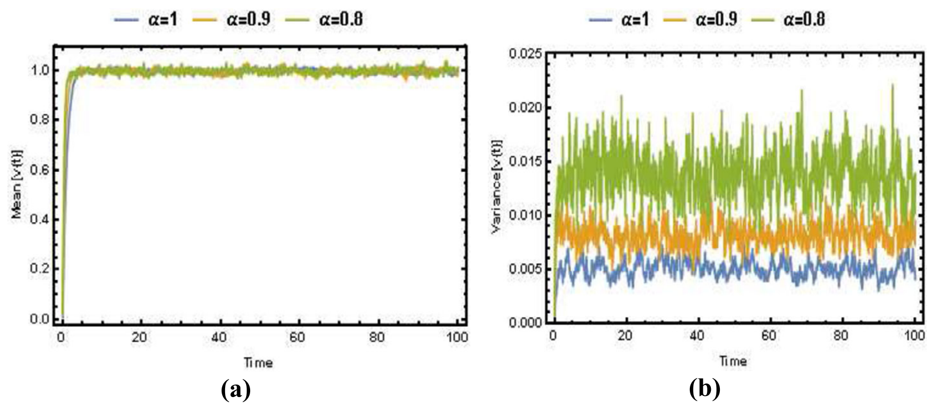

Figure 6 The mean and variance of a fractional stochastic RC Eq. (37) for several values of α by 100 iterations of CDTAM

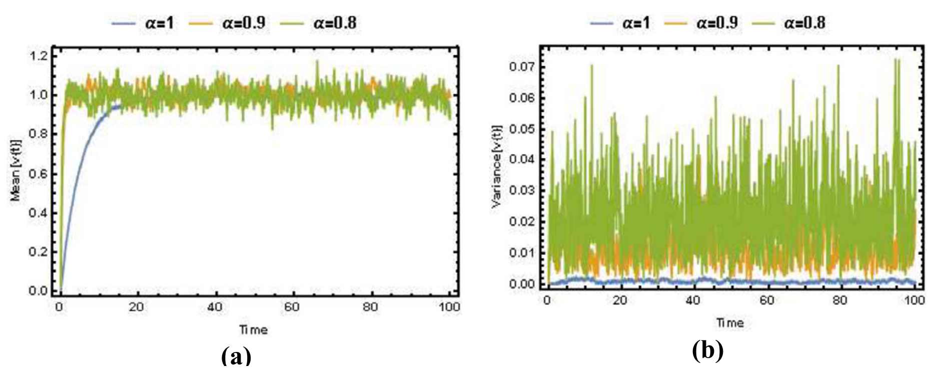

Figure 7 The mean and variance of a Fractional stochastic RC Eq. (37) for several values of α by 100 iterations of CFTAM

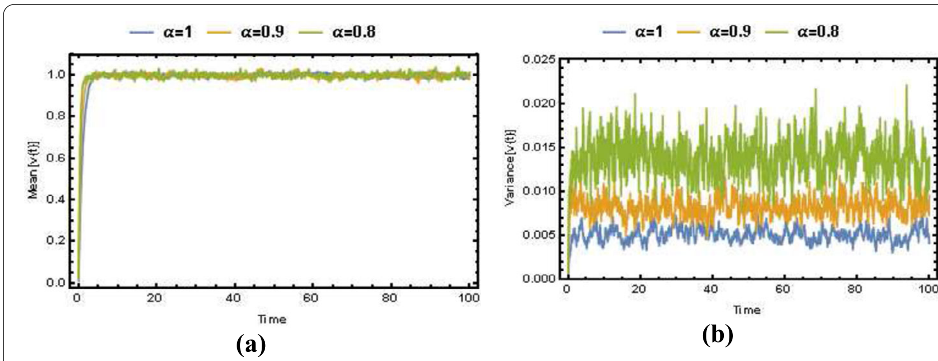

Figure 8 The mean and variance of a Fractional stochastic RC Eq. (37) for several values of α by 100 iterations of ABCTAM

potential source and resistance of fractional linear electrical circuits and fractional operator with local singular kernel of Caputo–Fabrizio and ABC types has been developed. Tables 1 and 2 show time comparisons between an approximate solution with CD, CFD, ABC, and the Stochastic Runge–Kutta method for the stochastic RC circuit model and

Table 1 Time comparison of the approximate solutions of fractional stochastic RC model with external noise for CD, CF, and ABC fractional derivatives at $\alpha = 1$, $\alpha = 0.9$, and $\alpha = 0.8$ and the solution of the Stochastic Runge–Kutta (SRK) method

Time	CD			CF			ABC			SRK
	$\alpha = 1$	$\alpha = 0.9$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0.9$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0.9$	$\alpha = 0.8$	
10	0.999304	0.9906497	1.050293	1.013083	1.202816	1.0074569	1.011102	0.806871	1.118561	1.0121553
20	1.013781	1.0073171	1.081426	1.059922	1.014441	1.2262395	0.891621	0.964927	0.963199	1.0023026
30	1.003784	0.9606641	1.036778	0.940042	1.095935	1.2146484	0.941030	1.117160	0.982169	1.0042459
40	0.973298	1.0492959	0.938137	1.009827	0.900094	1.5188262	0.989640	0.749823	0.680580	1.0014909
50	1.025356	1.0147813	0.985297	1.028005	0.672779	1.0174633	1.075177	1.124972	0.856008	1.0027986
60	1.015256	1.0591548	1.048250	0.875409	1.266037	1.0606401	1.080171	0.944387	1.062668	0.9892436
70	0.918271	1.0319845	0.937035	0.957743	0.779292	1.0886256	1.109512	0.948810	1.258928	1.0041955
80	0.926106	0.9709640	0.945952	0.975902	1.141334	0.9260061	1.004276	1.095910	1.062043	1.0013826
90	1.005957	1.0632894	1.013453	1.000967	0.674245	0.8481464	0.971001	0.857424	1.007927	1.0048435
100	0.906339	0.9907657	1.015398	0.987498	0.749267	0.9955613	1.009571	1.219733	1.110964	1.0009460

Table 2 Time comparison of the approximate solutions of fractional stochastic RC model with external and internal noise for CD, CF, and ABC fractional derivatives at $\alpha = 1$, $\alpha = 0.9$, and $\alpha = 0.8$

Time	CD			CF			ABC		
	$\alpha = 1$	$\alpha = 0.9$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0.9$	$\alpha = 0.8$	$\alpha = 1$	$\alpha = 0.9$	$\alpha = 0.8$
10	1.0417211	0.9586769	0.924926	0.852404	1.007488	1.1219353	1.0240777	1.1515323	1.101921
20	1.0431299	0.9238934	1.054235	0.994936	0.990548	1.0672070	0.8622823	1.2555819	0.583912
30	0.9610292	0.8936729	0.839914	0.976311	0.864789	0.8589190	0.9435111	1.2279188	1.071219
40	0.9945418	0.9867584	1.098535	0.960077	0.966953	0.8607355	0.9581774	0.7981606	1.140971
50	0.9578704	0.9875809	0.917085	0.985015	0.957769	1.0081455	0.9893450	0.9361715	1.254575
60	0.9501872	0.9148090	1.085565	1.027849	1.040610	1.1842127	0.9770676	1.2336195	0.933935
70	0.9406390	0.9989409	0.930192	0.997489	0.927629	0.9661507	1.0473299	1.2598179	1.181909
80	0.9630779	0.9705497	0.998849	0.984759	0.789988	1.0511748	0.9625619	0.9911711	0.979103
90	0.8660355	1.0325249	1.004245	1.025733	0.976182	1.1151639	0.9282355	0.9930742	0.402819
100	1.0123927	1.0645085	1.072624	0.969628	1.066534	1.0733375	1.0349394	1.4951479	0.840118

the newly presented numerical schemes for two different types of noise. The results show that the proposed method is trustworthy and can be applied to more complex stochastic problems in engineering sciences.

4 Conclusions

To overcome the limitations of the traditional Riemann–Liouville and Caputo fractional derivatives, new types of fractional differentiation with nonlocal and nonsingular kernels have recently been implemented. This paper introduces and applies a new numerical scheme to solve the linear fractional stochastic RC circuit model for a new fractional operator with Caputo local singular kernel and nonsingular kernel of Caputo–Fabrizio CF and Atangana–Baleanu ABC types. This article is dedicated to the development of a new numerical scheme that combines the fundamental theorem of fractional calculus and the recently developed DTAM approach. Both white and colored noise have been successfully implemented using the new proposed numerical approach. Unquestionably, the new numerical scheme is very efficacious and converges toward a solution very rapidly compared with approximate and stochastic Runge–Kutta solutions.

Acknowledgements
Not applicable.

Funding
Funding Statement No fund is applicable.

Availability of data and materials
Data sharing did not apply to this article as no datasets were generated or analyzed during the current study.

Declarations

Competing interests
The authors declare that they have no competing interests.

Author contribution
AFF created the work's design and the stochastic results. The analysis was carried out by MSS. The software used in the work was created by MTMME. The authors collaborated on the final manuscript. The final manuscript was read and approved by all the authors.

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Received: 30 May 2022 Accepted: 22 November 2022 Published online: 18 January 2023

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